

QUANTUM TELEPORTATION

A Seminar Report Submitted
in Partial Fulfillment of the Requirements
for the Degree of

BACHELOR OF SCIENCE
in
Physics

by

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I, MURSHED SK (Roll No.: 20PHB112, Enrollment No.: GM2821), hereby declare that, this report entitled “**QUANTUM TELEPORTATION**” submitted to ALIGARH MUSLIM UNIVERSITY towards the partial requirement of **Bachelor of Science** in **Physics**, is a work taken from many references carried out by me under the supervision of **Dr. Haider Hasan Jafri** and has not formed the basis for the award of any degree or diploma, in this or any other institution or university. I have sincerely tried to uphold academic ethics and honesty.

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CERTIFICATE

This is to certify that the work contained in this Seminar report entitled "**QUANTUM TELEPORTATION**" submitted by **MURSHED SK** (Roll No.: **20PHB112**) to ALIGARH MUSLIM UNIVERSITY towards the partial requirement of Bachelor of Science in Physics has been carried out by him under my supervision and that it has not been submitted elsewhere for the award of any degree.



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ACKNOWLEDGEMENT

I thank everyone who helped me see this project through to completion. I would like to first express my profound gratitude and deep regard to Dr. Haider Hasan Jafri, Aligarh Muslim University and sincerely wish to acknowledge his vision, guidance, valuable feedback and constant support throughout the duration of this project.

I am highly indebted to Shahid Ul Islam(20PHB101) and indebted to Md. Shafi Alam Reja(20PHB126), Molla Suman Rahaman(20PHB049), Noyon Ahammad Khan (20PHB140) for their steadfast encouragement and time. I am lastly grateful to the [IBM Quantum](#), [QWORLD](#), edX/University of Chicago ([UChicagoX QUAN12000](#)), [Qiksit](#), [MIT Open Learning](#), [Meduim](#), [GitHub](#), [Stack Exchange](#), [Overleaf](#) and many other resources and authors for providing the necessary resources and facilities to complete this project to the best of my ability.

I show my deepest appreciation towards my close friends and family for their pivotal care and well wishes and for encouraging me every step of the way.

Aligarh - 202002

December 28, 2022

MURSHED SK

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Introduction

Ever since the wheel was invented more than 5,000 years ago, people have been inventing new ways to travel faster from one point to another. The chariot, bicycle, automobile, airplane and rocket have all been invented to decrease the amount of time we spend getting to our desired destinations. Yet each of these forms of transportation shares the same flaw: They require us to cross a physical distance, which can take anywhere, from minutes to many hours depending on the starting and ending points. There are scientists working right now on such a method of travel, combining properties of telecommunications and transportation to achieve a system called **Teleportation**. Teleportation makes communications faster than ever existed communications along with transportation.

Many works of science fiction(like ‘Star Trek’) have enticed our imagination with human teleportation. But does it have any basis in reality? Teleportation is the hypothetical transfer of matter or energy from one point to another without traversing the physical space between them [1]. An actual teleportation of matter has never been realized by modern science which is based entirely on mechanistic methods and it is questionable if it can ever be achieved because any transfer of matter from one point to another without traversing the physical space between them violates Newton’s laws. However teleportation in the quantum world is possible. Quantum teleportation is not the same as shown in science fiction. The unique nature of quantum mechanics does not allow to transfer the actual object from one point to another point. It actually transfer the information(more precisely quantum information) of the objects. It involves two distant entangled particles

which the state of a third particle instantly teleport its state to the two entangled particles. Precisely, **Quantum Teleportation** is a technique for transferring quantum information from a sender at one place to a receiver at another place.

To happen a quantum teleportation possible we need mainly communication medium such as quantum computer, entangled state of quantum information and informations which are to be teleported. In order to understand quantum teleportation we need to know its historical background, many terminologies and concepts and their mathematical foundations associated with it. It is a application of quantum computation and information science, so many programming terminologies are used in this report. We will visualize this phenomenon by doing some kind of coding in [Qiksit](#)(a python library or SDK) as well mathematically later. As it is an application of quantum computation and quantum computation is a vast field merging of many fields such as quantum mechanics, mathematics, computer science and information science, we will restrict our discussion which are essential only to explain this particular topic.

Chapter 1

Impossibility

Quantum mechanics is still not fully understood! Quantum systems have various unique traits such as the observer effect (an observer affects the observed), the uncertainties (we cannot simultaneously know the position of a quantum particle if know its position), etc. The observation and hence measurement that can be performed on a quantum particle are limited. There is another limitation called No Cloning Theorem Which states that any qubit or quantum information can not simply be copied or cloned. EPR experiments suggest that if entanglement exist between two quantum states how can one violate the law of relativity. Precisely, information did not travel faster than speed of light. Einstein derisively called this "spooky-action-at-a-distance". But teleportation is an instantaneous process. This arises many questions like- did information really travel faster than light in order to teleport information? Was Einstein wrong about the speed of light being the speed limit of the universe? Or, concept of entanglement, teleportation are just wrong?

Quantum mechanics shows that entanglement is the unique property of quantum mechanics and quantum teleportation is also possible.

So, was theory of relativity wrong or quantum mechanics wrong ? This question arises a paradox called EPR Paradox. We know neither quantum mechanics nor relativity is wrong but both has great impact in physics. Later we will see how to solve this paradox.

1.1 Can information travel faster than light?

Information can travel faster than the speed of light, but the information was random, and hence useless. You cannot send a real message, or Morse code, via the EPR experiment even if information is traveling faster than light. If we have a quantum system having two electrons with opposite spins and we know the spin of an electron is up. Then Knowing that the electron on the other side of the universe is spinning down is useless information. Another example, let's say that a friend always wears one red and one green sock, in random order. Let's say you examine one leg, and the leg has a red sock on it. Then you know, faster than the speed of light, that the other sock is green. Informations actually travel faster than light, but this information is useless [2]. But there is a question still nagging.

1.2 Is teleportation possible?

In order to answer this question from the present development of quantum teleportation, a clear distinction between teleportation and quantum teleportation should be made. While teleportation is commonly portrayed in science fiction as a means to transfer physical objects from one location to the next, quantum teleportation only transfers quantum information. If you have watched movies X-men and Star trek, the concept becomes clear. In X-men, professor X can communicate through telepathy to the mutants, this sharing of information can be analogous to quantum teleportation. In movie Star trek, the individuals could move from a spaceship to planets instantly, without traversing the space between, this actually is teleportation. So we can say that quantum teleportation is some sort of telepathy in quantum particles held together by quantum entanglement and completely different from teleportation. As stated above, teleportation and quantum teleportation has different meaning since the former refers to matter transfer and the latter refers to information exchange [3]. In the quantum world, teleportation involves the transportation

of information, rather than the transportation of matter.

In 1993, scientists at IBM, led by Charles Bennett, showed that it was physically possible to teleport objects, at least at the atomic level, using the EPR experiment. (More precisely, they showed that you could teleport all the information contained within a particle.) Since then physicists have been able to teleport photons and even entire cesium atoms. Later many experiments were performed which leads to us that quantum teleportation is possible.

So teleportation exists at the atomic level, and we may eventually teleport complex and even organic molecules within a few decades(hope so). But the teleportation of a macroscopic object will have to wait for several decades to centuries beyond that, or longer, if indeed it is even possible. But teleporting a human being, although it is allowed by the laws of physics, may take many centuries beyond that, assuming it is possible at all [2].

Chapter 2

Historical Background

After the foundation of quantum mechanics many development in this field are happened theoretically and it has various applications in many modern devices such as the laser the transistor the electron microscope and magnetic resonance imaging or MRI. Another goal is the development of quantum computers which are expected to perform certain computational tasks exponentially faster than classical computers.

Though quantum mechanics has many applications in daily life, it is quite difficult to understand this subject as it is not deal with our macroscopic level physics which are governed by Newton's classical mechanics. This weirdness makes quantum mechanics special and precisely describe the phenomenon such as photoelectric effect, black body radiation, Compton effect which can not be described by classical mechanics.

Quantum teleportation is a demonstration of what Albert Einstein famously called "spooky action at a distance" also known as 'quantum entanglement'. The key to quantum teleportation lies in a celebrated 1935 paper by Albert Einstein and his colleagues Boris Podolsky and Nathan Rosen, who, ironically, proposed the **EPR experiment** which will be discussed later. According to EPR experiment and relativity an information can not moves faster than light and quantum teleportation is an instantaneous process no matter how the distance between the sender and the receiver(it may be the distance larger than

light year or two opposite position of the the universe). So the question raised, “is quantum teleportation really possible”?

2.1 First Breakthrough

Everything changed in 1993, when scientists at IBM, led by Charles Bennett, showed that it was physically possible to teleport objects, at least at the atomic level, using the EPR experiment. (More precisely, they showed that you could teleport all the information contained within a particle.) Since then physicists have been able to teleport photons and even entire cesium atoms.

In these teleportation experiments physicists start with two atoms, A and C. Let’s say we wish to teleport information from atom A to atom C. We begin by introducing a third atom, B, which starts out being entangled(will be discussed later) with C, so B and C are coherent. Now atom A comes in contact with atom B. A scans B, so that the information content of atom A is transferred to atom B. A and B become entangled in the process. But since B and C were originally entangled, the information within A has now been transferred to atom C. In conclusion, atom A has now been teleported into atom C, that is, the information content of A is now identical to that of C.

The group showed how, in principle, entangled particles might serve as “transporters” of sorts. By introducing a third “message” particle to one of the entangled particles, one could transfer its properties to the other one, without ever measuring those properties.

2.2 The Innsbruck Experiment

Bennett’s ideas were not verified experimentally.Though the original announcement of this breakthrough, progress has been fiercely competitive as different groups have attempted to outrace each other. The first historic demonstration of quantum teleportation in which photons of ultraviolet light were teleported occurred in 1997 at the University of

Innsbruck.

The researchers produced pairs of entangled photons and showed they could transfer the polarization state from one photon to another.

2.3 Further Progresses

After the Innsbruck Experiment the next year by experimenters at Cal Tech did an even more precise experiment involving teleporting photons. Another team headed by Francesco De Martini in Rome has submitted similar evidence to Physical Review Letters for publication.

