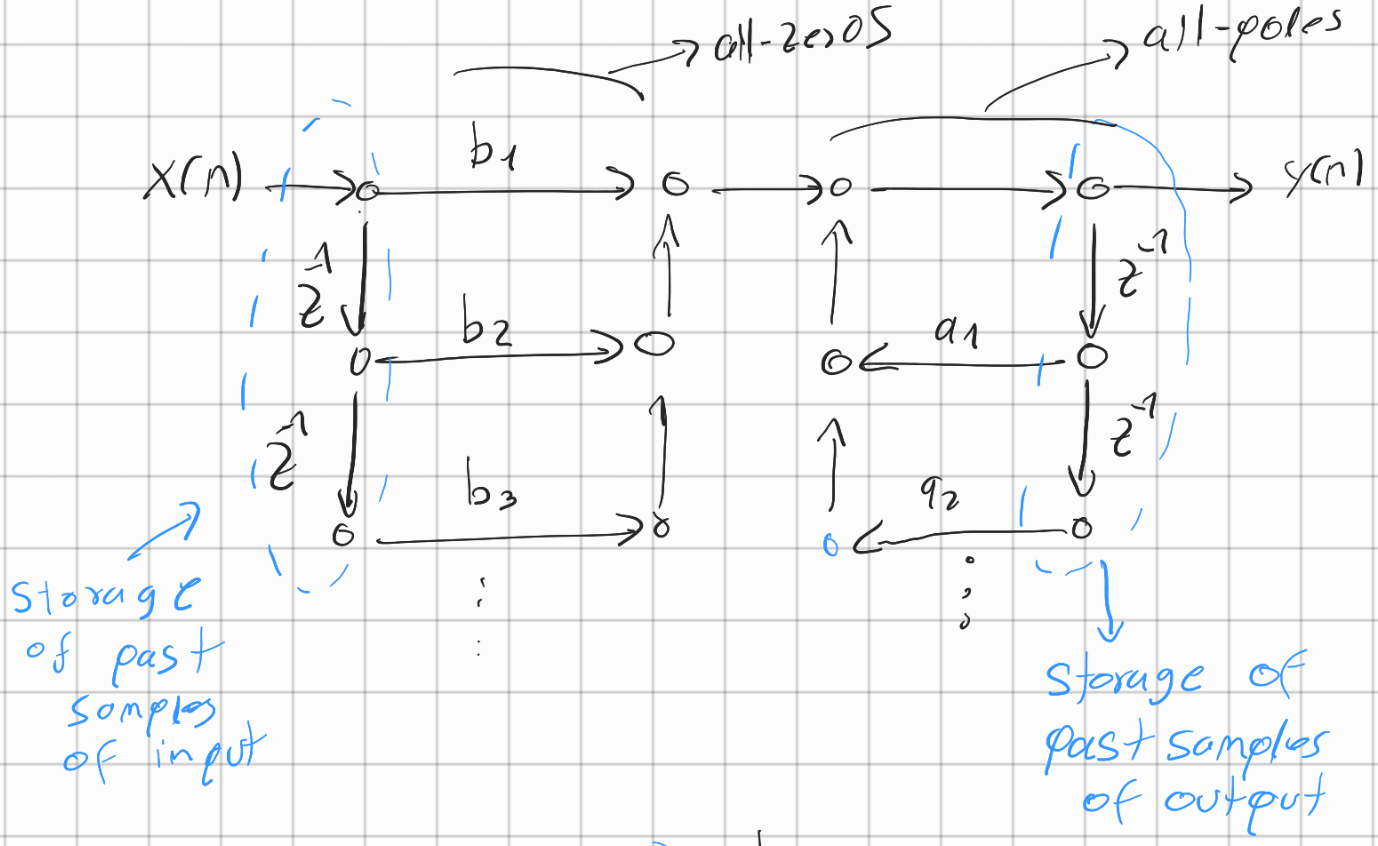
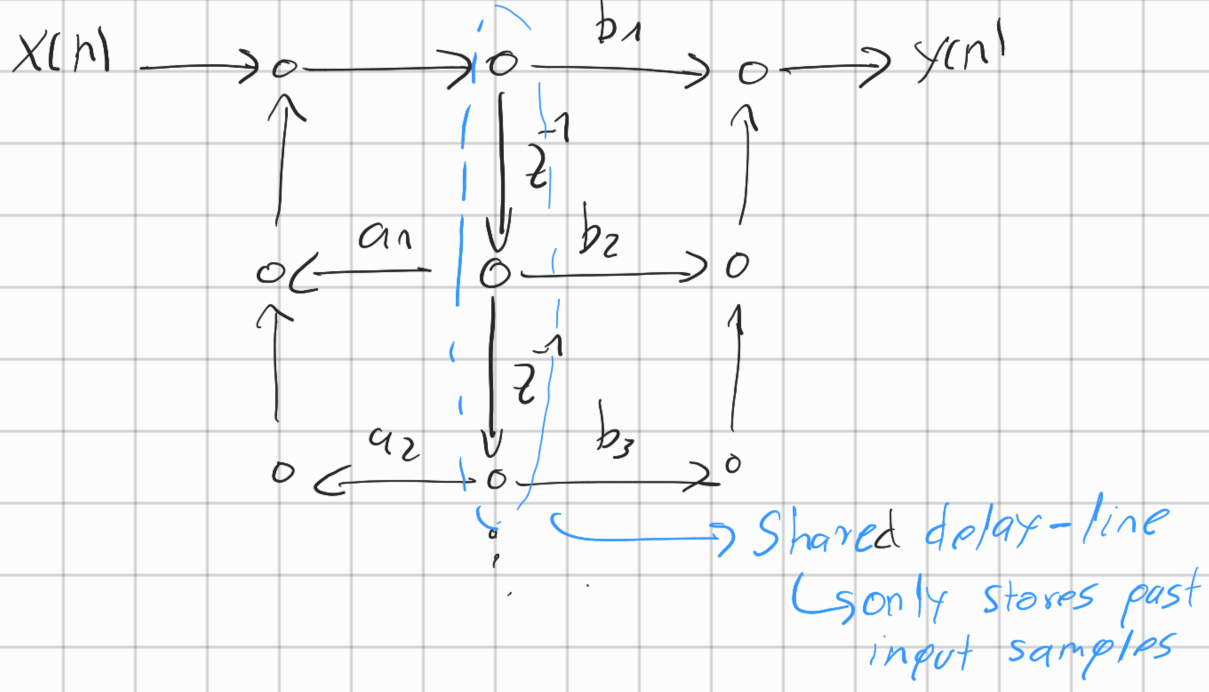
Exam TNA 2020-2021

