$\mathbf{secrBVN}$ - simulation of spatially explicit capture-recapture with bivariate normal home ranges

Murray Efford 2018-08-15

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This small package evaluates SECR when home ranges are BVN or uniform (flat-topped) ellipses. Here we assume detection hazard is directly proportional to home range utilisation (activity). Code to use **secrBVN** for the simulations of Efford (in prep.) is provided in the Appendix.

The key user-visible functions are simpopn.bvn, plotpopn.bvn, simcapt.bvn, runEllipseSim, simsum, simplot and anisotropic.fit.

Generating and plotting elliptical home ranges

simpopn.bvn is a wrapper for the secr function sim.popn that adds attributes specifying a bivariate normal home range shape, size and orientation for each individual. By default, shape and size are the same for all

individuals, but the resulting popn object may be modified so they vary individually (see Heterogeneous elliptical home ranges).

First, load the package.

```
library(secrBVN)
## Loading required package: secr
## This is secr 3.1.7. For overview type ?secr
vignettefolder <- "c:/density secr 3.1/secrBVN/vignettes/"</pre>
simfolder <- "c:/density communication/noncircularity/paper/simulations/"</pre>
runall <- FALSE # skip lengthy simulations
ncores <- 20
                 # for simulations
tempgrid <- make.grid(nx = 10, ny = 10)</pre>
par(mfrow = c(2,4), mar = c(2,2,2.6,2), xpd = TRUE)
for (i in 1:4) {
    s2xy \leftarrow 25^2 * c(1/i, i)
    random.pop <- simpopn.bvn(s2xy = s2xy, core = tempgrid, buffer = 100, D = 1)
    plotpopn.bvn(random.pop, col = 'lightblue')
    mtext(side=3, line=1.5, i)
for (i in 1:4) {
    s2xy < -25^2 * c(1/i, i)
    aligned.pop <- simpopn.bvn(s2xy = s2xy, core = tempgrid, buffer = 100, D = 1, theta = -1)
    plotpopn.bvn(aligned.pop, col = 'lightblue')
}
                                  2
                                                          3
```

Fig. 1. Elliptical home ranges with varying ratio of major to minor axes as shown. Upper row oriented randomly and independently, lower row with shared random orientation ('randomly aligned').

Simulating elliptical detection data

Generating detection histories

Normally in **secr** we use **sim.capthist** to generate capture histories, but that is limited to circular detection functions. The function **simcapt.bvn** is a partial replacement for **sim.capthist** that models detection with a bivariate normal. Specifically, the cumulative hazard of detection is a constant times the bivariate normal

probability density at the detector. The constant is $\lambda_0 2\pi \sigma_X \sigma_Y$. The user provides a 'popn' object that includes the BVN parameter values $(\sigma_x^2, \sigma_y^2, \theta)$ for each animal (row), as generated above by simpopn.bvn. The constant scales the BVN density so that the maximum hazard is λ_0 .

The preceding paragraph describes the default behaviour of simcapt.bvn. If type = uniform is selected then a uniform (flat-topped) elliptical home range is simulated. The uniform probability of detection within the ellipse is controlled by g0, not lambda0.

Fitting a circular detection model to BVN data

The function runEllipseSim is a wrapper for the preceding steps (simpopn.bvn, simcapt.bvn) that also fits a standard (circular) SECR model with secr.fit. The default population has fixed number of individuals within the rectangular buffered area around the detectors (Ndist = 'fixed').

Here we use a 6×6 grid of binary proximity detectors with 50 metre spacing. The code in **secrBVN** does not allow for competition among detectors (secr detector type 'multi') or other other secr detector types. A density of 4/ha gives exactly 169 animals in the buffered area. The intercept parameter g0 or lambda0 is varied to reduce the effect of type on total number of captures and precision. Conditional likelihood is used for speed; the default extractfn = derived is compatible with both CL = TRUE and CL = FALSE. The 200-m buffer allows for the longest ranges ($\sigma_y = 50$ m).

```
nrepl <- 500
tr <- make.grid(6,6, spacing = 50, detector = 'proximity')</pre>
simrandom <- vector('list')</pre>
simrandomBVN <- vector('list')</pre>
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    details <- list(distribution = 'binomial')</pre>
    # uniform
    simrandom[[i]] <- runEllipseSim (nrepl, sigmaX, sigmaY, buffer = 200,</pre>
        ncores = ncores, traps = tr, g0 = 0.2, D = 4, type = 'uniform',
        CL = TRUE, details = details)
    # bun
    simrandomBVN[[i]] <- runEllipseSim (nrepl, sigmaX, sigmaY, buffer = 200,</pre>
        ncores = ncores, traps = tr, lambda0 = 0.4, D = 4, type = 'BVN',
        CL = TRUE, details = details)
}
save(simrandom, file = paste0(vignettefolder, 'simrandom36.2.RData'))
save(simrandomBVN, file = paste0(vignettefolder, 'simrandomBVN36.2.RData'))
```

The package function simplot is used to summarize the results

```
load(file = paste0(vignettefolder, 'simrandom36.2.RData'))
load(file = paste0(vignettefolder, 'simrandomBVN36.2.RData'))
par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = FALSE)
output <- simplot(list(Uniform = simrandom36.2[1:4], BVN = simrandomBVN36.2[1:4]))</pre>
```

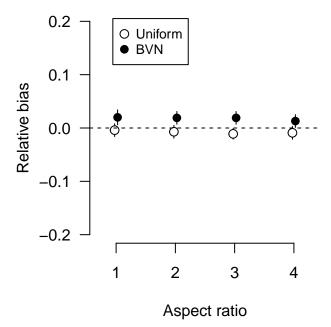


Fig. 2. Relative bias of density estimated by fitting circular SECR detection model to elliptical data, with 95% confidence limit for simulated values. 'Uniform' home ranges were truncated at the 95% activity contour of an equivalent bivariate normal, and detection probability was uniform inside the boundary. 'BVN' home ranges extended indefinitely in all directions.

output

##	\$Uniform				
##		1	2	3	4
##	av.npop	169.000	169.000	169.000	169.000
##	av.nCH	49.310	50.536	52.198	53.946
##	nvalid	500.000	500.000	500.000	500.000
##	${\tt av.parmhat}$	3.986	3.972	3.954	3.963
##	${\tt md.parmhat}$	3.979	3.951	3.939	3.943
##	${\tt sd.parmhat}$	0.497	0.497	0.490	0.501
##	RB	-0.004	-0.007	-0.011	-0.009
##	seRB	0.006	0.006	0.005	0.006
##	RSE	0.122	0.120	0.118	0.117
##	seRSE	0.000	0.000	0.000	0.000
##	rRMSE	0.124	0.124	0.123	0.126
##	COV	0.944	0.942	0.952	0.928
##					
##	\$BVN				
##		1	2	3	4
##	av.npop	169.000	169.000	169.000	169.000
##	av.nCH	42.184	42.942	43.950	44.704
##	nvalid	500.000	500.000	500.000	500.000
##	${\tt av.parmhat}$	4.081	4.076	4.074	4.052
##	${\tt md.parmhat}$	4.051	4.067	4.067	4.028
##	${\tt sd.parmhat}$	0.593	0.581	0.569	0.565

```
## RB
                 0.020
                         0.019
                                  0.019
                                          0.013
                 0.007
                         0.006
                                  0.006
                                          0.006
## seRB
## RSE
                 0.140
                         0.139
                                  0.139
                                          0.139
## seRSE
                 0.000
                         0.000
                                  0.000
                                          0.000
## rRMSE
                 0.149
                         0.146
                                  0.143
                                          0.142
## COV
                 0.940
                         0.934
                                  0.946
                                          0.948
```

There is no apparent effect of range elongation itself on the bias of the estimates. Fitting a halfnormal detection function (detectfn = 0) to data from 'hard-edged' (uniform) home ranges (detectfn = 4) appears to result in negative bias on the order of -1% to -2% (Efford 2004 noted a relative bias of -1.2%, SE 0.8% for a small sample of 100 simulations fitting a model by inverse prediction in the circular case).