In 2004 physicists at the University of Vienna were able to teleport particles of light over a distance of 600 meters beneath the River Danube, using a fiber-optic cable, setting a new record. (The cable itself was 800 meters long and was strung underneath the public sewer system beneath the River Danube. The sender stood on one side of the river, and the receiver was on the other.) In this experiment quantum teleportation was demonstrated not with photons of light, but with actual atoms, bringing us a step closer to a more realistic teleportation device.

The physicists at the National Institute of Standards and Technology in Washington, D.C., successfully entangled three beryllium atoms and transferred the properties of one atom into another. This achievement was so significant that it made the cover of Nature magazine. Another group was able to teleport calcium atoms as well.

In 2006 yet another spectacular advance was made, for the first time involving a macroscopic object. Physicists at the Niels Bohr Institute in Copenhagen and the Max Planck Institute in Germany were able to entangle a light beam with a gas of cesium atoms, a feat involving trillions upon trillions of atoms. Then they encoded information contained inside laser pulses and were able to teleport this information to the cesium atoms over a distance of about half a yard.

2.4 Recent Breakthroughs

The collaborative team, which includes NASA's jet propulsion laboratory, successfully demonstrated sustained, long-distance teleportation of qubits of photons (quanta of light) with fidelity greater than 90%. The qubits were teleported 44 kilometers (27 miles) over a fiber-optic network using state-of-the-art single-photon detectors and off-the-shelf equipment [5].

In 2017 China's growing mastery of both the quantum world and space science, a team of physicists reported that it sent eerily intertwined quantum particles from a satellite to ground stations separated by 1200 kilometers, smashing the previous world record. The result is a stepping stone to ultra secure communication networks and, eventually, a space based quantum internet [6].

In 2019, researchers at the University of Bristol and the Technical University of Denmark were able to successfully carry out quantum teleportation of information between two programmable computer chips for the first time in history. It was done through quantum entanglement; the two computer chips were able to link together without any wired electronic connection over a large distance, then pass information from one chip to the other. It resulted in a 91% information transfer accuracy rate with low data loss and high communication stability.

Also in 2019, two separate research teams were able to quantum teleport a qutrit amount of information. A qutrit is a data unit that is larger than the base data unit that is used in quantum computing, qubit.

Previous experiments were only able to transport a qubit of information. These experiments are huge advancements for the fields of quantum communication and quantum computing.

Chapter 3

Theoretical Background

After 1925, the birth of the quantum mechanics, many breakthrough happened in this field. Now a days almost most of the branch of physics are described by quantum mechanical view precisely and many development is doing with the help of this tool. As discussed earlier that the first historic demonstration of quantum teleportation in which photons of ultraviolet light were teleported occurred in 1997 at the University of Innsbruck. So Quantum teleportation is relatively a new field of quantum mechanics.

Therefore many quantum mechanical concepts are needed to understand in order to understand quantum teleportation. We will restrict our topic to main theoretical concepts such as Superposition, Quantum State, Entanglement, EPR Experiment, No Cloning Theorem, Quantum Decoherence etc. Now we discuss this concept one by one from the next page.

3.1 Superposition

Imagine a system that consists of two electrons. it's important to note though that we don't necessarily want these electrons to be close enough to each other to be interacting with each other. In fact we don't even care if they're that close to each other or whether they're

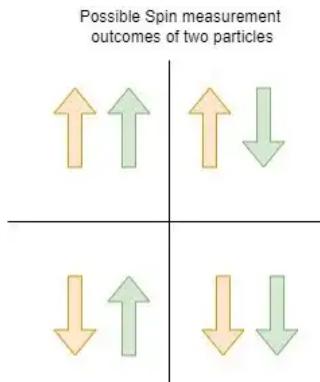


Figure 3.1: Superposition

on the opposite ends of the universe. We're not particularly interested in the behavior of these electrons in terms of where they are or how they're moving. What we care about is a property that's inherent to these electrons as well as other subatomic particles known as spin.

This system of two electrons have four possible combinations of spins. That means both of them are in spin down state, or both of them have spin up state, or the first one has the spin up state when the other electron has spin down state and vice versa. So if we measure the spins of both of these particles we will find one of these four possibilities. The possible spin combinations are shown in Figure 3.1.

Now quantum mechanics being the weird mind-bending set of ideas it is, tells us that when no external system is interacting with our original system, so for example when I'm not making a measurement or when nothing else from the external world is interacting with our system then our system is in a **superposition** of all four possible states. Most famous example of superposition is Schrödinger's Cat. In short if we don't open the box where the cat and the radioactive element are situated, the cat will both alive and dead.

3.2 Measurements

Now another really weird and interesting thing about quantum mechanics is that it tells us that if something external interacts with our system, so for example if we go and we measure the spins of these electrons then we cause the system to collapse into one of the four possible combinations. We will never measure these electrons to be in a superposition of up and down States. We will always measure them to either be spin up or spin down but whilst we're not measuring them they're in a superposition. Why does this type of collapse occur? Nobody knows. This behavior is simply one of the fundamental postulates of quantum mechanics.

3.3 Separable Quantum State

In quantum mechanics, separable states are quantum states belonging to a composite space that can be factored into individual states belonging to separate sub-spaces. Simply if we separate one electron from the other one while in the superposition state, then we can say the quantum system is separable into two individual state both having spin up and down.

Mathematically, for two quantum systems, A and B, the combined state, $|\psi\rangle_{AB}$, is separable if

$$|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$$

We will discuss those above notations in the later chapter. $|\rangle$ is called Dirac ket notation, ψ is the quantum state and \otimes is called tensor product.

So mathematically two quantum state is called separable if system of quantum state can be written as tensor product of two individual quantum state.

3.4 Entanglement

Let suppose we have a quantum system having zero spin. It emits two electrons. We know that spin of an electron is $\pm\frac{1}{2}$. Then getting the spin up in an electron is 50% and spin down is 50%, because we don't know which electron is in spin up state and which is one is in spin down state. If we measure the spin of first electron and get spin up ($+\frac{1}{2}$) state then we can immediately say that the spin of the other electron is down ($-\frac{1}{2}$) no matter how far they are situated. Also notice that probability of getting the down state of the second electron is now changed to 100% from 50%. This suggest that the measurement affects the system and the state of electrons are correlated that measuring one state is affects the other state.

If two states are dependent to each other or disturbing one state affects the other state the the states are said to be entangled and the phenomenon is called **Entanglement**. This is the lucid explanation of entanglement.

Let's discuss mathematically. The up spin of the first electron (A) is $|\uparrow_A\rangle$ and down spin of the first electron (A) is $|\downarrow_A\rangle$. Similarly, the up spin of the second electron (B) is $|\uparrow_B\rangle$ and down spin of the second electron (B) is $|\downarrow_B\rangle$. Let's imagine that's something interacts with just one of the electrons in our system not both because we're imagining that actually quite far apart from each other. So let's imagine we measure the spin of one of the electrons, let's say we measure the spin of particle A. If we have an entangled system then the probability distribution of spins or finding a particular spin whether that's up or down for particle B will change after we measure the spin of particle a compared to the probability distribution that we had before. And this is what we mean by an entangled system. If something interacts with just one part of our system for example particle A then the whole system is affected specifically the probability distribution of finding an up or down spin for the other particle in this particular case. We can say there's quite generally the interaction of something external with one part of our entangled system causes the entire system to be affected. In fact when discussing quantum entanglement people often use very special

cases to illustrate what they're talking about entanglement. They use states known as **Bell States**.

$$\frac{1}{\sqrt{2}} |\uparrow_A\rangle |\downarrow_B\rangle + \frac{1}{\sqrt{2}} |\downarrow_A\rangle |\uparrow_B\rangle$$

This Bell State for example is a very special case of the general quantum state that we talked about earlier when dealing with two electrons each of which has a spin and with this particular bell state we can see that the probability of both electrons being in the spin up state was 0 and the probability of both electrons being in the spin down state was 0 before we made our measurements and this state they're really brilliantly illustrates what quantum entanglement is all about.

Because let's say we are now to go and measure the spin of particle A. Let's say we found it to be spin up. In that case we now know that the system has collapsed into particular state $|\uparrow_A\rangle |\downarrow_B\rangle$ which means that the spin of particle B must be spin down and vice-versa. Let's say we measure the spin of particle A and we found it to be spin down. Then we know that the system is collapsed into the state $|\downarrow_A\rangle |\uparrow_B\rangle$ and the spin now must be spin up for particle B. In other words before we did any measuring to our system there was an equal probability of our particle B being spin-up or spin-down states. But after the measurement, we caused the system to collapse into a particular state and the probability of the other particle the one that we haven't measured, of being in a particular spin state was now 100%. We knew with certainty without measuring what the spin state of that particle was. Now we've just talked about an entangled state but we said earlier that if we could mathematically separate them into two individual state then this was a separable state and this was the opposite of entangled state.

Mathematically we can say that if the state $|\psi\rangle_A$ and $|\psi\rangle_B$ are entangled then we can not write the state of the system $|\psi\rangle_{AB}$ as tensor product of individual state i.e.

$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B$$

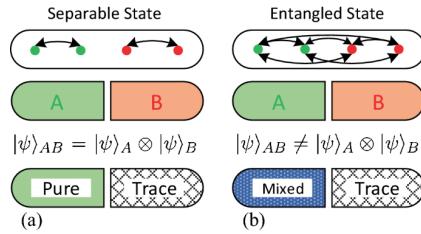


Figure 3.2: Separable and Entangled State [7]

Figure 3.2 illustrate the distinction between separable state and entangled state.

More than any other feature, it is entanglement that gives quantum theory its distinctive character. It is the sources of paradoxes like EPR paradox, of test of distinctive quantumness such as the violation of Bell's famous inequality, and it underlies some of the unexpected features of quantum information such as quantum teleportation and super dense coding.