1. Structures

Q1. Form II includes 2 shared delays between all-zero and the all-pole structure. It implies a reduction in the required memory to store past samples





92. Parallel structures may be useful when the latency effects are importat

→ The delay may be reduced using the polyphase structure → it's parallel

→ also it may be advisable to use when implementing high-order filters since separating the transfer fuction in parallel low-order system reduces the cumulative error effect that might be present and cascade series structures

Series: $H(z) = H_1(z)\,H_2(z)\cdots H_N(z)$

$\downarrow \quad \downarrow \quad \cdots \quad \downarrow$

$e_1 \quad e_2 \quad \cdots \quad e_n$

$e = e_1 e_2 \cdots e_n$ → acumulated error → amplification

Parallel: $H(z) = H_1(z) + H_2(z) + \cdots + H_N(z)$

$\downarrow \quad \downarrow \quad \cdots \quad \downarrow$

$e_1 \quad e_2 \quad \cdots \quad e_N$

$e = e_1 + e_2 + \cdots + e_N$ → accumulated error → addition

probably wrong
→ just an idea

93. The bi-quad is commonly used because of stability and sensivity purposes when dealing with the quantization effects.

↳ The higher the order of the system, the more sensitive to quantization error it becomes

↳ small variations in coefficients lead to large variations in the zeros and poles

bi-quad

Second-order Transfer fuctions are not that sensitive to quantization effects

↓

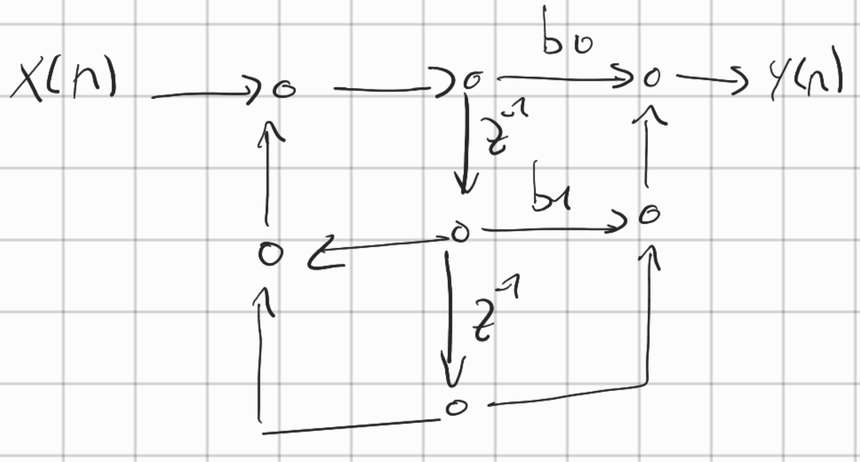
bi-quad

so the cascade of second-order subsystems is much less sensitive to coefficient quantization than a direct implementation

→ also as it was stated before this cell is efficient in terms of the storage requiered for past samples

→ Reduction on the number of required buffers/registers

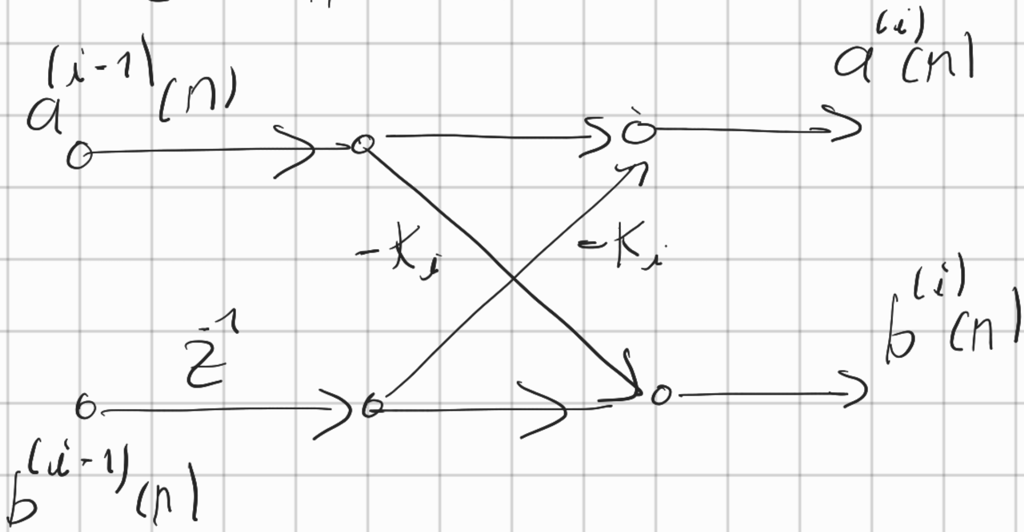
<u>bi-quad Cell:</u>



Q4

$\underbrace{\text{An alternative to the bi-quad cell}}_{\text{alternative structure for coscading}}$

is the lattice structure



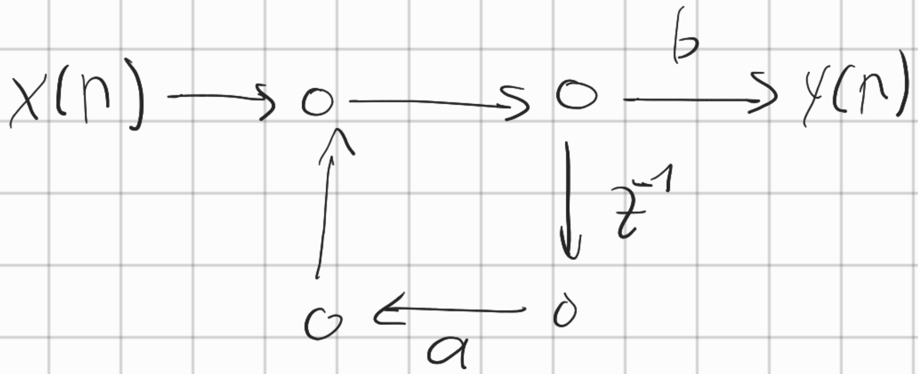
→ Advantages :→ more robust to coefficient quantization and computation noise