Check circular using alternate method (sim.capthist)

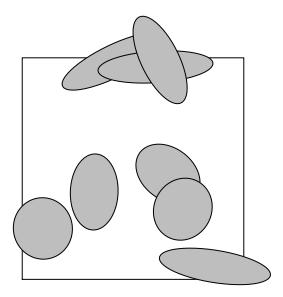
```
nrepl <- 500
tr <- make.grid(6,6, spacing = 50, detector = 'proximity')</pre>
simcirc <- vector('list')</pre>
simcircBVN <- vector('list')</pre>
    sigmaX <- 25
    details <- list(distribution = 'binomial')</pre>
    # uniform
    simcirc[[1]] <- runEllipseSim (nrepl, sigmaX, sigmaY=NULL, buffer = 200, ncores = 2,</pre>
                                traps = tr, g0 = 0.2, D = 4, type = 'uniform', CL = TRUE,
                                details = details)
    # bun
    simcircBVN[[1]] <- runEllipseSim (nrepl, sigmaX, sigmaY=NULL, buffer = 200, ncores = 2,
                                traps = tr, lambda0 = 0.4, D = 4, type = 'BVN', CL = TRUE,
                                details = details)
save(simcirc, file = 'simcirc.RData')
save(simcircBVN, file = 'simcircBVN.RData')
load(file = paste0(vignettefolder, 'simcirc.RData'))
load(file = paste0(vignettefolder, 'simcircBVN.RData'))
par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = FALSE)
simsum(list(Uniform = simcirc, BVN = simcircBVN))
## $Uniform
                    RSE rRMSE
## av.nCH
              RB
                                   COV
## 48.756 -0.018 0.122 0.127
                                 0.924
##
## $BVN
                                   COV
## av.nCH
              RB
                    RSE rRMSE
## 42.678 0.006 0.137 0.150 0.938
```

Heterogeneous elliptical home ranges

simpopn.bvn usually generates a population with equal-sized home ranges. The size and elongation of each range may be varied by providing a function as the argument s2xy:

```
tr <- make.grid(6,6, spacing = 50, detector = 'proximity')
rs2xy <- function(N, scale = 25) {
   aspectratio <- 1 + runif(N) * 3
   cbind(scale^2 / aspectratio, scale^2 * aspectratio)</pre>
```

```
pop <- simpopn.bvn(s2xy = rs2xy, core = tr, buffer = 100, D = 1)
par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = TRUE)
plotpopn.bvn(pop, col = 'grey')</pre>
```



Heterogeneous populations may also be simulated in runEllipseSim by passing a function as the argument 'sigmaX'.

Anisotropic home ranges: a partial solution

In principle, we can deal with elongated ranges by replacing Euclidean distances with distances in a space transformed to render home ranges circular (Murphy et al. 2016). In effect home ranges are compressed along their major axis. The transformation uses two parameters: ψ_A for the shared orientation measured in radians and ψ_R for the degree of compression ($\psi_R \geq 1$; $\psi_R = 1$ corresponds to no compression). Thanks to Ben Augustine for pointing out the **geoR** function **coords.aniso** that lets us do this (Ribeiro and Diggle 2018)¹.

The method does not work if the detector array is strictly linear (2-D data are required to estimate elongation). It is tested in the Appendix on data from a hollow square array. In this version both transformation parameters are estimated (cf Murphy et al. 2016). This is the only place in this vignette that we attempt to fit elongated ranges rather than circular ranges.

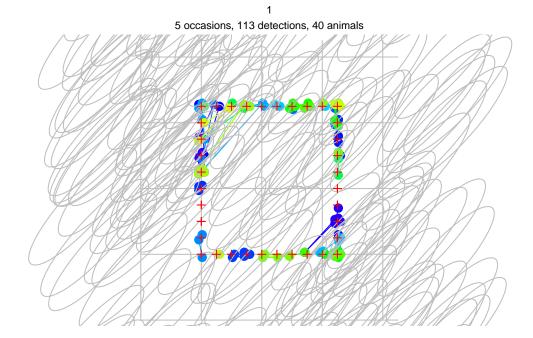
The code uses a user-defined distance function that computes a Euclidean distance in the transformed space. Transformation parameters are handled in the undocumented 'miscparm' feature of 'secr'. This allows

 $^{^1}$ The package intamap (Pebesma et al. 2010) also offers anisotropic transformation in function rotateAnisotropicData.

supernumerary unmodelled parameters to be passed to the distance function. The parameters are included in the vector of coefficients (beta parameters) over which the likelihood is maximised. 'Unmodelled' here means that the parameter takes a single value for all animals, times and places. The link function is 'identity' for ψ_A and 'log' for $\psi_R - 1$.

```
anisodistfn <- function (xy1, xy2, mask) {</pre>
    if (missing(xy1)) return(character(0))
    xy1 <- as.matrix(xy1)</pre>
    xy2 <- as.matrix(xy2)</pre>
    miscparm <- attr(mask, 'miscparm')</pre>
    psiA <- miscparm[1]</pre>
                                    # anisotropy angle; identity link
    psiR <- 1 + exp(miscparm[2]) # anisotropy ratio; log link</pre>
    aniso.xy1 <- geoR::coords.aniso(xy1, aniso.pars = c(psiA, psiR))</pre>
    aniso.xy2 <- geoR::coords.aniso(xy2, aniso.pars = c(psiA, psiR))</pre>
    secr::edist(aniso.xy1, aniso.xy2) # nrow(xy1) x nrow(xy2) matrix
}
Here is a simple application with oblique elliptical home ranges (theta = pi/4).
tr <- make.grid(10, 10, spacing = 25, hollow = TRUE, detector = 'proximity')</pre>
pop <- simpopn.bvn(s2xy = (25*c(0.5,2))^2, theta = pi/4, core = tr, buffer = 200,
                    D = 4, Ndist = 'fixed')
CH <- simcapt.bvn(tr, pop, type = 'BVN', lambda = 0.4, noccasions = 5)
par(mfrow = c(1,1))
plot(CH, tracks = TRUE)
## Warning in plot.capthist(CH, tracks = TRUE): track for repeat detections on
## same occasion joins points in arbitrary sequence
plotpopn.bvn(pop, border='grey', add = TRUE)
```

plot(tr, add = TRUE)



