A measurement-based variational quantum eigen solver

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This talk

Hybrid quantum-classical algorithms

Circuit-based	→	Measurement-based
VQE		**MB-VQE**

Hybrid quantum-classical algorithms

Circuit-based

VQE

Entangling gates
+
Local gates
+
Local measurements

Measurement-based

MB-VQE

Efficient resource state
+
Local gates
+
Local measurements

Hybrid quantum-classical algorithms

Circuit-based	→	Measurement-based
VQE		**MB-VQE**
Entangling gates + Local gates + Local measurements		Local gates + Local measurements

Hybrid quantum-classical algorithms

Circuit-based

VQE

Entangling gates
+
Local gates
+
Local measurements

Measurement-based

MB-VQE

Local gates
+
Local measurements

Complementary approach
Different resource requirements
Advantageous for certain applications


Vision

Universal one-way computer

⬇

One-way quantum simulator

Quantum Interactions

Universal one-way computer

$\Downarrow$

Special purpose one-way quantum simulator

Quantum Interactions

A measurement-based variational quantum eigensolver arXiv:2010.13940

Ryan Ferguson, Luca Dellantonio, Abdulrahim Al Balushi, Karl Jansen, Wolfgang Dür, and Christine Muschik

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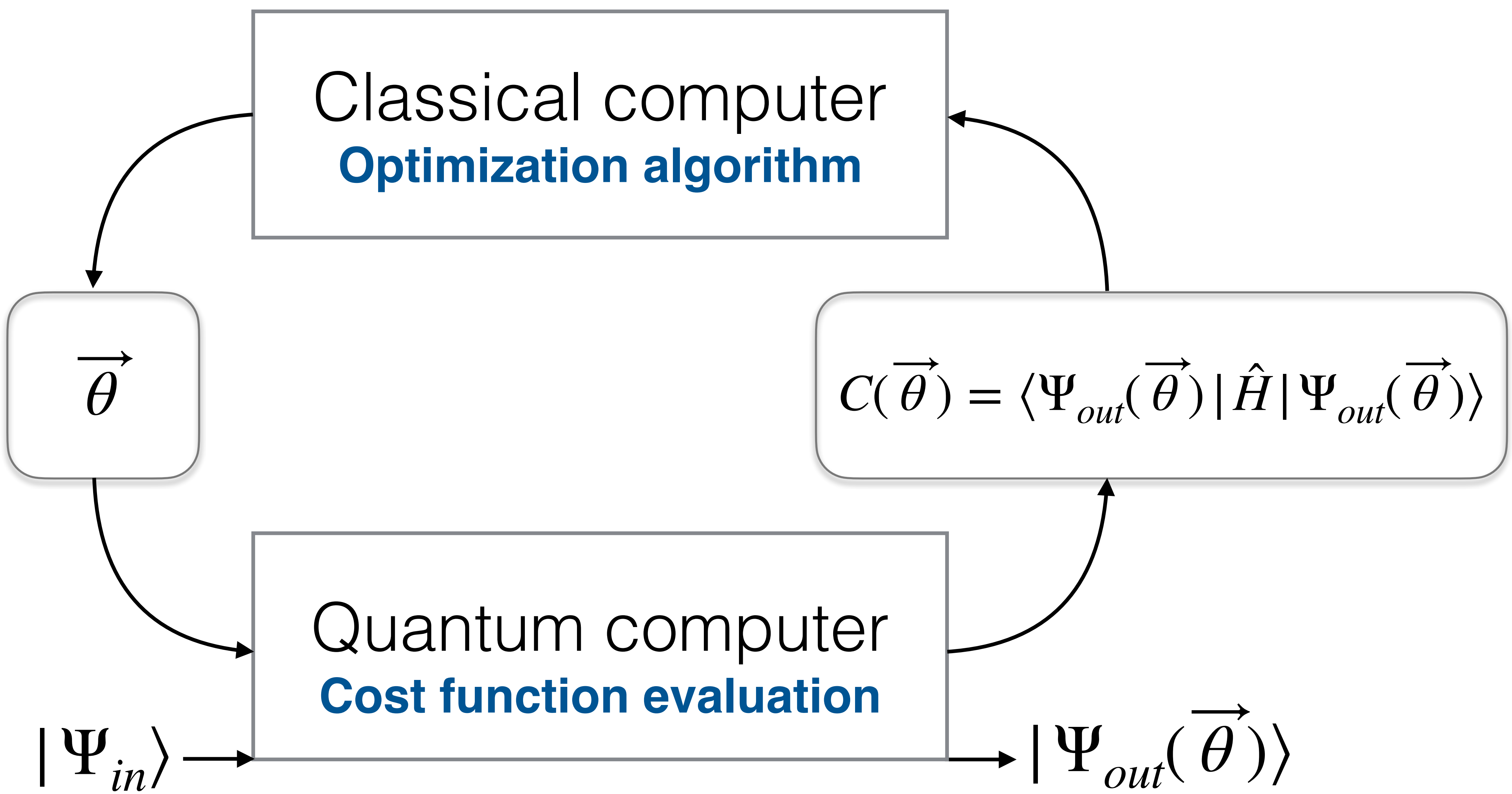
Wolfgang Dür

Overview

1. Introduction
2. Measurement-based quantum computing
3. Scheme I
4. Scheme II
5. Conclusions

Overview

Variational quantum eigensolvers



J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, New J. Phys. 18, 023023 (2016).

Variational quantum eigensolvers

Applications in

- Chemistry [1,2]
- Particle physics [3]
- Classical optimisation [4]
- Sensing [5]
- Many-body physics
- …

[1] F. Arute et. al.,Science 369 (6507), 1084, (2020).
[2] P. J. J. O'Malley et al. Phys. Rev. X 6, 031007 – Published 18 July 2016
[3] C. Kokail et. al., Nature 569, 355 (2019).
[4] E. Farhi, J. Goldstone, and S. Gutmann, MITCTP/4610 (2014).
[5] R. Kaubruegger et. al., Phys. Rev. Lett. 123, 260505 (2019).

Quantum computation

Circuit-based quantum computing

Typically limited by

- size of qubit register
- number of gates performed (circuit depth)

Measurement-based quantum computing

Based on an entangled input state.
Computation is performed by performing single-qubit measurements.

Typically limited by

- size of entangled input state that can be prepared

BM-VQE

New variational technique
→ based on the principles of measurement-based quantum computing.

Task:
→ prepare the ground state of a traget Hamiltonian.

Main idea:
→ use tailored entangled state as resource: small custom states.

Universal one-way computer
(large universal resource state)

Special purpose one-way quantum simulator
(small problem-specific state)

Quantum Interactions

BM-VQE

Scheme I

Direct translation
circuit-VQE to MB-VQE

Example: 1D-QED

Scheme II

New type of state variation
“edge decoration”

Example: toric code

Overview

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2. Measurement-based quantum computing
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Measurement-based quantum computing
a (very) quick introduction

Measurement-based quantum computing

A one-way quantum computer,
R. Raussendorf, and H. Briegel,
Phys. Rev. Lett. 86, 5188 (2001).

Measurement-based quantum computation with cluster states
R. Raussendorf, D. Browne, H. Briegel
Phys. Rev. A 68, 022312 (2003).

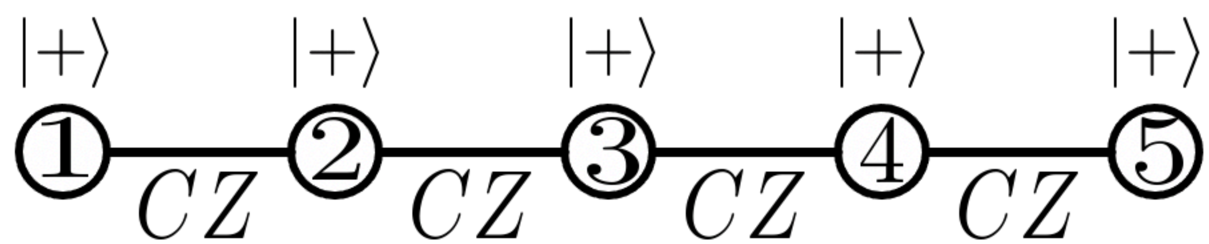
Universal set of gates in measurement-based quantum computing

1. **Arbitrary single-qubit rotation**
2. **CNOT**

Arbitrary single-qubit rotation

$$\hat{U}(\theta_1, \theta_2, \theta_3) = \hat{U}_x(\theta_3)\hat{U}_z(\theta_2)\hat{U}_x(\theta_1)$$

Resource state:



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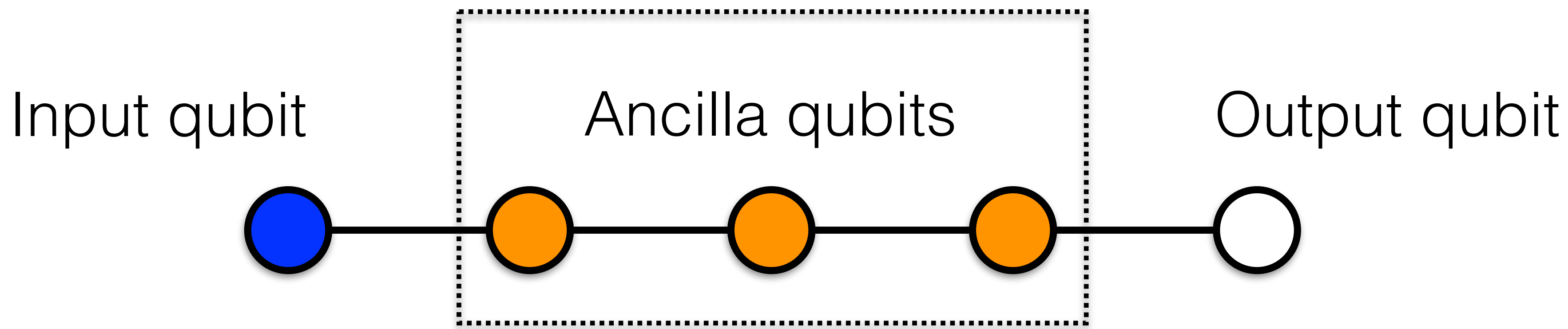

