Quantum Computing Essentials: Qubits, States, Circuits, and Measurements Explained

#Quantum30 Challenge Cohort 2 Day 3

In the quest to unlock the mysteries of quantum computing, we must first delve into the fundamental building blocks of this revolutionary field. From bits to qubits, Dirac notation to Bloch spheres, and measurements to quantum circuits, we embark on a journey to unravel the enigmatic world of quantum information processing. In this article, we will explore the key concepts that form the foundation of quantum computing, shedding light on the transition from classical bits to the intriguing realm of qubits.

From Bits to Qubits:

At the heart of quantum computing lies a profound shift from classical bits to qubits. While classical bits can exist in one of two states, 0 or 1, qubits introduce a new dimension. Quantum bits can exist in a superposition of both 0 and 1 simultaneously. This inherent duality is expressed using Dirac notation, where a qubit's state is represented as $|\psi\rangle$, with complex coefficients α and β:

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

The Basis of Quantum States:

To understand quantum states, we must grasp the concept of basis states. In quantum mechanics, the computational basis consists of $|0\rangle$ and $|1\rangle$. These states are orthogonal, forming the foundation upon which all quantum information is built. Any quantum state can be expressed as a linear combination of these basis states. While the computational basis states $|0\rangle$ and $|1\rangle$ are the most commonly used basis states, there are other basis states that can be employed in specific quantum computing contexts. These alternative basis states are particularly useful for certain quantum algorithms and operations. Here are some examples of other basis states:

1. **Hadamard Basis:**
— $|+\rangle$ (Hadamard $|0\rangle$): This state is created by applying a Hadamard gate to the $|0\rangle$ state, resulting in a superposition of $|0\rangle$ and $|1\rangle$ states with equal probability amplitudes.
— $|-\rangle$ (Hadamard $|1\rangle$): Similarly, this state is obtained by applying a Hadamard gate to

the |1⟩ state, resulting in a superposition of |-⟩ and |1⟩ states with equal probability amplitudes.

2. **Bell Basis:**
— $|\Phi^+\rangle$: One of the four Bell states, $|\Phi^+\rangle$ represents a maximally entangled state between two qubits.
— $|\Phi^-\rangle$, $|\Psi^+\rangle$, and $|\Psi^-\rangle$: These are the other three Bell states, each with its own unique entanglement properties.

3. **Fourier Basis:**
— These basis states are used in quantum Fourier transforms and quantum phase estimation algorithms.
— They are often represented as |k⟩, where k is an integer, and the states are defined based on the discrete Fourier transform.

4. **Energy Eigenstate Basis:**
— In certain quantum systems, the basis states correspond to the energy eigenstates of the system. These states are important in quantum chemistry and quantum physics simulations.

5. **Amplitude Basis:**
— Some quantum algorithms, like amplitude amplification, utilize basis states that maximize or minimize the amplitudes of specific states within a superposition.

The choice of basis states depends on the specific problem being addressed and the quantum algorithm being employed. Different basis states may offer advantages in terms of simplifying calculations, enhancing entanglement, or enabling specific quantum transformations.

Measurements and the Born Rule:

Measurements and the Born Rule are fundamental concepts in quantum mechanics that govern how quantum systems behave when observed. They are crucial for understanding how quantum states transition to classical information upon measurement. Let's explore these concepts in more detail:

1. **Quantum Measurements:**
— In quantum mechanics, measurements are the means by which we extract information from a quantum system. When we measure a quantum system, it collapses from a superposition of possible states into a definite state.

— Quantum measurements are represented by a set of projection operators. These operators correspond to the observable properties we want to measure. For example, if we want to measure whether a qubit is in the $|0\rangle$ state, we use a projection operator associated with $|0\rangle$, typically denoted as $|0\rangle\langle 0|$.
— The outcome of a quantum measurement is probabilistic. When a quantum state $|\psi\rangle$ is measured, it has a certain probability of collapsing into each of its possible basis states. The probabilities are determined by the amplitudes of the basis states.