First we fit a naive circular model.

```
fit0 <- secr.fit(CH, buffer = 200, detectfn = 'HHN', trace = FALSE,
                details = list(distribution = 'binomial'))
predict(fit0)
##
           link estimate SE.estimate
                                              lcl
                                                         ucl
            log 4.4223537 0.70480154 3.2422397 6.0320068
## lambda0 log 0.3637934 0.06910876 0.2515275 0.5261676
## sigma
            log 26.4904328 2.09136282 22.6982762 30.9161374
Next, the model with transformation to isotropy.
details <- list(distribution = 'binomial', userdist = anisodistfn,</pre>
                miscparm = c(psiA = 0.5, psiR = 1.5)) # initial values
fit1 <- secr.fit(CH, buffer = 200, detectfn = 'HHN', trace = FALSE,
                details = details)
predict(fit1)
##
           link estimate SE.estimate
                                             lcl
                                                        ucl
            log 3.3876558 0.63526156 2.3531900 4.8768743
## lambda0 log 0.4924023 0.08259622 0.3552448 0.6825153
## sigma
            log 11.3313969 0.98536729 9.5587951 13.4327135
coef(fit1)
##
                         SE.beta
                 beta
                                       1c1
                                                   ucl
            1.2201382 0.18590460 0.8557719 1.5845045
## lambda0 -0.7084592 0.16657904 -1.0349481 -0.3819703
```

```
## sigma 2.4275774 0.08679531 2.2574617 2.5976930
## psiA 0.7051085 0.04453409 0.6178233 0.7923937
## psiR 1.3705119 0.25663404 0.8675184 1.8735053
```

The estimated bearing is 40.4 degrees. The estimated aspect ratio is 4.94, (95% CI 3.38, 7.51).

The function anisotropic.fit streamlines model fitting and does not require anisodistfn to be defined externally. Use it as you would use secr.fit. For example,

```
## estimate SE.estimate 1c1 ucl
## psiA 40.39978 2.551621 35.398695 45.40087
## psiR 4.93735 1.027328 3.380984 7.51106
```

Package limitations

This package has the limited goal of determining how range elongation affects estimates of density in simple SECR models, and these specific limitations:

- 1. Only binary proximity detectors are supported.
- 2. The spatial distribution of activity centres is assumed to be homogeneous Poisson.
- 3. Ellipses are specified using either 'sigmaX' and 'sigmaY' as separate arguments (runEllipseSim) or as a vector of the two values, squared ('s2xy' in simpopn.bvn). This is confusing but it's better at this point not to change.

References

Efford, M. G. (2004) Density estimation in live-trapping studies. Oikos 106, 598–610.

Efford, M. G. In prep. Non-circular home ranges and the estimation of population density.

Huggins, R. M. (1989) On the statistical analysis of capture experiments. Biometrika 76, 133-140.

Ivan, J. S., White, G. C. and Shenk, T. M. (2013) Using simulation to compare methods for estimating density from capture–recapture data. *Ecology* **94**, 817–826.

Murphy, S. M., Cox, J. J., Augustine, B. C., Hast, J. T., Guthrie, J. M., Wright, J., McDermott, J., Maehr, S. C. and Plaxico, J. H. (2016) Characterizing recolonization by a reintroduced bear population using genetic spatial capture—recapture. *Journal of Wildlife Management* 80, 1390–1407.

Pebesma, E., Cornford, D., Dubois, G., Heuvelink, G.B.M., Hristopoulos, D., Pilz, J., Stoehlker, U., Morin, G. and Skoien, J.O. (2010) INTAMAP: the design and implementation of an interoperable automated interpolation web service. *Computers & Geosciences* 37, 343–352.

Ribeiro, P. J. Jr and Diggle, P. J. (2018). geoR: Analysis of Geostatistical Data. R package version 1.7-5.2.1. https://CRAN.R-project.org/package=geoR.

Appendix. Code for simulations of Efford (in prep)

Functions for heterogeneous aspect ratio.

```
# Uniform on 1-4
rs2xy <- function(N, scale = 25) {
    i <- 1 + runif(N) * 3
    cbind(scale^2 /i, scale^2 * i)
}
# 2 classes 1,4
rs2xy2 <- function(N, scale = 25, prob = c(0.5,0.5)) {
    i <- sample(c(1,4), size = N, replace = TRUE, prob = prob)
    cbind(scale^2 /i, scale^2 * i)
}</pre>
```

Main simulations

$10 \times 10 \text{ grid}$

Elongated ranges are oriented at random with respect to the grid (default theta = NULL).