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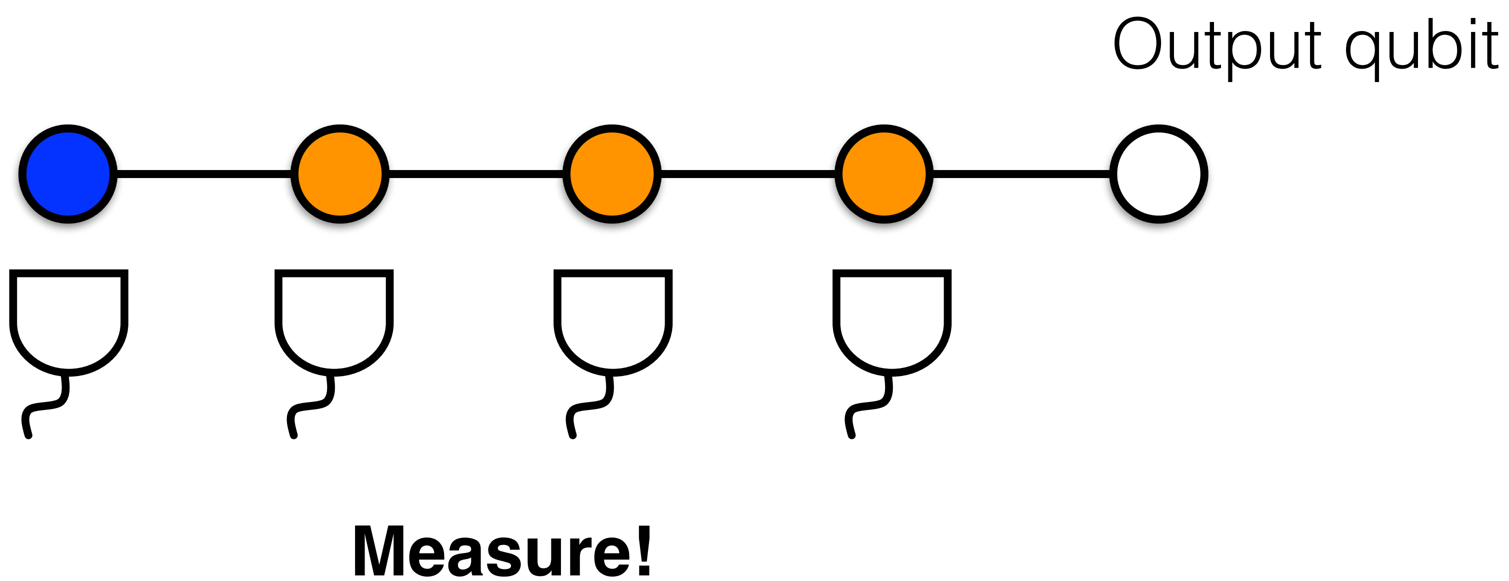

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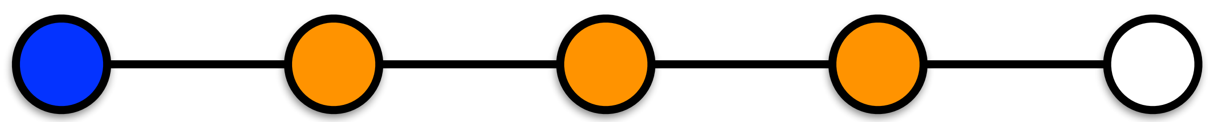
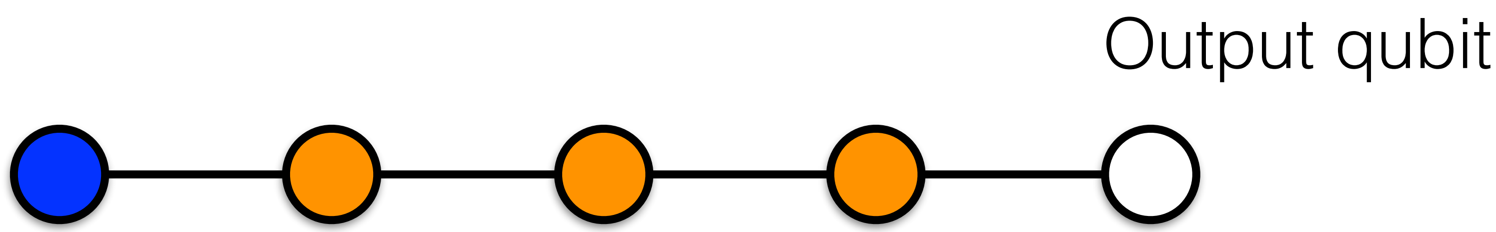


Measurement bases: X $\quad R(\Phi_1)$ $\quad R(\Phi_2)$ $\quad R(\Phi_3)$

$$R(\phi) = \left\{ \frac{|0\rangle + e^{i\phi}|1\rangle}{2}, \frac{|0\rangle - e^{i\phi}|1\rangle}{2} \right\}$$

Arbitrary single-qubit rotation

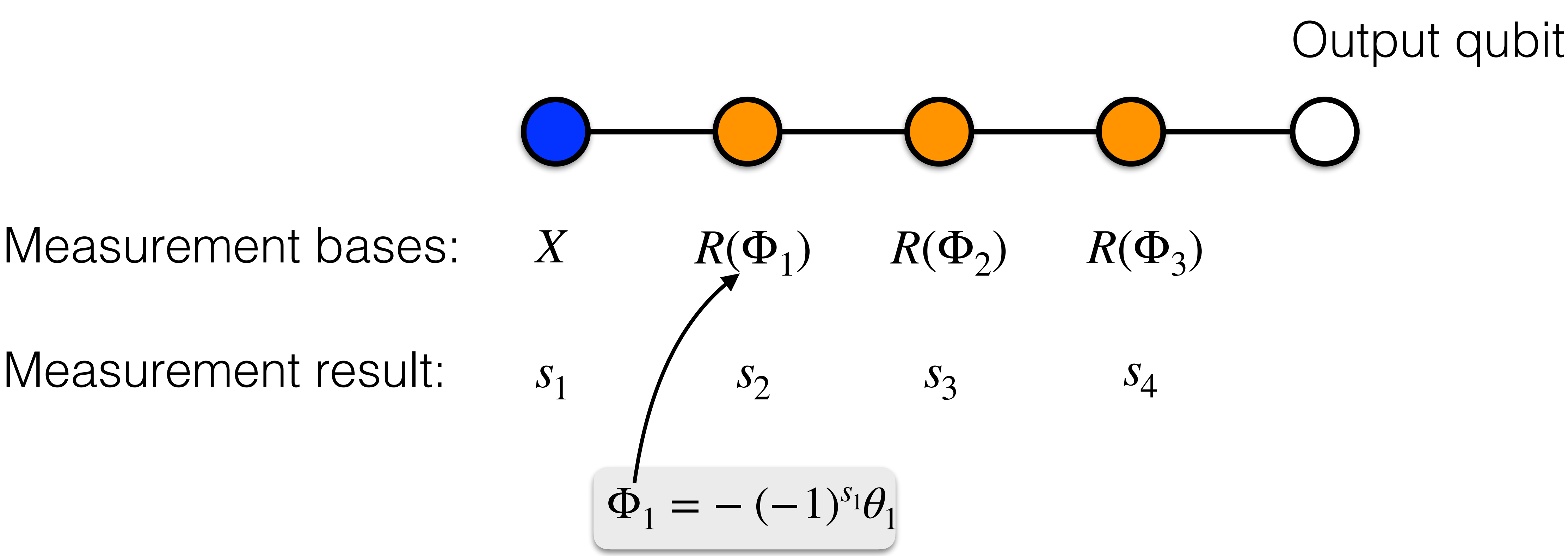
$$\hat{U}(\theta_1, \theta_2, \theta_3) = \hat{U}_x(\theta_3)\hat{U}_z(\theta_2)\hat{U}_x(\theta_1)$$



Measurement bases:	X	$R(\Phi_1)$	$R(\Phi_2)$	$R(\Phi_3)$
Measurement result:	s_1	s_2	s_3	s_4

Arbitrary single-qubit rotation

$$\hat{U}(\theta_1, \theta_2, \theta_3) = \hat{U}_x(\theta_3)\hat{U}_z(\theta_2)\hat{U}_x(\theta_1)$$