2. **The Born Rule:**
— The Born Rule, named after physicist Max Born, provides the mathematical formula to calculate the probabilities of different measurement outcomes in quantum systems.
— For a quantum state $|\psi\rangle$ and a measurement operator M associated with a particular outcome, the probability P(M) of obtaining that outcome is given by:

$$P(M) = |\langle M|\psi\rangle|^2$$

- Here, $\langle M|$ is the complex conjugate transpose (Hermitian adjoint) of the measurement operator M. The notation $|\langle M|\psi\rangle|^2$ represents the absolute square of the inner product between the measurement operator and the quantum state.

Interpretation:
— The Born Rule implies that the probability of measuring a particular outcome is determined by the overlap or inner product between the quantum state $|\psi\rangle$ and the basis state associated with the measurement.
— The square of the amplitude of the basis state in the superposition represents the probability of finding the system in that state upon measurement.

3. **Normalization:**
— It's important to note that the probabilities obtained from the Born Rule must sum to 1. This reflects the conservation of probability in quantum mechanics. In other words, upon measurement, the quantum system must collapse to one of its possible states with certainty.

Bloch Sphere Representation:

The Bloch sphere is a geometric representation used in quantum mechanics to visualize the state of a single qubit, which is the basic unit of quantum information. It provides an intuitive way to understand and describe the quantum state of a qubit, including superposition and phase information. The Bloch sphere is a unit sphere, typically represented as a sphere with a radius of 1. The surface of the sphere represents all possible quantum states of a qubit. Let's delve into the Bloch sphere representation:

Basis States:

- The Bloch sphere has two special points on its surface: the north pole and the south pole.
- The north pole represents the state $|0\rangle$ (zero state), which corresponds to the qubit being in the computational basis state $|0\rangle$.
- The south pole represents the state $|1\rangle$ (one state), corresponding to the qubit being in the computational basis state $|1\rangle$.

Superposition:

- States that are in a superposition, such as $\alpha|0\rangle + \beta|1\rangle$, are represented as points on the surface of the Bloch sphere. The angle θ (polar angle) determines how far the point is from the north pole, and the angle ϕ (azimuthal angle) determines the point's position around the sphere.
- The amplitudes α and β of the superposition state determine the coordinates of the point on the sphere.

Bloch Vector:

- The position of a qubit's state on the Bloch sphere is described by a vector known as the Bloch vector. The Bloch vector's direction points to the corresponding point on the sphere's surface, and its length represents the probability amplitude of measuring the qubit in the $|0\rangle$ state.
- The Bloch vector can also be used to calculate the probabilities of measuring the qubit in either the $|0\rangle$ or $|1\rangle$ state.

Phases:

- The Bloch sphere allows us to visualize the phase information of a qubit's state. The angle ϕ represents the phase difference between the $|0\rangle$ and $|1\rangle$ components of the state.

- The phase can affect the interference patterns and outcomes of quantum operations.

Measurement:

- When a qubit is measured along a specific axis, the measurement outcome corresponds to the projection of the Bloch vector onto that axis. The probability of obtaining a particular measurement outcome is related to the angle between the Bloch vector and the measurement axis.