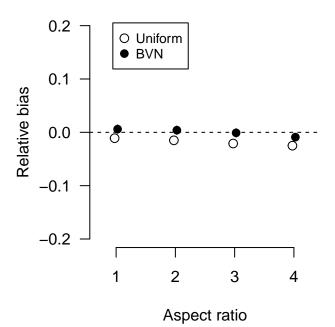
```
tr <- make.grid(10,10, spacing = 50, detector = 'proximity')</pre>
simrandom100.3 <- vector('list', 6)</pre>
names(simrandom100.3) <-c(1:4,'1-4','1,4')
simrandomBVN100.3 <- simrandom100.3</pre>
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr, theta = NULL,
                  D = 4, CL = TRUE, detectfn = 'HHN', details = details, seed = 347)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5</pre>
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    # uniform
    args$type <- 'uniform'; args$g0 <- 0.2</pre>
    simrandom100.3[[i]] <- do.call(runEllipseSim, args)</pre>
    # bvn
    args$type <- 'BVN'; args$lambda0 <- 0.4</pre>
    simrandomBVN100.3[[i]] <- do.call(runEllipseSim, args)</pre>
    message ('Completed aspect ratio', i)
}
# heterogeneous aspect ratio 1,4, uniform
args$type <- 'uniform'; args$sigmaX <- rs2xy</pre>
simrandom100.3[[5]] <- do.call(runEllipseSim, args)</pre>
# heterogeneous aspect ratio 1,4, BVN
args$type <- 'BVN'; args$sigmaX <- rs2xy</pre>
simrandomBVN100.3[[5]] <- do.call(runEllipseSim, args)</pre>
# heterogeneous aspect ratio 1,4, uniform
args$type <- 'uniform'; args$sigmaX <- rs2xy2</pre>
simrandom100.3[[6]] <- do.call(runEllipseSim, args)</pre>
#heterogeneous aspect ratio 1,4, BVN
args$type <- 'BVN'; args$sigmaX <- rs2xy2; args$seed <- 347
```

```
simrandomBVN100.3[[6]] <- do.call(runEllipseSim, args)

save(simrandom100.3, file = paste0(simfolder, 'simrandom100.3.RData'))
save(simrandomBVN100.3, file = paste0(simfolder, 'simrandomBVN100.3.RData'))

load(file = paste0(simfolder, 'simrandom100.3.RData'))
load(file = paste0(simfolder, 'simrandomBVN100.3.RData'))

par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = FALSE)
# select only first 4 scenarios for plotting
simplot(list(Uniform = simrandom100.3[1:4], BVN = simrandomBVN100.3[1:4]), legend = TRUE)</pre>
```



```
# tabulate all
simsum(list(Uniform = simrandom100.3, BVN = simrandomBVN100.3), compact = NULL, dec = 4)
```

```
## $Uniform
                               2
##
                     1
                                        3
                                                         1 - 4
                                                                  1,4
              289.0000 289.0000 289.0000 289.0000 289.0000 289.0000
## av.npop
## av.nCH
              121.0620 123.2520 126.3220 129.2880 124.7580 125.4240
## nvalid
              500.0000 500.0000 500.0000 500.0000 500.0000 500.0000
## av.parmhat
                         3.9404
                                   3.9174
                                            3.9005
                3.9543
                                                      3.9168
                                                               3.9210
## md.parmhat
                3.9722
                         3.9454
                                   3.9324
                                            3.9116
                                                      3.9189
                                                               3.9181
## sd.parmhat
                0.2829
                         0.2852
                                   0.2760
                                            0.2695
                                                      0.2689
                                                               0.2712
               -0.0114
                        -0.0149
                                  -0.0207 -0.0249
                                                    -0.0208
## RB
                                                             -0.0197
## seRB
                0.0032
                         0.0032
                                   0.0031
                                            0.0030
                                                      0.0030
                                                               0.0030
## RSE
                                   0.0662
                0.0686
                         0.0676
                                            0.0650
                                                      0.0669
                                                               0.0666
## seRSE
                0.0001
                         0.0001
                                   0.0001
                                            0.0001
                                                      0.0001
                                                               0.0001
## rRMSE
                0.0716
                         0.0728
                                   0.0720
                                            0.0718
                                                      0.0703
                                                               0.0706
## COV
                0.9400
                         0.9320
                                   0.9280
                                            0.9000
                                                      0.9280
                                                               0.9380
```

```
##
## $BVN
##
                                       3
              289.0000 289.0000 289.0000 289.0000 289.0000 289.0000
## av.npop
## av.nCH
              108.3260 109.4820 110.9340 112.0540 110.0220 109.9820
## nvalid
              500.0000 500.0000 500.0000 500.0000 500.0000
## av.parmhat
               4.0257
                         4.0172
                                  3.9958
                                           3.9652
                                                     3.9895
                4.0378
                         4.0215
                                  3.9992
                                           3.9739
                                                    4.0094
## md.parmhat
                                                              3.9591
                0.3202
## sd.parmhat
                         0.3280
                                  0.3247
                                           0.3100
                                                    0.3116
                                                              0.3089
## RB
                         0.0043 -0.0011 -0.0087
                                                   -0.0026
                0.0064
                                                            -0.0067
## seRB
                0.0036
                         0.0037
                                  0.0036
                                           0.0035
                                                    0.0035
                                                              0.0035
## RSE
                0.0773
                         0.0769
                                  0.0765
                                           0.0762
                                                    0.0767
                                                              0.0767
## seRSE
                0.0001
                         0.0001
                                  0.0001
                                           0.0001
                                                    0.0001
                                                              0.0001
## rRMSE
                0.0802
                         0.0820
                                  0.0811
                                           0.0779
                                                    0.0779
                                                              0.0774
## COV
                0.9500
                         0.9420
                                  0.9360
                                           0.9400
                                                    0.9500
                                                              0.9440
# compact table for paper
simsum(list(Uniform = simrandom100.3, BVN = simrandomBVN100.3))
## $Uniform
##
       av.nCH
                   RB
                        RSE rRMSE
                                    COV
      121.062 -0.011 0.069 0.072 0.940
## 2
       123.252 -0.015 0.068 0.073 0.932
       126.322 -0.021 0.066 0.072 0.928
       129.288 -0.025 0.065 0.072 0.900
## 1-4 124.758 -0.021 0.067 0.070 0.928
## 1,4 125.424 -0.020 0.067 0.071 0.938
## $BVN
##
       av.nCH
                   RB
                        RSE rRMSE
## 1
       108.326 0.006 0.077 0.080 0.950
       109.482 0.004 0.077 0.082 0.942
## 3
       110.934 -0.001 0.077 0.081 0.936
      112.054 -0.009 0.076 0.078 0.940
## 1-4 110.022 -0.003 0.077 0.078 0.950
## 1,4 109.982 -0.007 0.077 0.077 0.944
# spatial scale parameter
output <- simsum(list(Uniform = simrandom100.3, BVN = simrandomBVN100.3),</pre>
    component = 'pred', parm = 'sigma', compact = c("av.parmhat", "sd.parmhat"))
lapply(output, '/', 50) # in units of detector spacing
## $Uniform
##
       av.parmhat sd.parmhat
## 1
          0.59046
                     0.01422
## 2
          0.65116
                     0.01836
## 3
          0.74018
                     0.02470
## 4
          0.82576
                     0.02966
## 1-4
          0.70118
                     0.02356
## 1,4
          0.71320
                     0.02736
##
## $BVN
       av.parmhat sd.parmhat
##
## 1
          0.50072
                     0.01780
## 2
          0.55270
                     0.02558
## 3
          0.62760
                     0.03130
```

```
## 4 0.69782 0.03806
## 1-4 0.59366 0.03278
## 1,4 0.60450 0.03652
```