Arbitrary single-qubit rotation

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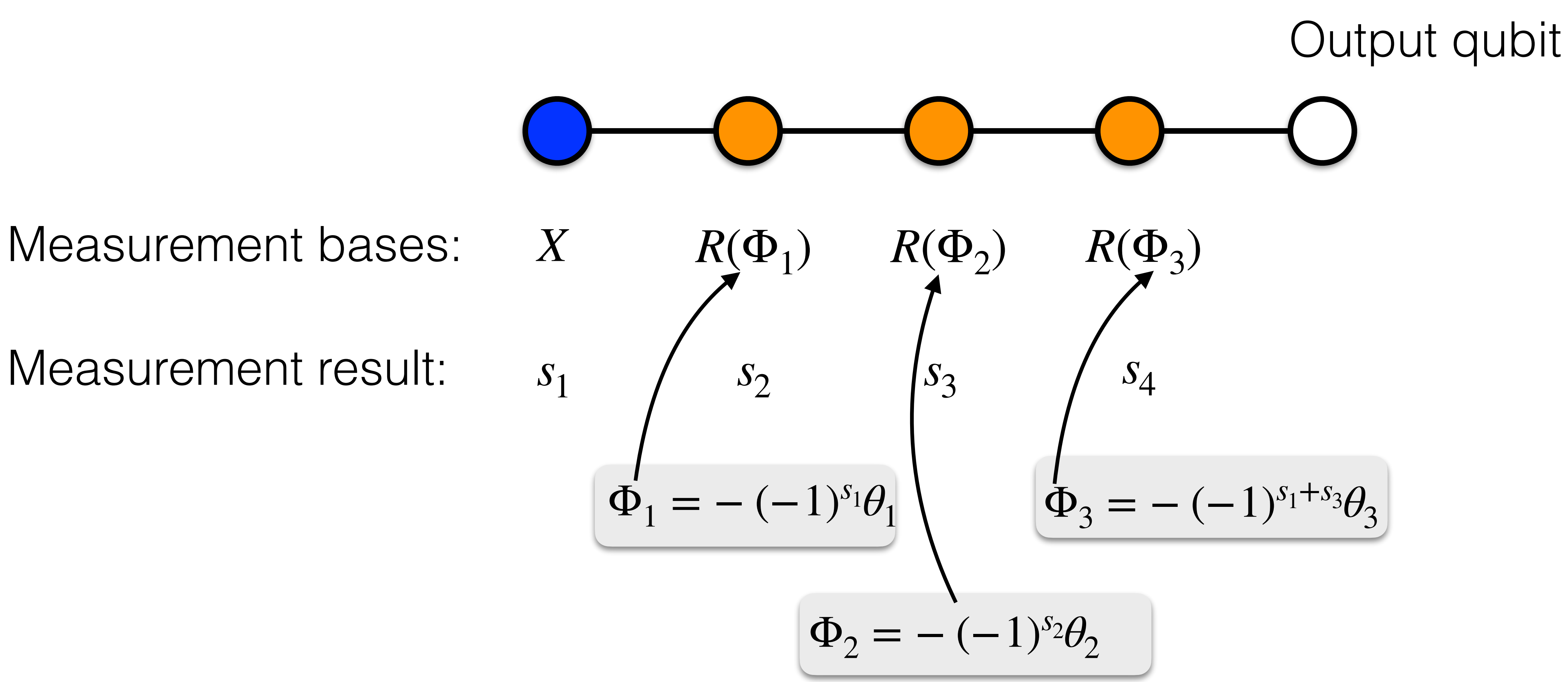

Arbitrary single-qubit rotation

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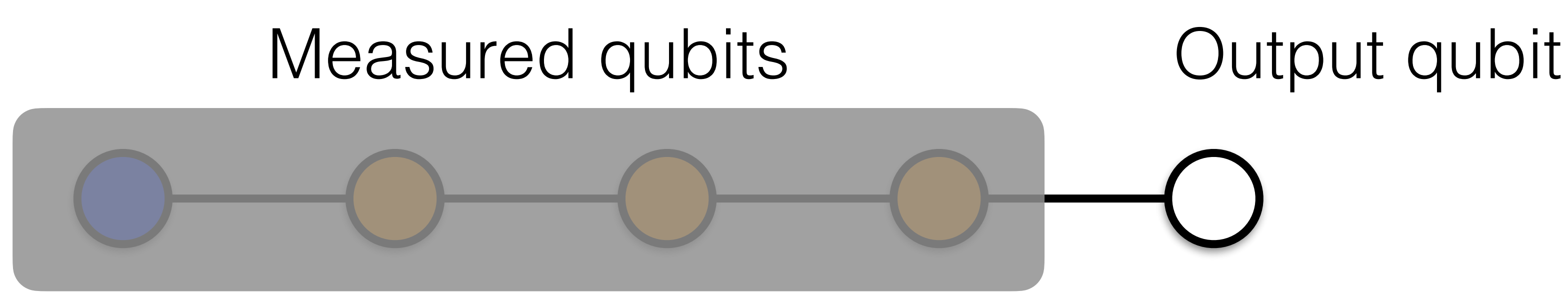


$\hat{U}_\Sigma \hat{U}(\theta_1, \theta_2, \theta_3)\,|\,in\rangle$

Byproduct operator
$\hat{U}_\Sigma = \hat{X}^{s_2+s_4}\hat{Z}^{s_1+s_3}$

Arbitrary single-qubit rotation

$$\hat{U}(\theta_1, \theta_2, \theta_3) = \hat{U}_x(\theta_3)\hat{U}_z(\theta_2)\hat{U}_x(\theta_1)$$

Adaptive measurement pattern

CNOT operation

Clifford-gate

Non-adaptive measurement pattern

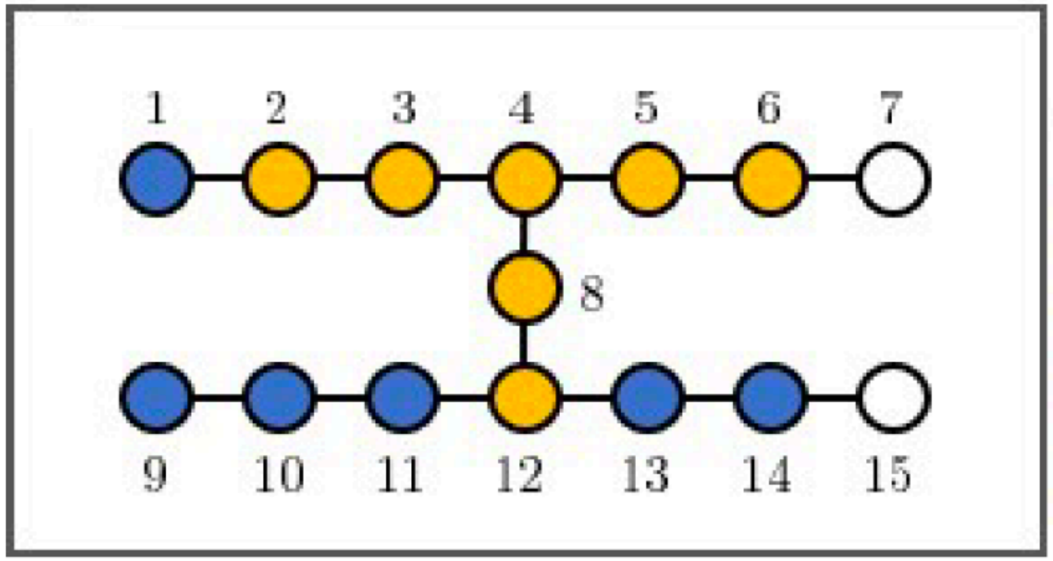
Measurements can be performed simultaneously

CNOT operation

Clifford-gate

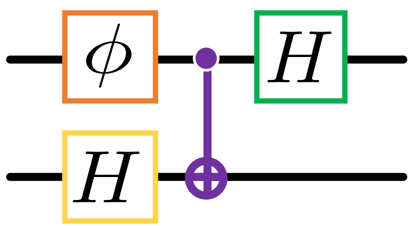
Non-adaptive measurement pattern

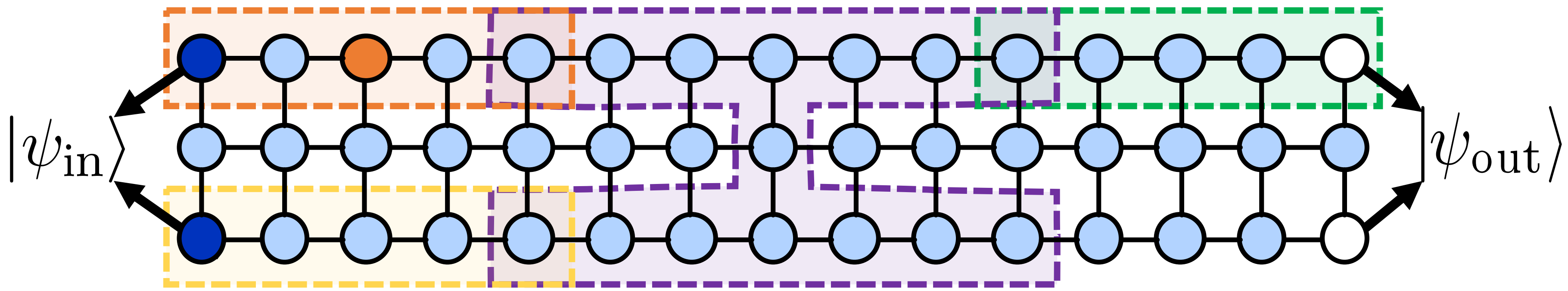
Measurements can be performed simultaneously