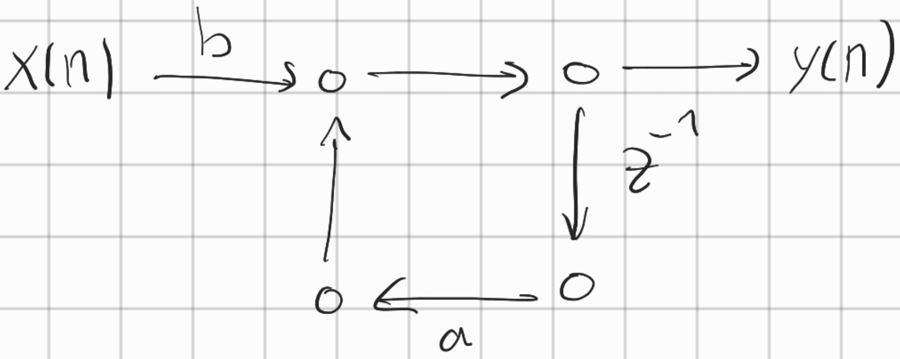
→ IIR always stable as far as $|k_i| < 1$

→ Disadvantage : $\underbrace{\text{Twice the number of multiplications per output sample as the direct form}}_{\text{more computational devices}}$

II. Rounding Effects

95





$$|y(n)| = \left| \sum_{m=-\infty}^{\infty} x(n-m)\,h(m) \right|$$

$$|y(n)| \leq X_{max} \sum_{m=-\infty}^{\infty} |h(m)|$$

without scaling x_m

for fixed-point representation: $|y(n)| < 1$

$$|y(n)| \leq X_{max} \sum_{m=-\infty}^{\infty} |h(m)| < 1$$

$$\boxed{X_{max} < \frac{1}{\sum_{m=-\infty}^{\infty} |h(m)|}}$$

for both systems:

$$H(z) = \frac{b}{1 - a z^{-1}} \xrightarrow{\text{Inverse TZ}} h(n) = b a^n u(n)$$

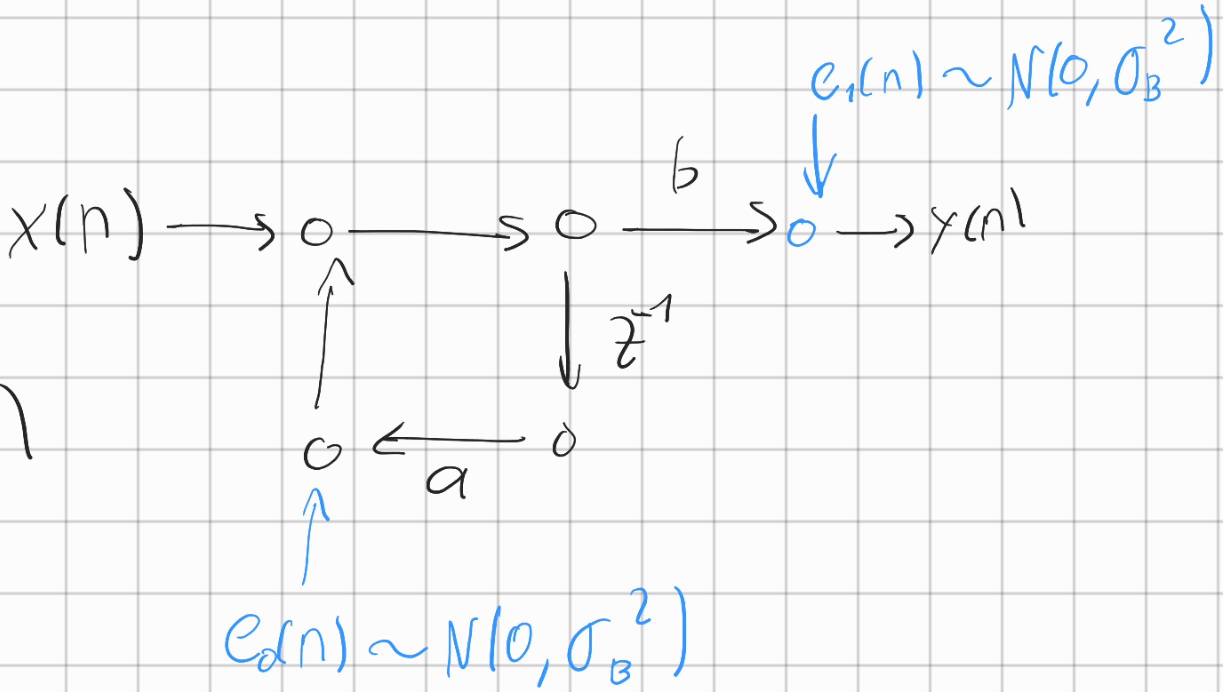
$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |b a^n u(n)| = |b| \sum_{n=0}^{\infty} |a|^n$$

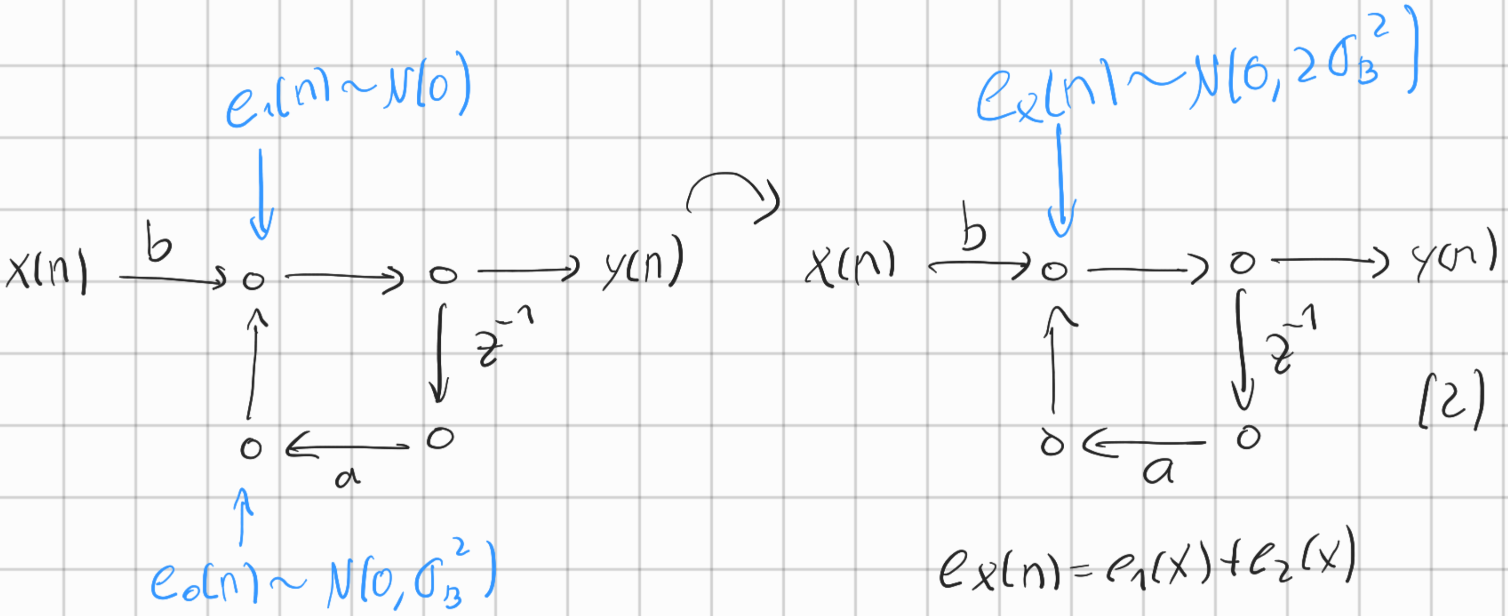
$|a| < 1$ Geometric sum $= \dfrac{|b|}{1-|a|}$

$$\boxed{X_{max} < \frac{1-|a|}{|b|}}$$

96

(1)





97

$f(n)$ depends on the input $e(n)$ which has infinite energy and non-deterministic.