3.5 EPR Experiment

The interesting thing is those entangled particles need not be in close proximity, they can be as far as one being in a physics lab on earth and the other being in the Andromeda galaxy. This information flow between two entangled particles takes place so instantaneously, that it seemed like quantum information can flow faster than the speed of light, and this idea greatly bothered famous physicist Albert Einstein. That is why he said the entanglement phenomena as ‘spooky-action-at-a-distance’. But it does not invalidate Einstein’s theory of relativity because we can do both measurements and outcomes can appear as random and only when we compare those outcomes we would know that they are opposite and this information is not being transferred any faster than the speed of light. So how does the information flow in quantum mechanics travels faster than the speed of light? Einstein along with his co-workers (Boris Podolsky, Nathan Rosen)conceived a thought experiment called the **EPR experiment**. The experiment pointing out that the special theory of

relativity implied that information could not travel faster than light, but the instantaneous action at a distance would mean that information could be sent instantaneously. They thought that there must be something and that something was named “Hidden Variables” that fixes the orientation of spin all along meaning spinning direction for the two entangled particles is predetermined from their creation and this information is contained locally so when they moved apart no communication was required. But physicist Neil Bohr just rejected the idea of hidden variables and the fact that entanglement seemed to create these weird correlations between particles was something that we had to accept as reality. There was no obvious way to tell who was right and who was wrong until 1964 physicist John Bell set up an experiment to determine who was right.

Let’s go into the detail of the measurements of this experiment. We are now aware that if we measure the spin direction of one particle, the other particle will be in the opposite direction. Now we make a little tweak here, suppose we measure the spin direction of the first particle in the vertical direction but the second one in the horizontal direction. That means the spin of the second particle would either be left or right with a 50% probability for each outcome. For better understanding, we can look into only those situations where the spin direction of the first particle is just up. Now we will look at what does quantum mechanics predicts in those situations and what does hidden variables predict. So the measurement in the vertical direction gives a spin-up for the first particle. Now when you measure the second particle, you measure in the horizontal direction, it can give left or right spin direction with equal probability. So we can say that the first particle’s measurement gave no predictive information about the measurement of the second particle. So in this scenario, quantum mechanics and hidden variables make the same prediction. But now if you make the measurement in the vertical direction, you will get a downward spin for the second particle 100% of the time. And this is again the same for quantum mechanics and hidden variables. Now we will see all possible measurement directions for the second particle and see if the predictions are different for quantum mechanics and hidden variables.

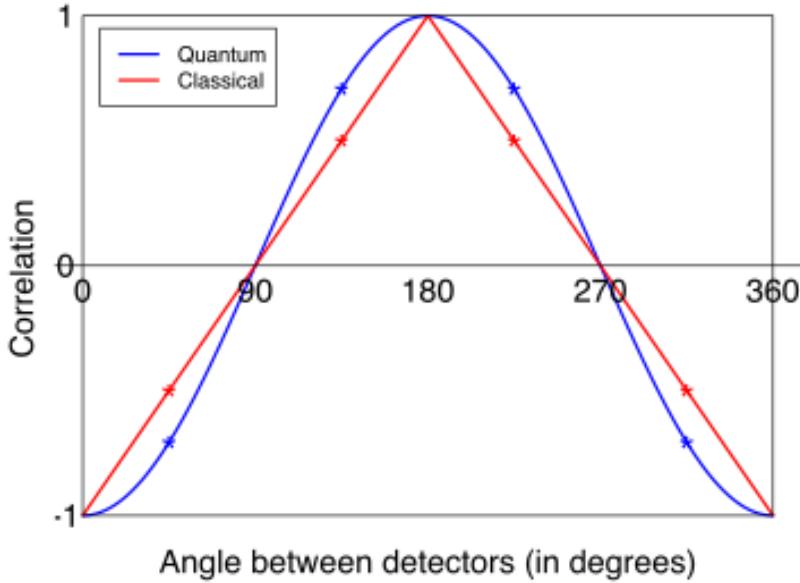


Figure 3.3: Hidden Variable [8]

Figure 3.3 shows the prediction of how often the second measurement will be the same as the measurement direction. And we are still focusing on the scenarios where the first particle's direction is up. So we start from the measuring direction being up, we know the second measurement is always down so that means that the second measurement agrees with the measurement direction zero percent of the time. And if the second measurement direction is to the right(making a 90-degree angle to the first measured direction) then we know that the second measurement could be either left or right, with a 50% probability for both. So it will be right for 50% of the time, and we can see from the graph at a 90-degree angle we have 50% of the time the second measurement agrees with the measuring direction. Now if the measuring direction goes down, then we know the second measurement agrees 100% of the time and this is visible in the graph as well. And as we go 360-degrees the pattern is reversed. We can see the predictions from quantum mechanics and hidden variables are different. And when we look at the actual measurements(the black dots) we

see that they follow the quantum mechanics predictions. So this way Einstein's idea of hidden variables was ruled out meaning that the final measurement is predetermined is at the moment two particles entangled is false.

So overall the classical physics and hidden variables just don't apply to quantum mechanics and quantum mechanics always violates Bell's Inequality. This meant that nature itself is fundamentally nonlocal.

3.6 No Cloning Theorem

In our daily life classically we copy information for many reasons such as computation, data storage, error determination correction. When we copy classical information, we can first learn what those bits are by measuring them and making copies of those bits.

We can not do this for quantum information because measuring a qubit only give us the outcome of the quantum state. Thus, measurement can not give us complete information about the qubit for us to make a copy.

No-Cloning theorem states that it is impossible to clone an exact quantum state (or qubit). In other words, it is impossible to make a copy of an arbitrary quantum state without first destroying the original. In computer language, this says that we can cut and paste a quantum state, but we cannot copy and paste it.

We can prove the no cloning theorem by the method of contradiction. To prove this theorem, let us pretend that a machine exists that can do just that: copy arbitrary quantum states. Given an input $|\psi\rangle$ and an ancillary bit $|0\rangle$, this machine turns $|0\rangle$ into $|\psi\rangle$. The schematic diagram is shown for the cloning process.

The following figure depicts the diagram of a cloning transformation gate.

1. It clones a qubit in the state $|0\rangle$.

$$U(|0\rangle|0\rangle) = |0\rangle|0\rangle$$

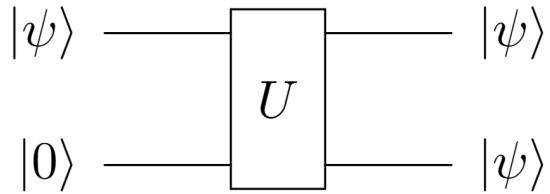


Figure 3.4: Cloning Gate

2. It clones a qubit in the state $|1\rangle$.

$$U(|1\rangle|1\rangle) = |1\rangle|1\rangle$$

3. It clones an arbitrary qubit $|\psi\rangle$.

$$U(|\psi\rangle|\psi\rangle) = |\psi\rangle|\psi\rangle$$

Now, let the superposition state of a arbitrary single-qubit consists of two basis states $|0\rangle$ and $|1\rangle$ and two corresponding probability amplitudes α and β , such that $|\alpha|^2 + |\beta|^2 = 1$.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Before applying the cloning gate the state of the system is:

$$(|\psi\rangle \otimes |0\rangle) = |\psi\rangle|0\rangle = |\psi 0\rangle$$

Now, apply the clone gate:

$$U(|\psi 0\rangle) = U((\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle) = U(\alpha|00\rangle + \beta|01\rangle)$$

Since U is a clone gate i.e. a quantum gate and quantum gates are linear, then clone gate

will be applied to each state individually. Therefore,

$$U(\alpha | 00\rangle + \beta | 01\rangle) = U(\alpha | 00\rangle) + U(\beta | 10\rangle)$$

Now applying clone gate the first bit be copied(cloned) to the second bit. Therefore $| 10\rangle$ transform to $| 11\rangle$. Hence,

$$U(\alpha | 00\rangle + \beta | 10\rangle) = \alpha | 00\rangle + \beta | 11\rangle$$

If, the clone be done after applying the clone machine(gate) then $|\psi 0\rangle$ transform into $|\psi\psi\rangle$. Now,

$$|\psi\psi\rangle = |\psi\rangle \otimes |\psi\rangle = (\alpha | 0\rangle + \beta | 1\rangle) \otimes (\alpha | 0\rangle + \beta | 1\rangle)$$

Simplification gives,

$$(\alpha | 0\rangle + \beta | 1\rangle) \otimes (\alpha | 0\rangle + \beta | 1\rangle) = \alpha^2 | 00\rangle + \alpha\beta | 01\rangle + \beta\alpha | 10\rangle + \beta^2 | 11\rangle$$

Clearly, clone state $(\alpha^2 | 00\rangle + \alpha\beta | 01\rangle + \beta\alpha | 10\rangle + \beta^2 | 11\rangle)$ is not same as the state after applying the clone gate $(\alpha | 00\rangle + \beta | 11\rangle)$.

$$\begin{aligned} \alpha^2 | 00\rangle + \alpha\beta | 01\rangle + \beta\alpha | 10\rangle + \beta^2 | 11\rangle &\neq \alpha | 00\rangle + \beta | 11\rangle \\ \implies |\psi\psi\rangle &\neq U(|\psi 0\rangle) \end{aligned}$$

Then we can say if the cloning transformation gate U exists, then two terms that are not equal must be equal. This is a contradiction. The only logical conclusion is that U can't exist. Therefore, it is impossible to clone a qubit of an arbitrary state, no such clone machines(quantum gates) exist which can copy an arbitrary quantum state. So, qubit can not be copied, this is called No-Cloning Theorem [9].

It has important consequences. In classical computing, we rely heavily on the ability

to copy. Even the simplest classical operation, the addition, relies on copying bits. But in quantum computing, it is impossible. In quantum computing, we simply can't use the information stored in a qubit as many times as we want to. In fact, the idea of cloning a qubit in an arbitrary state would contradict the underlying concept of superposition. Measuring a qubit collapses its state of superposition. But when we could clone a qubit, we could measure its state indirectly. We could measure the clones without collapsing the original qubit.

So, question come up to mind that ‘if we can not clone a quantum state then how quantum teleportation be possible?’ Well quantum teleportation by means transfer to quantum information of a quantum state from one place to another but not actual transfer of the quantum state.