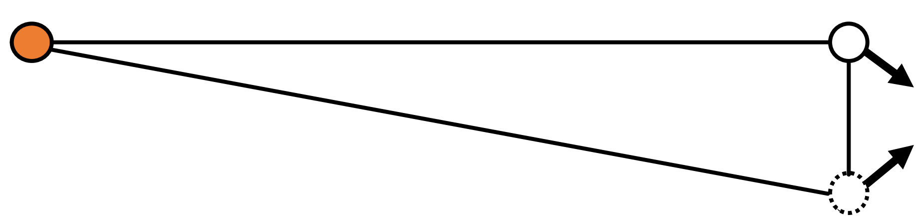
● → measure in X ● → measure in Y

Concatenation of measurement patterns:





Gottesmann Knill theorem

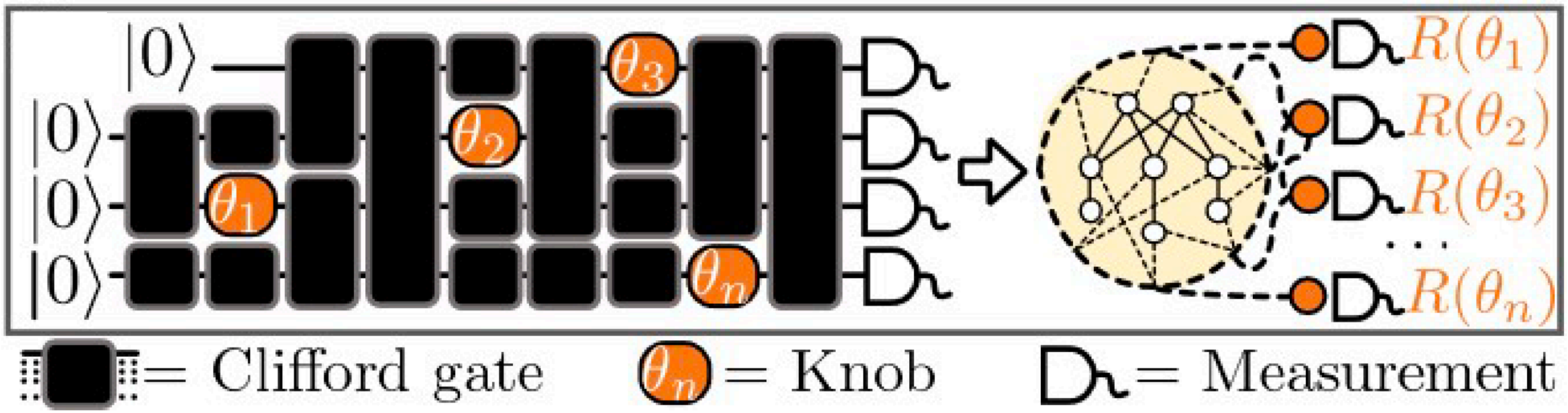


Overview

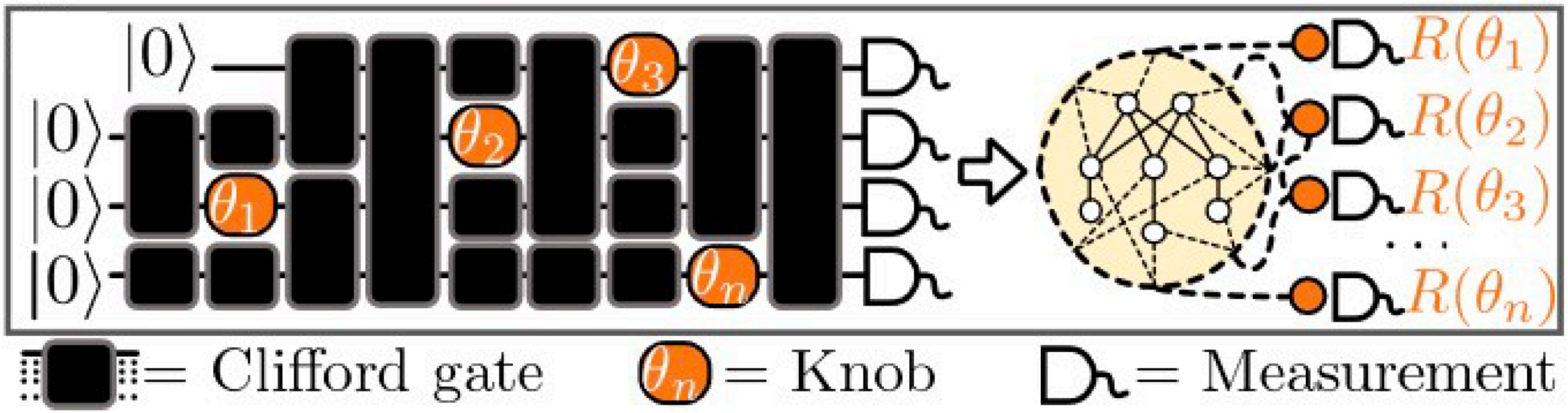
Translating VQEs into MB-VQEs

1. **General idea**
2. **Concrete example**

General idea



General idea



Useful for circuits with a large Clifford part.

Concrete example

Schwinger model: Quantum electrodynamics on a 1D lattice

Benchmarking model for quantum simulations of lattice gauge theories/high energy physics [1-4]

[1] J. Schwinger, Phys. Rev. 82, 914 (1951).
[2] M. C. Bañuls et. al, Rep. Prog. Phys. 83, 024401 (2020).
[3] C. Kokail et. al., Nature 569, 355 (2019).
[4] N. Klco et. al., Phys. Rev. A 98, 032331 (2018).

Concrete example

Schwinger model: Quantum electrodynamics on a 1D lattice

Benchmarking model for quantum simulations of lattice gauge theories/high energy physics [1-4]

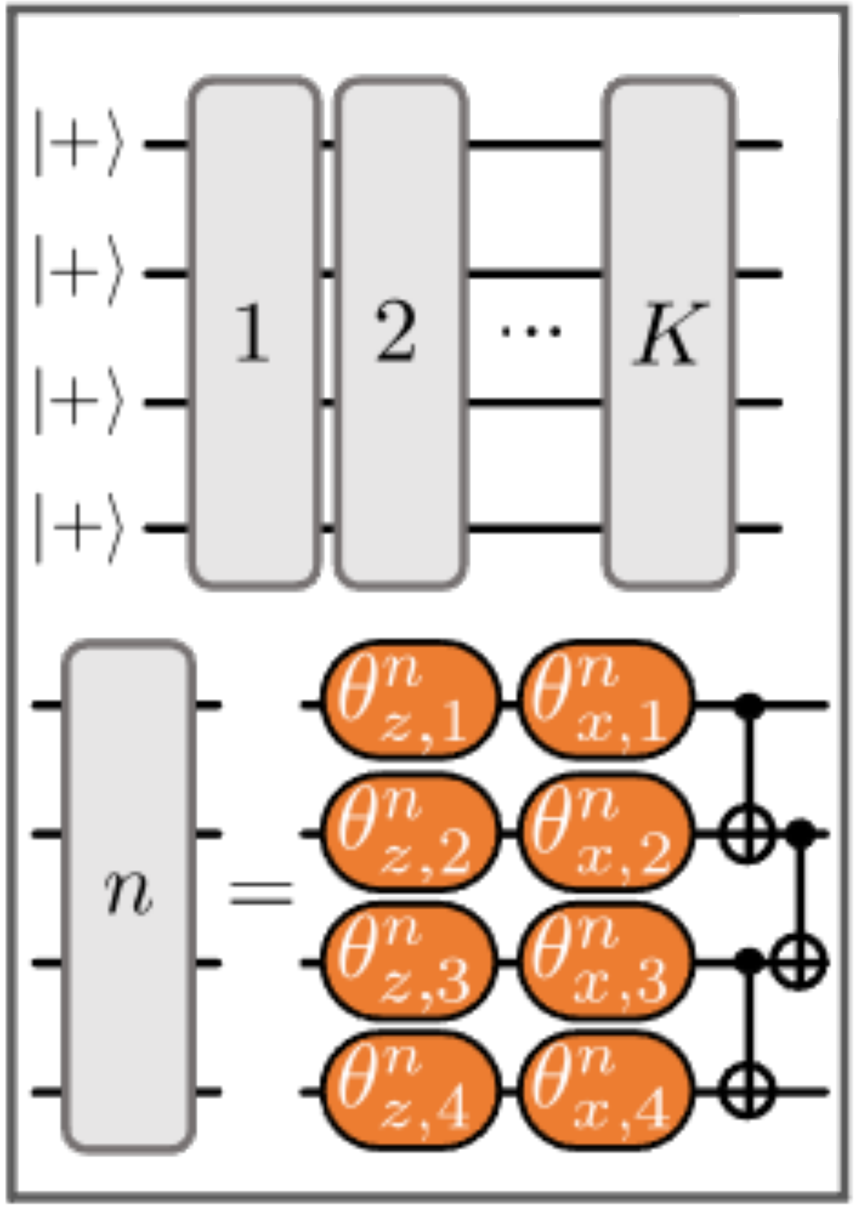
Schwinger model as spin model [5-8]:

$$\hat{H} = \frac{J}{2}\sum_{n=1}^{S-2}\sum_{k=n+1}^{S-1}(S-k)\hat{Z}_n\hat{Z}_k - \frac{J}{2}\sum_{n=1}^{S-1} n\mathrm{mod}2\sum_{k=1}^{n}\hat{Z}_k$$
$$+ w\sum_{n=1}^{S-1}\left(\hat{\sigma}_n^{+}\hat{\sigma}_{n+1}^{-} + \mathrm{H.C.}\right) + \frac{\mu}{2}\sum_{n=1}^{S}(-1)^n\hat{Z}_n,$$