Straight line 36-detectors

```
tr <- make.grid(36, 1, spacing = 50, detector = 'proximity')</pre>
simgridalignlinw1.1 <- vector('list', 6)</pre>
names(simgridalignlinw1.1) <- c(1:4,'1-4','1,4')</pre>
simgridalignlinw4.1 <- simgridalignlinw3.1 <- simgridalignlinw2.1 <- simgridalignlinw1.1
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                 lambda0 = 0.4, D = 4, type = 'BVN',
                 CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    # bun parallel to traps
    args$theta <- 0; simgridalignlinw1.1[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun oblique to traps
    args$theta <- pi/4; simgridalignlinw2.1[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun perpendicular to traps
    args$theta <- pi/2; simgridalignlinw3.1[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun random orientation
    args$theta <- NULL; simgridalignlinw4.1[[i]] <- do.call(runEllipseSim, args)</pre>
}
args <- c(baseargs, list(sigmaX = rs2xy))</pre>
args$theta <- 0; simgridalignlinw1.1[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignlinw2.1[[5]] <- do.call(runEllipseSim, args)
args$theta <- pi/2; simgridalignlinw3.1[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignlinw4.1[[5]] <- do.call(runEllipseSim, args)</pre>
args <- c(baseargs, list(sigmaX = rs2xy2))</pre>
args$theta <- 0; simgridalignlinw1.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignlinw2.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/2; simgridalignlinw3.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignlinw4.1[[6]] <- do.call(runEllipseSim, args)
save(simgridalignlinw1.1, file = paste0(simfolder, 'simgridalignlinw1.1.RData'))
save(simgridalignlinw2.1, file = paste0(simfolder, 'simgridalignlinw2.1.RData'))
save(simgridalignlinw3.1, file = paste0(simfolder, 'simgridalignlinw3.1.RData'))
save(simgridalignlinw4.1, file = paste0(simfolder, 'simgridalignlinw4.1.RData'))
# compact table linear
load(file = paste0(simfolder, 'simgridalignlinw1.1.RData'))
load(file = paste0(simfolder, 'simgridalignlinw2.1.RData'))
load(file = paste0(simfolder, 'simgridalignlinw3.1.RData'))
load(file = paste0(simfolder, 'simgridalignlinw4.1.RData'))
simsum(list(Parallel
                           = simgridalignlinw1.1,
                           = simgridalignlinw2.1,
             Oblique
             Perpendicular = simgridalignlinw3.1,
             Random
                           = simgridalignlinw4.1))
```

```
## $Parallel
##
       av.nCH
                 R.B
                      RSE rRMSE
                                   COV
## 1
       56.452 0.011 0.171 0.179 0.944
       66.474 0.867 0.159 0.926 0.052
## 2
## 3
       70.976 1.447 0.161
                             NA 0.000
## 4
      73.130 1.791 0.168
                             NA 0.000
## 1-4 67.740 1.008 0.160 1.057 0.008
## 1,4 64.686 0.643 0.167 0.710 0.206
##
## $Oblique
       av.nCH
                 RB
                      RSE rRMSE
                                   COV
       56.452 0.011 0.171 0.179 0.944
## 1
       59.770 0.262 0.168 0.337 0.718
## 3
       64.032 0.619 0.162 0.674 0.206
       67.228 0.942 0.157 0.987 0.022
## 1-4 61.960 0.423 0.167 0.483 0.468
## 1,4 61.666 0.402 0.168 0.475 0.488
##
## $Perpendicular
##
       av.nCH
                  RB
                       RSE rRMSE
                                    COV
## 1
       56.452 0.011 0.171 0.179 0.944
       46.480 -0.490 0.167 0.499 0.018
       40.924 -0.661 0.163 0.664 0.000
## 3
       37.206 -0.746 0.160 0.748 0.000
## 1-4 44.372 -0.578 0.166 0.583 0.000
## 1,4 46.732 -0.553 0.166 0.559 0.008
##
## $Random
##
       av.nCH
                      RSE rRMSE
                                   COV
                 R.B
       56.400 0.029 0.172 0.181 0.954
## 2
       58.290 0.075 0.173 0.207 0.908
## 3
       60.340 0.124 0.175 0.277 0.832
       61.762 0.171 0.176 0.343 0.764
## 1-4 59.258 0.106 0.173 0.263 0.816
## 1,4 59.360 0.094 0.173 0.243 0.862
```

Hollow square 36 detectors

```
args$theta <- NULL; simgridalignsqw3.1[[i]] <- do.call(runEllipseSim, args)
}
args <- c(baseargs, list(sigmaX = rs2xy))</pre>
args$theta <- 0;</pre>
                    simgridalignsqw1.1[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignsqw2.1[[5]] <- do.call(runEllipseSim, args)
args$theta <- NULL; simgridalignsqw3.1[[5]] <- do.call(runEllipseSim, args)
args <- c(baseargs, list(sigmaX = rs2xy2))</pre>
args$theta <- 0;</pre>
                    simgridalignsqw1.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignsqw2.1[[6]] <- do.call(runEllipseSim, args)
args$theta <- NULL; simgridalignsqw3.1[[6]] <- do.call(runEllipseSim, args)
save(simgridalignsqw1.1, file = paste0(simfolder, 'simgridalignsqw1.1.RData'))
save(simgridalignsqw2.1, file = paste0(simfolder, 'simgridalignsqw2.1.RData'))
save(simgridalignsqw3.1, file = paste0(simfolder, 'simgridalignsqw3.1.RData'))
# compact table square
load(file = paste0(simfolder, 'simgridalignsqw1.1.RData'))
load(file = paste0(simfolder, 'simgridalignsqw2.1.RData'))
load(file = paste0(simfolder, 'simgridalignsqw3.1.RData'))
simsum(list(Aligned = simgridalignsqw1.1,
            Oblique = simgridalignsqw2.1,
            Random = simgridalignsqw3.1))
## $Aligned
     av.nCH
                RB RSE rRMSE
                                  COV
## 1 55.580 0.021 0.166 0.177 0.958
## 2 55.792 -0.067 0.165 0.189 0.882
## 3 55.724 -0.191 0.165 0.248 0.658
## 4 55.220 -0.297 0.163 0.336 0.368
##
## $Oblique
                    RSE rRMSE
                                 COV
     av.nCH
               RB
## 1 55.580 0.021 0.166 0.177 0.958
## 2 58.794 0.198 0.168 0.284 0.800
## 3 62.406 0.421 0.171 0.513 0.492
## 4 65.362 0.616 0.175 0.733 0.306
##
## $Random
   av.nCH RB RSE rRMSE
                                 COV
##
## 1 55.468 0.020 0.166 0.170 0.940
## 2 57.068 0.061 0.167 0.212 0.906
## 3 59.110 0.108 0.169 0.275 0.834
## 4 60.814 0.140 0.170 0.314 0.808
Reduced spacing (\sigma instead of 2\sigma)
10 x 10 grid
tr <- make.grid(10,10, spacing = 25, detector = 'proximity')</pre>
simrandomBVNs100.1 <- vector('list', 4)</pre>
```

Straight line

```
tr <- make.grid(36, 1, spacing = 25, detector = 'proximity')</pre>
simgridalignlinw1.1 <- vector('list', 6)</pre>
names(simgridalignlinw1.1) <- c(1:4,'1-4','1,4')</pre>
simgridalignlinw4.1 <- simgridalignlinw3.1 <- simgridalignlinw2.1 <- simgridalignlinw1.1
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                 lambda0 = 0.4, D = 4, type = 'BVN', theta = 0,
                 CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    # bun parallel to traps
    args$theta <- 0; simgridalignlin1.3[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun oblique to traps
    args$theta <- pi/4; simgridalignlin2.3[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun perpendicular to traps
    args$theta <- pi/2; simgridalignlin3.3[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun random orientation
    args$theta <- NULL; simgridalignlin4.3[[i]] <- do.call(runEllipseSim, args)
args <- c(baseargs, list(sigmaX = rs2xy))</pre>
args$theta <- 0; simgridalignlin1.3[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignlin2.3[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/2; simgridalignlin3.3[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignlin4.3[[5]] <- do.call(runEllipseSim, args)
args <- c(baseargs, list(sigmaX = rs2xy2))</pre>
                    simgridalignlin1.3[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- 0;
args$theta <- pi/4; simgridalignlin2.3[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/2; simgridalignlin3.3[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignlin4.3[[6]] <- do.call(runEllipseSim, args)</pre>
save(simgridalignlin1.3, file = paste0(simfolder, 'simgridalignlin1.3.RData'))
save(simgridalignlin2.3, file = paste0(simfolder, 'simgridalignlin2.3.RData'))
save(simgridalignlin3.3, file = paste0(simfolder, 'simgridalignlin3.3.RData'))
save(simgridalignlin4.3, file = paste0(simfolder, 'simgridalignlin4.3.RData'))
```

