

1.

证: 若  $d$  整除  $m, n$ , 则  $d$  整除  $m \pm n$

若  $d$  整除  $n$ , 则  $d$  整除  $kn$

$\therefore$  若  $d$  整除  $m, n$

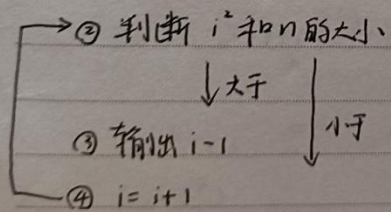
则  $d$  整除  $m - kn (m \bmod n)$

$\therefore (m, n)$  和  $(m \bmod n, n)$  的公约数相同

$\therefore \gcd(m, n) = \gcd(n, m \bmod n)$

2.

①  $i = 0$



### 3. 主定理

$$T(n) = aT(n/b) + f(n)$$

1. 若  $f(n) = O(n^{\log_b a - \epsilon})$ ,  $\epsilon > 0$  则  $T(n) = O(n^{\log_b a})$

2. 若  $f(n) = O(n^{\log_b a})$ , 则  $T(n) = O(n^{\log_b a} \log n)$

3. 若  $f(n) = \Omega(n^{\log_b a + \epsilon})$ ,  $\epsilon > 0$  且对某个常数  $c < 1$  和充分大的  $n$  有  $a f(n/b) \leq c f(n)$ , 则  $T(n) = O(f(n))$

~~即递归树各层下标为  $n/b^i$  的问题, 且对层数  $k$  有  $n/b^k = 1$~~   
叶节点数为  $a^k$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

$$= a[aT(\frac{n}{b^2}) + f(\frac{n}{b})] + f(n)$$

$= \dots$

$$= a^k T(1) + \sum_{j=0}^{k-1} a^j f(\frac{n}{b^j})$$

$$(n = b^k)$$

$$= T(1) n^{\log_b a} + \sum_{j=0}^{k-1} a^j f(\frac{n}{b^j})$$

当  $f(n) = O(n^{\log_b a - \epsilon})$ :

$$T(n) = T(1) n^{\log_b a} + O(n^{\log_b a - \epsilon} n^{\epsilon}) = O(n^{\log_b a})$$

当  $f(n) = O(n^{\log_b a})$ :

$$T(n) = T(1) n^{\log_b a} + O(n^{\log_b a} \log n) = O(n^{\log_b a} \log n)$$

当  $f(n) = \Omega(n^{\log_b a + \epsilon})$  且  $a f(n/b) \leq c f(n)$

$$T(n) \leq T(1) n^{\log_b a} + O(f(n)) = O(f(n))$$

1-b.

$$(1) f(n) = 2 \log n \quad g(n) = \log n + 5$$

$$\therefore f(n) = O(g(n))$$

$$(2) ~~f(n) = 2 \log n~~ \quad f(n) = 2 \log n \quad g(n) = \sqrt{n}$$

$$\therefore f(n) = O(g(n))$$

$$(3) f(n) = n \quad g(n) = \log^2 n$$

$$\therefore f(n) = \Omega(g(n))$$

$$(4) f(n) = \Omega(g(n))$$

$$(5) f(n) = \Theta(g(n))$$

$$(6) f(n) = \Omega(g(n))$$

$$(7) f(n) = \Omega(g(n))$$

$$(8) f(n) = O(g(n))$$