# EQUIVALENCE CLASSES OF MESH PATTERNS WITH A DOMINATING PATTERN

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Abstract. Two mesh patterns are coincident if they are avoided by the same set of permutations, and are Wilf-equivalent if they have the same number of avoiders at each length. We provide sufficient conditions for coincidence among mesh patterns, whilst also avoiding a longer classical pattern. Using these conditions we completely classify coincidences between families containing a mesh pattern of length 2 and a classical pattern of length 3. Furthermore, we completely Wilf-classify equivalences of mesh patterns of length 2 whilst also avoiding the classical pattern 231.

*Keywords:* permutation, pattern, mesh pattern, pattern coincidence

## 1. Introduction

The study of permutation patterns began as a result of Knuth's statements on stack sorting in *The Art of Computer Programming*[6, p. 243, Ex. 5,6]. This original concept—a subsequence of symbols having a particular relative order, now known as classical patterns—has been expanded to a variety of definitions. Mesh patterns Babson and Steingrímsson [1] considered *vincular* patterns—also known as *generalised* or *dashed* patterns—where two adjacent entries in the pattern must also be adjacent in the permutation. Bousquet-Mélou, Claesson, Dukes, *et al.* [3] look at classes of pattern where both columns and rows can be shaded, these are called *bivincular* patterns. *Bruhat-restricted* patterns were studied by Woo and Yong [7] in order to establish necessary conditions for a Schubert variety to be Gorenstein. All of these definitions are subsumed under the definition of *mesh patterns*, introduced by Brändén and Claesson [4] to capture explicit expansions for certain permutation statistics.

When considering permutation patterns some of the main questions posed relate to how and when a pattern is avoided by, or contained in, a arbitrary set of permutations. Two patterns $\pi$ and $\sigma$ are *Wilf-equivalent* if the number of permutations that avoid $\pi$ of length $n$ is equal to the number of permutations that avoid $\sigma$ of length $n$. A stronger equivalence condition is that of *coincidence*, where the set of permutations avoiding $\pi$ is exactly equal to the set of permutations avoiding $\sigma$. Avoiding pairs of patterns of the same length with certain properties has been studied previously, Claesson

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and Mansour [5] considered avoiding a pair of vincular patterns of length 3. Bean, Claesson, and Ulfarsson [2] study avoiding a vincular and a covincular pattern simultaneously in order to achieve some interesting counting results. However, very little work has been done on avoiding a mesh pattern and a classical pattern simultaneously.

In this work we aim to establish some ground in this field by computing coincidences and Wilf-classes and calculating some of the enumerations of avoiders of a mesh pattern of length 2 and a classical pattern of length 3. We begin by establishing coincidences between mesh patterns of length 2 while avoiding a classical pattern of length 3, this is used to establish sufficient conditions for coincidence. We then establish Wilf-equivalence classes of these coincidence classes who avoid the classical pattern 231

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