$\Downarrow$

The z-Transform does not exist for $f(n)$

→ Existance only of the TZ of the autocorrelation of $f(n)$

$$\overset{\circ}{\gamma}_f(z) = H(z)H^*(z^{-1})\overset{\circ}{\gamma}_e(z)$$

superposition

due to $e_1(1)$

$e_0(n)$

(1)

$$\overset{\circ}{\gamma}_{f_1}(z) = \left(\frac{b}{1 - a z^{-1}}\right)\left(\frac{b}{(1-az)}\right)\sigma_B^2 + \sigma_B^2$$

$$(1) \quad \mathring{\gamma}_{f_1}(z) = \frac{b^2 \sigma_B^2}{(1-az^{-1})(1-az)} + \sigma_B^2$$

$$(2) \quad \mathring{\gamma}_{f_2}(z) = \frac{2 b^2 \sigma_B^2}{(1-az^{-1})(1-az)}$$

$$\downarrow \quad z = e^{j\omega} = \nu$$

98

$$\mathring{\gamma}_f(\nu) = |H(\nu)|^2 \mathring{\gamma}(\nu)$$

$$\mathring{\gamma}_{f_1}(\nu) = \frac{|b|^2}{|1-ae^{-j\omega}|^2} \sigma_B^2 + \sigma_B^2$$

$$|1-ae^{-j\omega}| = (1-ae^{-j\omega})(1-ae^{j\omega}) = 1 \underbrace{- ae^{j\omega} - ae^{-j\omega}}_{-a(e^{j\omega}+e^{-j\omega})} + a^2$$

$$= 1 - 2a\cos(\omega) + a^2$$

$$\mathring{\gamma}_{f_1}(\nu) = \frac{|b|^2}{1-2a\cos(\omega)+a^2} \sigma_B^2 + \sigma_B^2$$

$$\boxed{\mathring{\gamma}_{f_1}(\nu) = \sigma_B^2 \left(1 + \frac{|b|}{1-2a\cos(\omega)+a^2}\right)}$$

$\omega = \nu$

$\downarrow$

confusion notation

$$\boxed{\overset{\circ}{\gamma}_{f_2}(v) = \frac{2\sigma_B^2 |b|}{1 - 2a\cos(\omega) + a^2}}$$

9.9

Noise power = variance = $\gamma_f(0)$

$$\gamma_f(p) = \underbrace{\int_{-1}^{1} \overset{\circ}{\gamma}_f(v) e^{jvp} dv}_{\text{Inverse FT}}$$

$$\gamma_f(0) = \int_{-1}^{1} \overset{\circ}{\gamma}_f(v) dv = \int_{-1}^{1} |H(v)|^2 \sigma_B^2 dv$$

$$\text{Power Noise} = \sigma_B^2 \int_{-1}^{1} |H(v)|^2 dv \longrightarrow \text{formula to use}$$

→ by Parseval's Theorem:

$$\int_{-1}^{1} |H(v)|^2 = \sum_{n=-\infty}^{\infty} |h(n)|^2$$

In our case:

$$h(n) = ba^n u(n)$$

if $|a| < 1 \Rightarrow |a|^2 < 1$

$$|b|^2 \sum_{n=-\infty}^{\infty} |a|^{2n} u(n) = |b|^2 \sum_{n=0}^{\infty} (|a|^2)^n = \frac{|b|^2}{1 - |a|^2} = \frac{b^2}{1 - a^2}$$

Then:

$$\boxed{\sigma_{f_1}^2 = \frac{\sigma_B^2 b^2}{1-a^2} + \sigma_B^2}$$

→ by superposition $e_0(n)$, $e_1(n)$

$$\boxed{\sigma_{f_2}^2 = \frac{2\sigma_B^2 b^2}{1-a^2}}$$

Polyphase Resampler

9.10 The model increase the sampling frequency by $3/2$, then it might be equivalent to:

$$x(n) \longrightarrow \boxed{\uparrow 3} \longrightarrow \boxed{H_{LP}(z)} \longrightarrow \boxed{\downarrow 2} \longrightarrow y(n)$$

$H_{LP}(z)$: - Lowpass filter
- Stopband at $\omega_s = \min\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

normalized angular freq. $\left(\omega_s = \pi/3 \right.$

in real frequencies

$$f_s = \omega_s \left(\frac{F_s}{2\pi}\right)$$

$$f_s = \frac{F_s}{6}$$

- Attenuation in the stopband

$$A_{dB} = 6{,}02\,dB \cdot 13\,bits$$

$$A_{dB} = 78{,}26\,dB$$

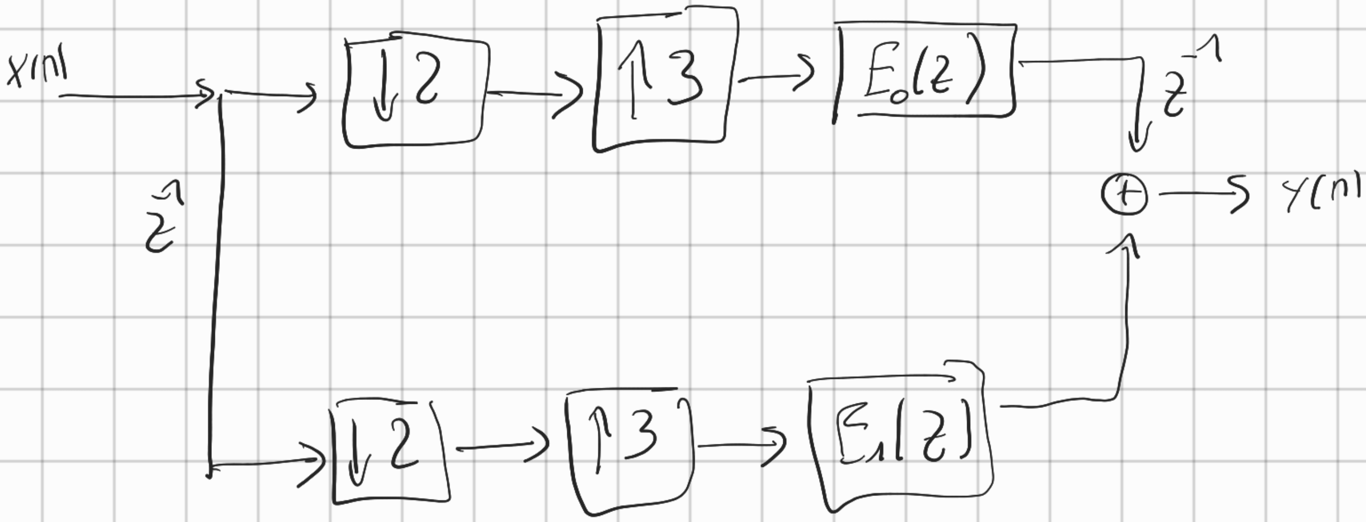
don't know what else ??