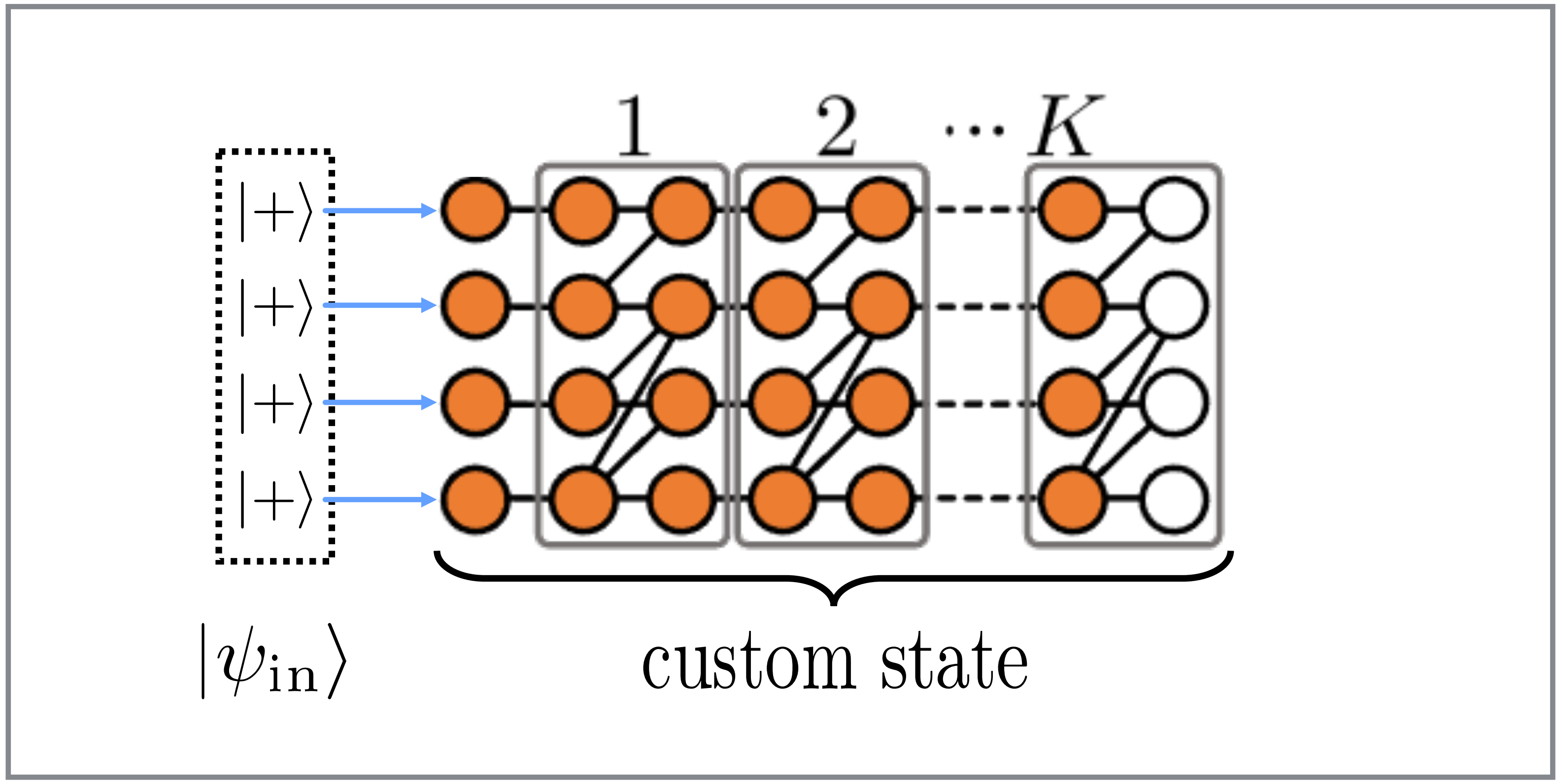
[1] J. Schwinger, Phys. Rev. 82, 914 (1951).
[2] M. C. Bañuls et. al, Rep. Prog. Phys. 83, 024401 (2020).
[3] C. Kokail et. al., Nature 569, 355 (2019).
[4] N. Klco et. al., Phys. Rev. A 98, 032331 (2018).
[5] C. J. Hamer, Z. Weihong, and J. Oitmaa, Phys. Rev. D 56, 55 (1997).
[6] M. C. Banuls et. al., 332, H 2013 POS(LATTICE 2013).
[7] E. A. Martinez, et al., Nature 534, 516 (2016).
[8] C. Muschik, et. al., New J. Phys. 19, 103020 (2017).

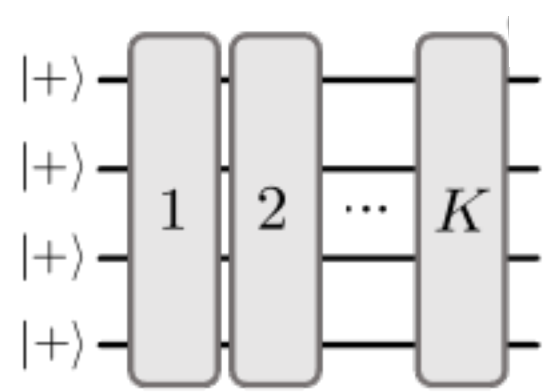
Generic VQE circuit for the Schwinger model

Fixed CNOT gates + parametric single qubit gates

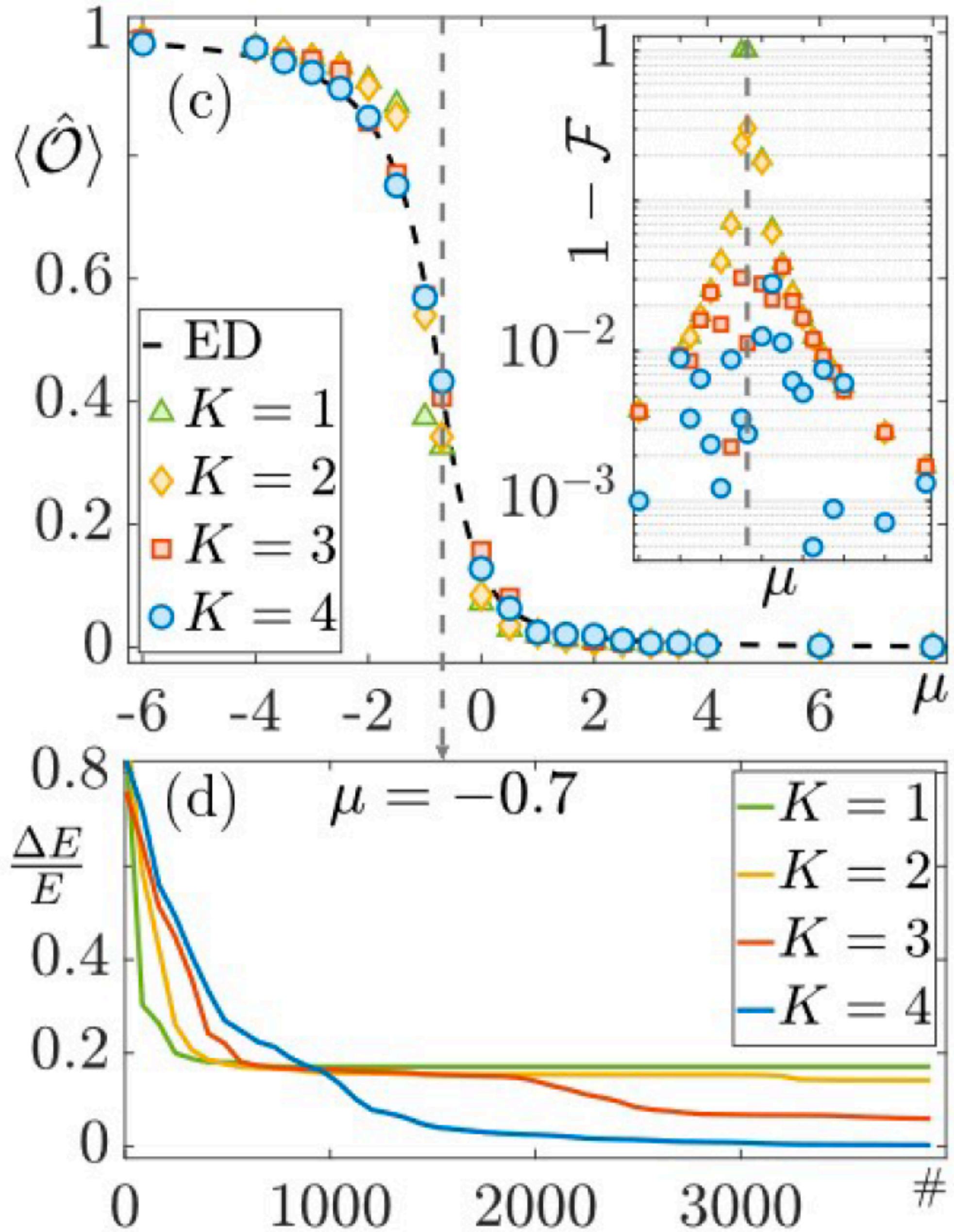


Translation into a MB-VQE





Results



(c)
$\langle\hat{\mathcal{O}}\rangle$
ED
$K=1$
$K=2$
$K=3$
$K=4$
$1-\mathcal{F}$
10^{-2}
10^{-3}
μ
(d)
$\mu=-0.7$
$\frac{\Delta E}{E}$
#

Different resource requirements

VQE

- K(S-1) entangling gates
- S qubits
- 2KS single-qubit rotations

MB-VQE

- S(2K+1) qubits
- 2KS single-qubit rotations (measurements)

K = # of layers
S = # of lattice sites in the model

MB-VQE

Required coherence time

(1)
The required coherence time only depends on the number of non-Clifford operations (i.e.“knobs” = parametric local operations)

(2)
The resource state preparation time is constant (independent of the number of qubits and the number of Clifford operations), i.e. the state preparation has “circuit depth one”.