Hollow square

```
tr <- make.grid(10, 10, spacing = 25, detector = 'proximity', hollow = TRUE)
simgridalignsq1.1 <- vector('list', 4) # 'w' for wide
names(simgridalignsq1.1) <- 1:4</pre>
simgridalignsq3.1 <- simgridalignsq2.1 <- simgridalignsq1.1</pre>
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                  lambda0 = 0.4, D = 4, type = 'BVN',
                  CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    # bun parallel to traps
    args$theta <- 0; simgridalignsq1.1[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun oblique to traps
    args$theta <- pi/4; simgridalignsq2.1[[i]] <- do.call(runEllipseSim, args)</pre>
    # bun random orientation
    args$theta <- NULL; simgridalignsq3.1[[i]] <- do.call(runEllipseSim, args)</pre>
}
args <- c(baseargs, list(sigmaX = rs2xy))</pre>
args$theta <- 0; simgridalignsq1.1[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignsq2.1[[5]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignsq3.1[[5]] <- do.call(runEllipseSim, args)</pre>
args <- c(baseargs, list(sigmaX = rs2xy2))</pre>
args$theta <- 0; simgridalignsq1.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- pi/4; simgridalignsq2.1[[6]] <- do.call(runEllipseSim, args)</pre>
args$theta <- NULL; simgridalignsq3.1[[6]] <- do.call(runEllipseSim, args)</pre>
save(simgridalignsq1.2, file = paste0(simfolder, 'simgridalignsq1.2.RData'))
save(simgridalignsq2.2, file = paste0(simfolder, 'simgridalignsq2.2.RData'))
save(simgridalignsq3.2, file = paste0(simfolder, 'simgridalignsq3.2.RData'))
```

Spacing comparisons

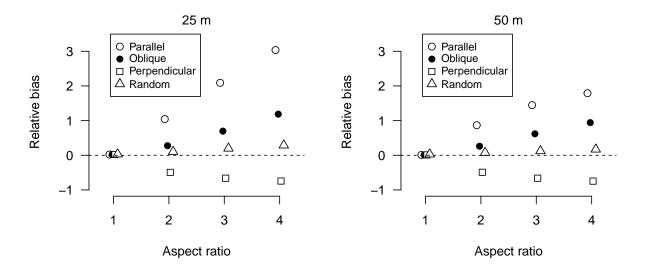
$10 \times 10 \text{ grid}$

```
load(file = paste0(simfolder, 'simrandomBVN100.3.RData'))
load(file = paste0(simfolder, 'simrandomBVNs100.1.RData'))
par(mfrow=c(1,2))
simplot(list(BVN = simrandomBVNs100.1), legend = FALSE)
mtext(side=3, text = '25 m')
simplot(list(BVN = simrandomBVN100.3[1:4]), legend = FALSE)
mtext(side=3, text = '50 m')
```

Straight line

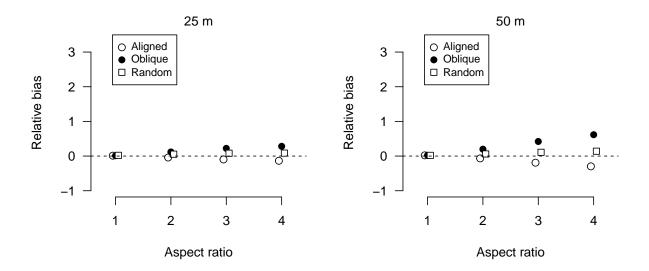
```
load(file = paste0(simfolder, 'simgridalignlin1.3.RData'))
load(file = paste0(simfolder, 'simgridalignlin2.3.RData'))
load(file = paste0(simfolder, 'simgridalignlin3.3.RData'))
load(file = paste0(simfolder, 'simgridalignlin4.3.RData'))
load(file = paste0(simfolder, 'simgridalignlinw1.1.RData'))
```

```
load(file = paste0(simfolder, 'simgridalignlinw2.1.RData'))
load(file = paste0(simfolder, 'simgridalignlinw3.1.RData'))
load(file = paste0(simfolder, 'simgridalignlinw4.1.RData'))
par(mfrow=c(1,2))
simplot(list(Parallel
                           = simgridalignlin1.3[1:4],
                           = simgridalignlin2.3[1:4],
             Perpendicular = simgridalignlin3.3[1:4],
             Random
                           = simgridalignlin4.3[1:4]),
        legend = TRUE, ylim = c(-1,3.5))
mtext(side=3, text = '25 m')
simplot(list(Parallel
                           = simgridalignlinw1.1[1:4],
                           = simgridalignlinw2.1[1:4],
             Oblique
             Perpendicular = simgridalignlinw3.1[1:4],
             Random
                           = simgridalignlinw4.1[1:4]),
        legend = TRUE, ylim = c(-1,3.5))
mtext(side=3, text = '50 m')
```



Hollow square

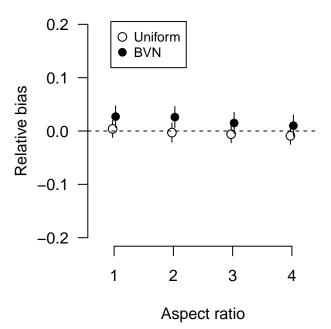
```
Random = simgridalignsqw3.1[1:4]),
legend = TRUE, ylim = c(-1,3.5))
mtext(side=3, text = '50 m')
```



Variations

Low density (0.5/ha)

```
tr <- make.grid(10, 10, spacing = 50, detector = 'proximity')</pre>
simrandom100low.2 <- vector('list', 4)</pre>
names(simrandom100low.2) <- 1:4</pre>
simrandomBVN100low.2 <- simrandom100low.2</pre>
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                  D = 0.5, CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    args$type <- 'uniform'; args$g0 <- 0.2</pre>
    simrandom100low.2[[i]] <- do.call(runEllipseSim, args)</pre>
    args\$type <- 'BVN'; args\$lambda0 <- 0.4
    simrandomBVN100low.2[[i]] <- do.call(runEllipseSim, args)</pre>
}
save(simrandom100low.2, file = paste0(simfolder, 'simrandom100low.2.RData'))
save(simrandomBVN100low.2, file = paste0(simfolder, 'simrandomBVN100low.2.RData'))
load(file = paste0(simfolder, 'simrandom100low.2.RData'))
load(file = paste0(simfolder, 'simrandomBVN100low.2.RData'))
par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = FALSE)
simplot(list(Uniform = simrandom100low.2, BVN = simrandomBVN100low.2),
    trueval = 0.5, legend = TRUE)
```