3.7 Quantum Decoherence

Quantum decoherence is the loss of quantum coherence. In quantum mechanics, particles such as electrons are described by a wave function, a mathematical representation of the quantum state of a system; a probabilistic interpretation of the wave function is used to explain various quantum effects. As long as there exists a definite phase relation between different states, the system is said to be coherent. A definite phase relationship is necessary to perform quantum computing on quantum information encoded in quantum states. Coherence is preserved under the laws of quantum physics [10].

If a quantum system were perfectly isolated, it would maintain coherence indefinitely, but it would be impossible to manipulate or investigate it. If it is not perfectly isolated, for example during a measurement, coherence is shared with the environment and appears to be lost with time; a process called **Quantum Decoherence**. As a result of this process, quantum behavior is apparently lost, just as energy appears to be lost by friction in classical mechanics.

Decoherence was first introduced in 1970 by the German physicist H. Dieter Zeh and has

been a subject of active research since the 1980s. Decoherence has been used to understand the possibility of the collapse of the wave function in quantum mechanics. Decoherence does not generate actual wave-function collapse. It only provides a framework for apparent wave-function collapse, as the quantum nature of the system "leaks" into the environment. That is, components of the wave function are decoupled from a coherent system and acquire phases from their immediate surroundings.

Decoherence represents a challenge for the practical realization of quantum computers, since such machines are expected to rely heavily on the undisturbed evolution of quantum coherences. Simply put, they require that the coherence of states be preserved and that decoherence be managed, in order to actually perform quantum computation. Therefore coherence is also necessary for quantum teleportation. The preservation of coherence, and mitigation of decoherence effects, are thus related to the concept of quantum error correction.

Chapter 4

Mathematical and Computational Aspects

In this chapter we will make some intuition about the mathematical and computational terminologies like Tensor Products, Density Matrix, Bloch Sphere, Bits and Qubits, Quantum Gates etc.

Well Quantum Computation is a vast field merging mathematics, quantum mechanics, information science, engineering etc. Therefore there are much much things one should know to understand quantum computation. But we will restrict only to those topics which are mentioned above to properly understand quantum teleportation. These should be minimum requirement for understanding quantum teleportation and we will discuss those briefly.

4.1 Tensor Product and Density Matrix

Tensor Product:

In mathematics, a tensor refers to objects that have multiple indices. Roughly speaking this can be thought of as a multidimensional array. Generally we mean array as an two

dimensional structure of data. But Tensor have array of multi-dimension.

In mathematics, the **Tensor Product** $V \otimes W$ of two vector spaces V and W (over the same field) is a vector space to which is associated a bilinear map $V \times W \rightarrow V \otimes W$ that maps a pair (v, w) , $v \in V, w \in W$ to an element of $V \otimes W$ denoted $v \otimes w$.

If V is a vector space of dimension n and W is a vector space of dimension m, then $V \otimes W$ is a vector of dimension nm. Thus, $C^2 \otimes C^3 = C^6$.

Let's a quantum state is $|\psi\rangle$ and another state is $|\phi\rangle$, then the inner product of these two quantum state is defined as tensor product of $|\psi\rangle$ and $|\phi\rangle$ i.e. $|\psi\rangle \otimes |\phi\rangle = |\psi\phi\rangle$.

For example, The quantum state $|0\rangle$ in terms of matrix is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ then the composite system $|00\rangle$ is the tensor product of $|0\rangle$ and $|0\rangle$ i.e. $|0\rangle \otimes |0\rangle$. The tensor product

$$|0\rangle \otimes |0\rangle \text{ in terms of matrix is } \begin{bmatrix} 1 & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0 & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Tensor product obeys distributive property w.r.t. matrix addition. Therefore, we have seen in No Cloning theorem that $|\psi\rangle \otimes |0\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes |0\rangle = \alpha |00\rangle + \beta |10\rangle$.

Density Matrix:

Quantum states are usually denoted as dirac's ket notation($|\rangle$). Quantum states are also called state vector which is a column matrix and belongs to Hilbert Space(an inner product space that is complete with respect to the norm defined by the inner product).

Similarly bra notation ($\langle|$) is belongs to dual Hilbert space and is represented by a row matrix.

In quantum mechanics, a density matrix (or density operator) is a matrix that describes the quantum state of a physical system. It allows for the calculation of the probabilities of the outcomes of any measurement performed upon this system,

For a quantum state the product of $|\psi\rangle\langle\psi|$ is called Density Matrix. The density matrix is a representation of a linear operator called the density operator. The density matrix is obtained from the density operator by choice of basis in the underlying space. In practice, the terms density matrix and density operator are often used interchangeably.

We will see density matrix and it's city diagram in quantum teleportation with qiskit [11].

4.2 Bloch Sphere

The Bloch sphere is a geometric representation of qubit states as points on the surface of a unit sphere. Many operations on single qubits that are commonly used in quantum information processing can be neatly described within the Bloch sphere picture.

It turns out that an arbitrary single qubit state can be written:

$$|\psi\rangle = e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \right)$$

here, $e^{i\gamma}$ is a global phase and has no observable consequences so we can ignore that term from the above equation.

Therefore equation of the Bloch sphere is:

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$

The co-ordinates in the Bloch sphere are chosen as spherical co-ordinates. A sphere has infinite number of points at its surface, therefore any qubit(quantum bit) can be represented in this sphere by changing the value of θ and ϕ , where $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$ are the coordinates of points on the Bloch Sphere.

A Bloch sphere along with some superposition states of qubits are shown in the below figure.

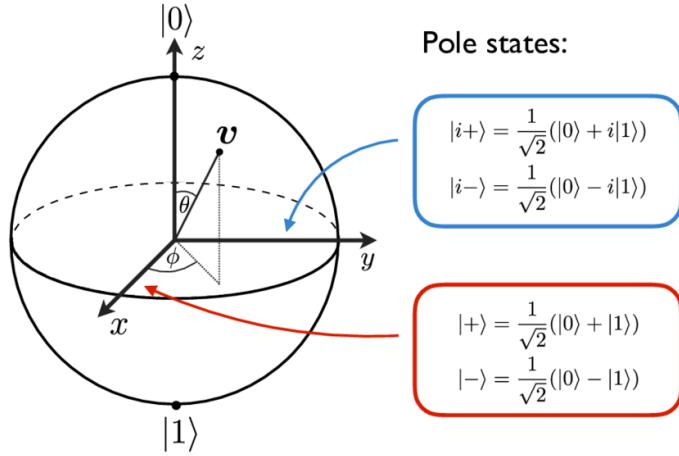


Figure 4.1: Bloch Sphere

4.3 Bits and Qubits

A bit is a unit of information describing a two-dimensional classical system, that is, a bit is a way of describing a system whose set of states is of size 2. We may use matrices to represent a bit. We represent the state $|0\rangle$ and $|1\rangle$ as $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A classical bit can be either in state $|0\rangle$ or in state $|1\rangle$. A simple example of bits is, a bit may denote the presence or absence of current. If there is a current then the bit is $|1\rangle$ and if there is no current then the bit is $|0\rangle$.

A quantum bit(qbit) can simultaneously be in both the states $|0\rangle$ and $|1\rangle$. A quantum bit or qubit can be represented as a 2 by 1 matrix with complex numbers c_0 and c_1 such as $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ where $|c_0|^2 + |c_1|^2 = 1$. Notice that a classical bit is a special type of qubit. $|c_0|^2$ is to be interpreted as the probability that after measuring the qubit, it will be found in state $|0\rangle$. $|c_1|^2$ is to be interpreted as the probability that after measuring the qubit will be found in state $|1\rangle$. Whenever we measure a qubit it automatically becomes a bit. So we can never observe a general qubit, nevertheless, they do exist and are main objects of

quantum computing.

4.4 Quantum Gates

A quantum logic gate is simply Such operators are represented by unitary matrices because Quantum gates are reversible. Except for the NOT gate all other familiar classical gates like the OR, AND, NOR, NAND, etc are irreversible in the sense that we cannot uniquely reconstruct the input states from the output states. Reversible gates and quantum gates are similar but there exists a fundamental difference that reversible gates cannot accept superposition states as input states, whereas quantum gates can. Thus all quantum gates are essentially reversible but the converse is not true. There area many multi-qubit quantum gates along with single qubit gate. But we will use only those quantum gates which are used in quantum teleportation protocol . There are mainly two type of quantum gates which are used in the process of creating entanglement, they are H(Hadamard) gate and C-NOT gate and also have two gates to retrieve the data after the quantum teleportation, they are Z gate and X(NOT) gate.

Now, these gates are introduced next one by one.

Hadamard gate(H gate):

Hadamard gate is single qubit quantum self inverse gate [12]. A self-inverse always maps the input states into mutually orthogonal output states and we can check that H maps the standard basis vectors $| 0\rangle$ and $| 1\rangle$ to mutually orthogonal output superposition states $| +\rangle$ and $| -\rangle$.

$$H | 0\rangle = \frac{1}{\sqrt{2}}(| 0\rangle + | 1\rangle) = | +\rangle$$

$$H | 1\rangle = \frac{1}{\sqrt{2}}(| 0\rangle - | 1\rangle) = | -\rangle$$

The unitary matrix corresponding to this gate can be given as: $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Controlled NOT gate(C-NOT gate):

Controlled-NOT gate, the SWAP gate, Controlled-U gates are examples of two qubit gates. The most popular and the most interesting of these two qubit gates is the Controlled-NOT or CNOT gate, which complements the second qubit if the first qubit is in the state $| 1 \rangle$ and leaves the second qubit unchanged otherwise. Thus the matrix representation for this gate is :

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

As the state of the first qubit controls whether the second qubit will be flipped or not, the first qubit is referred to as control qubit and the second qubit is called target qubit. The same convention is used in other controlled gates. The basic reason of the popularity of the CNOT gate is twofold, first it is used in many circuits of practical importance and secondly we can construct any other unitary quantum operation with suitable combination of these gates.