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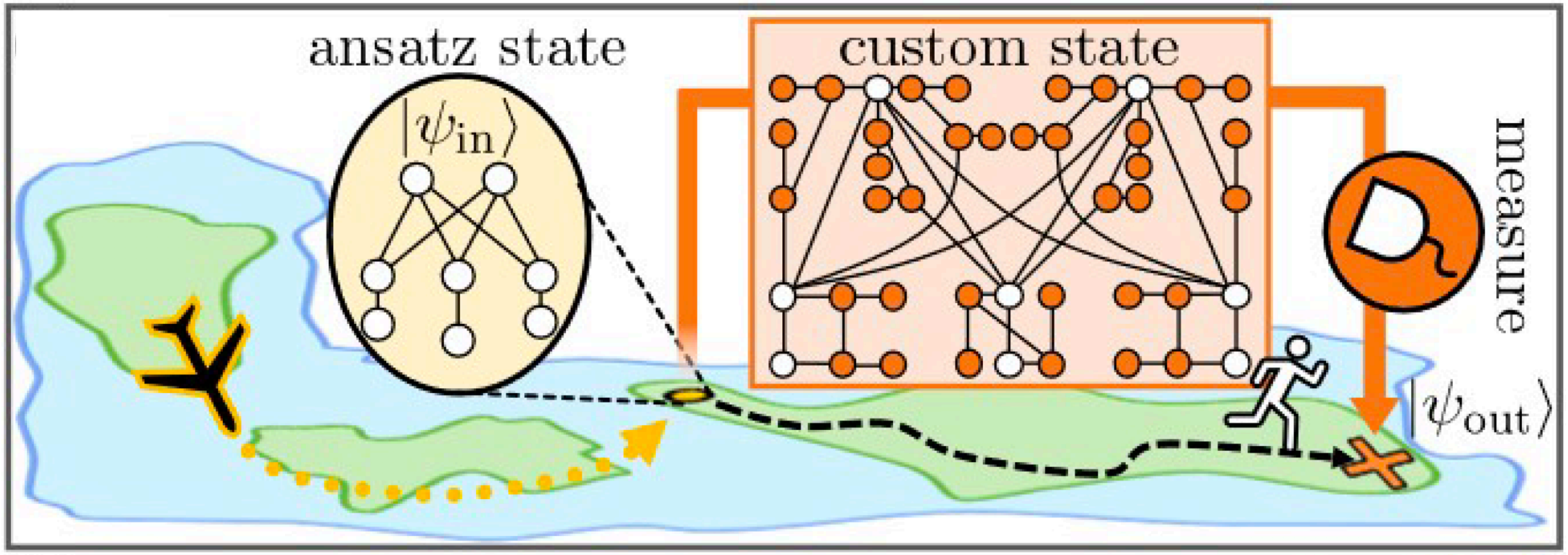
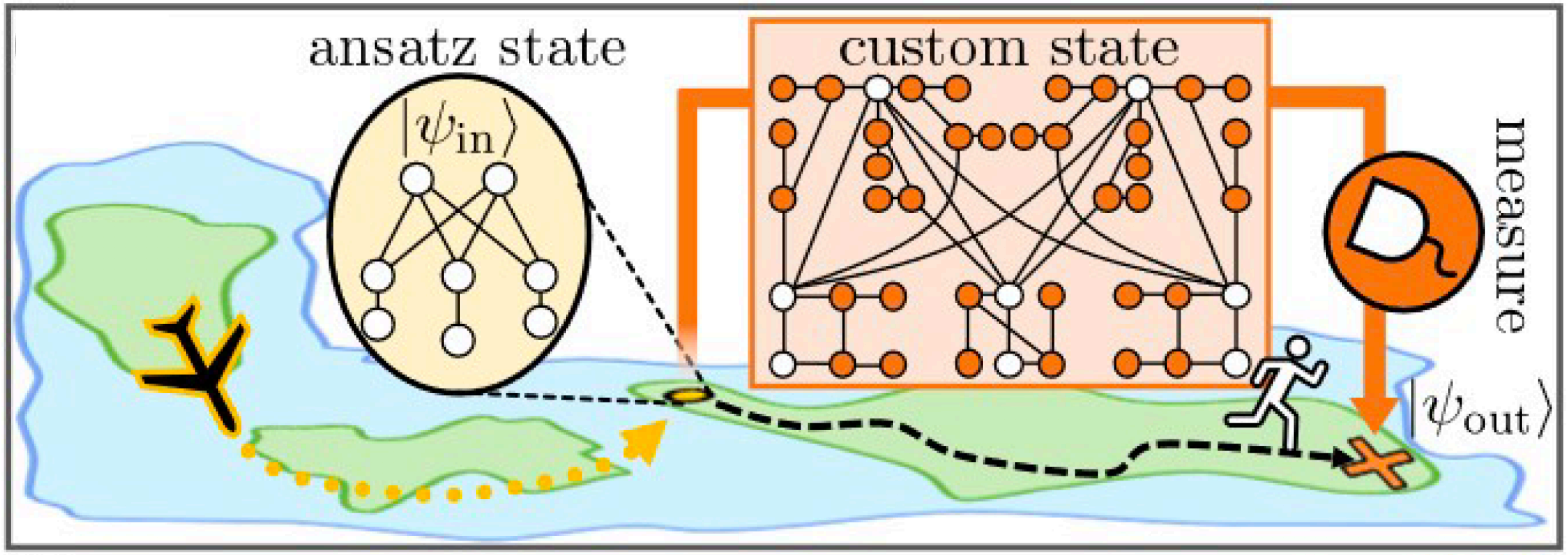
State variation by edge-modification

1. **General idea**
2. **Concrete example**

General idea

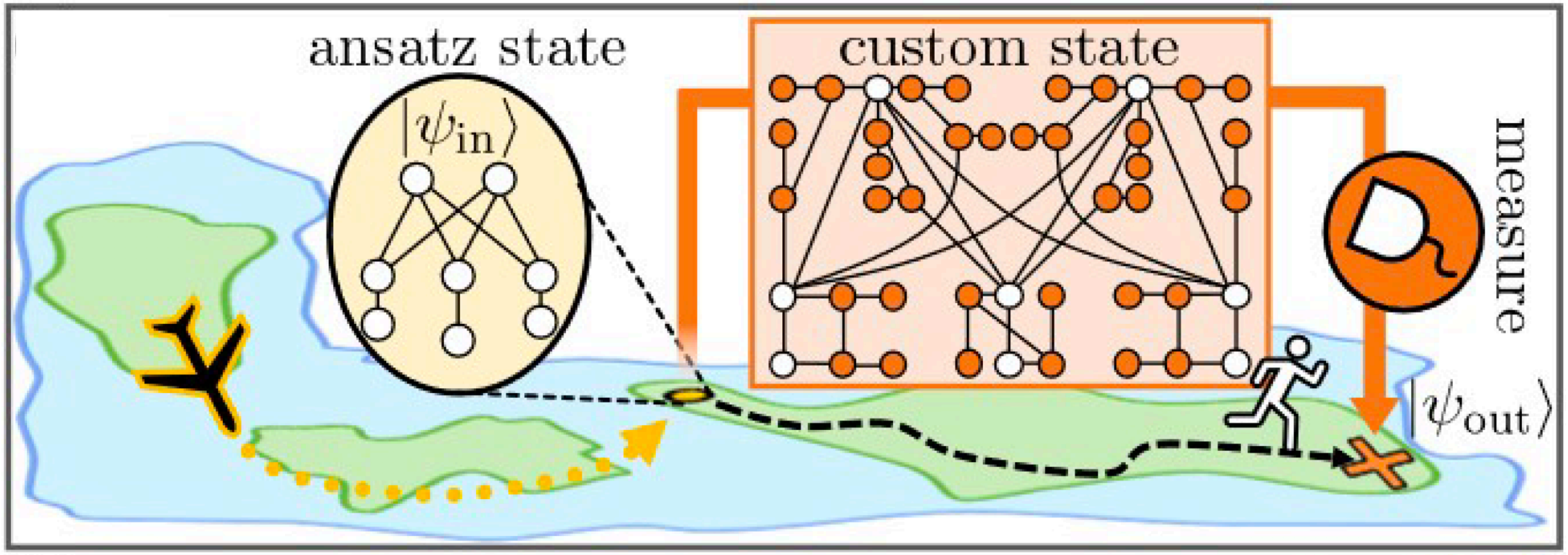


General idea



- Start in a suitable corner of the Hilbert space.
- Ansatz state = problem-specific graph state.
- Classical algorithm optimizes locally in this corner