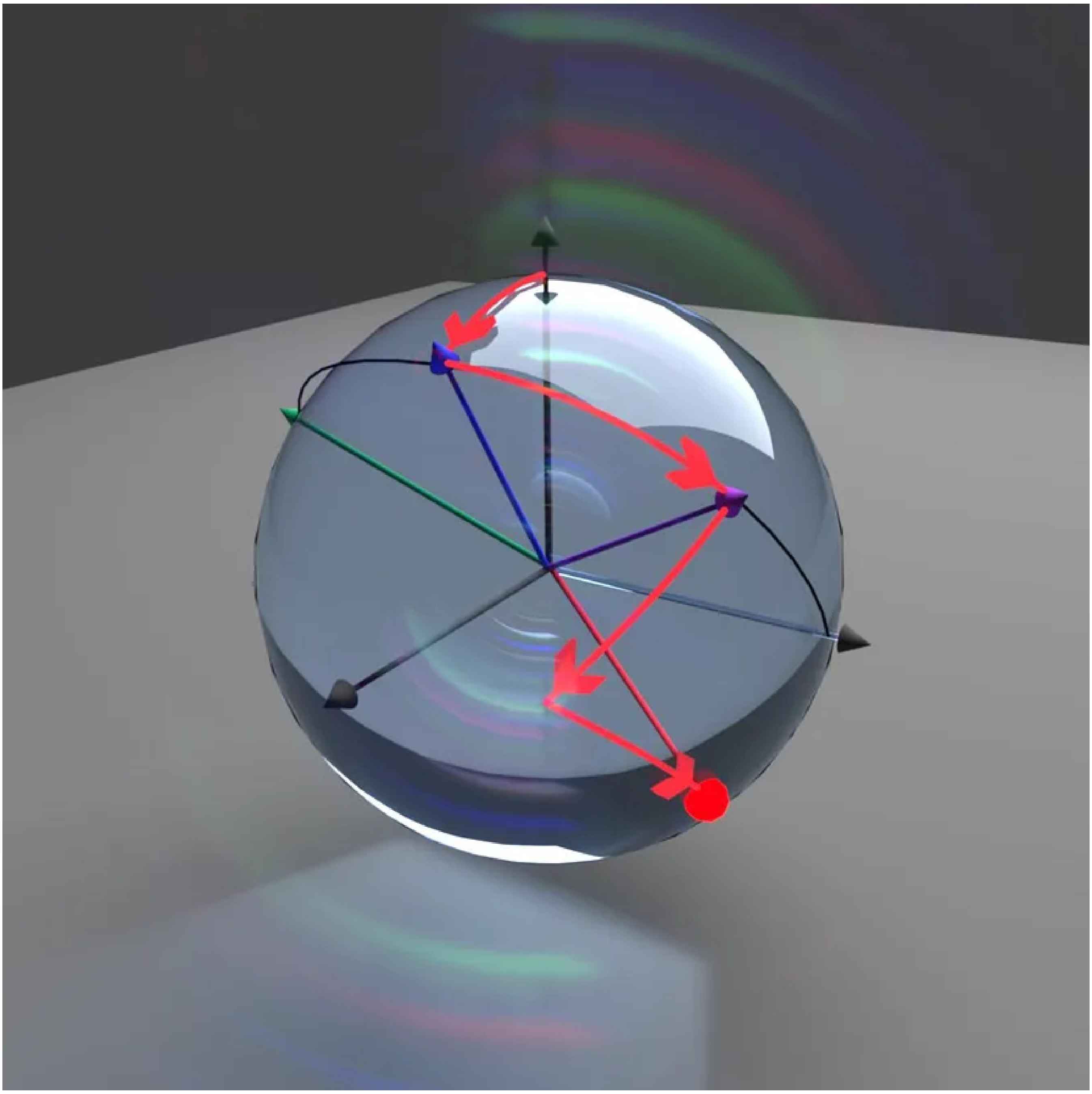

Image Credit: **Bloch Sphere**

Local Phase vs. Global Phase:

Qubits can have both local and global phases. The local phase is a phase factor that can be applied to either $|0\rangle$ or $|1\rangle$ basis states, while the global phase applies to the entire qubit state. Local phases do not affect measurements, as they cancel out when squared. However, global phases can impact interference effects in quantum circuits.

Quantum Circuits:

Quantum circuits are the blueprints for quantum computations. They consist of quantum gates that manipulate qubits to perform specific operations. Quantum gates are analogous to classical logic gates but operate on quantum states. Some common quantum gates include the Hadamard gate, Pauli-X, Pauli-Y, and Pauli-Z gates, which can create superposition, flip qubit states, and introduce phase changes.

Conclusion:

In the fascinating world of quantum computing, qubits, quantum states, quantum circuits, and measurements are the cornerstones of information processing. The transition from classical bits to qubits introduces new dimensions of computation, with Dirac notation, basis states, the Born rule, Bloch spheres, and local and global phases shaping the quantum landscape. As we continue to explore the potential of quantum computing, these foundational concepts will remain at the forefront, paving the way for groundbreaking advancements in science, technology, and computation.

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The reference for this article is given below —

1. Qubits and Quantum States, Quantum Circuits, Measurements - Part 1

Video Credit: Qiskit Youtube Channel

The video is provided by QuantumComputingIndia, as a part of the #Quantum30 learning challenge cohort 2.

I want to take a moment to express my gratitude to **Dr. Manjula Gandhi** for this initiative and encouragement and sincere thanks to **Moses Sam Paul Johnraj** for providing the 30-day schedule.

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Qubit | Quantum Computing | Bloch Sphere | Born Rule | Dirac Notation

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