NOT gate:

This gate invert the state. If the state is $| 0 \rangle$ then the output after applying this gate will be $| 1 \rangle$ and if the state is $| 1 \rangle$ then the output after applying this gate will be $| 0 \rangle$. the matrix representation for this gate is :

$$NOT = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Z gate:

This gate is also known as Phase Shifting gate. if the input state is $| 0 \rangle$ then there will be no change in phase. But if the input is $| 1 \rangle$ then the output will be $-| 1 \rangle$. the matrix

representation for this gate is :

$$Z = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Creating an Entanglement state using a H gate and a CNOT gate:

Here we use a $|0\rangle$ state as initial state and create a same entangled state without phase.

One can take any single bit , qubit or multi qubit and first apply H gate then apply CNOT gate to make an entangled state.

First applying H gate create a superposition state of the state $|0\rangle$ and $|1\rangle$ i.e.

$$\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

Then in this qubit first bit is control and second bit is target for CNOT gate. CNOT gate invert the bit if 1 bit is in control. Then the state $|01\rangle$ transformed to $|11\rangle$. Therefore the final state after applying the CNOT gate will be:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

which is a same entangled state without phase.

Chapter 5

Quantum Teleportation

Finally we come to our main topic. Now we are able to know what actually quantum teleportation is rather than what we have seen in sci-fi movie. We have already discuss what quantum teleportation is in introduction section. We have discussed historical background, theoretical background, mathematics related to this so that we have a clear picture build in our mind about quantum teleportation. We will now understand how actually quantum teleportation works mathematically as well as we will see the protocol via qiskit(a software development kit developed by IBM for quantum computing). But before going to the Protocol and coding stuff we will discuss some experiments which were done earlier and proved that quantum teleportation is possible at least at quantum level.

5.1 The Innsbruck Experiment

Image depicts the University of Innsbruck experimental setup for quantum teleportation. In the quantum teleportation process, physicists take a photon (or any other quantum-scale particle, such as an electron or an atom) and transfer its properties (such as its polarization - the direction in which its electric field vibrates) to another photon - even if the two photons are at remote locations. The scheme does not teleport the photon itself; only its properties

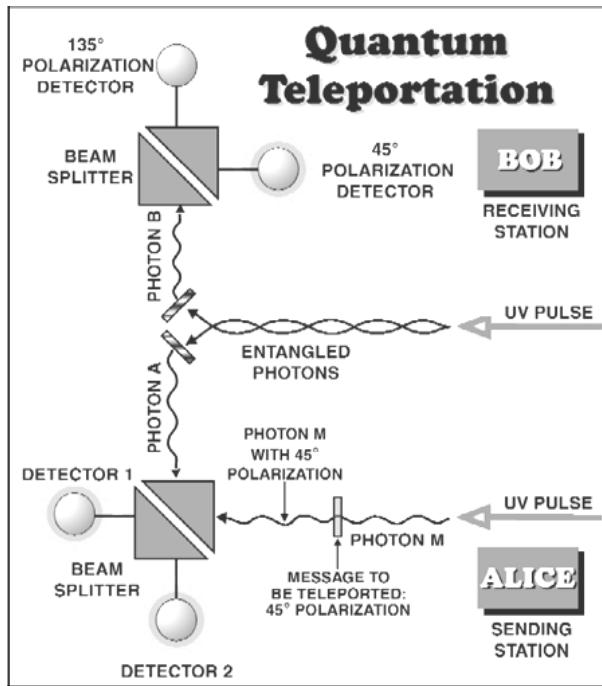


Figure 5.1: The Innsbruck Experiment

are imparted to another, remote photon.

Here is how it works: At the sending station of the quantum teleporter, Alice encodes a "messenger" photon (M) with a specific state: 45 degrees polarization. This travels towards a beam splitter. Meanwhile, two additional "entangled" photons (A and B) are created. The polarization of each photon is in a fuzzy, undetermined state, yet the two photons have a precisely defined interrelationship. Specifically, they must have complementary polarizations. For example, if photon A is later measured to have horizontal (0 degrees) polarization, then the other photon must "collapse" into the complementary state of vertical (90 degrees) polarization.

Entangled photon A arrives at the beam splitter at the same time as the message photon M. The beam splitter causes each photon to both continue toward detector 1 or change course and travel to detector 2. In 25% of all cases, in which the two photons go off

into different detectors, Alice does not know which photon went to which detector. This inability of Alice to distinguish between the two photons causes quantum weirdness to kick in. Just by the very fact that the two photons are now indistinguishable, the M photon loses its original identity and becomes entangled with A. The polarization value for each photon is now indeterminate, but since they travel toward different detectors Alice knows that the two photons must have complementary polarizations.

Since message photon M must have complementary polarization to photon A, then the other entangled photon (B) must now attain the same polarization value as M. Therefore, teleportation is successful. Indeed, Bob sees that the polarization value of photon B is 45 degrees: the initial value of the message photon.

5.2 The Photon Experiment

In 1998, physicists at the California Institute of Technology (Caltech), along with two European groups, turned the IBM ideas into reality by successfully teleporting a photon, a particle of energy that carries light. The Caltech group was able to read the atomic structure of a photon, send this information across 1 meter (3.28 feet) of coaxial cable and create a replica of the photon. As predicted, the original photon no longer existed once the replica was made.

In performing the experiment, the Caltech group was able to get around the Heisenberg Uncertainty Principle, the main barrier for teleportation of objects larger than a photon. This principle states that you cannot simultaneously know the location and the speed of a particle. But if you can't know the position of a particle, then how can you teleport it? In order to teleport a photon without violating the Heisenberg Principle, the Caltech physicists used a phenomenon known as entanglement. In entanglement, at least three photons are needed to achieve quantum teleportation:

- Photon A: The photon to be teleported
- Photon B: The transporting photon
- Photon C: The photon that is entangled with photon B

If researchers tried to look too closely at photon A without entanglement, they would bump it, and thereby change it. By entangling photons B and C, researchers can extract some information about photon A, and the remaining information would be passed on to B by way of entanglement, and then on to photon C. When researchers apply the information from photon A to photon C, they can create an exact replica of photon A. However, photon A no longer exists as it did before the information was sent to photon C.

A more recent teleportation success was achieved at the Australian National University, when researchers successfully teleported a laser beam.

While the idea of creating replicas of objects and destroying the originals doesn't sound too inviting for humans, quantum teleportation does hold promise for quantum computing. These experiments with photons are important in developing networks that can distribute quantum information. Professor Samuel Braunstein, of the University of Wales, Bangor, called such a network a "quantum Internet." This technology may be used one day to build a quantum computer that has data transmission rates many times faster than today's most powerful computers.

5.3 How does teleportation work?

Six scientists found a way to teleport quantum information using the celebrated and paradoxical feature of quantum mechanics known as the Einstein-Podolsky-Rosen effect (EPR Theory). In brief, they found a way to scan out part of the information from an object A, which one wishes to teleport, while causing the remaining, unscanned, part of the information to pass, via the Einstein-Podolsky-Rosen effect, into another object C which has never

been in contact with A. Later, by applying to C a treatment depending on the scanned-out information, it is possible to maneuver C into exactly the same state as A was in before it was scanned. A itself is no longer in that state, having been thoroughly disrupted by the scanning, so what has been achieved is teleportation, not replication.

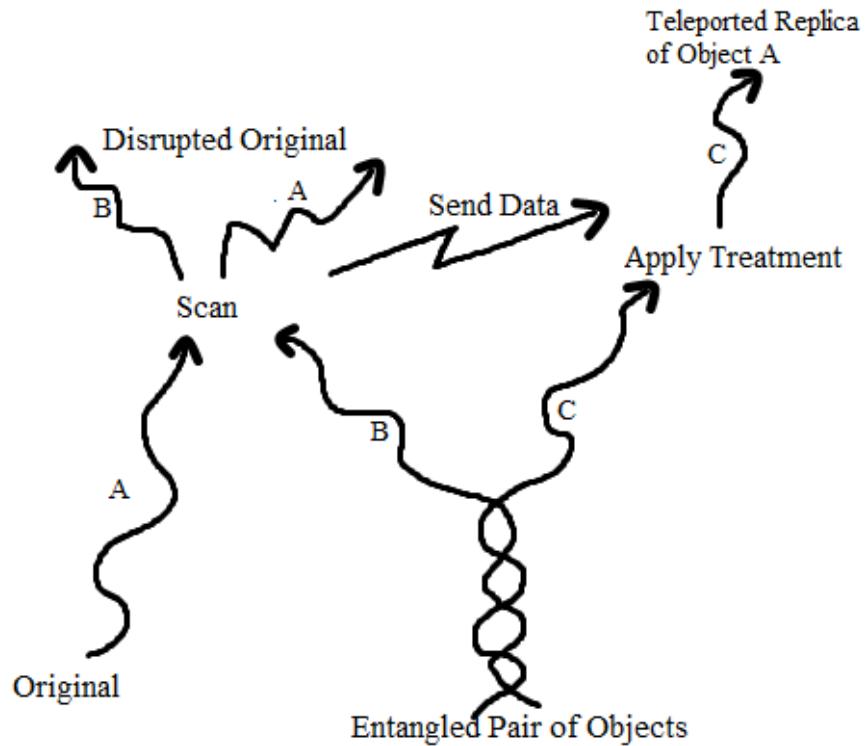


Figure 5.2: Protocol of Quantum Teleportation

As the figure suggests, the unscanned part of the information is conveyed from A to C by an intermediary object B, which interacts first with C and then with A. What? Can it really be correct to say “first with C and then with A”? Surely, in order to convey something from A to C, the delivery vehicle must visit A before C, not the other way around. But there is a subtle, unscannable kind of information that, unlike any material cargo, and even unlike ordinary information, can indeed be delivered in such a backward fashion. This subtle kind of information, also called ”Einstein-Podolsky-Rosen (EPR) correlation” or

”entanglement”, has been at least partly understood since the 1930s when it was discussed in a famous paper by Albert Einstein, Boris Podolsky, and Nathan Rosen. In the 1960s, John Bell showed that a pair of entangled particles, which were once in contact but later move too far apart to interact directly, can exhibit individually random behavior that is too strongly correlated to be explained by classical statistics. Experiments on photons and other particles have repeatedly confirmed these correlations, thereby providing strong evidence for the validity of quantum mechanics, which neatly explains them.