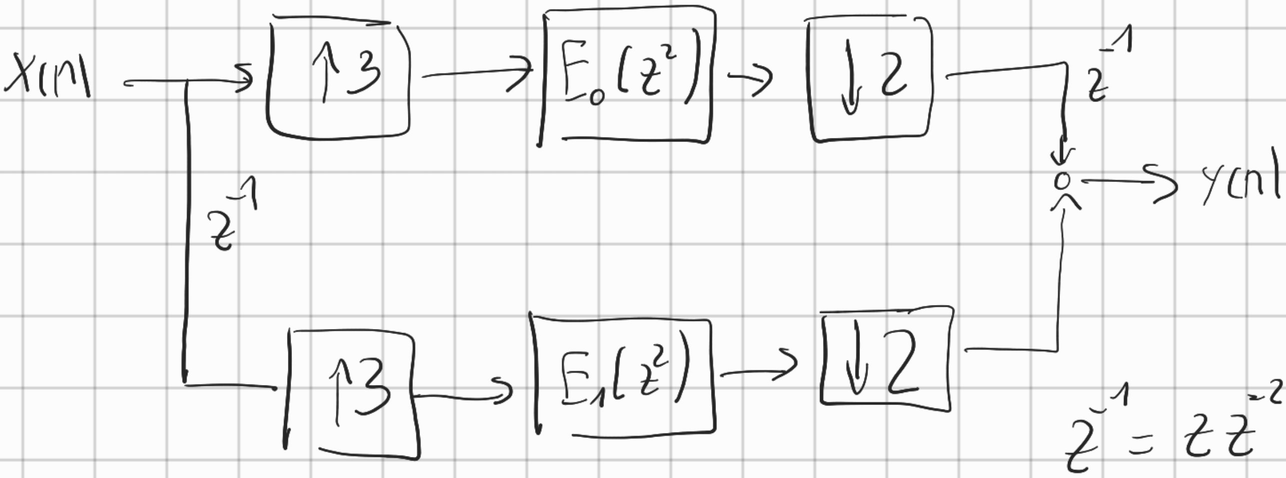
→ Approximattely linear or linear phase

→ flat passband

9.11 FIR $h[n] = [h_0, h_1, \dots, h_{N-1}]$ $N=12$

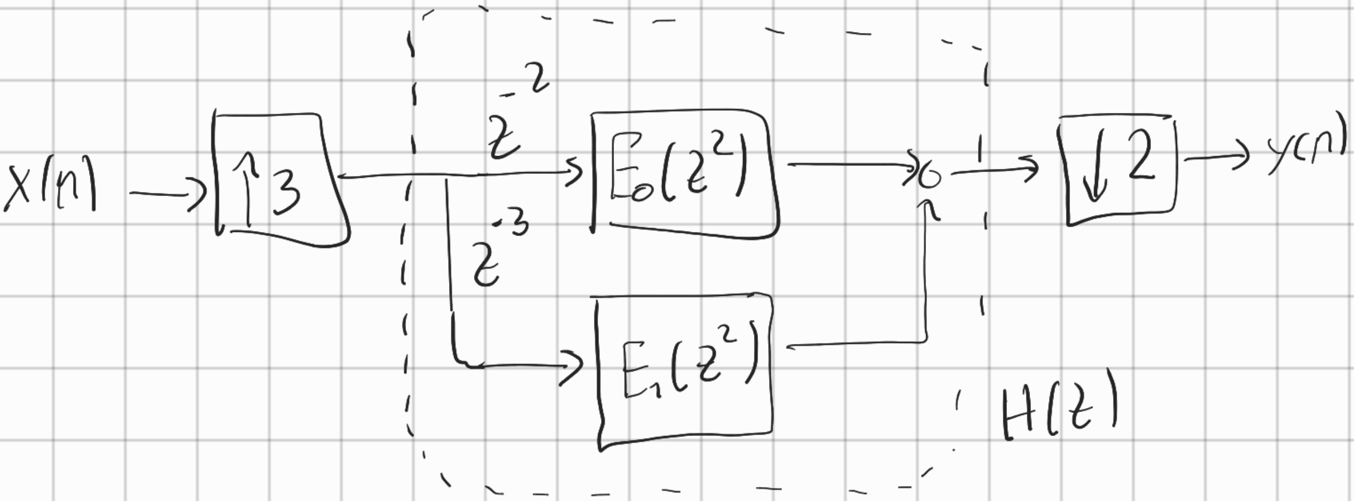
symplifing the schema:





$$z^{-1} = z\, z^{-2}$$

$$z^{-1} = z^{-3} z^{2}$$



$$H(z) = z^{-2}E_0(z^2) + z^{-3}E_1(z^{-2})$$

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + \dots + h_{11} z^{-11}$$ for a FIR

$$= z^{-2}(\underbrace{h_0 z^2}_{\text{Non-causal}} + h_2 + h_4 z^{-2} + h_6 z^{-4} + h_8 z^{-6} + \dots + h_{10} z^{-8})$$

