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Tp4 - Advanced Signal Methods
Kalman Filters

1. Introduction

Preparation

Here are the Kalman filter equations:

- $\mathbf{x_{n+1} = A_n\, x_n + b_n}$, $\mathbf{A_n} \in R^{N \times N}$
- $\mathbf{y_n = C_n\, x_n + v_n}$, $\mathbf{C_n} \in R^{M \times N}$

We started by initializing $\hat{z}_0$ and P_0 and applied the following update equations

For each n iteration:

$$\mathbf{e_n = x_n - C\hat{z}_n}$$

$$\mathbf{G_n = P_n\, C^{\dagger}(CP_nC^{\dagger} + Qb^2)^{-1}}$$

$$\mathbf{P_{n+1} = A(I - G_nC)P_n\, A^{\dagger} + Qv}$$

$$\mathbf{\hat{z}_{n+1} = A\hat{z}_n + AG_ne_n}$$

1.2. Implementation

We started by defining the ***update_filter*** function that updates G_{n+1} et P_{n+1} based on G_n , P_n, and other model parameters.

```
function [G, P] = update_filter(G_, P_, C_, C, Qv, Qb_, A_)

N = size(A_, 1);
P = A_*(eye(N) - G_*C_)*P_*A_'+Qb_;
G = P*C'/(C*P*C'+Qv);

end
```

We then defined the ***predict*** function that updates ẑn+1 based on zn and xn

```
function [x] = predict(x_, y_, C_, A_, G)

e = C_*x_ - y_;
x = A_*(x_ - G*e);

end
```

2. Estimation of the water level in a reservoir

2.1

Here we want to estimate the water level in a reservoir based on the position of a floating device. Every observation corresponds to the vertical position of the device under the influence of waves.

2.1.1 Preparation

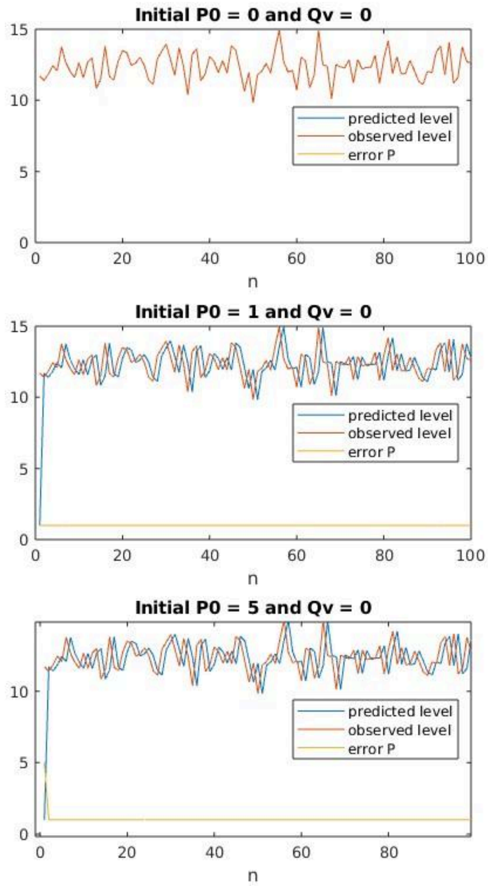
The water level is constant (**A** = **C** = 1), the following functions can describe the problem:

- $\mathbf{h_{n+1} = h_n + b_n}$
- $\mathbf{y_n = h_n + v_n}$

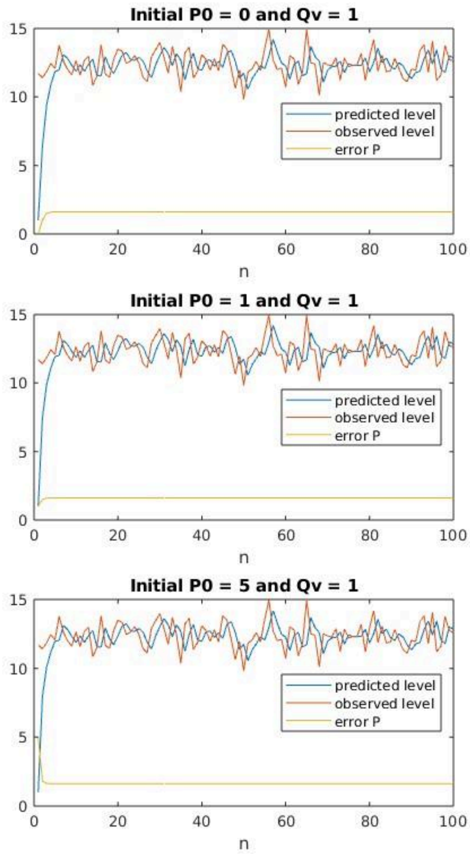
  ($\mathbf{v_n}$ and $\mathbf{b_n}$ are Gaussian white noises)