Another well-known fact about EPR correlations is that they cannot by themselves deliver a meaningful and controllable message. It was thought that their only usefulness was in proving the validity of quantum mechanics. Now it is known that, through the phenomenon of quantum teleportation, they can deliver exactly that part of the information in an object, which is too delicate to be scanned out, and delivered by conventional methods.

5.4 Quantum Teleportation Protocol

According to the No Cloning theorem, we can’t copy qubit state. So how do we transmit qubits from one point to another without destroying superposition or phase information? Suppose we’ll have two parties trying to communicate, Alice and Bob. Alice will have a qubit ($|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$) that she wishes to transfer to Bob. How can she do this? Quantum teleportation enables the transfer of qubits while perfectly preserving qubit state.

Requirements for Quantum Teleportation [13]:

A qubit can be perfectly transmitted using 3 qubits and 2 classical bits. So teleportation needs:

- Two qubits that are entangled and each party has one half of this entangled pair.
- A message qubit that is to be transported.
- A classical communication channel between both parties for the transmission of classical bits.

As a note, since teleportation requires that classical communication line, it cannot be faster than the speed of light. So let's walk through the process of quantum teleportation.

Step 1: Create an entangled pair of qubits

Here we're going to create same entanglement state without phase. Whenever we talk about this generation of entanglement, Alice and Bob can create the entangled pair or they can receive it from another party.

To create entanglement we can use the Hadamard gate and the CNOT gate. If both Alice and Bob has the bit $|0\rangle$ then applying H gate to Alice's bit and CNOT gate's control to Alice's bit and target to Bob's bit, we can create a same entangled pair of qubit without phase. Note, we are taking same entanglement state to understand this protocol easily. One can choose any other entanglement state to teleport the quantum information. Let's back to our point.

We know that a H gate and a CNOT gate crate an entanglement gate. Here we want to create same entanglement state without phase. So the entangled state is:

$$|e\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Alice and Bob each possess one qubit of the entangled pair (denoted as A and B respectively), so

$$|e\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

Step 2: Distribute entangled qubits to Alice and Bob

Once we have entangled qubit pair, we distribute them to Alice and Bob. Alice and Bob can be separated by a great distance, but they just have to have their entangled qubits with them.

Now we'll have Alice with the mystery qubit $|\psi\rangle$ that will act as our message. We'll have a total quantum state including the message qubit and the two entangled qubits as

follows:

$$|\psi\rangle \otimes |e\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes \left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$$

So, the total three-qubit quantum system where Alice has the first two qubits and Bob the last one is:

$$|\psi\rangle \otimes |e\rangle = \frac{1}{\sqrt{2}}(\alpha |000\rangle + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle)$$

Step 3: Alice applies CNOT gate to her message qubit

Her CNOT operation will have the message qubit as the control and her qubit that is one half of the entangled pair as the target. This will allow for the following state transformation.

state 1: (Before applying CNOT gate)

$$|\psi\rangle \otimes |e\rangle = \frac{1}{\sqrt{2}}(\alpha |000\rangle + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle)$$

↓

state 2: (After applying CNOT gate)

$$\frac{1}{\sqrt{2}}(\alpha |000\rangle + \alpha |011\rangle + \beta |101\rangle + \beta |110\rangle)$$

We can see that because Alice and Bob share entanglement, the whole quantum state has been changed, including the qubit that's in Bob's possession.

Step 4: Alice applies an H gate to her message qubit $|\psi\rangle$

In this step Alice will apply a H gate or a superposition gate to the mystery message qubit $|\psi\rangle$. That will cause the following state transformation.

state 2:

$$\frac{1}{\sqrt{2}}(\alpha |000\rangle + \alpha |011\rangle + \beta |101\rangle + \beta |110\rangle)$$

↓

state 3:

$$\frac{1}{2}[\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|001\rangle + |010\rangle - |101\rangle - |110\rangle)]$$

Because we insert the Hadamard operation, which is our superposition operation, our state expands to include a lot more basis states. We have four for alpha and four for beta, with some phase introduced for the case of beta.

Step 5: Alice measures both of her qubits

In this step Alice measures both of her qubits and we shall look to the quantum state before the measurement and after the measurement. The state before the measurement is:

$$\frac{1}{2}[\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|001\rangle + |010\rangle - |101\rangle - |110\rangle)]$$

when she measures, she'll be observing the state of these first two qubits in all of these basis states. There are four possible outcomes for Alice's qubits upon measurement. And remember, when she measures, that information collapses into a classical bit so the result is classical.

The states that she could possibly observe are $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

Step 6: Process results of measurements

Next, we can come to conclusions about Bob's state by using those measurements.

Alice's measurement	00	01	10	11
Value of Bob's qubit	$\alpha 0\rangle + \beta 1\rangle$	$\alpha 1\rangle + \beta 0\rangle$	$\alpha 0\rangle - \beta 1\rangle$	$\alpha 1\rangle - \beta 0\rangle$

If Alice sees a 00, we narrowed down the state of the entire system to include those only that include the 00 state for the first two qubits. So with her measurement information, we're able to come to some conclusions about what the state of that final qubit will be. If we have a measurement of 00 for Alice, the value of Bob's qubit is $\alpha | 0 \rangle + \beta | 1 \rangle$. If Alice measures a 00, the value of Bob's qubit is $\alpha | 1 \rangle - \beta | 0 \rangle$.

Step 7: Alice transmits her two classical bits to Bob

After measurement, Alice transmits her two classical bits to Bob. Bob receives those classical bits and has to correct the state of his qubit in order to recover the original state of $|\psi\rangle$, which is $\alpha | 0 \rangle + \beta | 1 \rangle$. Remember, if Alice measures the following bit combinations shown in the above figure, the value of Bob's qubit will be reflected. So with that in mind, how does Bob correct his qubit? To recover $|\psi\rangle$, Bob has to do some operation which are described in the next step.

Step 8: Bob recovers $|\psi\rangle$

To recover $|\psi\rangle$, Bob has to do the following:

- If the first bit (b_1) is 1, then he apply a Z gate.
- If the second bit (b_0) is 1, then he apply a NOT(X) gate.

If Alice measures a 00, we don't need correction. Bob already has the recovered state for $|\psi\rangle$, the message qubit.

If Alice measures a 01, Bob has to apply a NOT gate in order to exchange the probability amplitudes associated with $|0\rangle$ and $|1\rangle$.

If Alice measures a 10, Bob must apply a Z gate in order to correct the phase associated with $|1\rangle$ and his qubit.

If Alice has measured a 11, Bob must apply a NOT gate or an X gate as well as a Z gate in order to correct the bit flip and the phase flip that occurred in his qubit, so that he can recover the state of $|\psi\rangle$.

The figure above shows the operation which Bob has to apply in order to get the message qubit.

Alice's measurement ($b_1 b_0$)	00	01	10	11
Value of Bob's qubit	$\alpha 0\rangle + \beta 1\rangle$	$\alpha 1\rangle + \beta 0\rangle$	$\alpha 0\rangle - \beta 1\rangle$	$\alpha 1\rangle - \beta 0\rangle$
Gates to be applied by Bob	Identity(I) gate	NOT(X) gate	Z gate	NOT(X) gate and Z gate

This is how we teleport the secret qubit message or quantum information from Alice to Bob via a classical channel. Alice never knows the state of the message, qubit $|\psi\rangle$. She transmits it without knowing it and performs operations on it without knowing it. The only thing she knows is the results of measurement. Communication here is not faster than the speed of light because of that dependence on the classical communication channel. And that's a common **misperception** about quantum teleportation. During the process of teleportation, Alice and Bob destroy their entangled qubits. So if we want to transmit more qubits, we need to generate more entanglement and distribute it.

5.4.1 Why is this protocol interesting?

To answer this, imagine Alice and Bob met a long time ago and each took one qubit of the entangled pair. Bob is now traveling around the world and can only communicate with Alice by phone or email. If Alice wanted to transfer quantum data to Bob without quantum teleportation, she would have to meet Bob and physically give Bob her qubit. Quantum teleportation allows Alice to send quantum information using a classical communications channel. All she has to do is make some measurements and email Bob the values. Bob can then apply the correct recipe to his qubit to bring it to the state of the original qubit. As well as sending information between two people, quantum teleportation is a useful way of causing interaction between different parts of a quantum computer (by teleporting a qubit to a different part of the quantum computer you want to interact with).

Chapter 6

Quantum Teleportation using Qiskit

Now we will see quantum teleportation using Qiskit in this chapter .
Qiskit stands for Quantum Information Software Kit for Quantum Computation. Qiskit is an open-source software development kit (SDK) for working with quantum computers at the level of circuits, pulses, and algorithms. It provides tools for creating and manipulating quantum programs and running them on prototype quantum devices on IBM Quantum Experience or on simulators on a local computer. It follows the circuit model for universal quantum computation, and can be used for any quantum hardware (currently supports superconducting qubits and trapped ions).