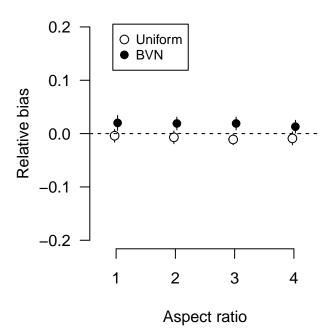
```
simsum(list(Uniform = simrandom100low.2, BVN = simrandomBVN100low.2),
    trueval = 0.5, compact = NULL)
```

```
## $Uniform
##
                     1
                              2
                                       3
                36.122
                        36.122
                                 36.122
                                          36.122
## av.npop
## av.nCH
                15.310
                         15.488
                                 15.906
                                          16.318
## nvalid
               500.000 500.000 500.000 500.000
## av.parmhat
                 0.502
                          0.498
                                  0.497
                                           0.495
## md.parmhat
                 0.495
                          0.496
                                  0.499
                                           0.497
   sd.parmhat
                 0.095
                          0.096
                                  0.094
                                           0.093
##
## RB
                 0.004
                         -0.003
                                 -0.006
                                          -0.009
   seRB
                 0.008
                                  0.008
                                           0.008
##
                          0.009
  RSE
                 0.196
                          0.194
                                  0.190
                                           0.186
##
                 0.001
##
   seRSE
                          0.001
                                  0.001
                                           0.001
## rRMSE
                 0.189
                          0.191
                                  0.188
                                           0.186
## COV
                 0.952
                          0.954
                                  0.932
                                           0.938
##
## $BVN
                              2
                                       3
##
                     1
## av.npop
                36.122
                         36.124
                                 36.122
                                          36.122
## av.nCH
                13.630
                         13.722
                                 13.904
                                          14.072
               500.000 499.000 500.000 500.000
## nvalid
## av.parmhat
                 0.513
                          0.513
                                  0.507
                                           0.505
## md.parmhat
                 0.509
                          0.505
                                  0.504
                                           0.497
## sd.parmhat
                 0.109
                          0.110
                                  0.110
                                           0.111
## RB
                 0.027
                                  0.015
                          0.026
                                           0.010
## seRB
                 0.010
                          0.010
                                  0.010
                                           0.010
## RSE
                 0.224
                          0.224
                                  0.222
                                           0.221
```

```
## seRSE 0.001 0.001 0.001 0.001
## rRMSE 0.219 NA 0.220 0.223
## COV 0.946 0.946 0.944 0.954
```

Small array (6 x 6 grid)

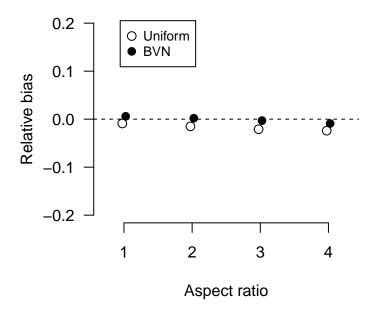
```
tr <- make.grid(6, 6, spacing = 50, detector = 'proximity')</pre>
simrandom36.2 <- vector('list', 4)</pre>
names(simrandom36.2) <- 1:4</pre>
simrandomBVN36.2 <- simrandom36.2</pre>
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                  D = 4, CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    args$type <- 'uniform'; args$g0 <- 0.2</pre>
    simrandom36.2[[i]] <- do.call(runEllipseSim, args)</pre>
    args$type <- 'BVN'; args$lambda0 <- 0.4</pre>
    simrandomBVN36.2[[i]] <- do.call(runEllipseSim, args)</pre>
}
save(simrandom36.2, file = paste0(simfolder, 'simrandom36.2.RData'))
save(simrandomBVN36.2, file = paste0(simfolder, 'simrandomBVN36.2.RData'))
load(file = paste0(simfolder, 'simrandom36.2.RData'))
load(file = paste0(simfolder, 'simrandomBVN36.2.RData'))
par(mfrow = c(1,1), mar = c(4,4,4,4), xpd = FALSE)
simplot(list(Uniform = simrandom36.2[1:4], BVN = simrandomBVN36.2[1:4]), legend = TRUE)
```



```
simsum(list(Uniform = simrandom36.2, BVN = simrandomBVN36.2), compact = NULL)
## $Uniform
##
                          2
                                 3
                                              1-4
                  1
                                                      1,4
## av.npop
            169.000 169.000 169.000 169.000 169.000 169.000
## av.nCH
             49.310 50.536 52.198 53.946 50.890 51.102
## nvalid
            500.000 500.000 500.000 500.000 500.000 500.000
                             3.954
## av.parmhat
              3.986
                      3.972
                                     3.963
                                            3.912
                                                    3.927
## md.parmhat
              3.979
                      3.951 3.939
                                     3.943
                                            3.901
                                                    3.937
## sd.parmhat
              0.497
                      0.497
                            0.490
                                     0.501
                                            0.486
                                                   0.466
## RB
             -0.004 -0.007 -0.011 -0.009 -0.022 -0.018
## seRB
              0.006
                      0.006 0.005
                                     0.006
                                           0.005
                                                   0.005
## RSE
              0.122
                      0.120 0.118
                                     0.117
                                            0.120
                                                   0.120
## seRSE
              0.000
                      0.000 0.000
                                     0.000
                                            0.000
                                                   0.000
## rRMSE
              0.124
                      0.124 0.123
                                     0.126
                                            0.123
                                                  0.118
## COV
              0.944
                      0.942
                             0.952
                                     0.928
                                            0.942
                                                    0.956
##
## $BVN
##
                          2
                                 3
                                         4
                                              1-4
                  1
                                                      1,4
            169.000 169.000 169.000 169.000 169.000
## av.npop
## av.nCH
             42.184 42.942 43.950 44.704 42.994 32.224
            500.000 500.000 500.000 500.000 500.000
## nvalid
## av.parmhat
             4.081
                      4.076
                             4.074
                                     4.052
                                            4.019
                                                   4.112
## md.parmhat
              4.051
                      4.067 4.067
                                     4.028
                                            4.018
                                                   4.027
                      0.581 0.569
## sd.parmhat
              0.593
                                     0.565
                                            0.573
                                                   0.837
## RB
              0.020
                      0.019 0.019
                                    0.013
                                           0.005
                                                   0.028
## seRB
              0.007
                      0.006 0.006
                                    0.006 0.006
                                                  0.009
## RSE
              0.140
                      0.139 0.139
                                     0.139 0.139
                                                  0.202
## seRSE
              0.000
                      0.000
                             0.000
                                     0.000
                                            0.000
                                                   0.001
## rRMSE
              0.149
                      0.146 0.143
                                     0.142 0.143 0.211
## COV
              0.940
                      0.934 0.946
                                     0.948 0.946 0.942
```

Common random orientation

```
tr <- make.grid(10, 10, spacing = 50, detector = 'proximity')</pre>
simrandom100C.2 <- vector('list', 4)</pre>
names(simrandom100C.2) <- 1:4</pre>
simrandomBVN100C.2 <- simrandom100C.2</pre>
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                  theta = -1, D = 4, CL = TRUE, detectfn = 'HHN', details = details)
for (i in 1:4) {
    sigmaX <- 25/i^0.5; sigmaY <- 25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    args$type <- 'uniform'; args$g0 <- 0.2</pre>
    simrandom100C.2[[i]] <- do.call(runEllipseSim, args)</pre>
    args$type <- 'BVN'; args$lambda0 <- 0.4</pre>
    simrandomBVN100C.2[[i]] <- do.call(runEllipseSim, args)</pre>
}
save(simrandom100C.2, file = paste0(simfolder, 'simrandom100C.2.RData'))
save(simrandomBVN100C.2, file = paste0(simfolder, 'simrandomBVN100C.2.RData'))
load(file = paste0(simfolder, 'simrandom100C.2.RData'))
load(file = paste0(simfolder, 'simrandomBVN100C.2.RData'))
```