2.1.2 Implementation

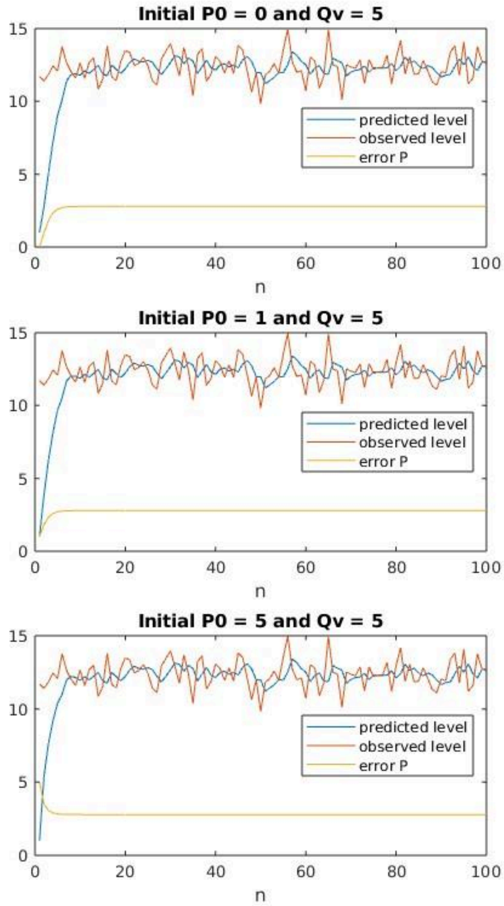
We tested different values for the parameters P and Qb, we started by testing Qv = 0 for different values of P (0,1,5):



We then tested for Qv = 1:



We then tested for Qv = 5:



The higher the Qb values, the slower the convergence of the estimation will be.

2.2

We suppose that the water level is rising at an unknown constant speed, we can only observe the position of the floating device.

2.2.1 Preparation

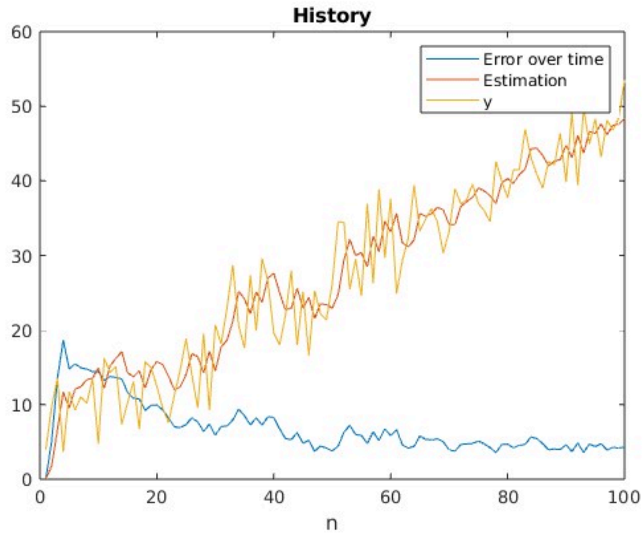
We suppose that **K** is the water level rising constant, which gives us the following bi-dimensional system

$$\left[K;\ h_{n+1}\right] = [1\ \ 0;1\ \ 1]\left[a\,;h_n\right] + \left[0\,;b_n\right]$$

$$y_n = [0;\ 1]\left[a\,;h_n\right] + v_n$$

2.2.2 Implementation

We tested different values for the parameters P and Qb:



After 20-30 iterations the filter converges and gives a good estimate of the increasing water level.

3. Following a moving object

Having an object that is moving on a straight line, we will approximate its movement using the following model:

$$\begin{cases} s_{n+1} = s_n + v_n \Delta t + \frac{1}{2} a_n (\Delta t)^2 \\ v_{n+1} = v_n + a_n \Delta t, \end{cases}$$

sn = s(tn) is the position and vn = v(tn) the speed at T = tn.

The acceleration **an** is a centered Gaussian white noise with a variance σ_a^2.

To follow the object, we dispose of the measurements of its position using a sensor for n≥1. We suppose that the observations are disturbed by a noise with a variance of σ_m^2.

- $\left[S_{n+1} ; \; v_{n+1}\right] = [1 \;\; \Delta t \; ; \; 0 \;\; 1] \left[s_n \; ; \; v_n\right] + \left[a_n (\Delta t)^2/2 \; ; \; a_n \Delta t\right]$

- $y_n = [1 \;\; 0]\left[s_n ; \;\; v_n\right] \; + \; m_n$

End.