For this we will discuss the whole process in several step for the easily understanding. Many comments are written in the codes to explain the next steps in brief. So, here we go -

Importing necessary libraries:

```
[1]: from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister
      from qiskit import Aer, execute
      from qiskit.visualization import plot_bloch_multivector, array_to_latex
      from qiskit.extensions import Initialize
      from qiskit.quantum_info import random_statevector
      import qiskit.quantum_info as qi
```

Initializing a random qubit state:

```
[2]: # Create random 1-qubit state
psi = random_statevector(2)

#print('psi state: {}'.format(psi))

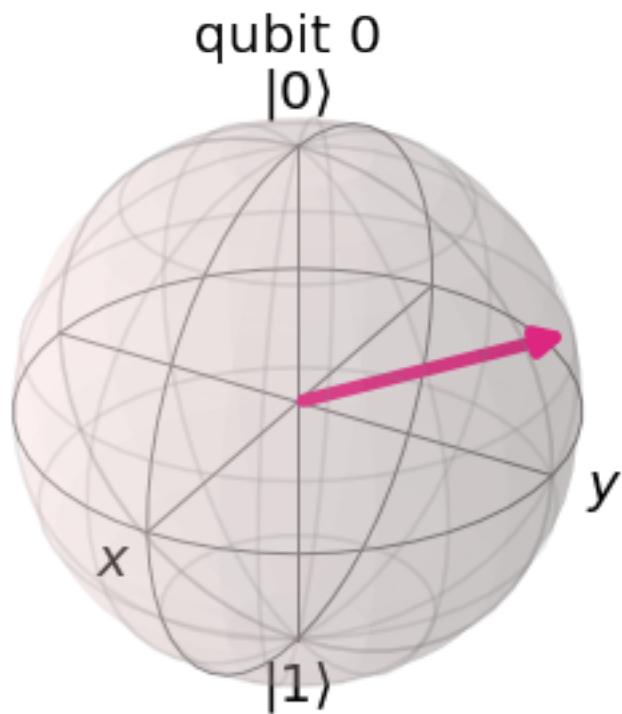
# Display in LaTeX
display(array_to_latex(psi,prefix = "|\\"psi\\rangle ="))
```

$$|\psi\rangle = [0.40416 - 0.6831i \quad 0.27831 + 0.54091i]$$

```
[3]: #/ $\psi$  vector in terms of standard basis
display(psi.draw('latex'))
```

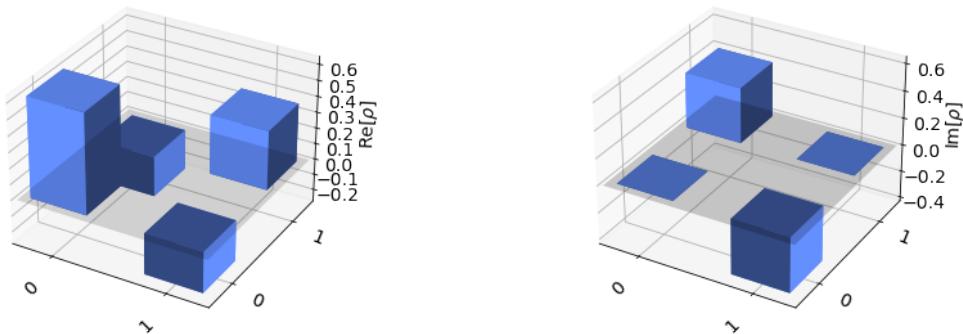
$$(0.404156195507 - 0.683098786632i)|0\rangle + (0.278307579725 + 0.540905452369i)|1\rangle$$

```
[4]: # Visualize the generated random qubit state in Bloch sphere
display(plot_bloch_multivector(psi))
```



```
[5]: # Visualize the generated random qubit state in 3D version of Density Matrix
psi.draw('city')
```

[5]:



```
[6]: # Visualize the generated random qubit state in Density Matrix
qq1 = qi.DensityMatrix(psi)
display(qq1.draw('latex'))
```

$$\begin{bmatrix} 0.62997 & -0.25701 - 0.40872i \\ -0.25701 + 0.40872i & 0.37003 \end{bmatrix}$$

```
[7]: # Define initialization gate to create |ψ⟩ from the state |0⟩
init_state = Initialize(psi)
init_state.label = "initial_state"
```

Essential functions for quantum teleportation protocol:

```
[8]: def create_bell_pair(qc, a, b):
    """Creates a bell pair in qc using qubits a & b"""
    qc.h(a) # Put qubit a into state |+>
    qc.cx(a,b) # CNOT with a as control and b as target
```

```
[9]: def alice_gates(qc, psi, a):
    qc.cx(psi, a)
    qc.h(psi)
```

```
[10]: def measure_and_send(qc, a, b):
    """Measures qubits a & b and 'sends' the results to Bob"""
    qc.barrier()
    qc.measure(a,0)
    qc.measure(b,1)
```

```
[11]: # This function takes a QuantumCircuit (qc), integer (qubit)
# and ClassicalRegisters (crz & crx) to decide which gates to apply
def bob_gates(qc, qubit, crz, crx):
    # Here we use c_if to control our gates with a classical
    # bit instead of a qubit
    qc.x(qubit).c_if(crx, 1) # Apply gates if the registers
    qc.z(qubit).c_if(crz, 1) # are in the state '1'
```

Executing Quantum Teleportation Protocol:

```
[12]: ## SETUP
qr = QuantumRegister(3, name="q")      # Protocol uses 3 qubits
crz = ClassicalRegister(1, name="crz") # and 2 classical registers
crx = ClassicalRegister(1, name="crx")
qc = QuantumCircuit(qr, crz, crx)

## STEP 0
# First, let's initialize Alice's q0
qc.append(init_state, [0])
qc.barrier()

## STEP 1
# Now begins the teleportation protocol
create_bell_pair(qc, 1, 2)
qc.barrier()

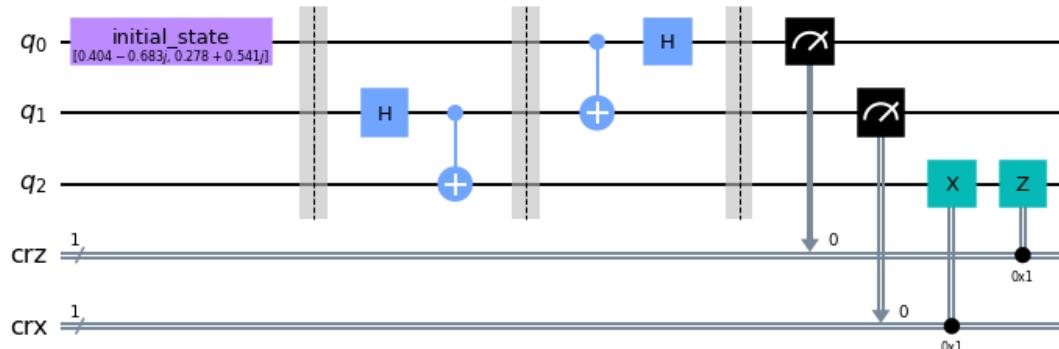
## STEP 2
# Send q1 to Alice and q2 to Bob
alice_gates(qc, 0, 1)

## STEP 3
# Alice then sends her classical bits to Bob
measure_and_send(qc, 0, 1)

## STEP 4
# Bob decodes qubits
bob_gates(qc, 2, crz, crx)

# Display the circuit
qc.draw('mpl')
```

[12]:



Simulating our Quantum Teleportation Circuit:

```
[13]: # Let's see the result

# To get the eigenvector you should use the statevector simulator in the core
# of the circuit (without measurements)
backend = Aer.get_backend('statevector_simulator')

# Execute the circuit
result = execute(qc, backend).result().get_statevector(qc, decimals=3)

# Printing the state after X gate
print("Quantum state is:")
display(array_to_latex(psi, prefix = "|\\psi\\rangle ="))
print()
print()
print("|\psi\rangle vector in terms of standard basis:")
display(result.draw('latex'))
print()
print()
# Visualize the result state in 3D version of Density Matrix
display(result.draw('city'))
print()
print()
print("The Density Matrix is:")
DensityMatrix = qi.DensityMatrix(result)
display(DensityMatrix.draw('latex'))
print()
print()
print("Plotting the Bloch Sphere:")
plot_bloch_multivector(result)
```

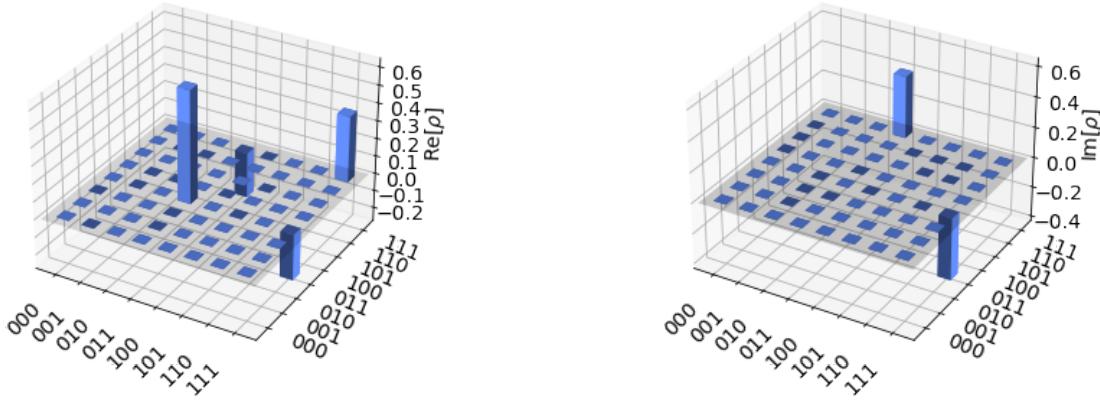
Quantum state is:

$$|\psi\rangle = [0.40416 - 0.6831i \quad 0.27831 + 0.54091i]$$

$|\psi\rangle$ vector in terms of standard basis:

$$(0.404 - 0.683i)|011\rangle + (0.278 + 0.541i)|111\rangle$$

The Density Matrix visualization is:

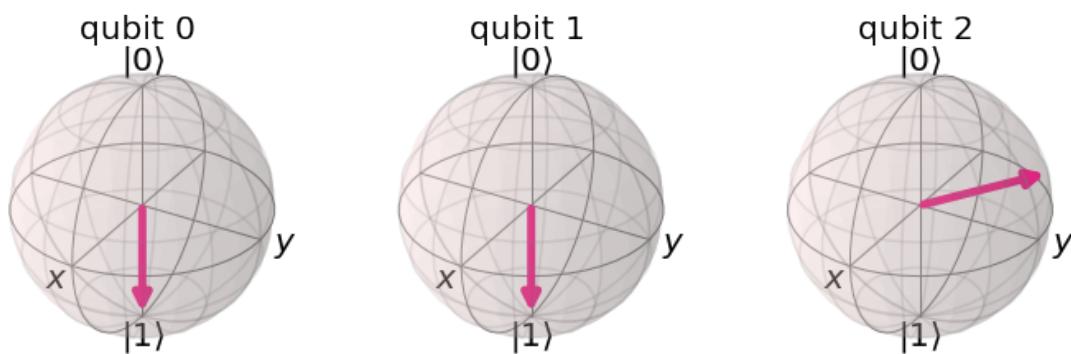


The Density Matrix is:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.62971 & 0 & 0 & 0 & -0.25719 - 0.40844i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.25719 + 0.40844i & 0 & 0 & 0 & 0.36997 \end{bmatrix}$$

Plotting the Bloch Sphere:

[13]:



Chapter 7

Human Teleportation!