$$+ z^{-3}(h_1 z^2 + h_3 + h_5 z^{-2} + \dots + h_{11} z^{-8})$$

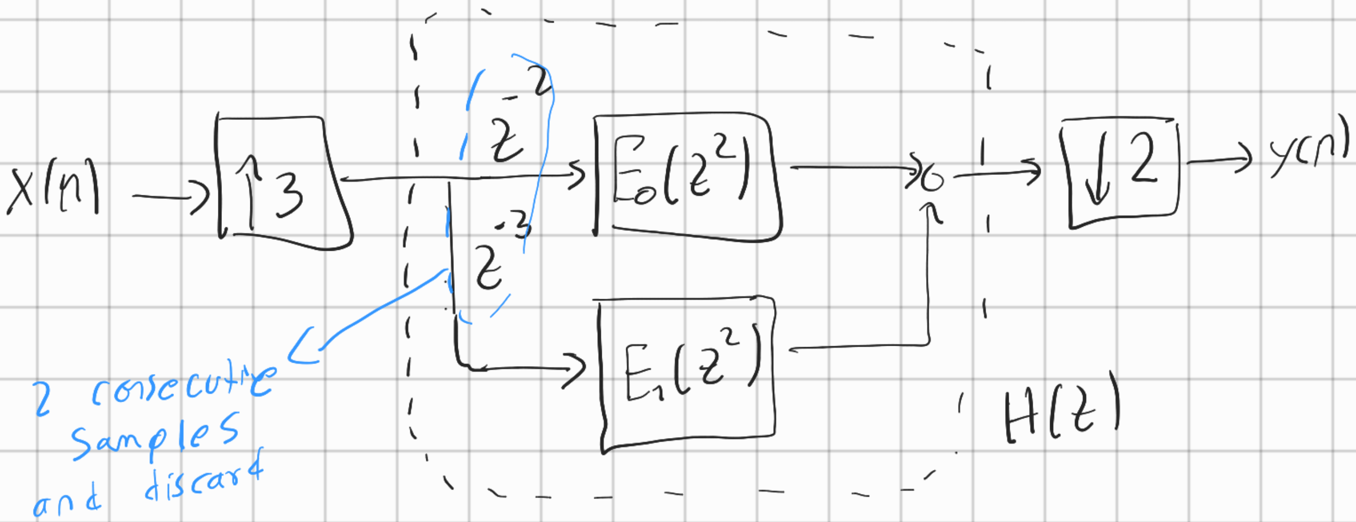
$$E_0(z) = h_0 z + h_2 + h_4 z^{-1} + h_6 z^{-2} + h_8 z^{-3} + h_{10} z^{-4}$$

$$E_0(z) = E_{00}(z^3) + z^{-1}E_{01}(z^3) + z^{-2}E_{02}(z^3)$$

$$= (h_2 + h_8 z^{-3}) + z^{-1}(h_4 + h_{10} z^{-3}) + z^{-2}\underbrace{(h_6 + h_0 z^3)}_{}$$

Non-causal
→ requires future samples

adjustmet

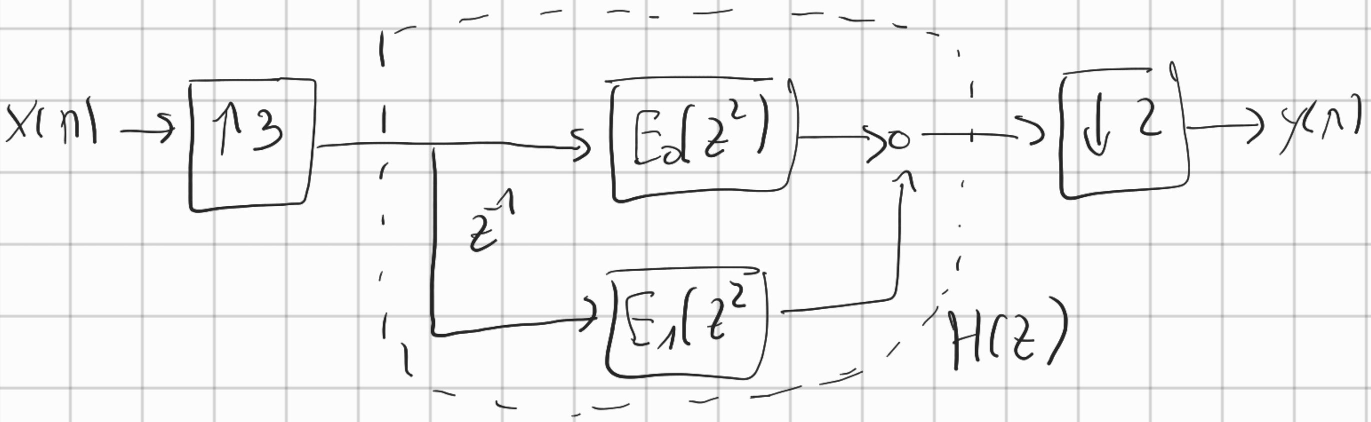


2 consecutive samples and discard first 2 samples

↓

h_0 and h_1 are the last samples

↳ adjusmant to not discard the first samples multiply by z^2 → shifts the initial sample to zero



$$H(z) = E_0(z^2) + z^{-1} E_1(z^2)$$

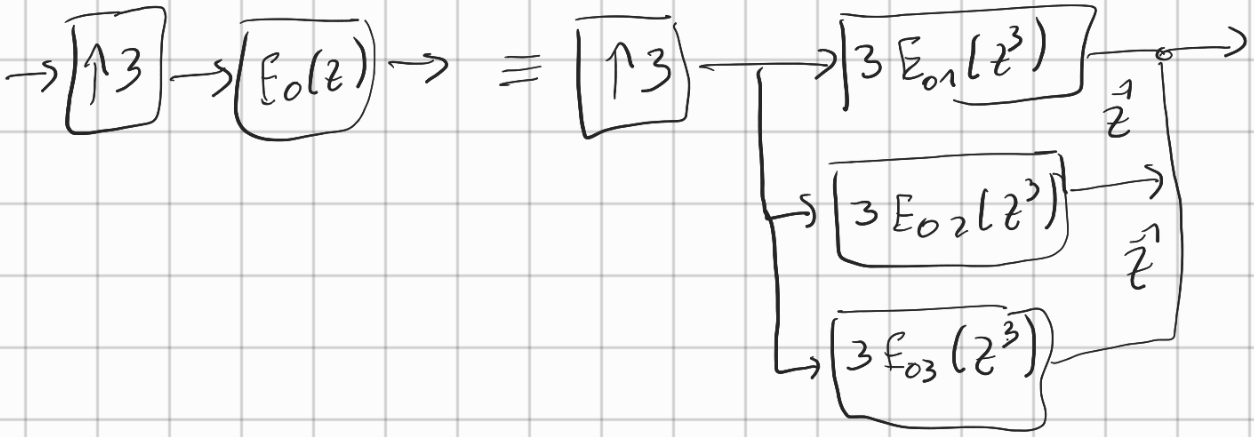
$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + \cdots + h_{11} z^{-11}$$