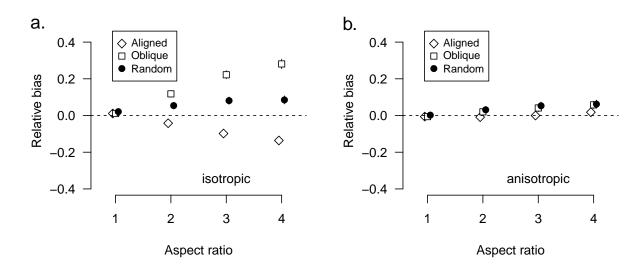


Anisotropic model for data from hollow square array

Using function anisotropic.fit. Extract the fitted coefficients corresponding to psiA and psiR with the coef method for secrobjects, and rely on direct estimation of density (CL = FALSE).

```
tr <- make.grid(10, 10, spacing = 25, detector = 'proximity', hollow = TRUE)
simaniso1 <- vector('list', 4); names(simaniso1) <- 1:4</pre>
simaniso4 <- simaniso3 <- simaniso2 <- simaniso1
baseargs <- list(nrepl = nrepl, buffer = 200, ncores = ncores, traps = tr,
                 lambda0 = 0.4, D = 4, type = 'BVN', CL = FALSE, detectfn = 'HHN',
                 details = list(distribution = 'binomial'), extractfn = coef,
                 secrfn = 'anisotropic.fit')
for (i in 1:4) {
    sigmaX < -25/i^0.5; sigmaY < -25*i^0.5
    args <- c(baseargs, list(sigmaX = sigmaX, sigmaY = sigmaY))</pre>
    args$theta <- 0
                         # parallel to detectors
    simaniso1[[i]] <- do.call(runEllipseSim, args)</pre>
    args$theta <- pi/4 # oblique to detectors
    simaniso2[[i]] <- do.call(runEllipseSim, args)</pre>
    args$theta <- NULL # random orientation</pre>
    simaniso3[[i]] <- do.call(runEllipseSim, args)</pre>
                         # common random orientation
    args$theta <- -1
    simaniso4[[i]] <- do.call(runEllipseSim, args)</pre>
}
save(simaniso1, file = paste0(simfolder, 'simaniso1.RData'))
save(simaniso2, file = paste0(simfolder, 'simaniso2.RData'))
```

```
save(simaniso3, file = paste0(simfolder, 'simaniso3.RData'))
save(simaniso4, file = paste0(simfolder, 'simaniso4.RData'))
load(file = paste0(simfolder, 'simaniso1.RData'))
load(file = paste0(simfolder, 'simaniso2.RData'))
load(file = paste0(simfolder, 'simaniso3.RData'))
load(file = paste0(simfolder, 'simaniso4.RData'))
par(mfrow = c(1,2))
simplot(list(Aligned = simgridalignsq1.2[1:4],
             Oblique = simgridalignsq2.2[1:4],
             Random = simgridalignsq3.2[1:4]),
        ylim = c(-0.4, 0.45), legend = TRUE, pchi = c(23, 22, 16))
text(-0.4, 0.5, 'a.', cex = 1.45, xpd = TRUE)
text(3, -0.35, 'isotropic')
simplot(list(Aligned = simaniso1,
             Oblique = simaniso2,
             Random = simaniso3),
        component = "pred",
        ylim = c(-0.4, 0.45), legend = TRUE, pchi = c(23, 22, 16))
text(-0.4, 0.5, 'b.', cex = 1.45, xpd = TRUE)
text(3, -0.35, 'anisotropic')
```



Tabular summary for anisotropic model.

```
## $Aligned

## av.nCH RB RSE rRMSE COV

## 1 34.428 -0.006 0.184 0.174 0.960

## 2 35.188 -0.009 0.186 0.168 0.970

## 3 36.572 0.000 0.186 0.165 0.968
```

```
## 4 37.898 0.018 0.182 0.171 0.970
##
## $Oblique
                                  COV
##
     av.nCH
                     RSE rRMSE
                RB
## 1 34.428 -0.006 0.184 0.174 0.960
## 2 36.468 0.020 0.196 0.191 0.960
## 3 39.392 0.041 0.200 0.221 0.936
## 4 41.842 0.058 0.197 0.228 0.938
##
## $Random
     av.nCH
               RB
                    RSE rRMSE
## 1 34.626 0.002 0.184 0.184 0.956
## 2 36.086 0.031 0.182 0.198 0.936
## 3 38.148 0.053 0.178 0.234 0.878
## 4 40.000 0.062 0.174 0.248 0.878
Tabular comparison of isotropic and anisotropic models for hollow grid.
load(file = paste0(simfolder, 'simgridalignsq1.2.RData'))
load(file = paste0(simfolder, 'simgridalignsq2.2.RData'))
load(file = paste0(simfolder, 'simgridalignsq3.2.RData'))
tab1 <- simsum(list(Aligned = simgridalignsq1.2[1:4],
             Oblique = simgridalignsq2.2[1:4],
             Random = simgridalignsq3.2[1:4]))
tab2 <- simsum(list(Aligned = simaniso1,</pre>
             Oblique = simaniso2,
             Random = simaniso3),
        component = "pred")
fn \leftarrow function(t1,t2) cbind(t1[,-1],t2[,-1])
mapply(fn, tab1, tab2, SIMPLIFY = FALSE)
## $Aligned
              RSE rRMSE
                          COV
##
         RB
                                   RB
                                        RSE rRMSE
## 1 0.011 0.173 0.173 0.948 -0.006 0.184 0.174 0.960
## 2 -0.042 0.170 0.184 0.904 -0.009 0.186 0.168 0.970
## 3 -0.098 0.166 0.205 0.846 0.000 0.186 0.165 0.968
## 4 -0.136 0.162 0.239 0.754 0.018 0.182 0.171 0.970
##
## $Oblique
##
        RB
             RSE rRMSE
                         COV
                                 RB
                                       RSE rRMSE
                                                   COV
## 1 0.011 0.173 0.173 0.948 -0.006 0.184 0.174 0.960
## 2 0.118 0.170 0.229 0.884
                              0.020 0.196 0.191 0.960
## 3 0.222 0.166 0.337 0.728 0.041 0.200 0.221 0.936
## 4 0.281 0.163 0.402 0.636
                              0.058 0.197 0.228 0.938
##
## $Random
##
       R.B
             RSE rRMSE
                         COV
                                RB
                                     RSE rRMSE
                                                  COV
## 1 0.021 0.172 0.182 0.938 0.002 0.184 0.184 0.956
## 2 0.054 0.170 0.200 0.924 0.031 0.182 0.198 0.936
## 3 0.081 0.166 0.241 0.854 0.053 0.178 0.234 0.878
## 4 0.085 0.163 0.254 0.854 0.062 0.174 0.248 0.878
```