General idea



For example: $\hat{H} = \hat{H}_0 + \hat{H}_P$

$\hat{H}_P$ = Perturbation

Ansatz state = ground state of the unperturbed Hamiltonian $\hat{H}_0$

Concrete Example

Perturbed toric code

No method known for calculating the ground state efficiently.

$$\hat{H}_0 = -\sum_s \hat{A}_s - \sum_p \hat{B}_p$$

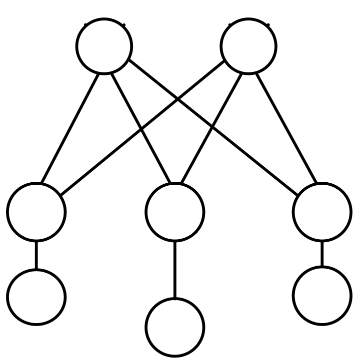
Toric code Hamiltonian

Ground state
- is a graph state
- can be determined efficiently

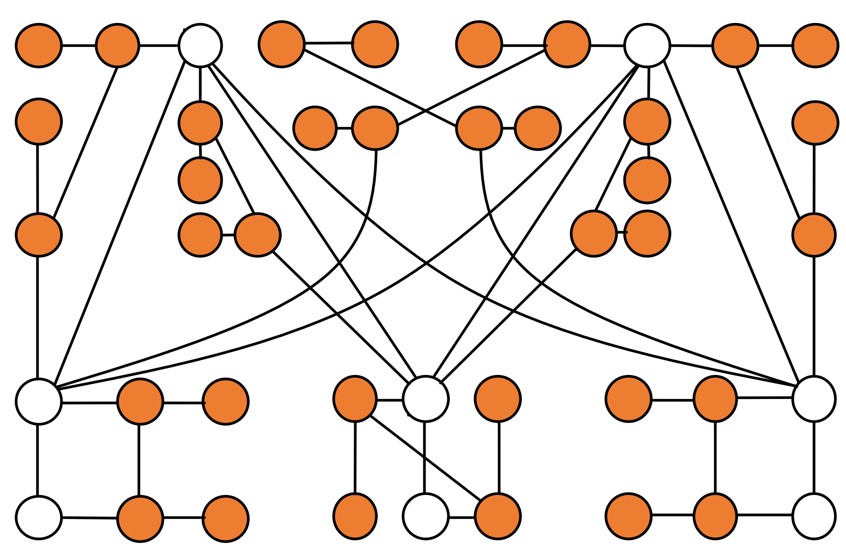
$$\boxed{\hat{H} = \hat{H}_0 + \hat{H}_P}$$

$$\hat{H}_\mathrm{p} = \sum_{n=1}^{2N_x N_y} \lambda_n \hat{Z}_n$$

Perturbation



Ansatz state



Custom state

Concrete Example

Perturbed toric code

No method known for calculating the ground state efficiently.

Concrete Example

Perturbed toric code

No method known for calculating the ground state efficiently.

$$\hat{H}_0 = -\sum_s \hat{A}_s - \sum_p \hat{B}_p$$

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Perturbation

Concrete Example

Perturbed toric code

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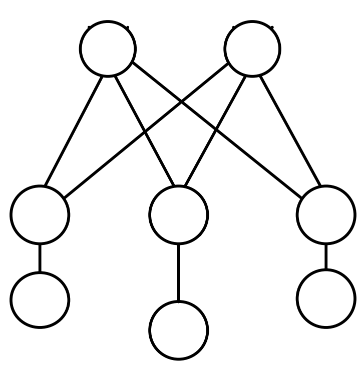
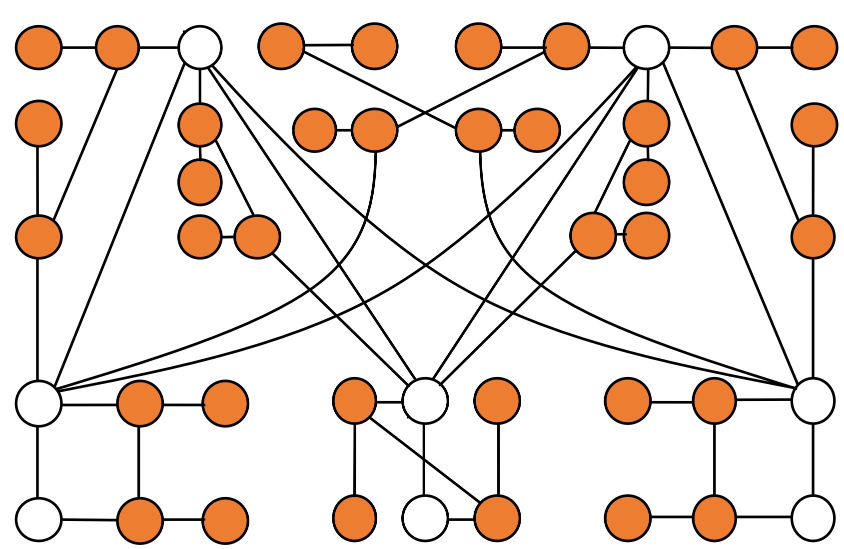
Perturbation



Ansatz state



Custom state

Edge decoration

Custom state = Ansatz state + **Auxiliary qubits**

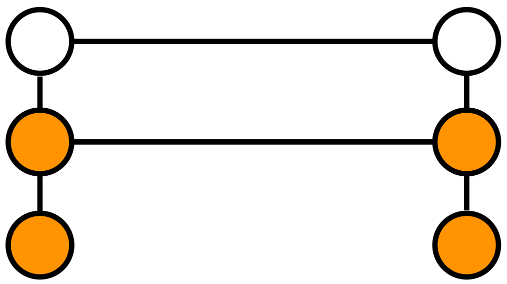
Edge decoration

Custom state = Ansatz state + **Auxiliary qubits**

Toric code example:

Original edge:

Decorated edge:



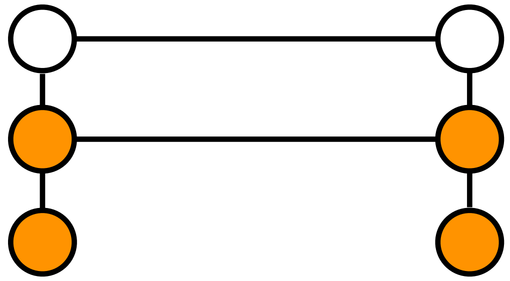
Edge decoration

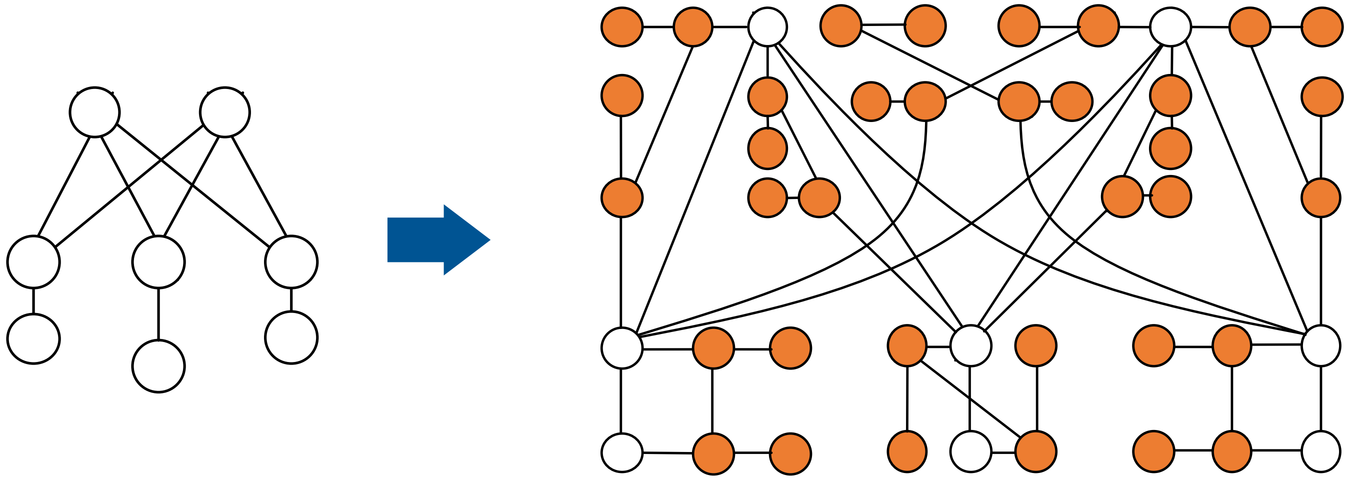
Custom state = Ansatz state + **Auxiliary qubits**

Toric code example:

Original edge:

Decorated edge:





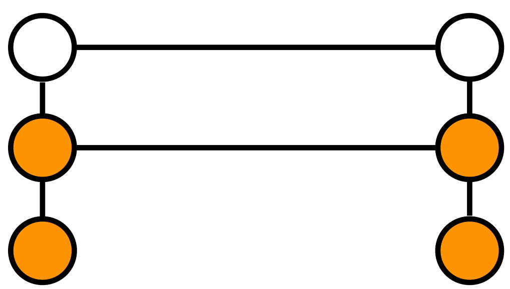
Edge decoration

Custom state = Ansatz state + **Auxiliary qubits**

Toric code example:

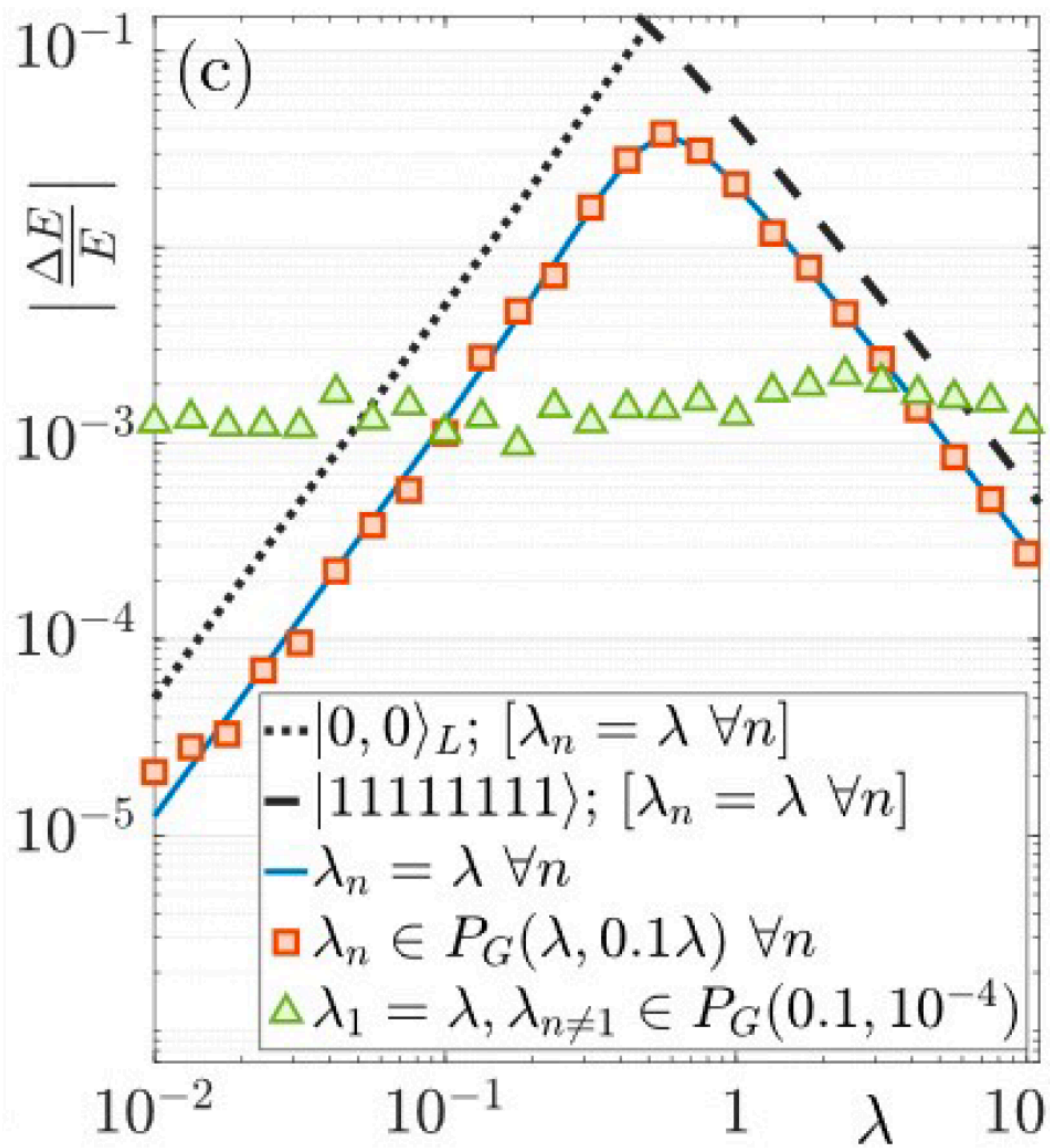
Original edge:

Decorated edge:



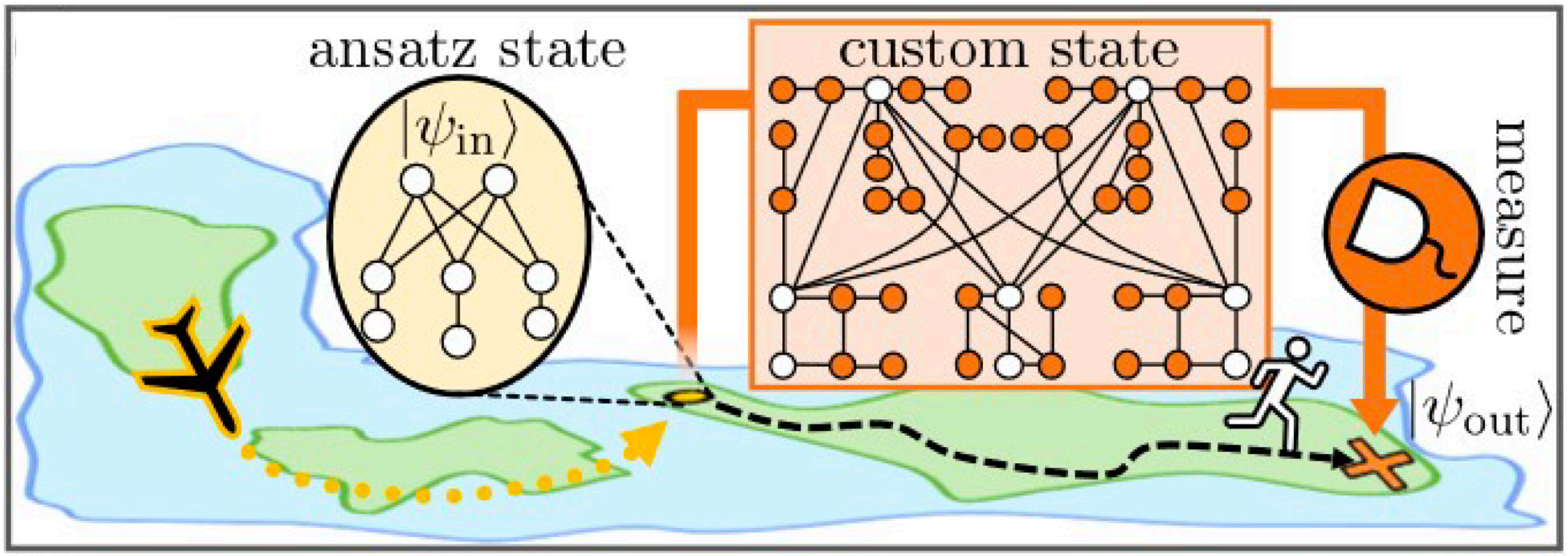
State variation protocol:
Auxiliary qubits are measured to modify the custom state.
The classical VQE algorithm selects rotated measurement bases $R(\theta)$ to minimize $\langle \hat{H}_0 + \hat{H}_P \rangle$.

Results



(c)
10^{-1}
10^{-3}
10^{-4}
10^{-5}
$\left|\frac{\Delta E}{E}\right|$
$|0,0\rangle_L$; $[\lambda_n = \lambda\ \forall n]$
$|11111111\rangle$; $[\lambda_n = \lambda\ \forall n]$
$\lambda_n = \lambda\ \forall n$
$\lambda_n \in P_G(\lambda, 0.1\lambda)\ \forall n$
$\lambda_1 = \lambda, \lambda_{n\neq 1} \in P_G(0.1, 10^{-4})$
10^{-2}
10^{-1}
1
λ
10

State variation by edge-modification



Overview

Measurement-based VQEs

1. Direct translation: circuit VQE → MB-VQE

Advantageous
- for circuits with large Clifford part (few “knobs’)
- if long entangling gate sequences are challenging

2. State variation by edge modification

Conceptually new way to construct variational state families

Measurement-based VQEs

Can be experimentally explored today

Full toric code example = 44 qubits,
but minimal instances can be realized with <u>much fewer qubits:</u>

- Toric code → planar code
- switching from deterministic to probabilistic protocols

New paths for VQEs on photonics platforms

See for example:

- Deterministic multi-mode gates on a scalable photonic quantum computing platform, arXiv:2010.14422 (2020).
- High-fidelity multi-photon-entangled cluster state with solid-state quantum emitters in photonic nanostructures, arXiv:2007.09295 (2020).
- Time-Domain Multiplexed 2-Dimensional Cluster State: Universal Quantum Computing Platform, Science 366, 373 (2019).
- Experimental one-way quantum computing, Nature 434, 169 (2005).

Future directions

Detailed analysis of noise robustness

- Will be platform dependent.
- Interesting: error thresholds for measurement-based quantum communication can be as high as 10%-23% per qubit [1].

Explore new tailored edge-modification schemes:
New way to think about state variation

- Edge decorations are not necessary unitary and can affect many qubits at once.
- New way to think about state variation: conceptually different VQE-design.

[1] M. Zwerger, H. Briegel, and W. Dür, App. Phys. B 122 (2015).

Universal one-way computer

Large universal resource state

Special purpose one-way quantum simulator

Small problem-specific state

Quantum Interactions

A measurement-based variational quantum eigensolver
arXiv:2010.13940

Ryan Ferguson, Luca Dellantonio, Abdulrahim Al Balushi, Karl Jansen,
Wolfgang Dür, and Christine Muschik

Ryan Ferguson

Luca Dellantonio

Wolfgang Dür

Thank you very much for your attention

Quantum Optics Theory

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

Quantum Interactions

Positions available

Overview

1. Introduction
2. Measurement-based quantum computing
3. Turning a VQE into a MB-VQE
4. New state variation techniques
5. Conclusions