$$= \underbrace{\left(h_0 + h_2 z^{-2} + h_4 z^{-4} + \cdots h_{10} z^{-10}\right)}_{E_0(z^2)} + z^{-1} \underbrace{\left(h_1 + h_3 z^{-2} + \cdots h_{11} z^{-10}\right)}_{E_1(z^2)}$$

$$E_0(z) = h_0 + h_2 z^{-1} + h_4 z^{-2} + \cdots + h_{10} z^{-5}$$

$$E_1(z) = h_1 + h_3 z^{-1} + h_5 z^{-2} + \cdots + h_{11} z^{-5}$$

$E_0(z)$ and $E_1(z)$ are decomposed as:



$$E_0(z) = 3\left(E_{01}(z^3) + z^{-1}E_{02}(z^3) + z^{-2}E_{03}(z^3)\right)$$

$$\frac{E_0(z)}{3} = h_0 + h_2 z^{-1} + h_4 z^{-2} + h_6 z^{-3} + h_8 z^{-4} + h_{10} z^{-5}$$

$$= \underbrace{(h_0 + h_6 z^{-3})}_{E_{00}} + z^{-1}\underbrace{(h_2 + h_8 z^{-3})}_{E_{01}} + z^{-2}\underbrace{(h_4 + h_{10} z^{-3})}_{E_{02}}$$

$$\frac{E_1(z)}{3} = E_{10}(z^3) + z^{-1}E_{11}(z) + z^{-2}E_{12}(z)$$

$$= h_1 + h_3 z^{-1} + h_5 z^{-2} + h_7 z^{-3} + h_9 z^{-4} + h_{11} z^{-5}$$

$$= \underbrace{(h_1 + h_7 z^{-3})}_{E_{10}} + z^{-1}\underbrace{(h_3 + h_9 z^{-3})}_{E_{11}} + z^{-2}\underbrace{(h_5 + h_{11} z^{-3})}_{E_{12}}$$

Then:

$$\boxed{\begin{array}{ll} E_{00} = [h_0, h_6] & E_{10} = [h_1, h_7] \\ E_{01} = [h_2, h_8] & E_{11} = [h_3, h_9] \\ E_{02} = [h_4, h_{10}] & E_{12} = [h_5, h_{11}] \end{array}}$$

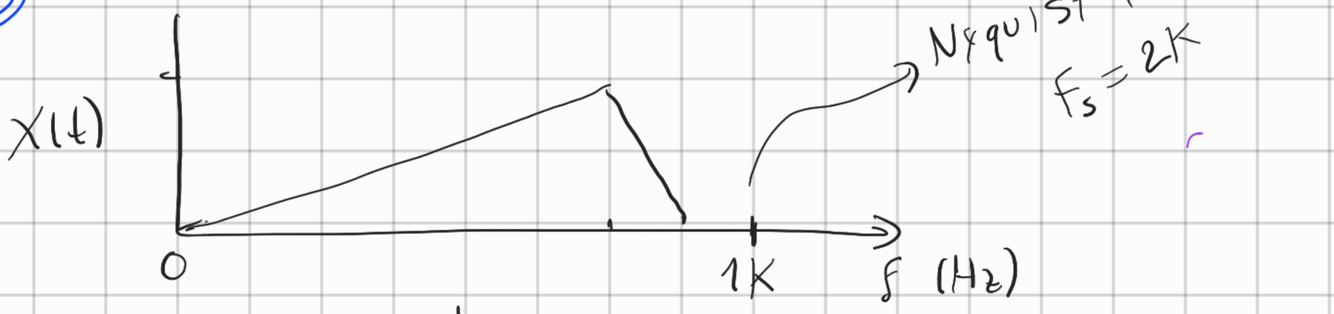
Q12

6 filters $\longrightarrow$ Divide all the freq sequence between those filters

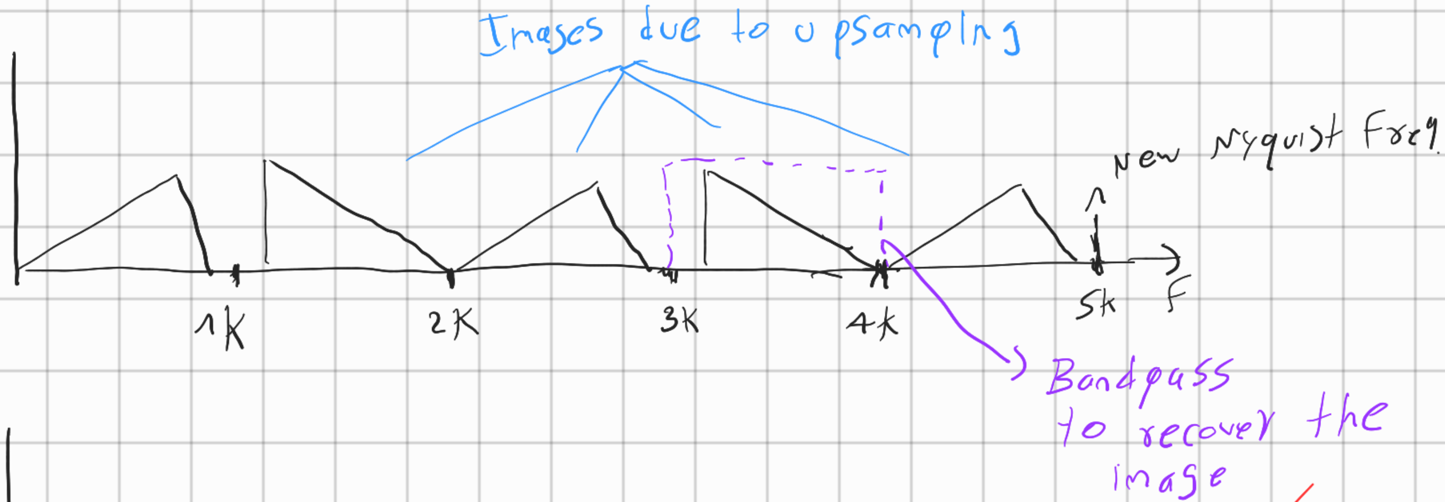
$$\frac{N}{6} = 30 \longrightarrow \text{size of each filter}$$