We are years away from the development of a teleportation machine like the transporter room on Star Trek's Enterprise spaceship. The laws of physics may even make it impossible to create a transporter that enables a person to be sent instantaneously to another location, which would require travel at the speed of light.

For a person to be transported, a machine would have to be built that can pinpoint and analyze all of the 10^{28} atoms(suppose) that make up the human body. That's more than a trillion atoms. This machine would then have to send this information to another location, where the person's body would be reconstructed with exact precision. Molecules couldn't be even a millimeter out of place, lest the person arrive with some severe neurological or physiological defect.

In the Star Trek episodes, and the spin-off series that followed it, teleportation was performed by a machine called a transporter. This was a platform that the characters stood on, while Scotty adjusted switches on the transporter room control boards. The transporter machine then locked onto each atom of each person on the platform, and used a transporter carrier wave to transmit those molecules to wherever the crew wanted to go. Viewers watching at home witnessed Captain Kirk and his crew dissolving into a shiny glitter before disappearing, re-materializing instantly on some distant planet.

If such a machine were possible, it's unlikely that the person being transported would actually be "transported." It would work more like a fax machine a duplicate of the person would be made at the receiving end, but with much greater precision than a fax machine. But what would happen to the original? One theory suggests that teleportation would combine genetic cloning with digitization.

In this bio-digital cloning, tele-travelers would have to die, in a sense. Their original mind and body would no longer exist. Instead, their atomic structure would be recreated in another location, and digitization would recreate the travelers' memories, emotions, hopes and dreams. So the travelers would still exist, but they would do so in a new body, of the same atomic structure as the original body, programmed with the same information.

But like all technologies, scientists are sure to continue to improve upon the ideas of teleportation, to the point that we may one day be able to avoid such harsh methods. One day, one of your descendants could finish up a work day at a space office above some far away planet in a galaxy many light years from Earth, tell his or her wristwatch that it's time to beam home for dinner on planet X below and sit down at the dinner table as soon as the words leave his mouth.

Chapter 8

Future Directions

As we have discussed earlier that it is impossible to deliver information simultaneously over a long-range. But it is feasible to take quantum entanglement as a protocol to encrypt information. In 1993, physicists Asher Peres and William Wootters, first proposed the concept of quantum teleportation. Then in that year IBM frontier Charles Bennett with his colleagues first showed that it was physically possible to teleport objects, at least at the atomic level, using the EPR experiment. Then the Innsbruck experiment provides the experimental proof. From then scientists did many experiments with different different atoms and proved that quantum teleportation is possible. So, many improvements occurred in this topic. In 2017 China claims that they have transmitted quantum information over 200 km distance and made a world record.

So, we can see many improvement happened. Therefore in this chapter we will discuss the future directions of quantum teleportation as well as quantum information science. Because without quantum information science and quantum computation, quantum teleportation is nothing. So some real time and future possibilities of quantum teleportation are as follows:

8.1 Applications of Quantum Teleportation

1. Not only does the quantum teleportation provide a complete secure information transmission but also it boosts the development of quantum technologies. From the perspective of traditional communication protocol, it is a revolutionary watershed. In the point of quantum technologies, it serves as an indispensable foundation. Lots of technologies like quantum gate teleportation, computing, port-based teleportation and quantum networks are derived from the basis of quantum teleportation. Quantum teleportation has been achieved in many laboratories with different approaches. With the great progress of the experiment, some extensions of quantum teleportation have been brought to the real world for commercial and scientific purposes.
2. Reducing computation errors in the form of noise-resistant quantum gates and error correcting codes.
3. Teleportation can also be used to unite quantum computers to form networks.
4. Teleportation can be applied to create ultra-secure communications channels. This is because when qubits are transferred, they're transferred with ultimate privacy. Eavesdroppers cannot read the messages.
5. Quantum Internet, which is now-a-days gets special attention to the quantum computer scientists. Those days are now far when we all will use this secure quantum internet.
6. Transportation becomes much easier and time effective.
7. Implementations of quantum teleportation algorithms can solve many real life complex problem such as optimisation problem, logistics problem and many more.

Progress in teleportation is rapidly accelerating. But to become teleportation possible the most essential thing is Entanglement. So, again question arises that is it ever be

possible that teleportation can be done without entanglement? For the sake of our queries, it is possible. There is proposal that scientists made that quantum teleportation is possible without entanglement.

8.2 Teleportation without entanglement!

In 2007 yet another breakthrough was made. Physicists proposed a teleportation method that does not require entanglement. We recall that entanglement is the single most difficult feature of quantum teleportation. Solving this problem could open up new vistas in teleportation. “We’re talking about a beam of about 5,000 particles disappearing from one place and appearing somewhere else,” says physicist Aston Bradley of the Australian Research Council Centre of Excellence for Quantum Atom Optics in Brisbane, Australia, who helped pioneer a new method of teleportation.

In their approach, he and his colleagues take a beam of rubidium atoms, convert all its information into a beam of light, send this beam of light across a fiber-optic cable, and then reconstruct the original beam of atoms in a distant location. If his claim holds up, this method would eliminate the number one stumbling block to teleportation and open up entirely new ways to teleport increasingly large objects. In order to distinguish this new method from quantum teleportation, Dr. Bradley has called his method “classical teleportation.” (This is a bit misleading, since his method also depends heavily on the quantum theory, but not on entanglement). The key to this novel type of teleportation is a new state of matter called a “Bose Einstein Condensate,” or BEC, which is one of the coldest substances in the entire universe [14].

8.2.1 Bose Einstein Condensate

We will just discuss about this topic to have a concept about BEC and do not deep dive into this topic.

In nature the coldest temperature is found in outer space; it is 3 K above absolute zero. (This is due to residual heat left over from the big bang, which still fills up the universe.) But a BEC is a millionth to a billionth of a degree above absolute zero, a temperature that can be found only in the laboratory. When certain forms of matter are cooled down to near absolute zero, their atoms all tumble down to the lowest energy state, so that all their atoms vibrate in unison, becoming coherent. The wave functions of all the atoms overlap, so that, in some sense, a BEC is like a gigantic “super atom,” with all the individual atoms vibrating in unison. This bizarre state of matter was predicted by Einstein and Satyendranath Bose in 1925, but it would be another seventy years, not until 1995, before a BEC was finally created in the lab at MIT and the University of Colorado.

The immediate practical application of BECs is to create “atomic lasers.” Lasers, of course, are based on coherent beams of photons vibrating in unison. But a BEC is a collection of atoms vibrating in unison, so it’s possible to create beams of BEC atoms that are all coherent. In other words, a BEC can create the counterpart of the laser, the atomic laser or matter laser, which is made of BEC atoms. The commercial applications of lasers are enormous, and the commercial applications of atomic lasers could also be just as profound. But because BECs exist only at temperatures hovering just above absolute zero, progress in this field will be slow, though steady [15].

Back to the topic.

Here’s how Bradley and company’s teleportation device works. First they start with a collection of super cold rubidium atoms in a BEC state. They then apply a beam of matter to the BEC (also made of rubidium atoms). These atoms in the beam also want to tumble down to the lowest energy state, so they shed their excess energy in the form of a pulse of light. This light beam is then sent down a fiber-optic cable. Remarkably the light

beam contains all the quantum information necessary to describe the original matter beam (e.g., the location and velocity of all its atoms). Then the light beam hits another BEC, which then converts the light beam into the original matter beam. This new teleportation method has tremendous promise, since it doesn't involve the entanglement of atoms. But this method also has its problems. It depends crucially on the properties of BECs, which are difficult to create in the laboratory.

Furthermore, the properties of BECs are quite peculiar, because they behave as if they were one gigantic atom. In principle, bizarre quantum effects that we see only at the atomic level can be seen with the naked eye with a BEC. This was once thought to be impossible.

Chapter 9

Conclusion

In conclusion, Quantum Teleportation is possible at least at quantum level.

Entanglement is the main key of this kind of teleportation. We can say that quantum teleportation is some sort of telepathy in quantum particles held together by quantum entanglement and completely different from teleportation. Teleportation and quantum teleportation has different meaning since the former refers to matter transfer and the latter refers to information exchange and matter and information are two independent entities. Though we have seen that quantum teleportation is also possible without entanglement. But for that, Bose Einstein Condensate(BEC) is essential. Also it is very hard to create a BEC state and also very difficult to convert all the information of a large object body into light and then teleport them to a certain distance without any loss of information(maintaining the coherent state).

More than two decades have passed since the concept of quantum teleportation was proposed and verified. Now branches of it like Quantum Gate Teleportation, Quantum Computing, Port-based Teleportation, Photonic Qubits, Optical modes, Nuclear Magnetic Resonance(NMR), Atomic Ensembles, and Trapped Atomic Qubits are achievable both theoretically and experimentally. The technologies mentioned above performed well in some aspects and failed in others, therefore certain technologies only correspond to a certain

kind of practical situation. Those technologies are imperfect more or less in some aspects, which raise lots of engineering questions as most of them are expected to be solved when more experiments are devised.

Quantum Teleportation will make transportation much more easier and time effective. Also this can solve many real life complex matters like Logistics Problems, Optimisation problems, Quantum Complexity, Cryptography and will be helpful to unite quantum computers to form networks, encrypts data more securely which leads to quantum internet and many more.

Last but not the least, Future aspects of quantum teleportation might focus on long-range quantum teleportation between light and macroscopic matter or even quantum energy teleportation proposed in recent years.

Quantum teleportation may lead one day to the understanding of quantum entanglement. This will make the picture of the quantum world much vivid and give birth enigmatic technologies.

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