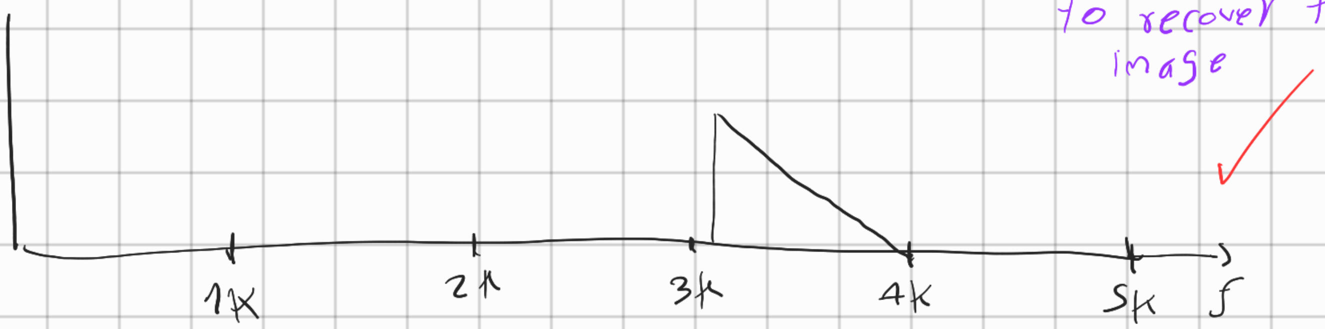
$$\boxed{N = 180}$$

Q13



Increase the sampling rate f_s until 10K $\equiv$ $\boxed{\uparrow 5}$



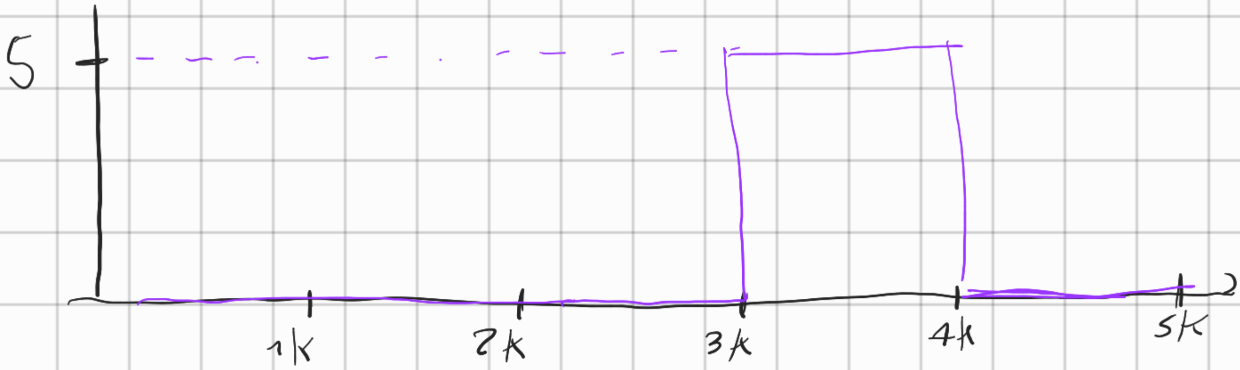


Proposed Device:

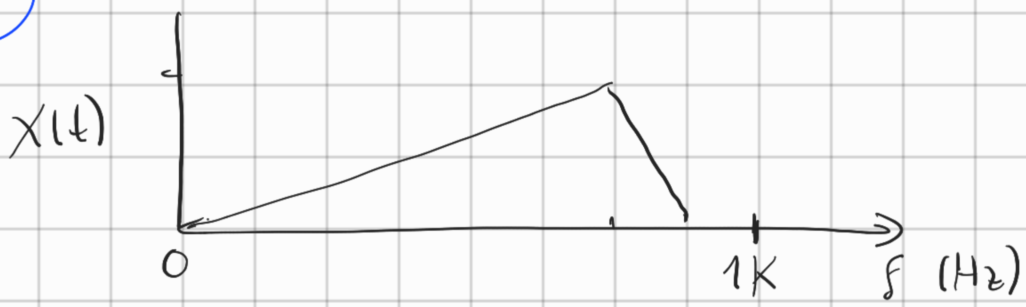
$$x(n) \longrightarrow \boxed{\uparrow 5} \longrightarrow \boxed{H_{BP}(z)} \longrightarrow y(n)$$

Q14 Assuming $H_a(z)$ refers to the proposed $H_{BP}(z)$:

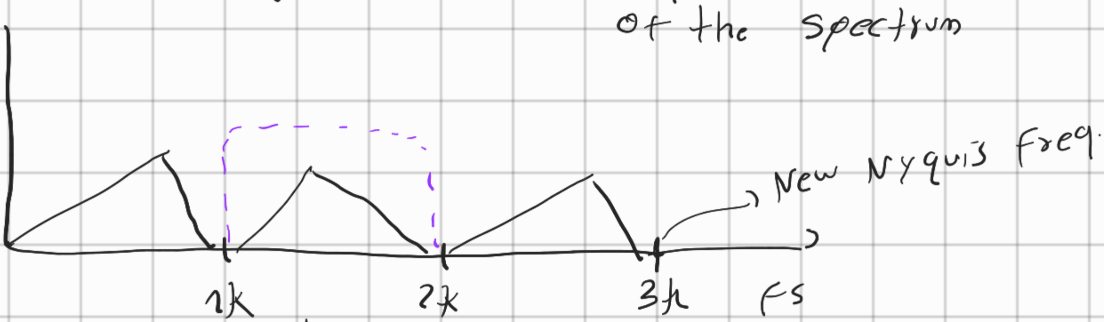
Gain of 5



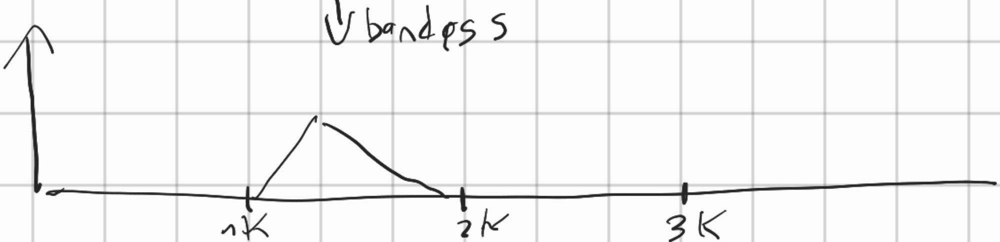
Q15



$\boxed{\uparrow 3}$ = so that one of the image gets positioned at the center of the spectrum



bandpass



↓ downsampling $\boxed{\downarrow 3}$ → effect of expanding in the normalized frequency

Proposed device:

$$x(n) \longrightarrow \boxed{\uparrow 3} \longrightarrow \boxed{H_{BP}(z)} \longrightarrow \boxed{\downarrow 3} \longrightarrow y(n)$$

Q16 Assuming that $H_b(z)$ refers to the proposed $H_{BP}(z)$

Gain of 3 to compensate the reduction by 1/M due to downsampling

↓

amplitude of output same as the input