Chapter 1

Part 3

Fundamental and Elements of Logic

Why Are We Studying Logic?

Some of the reasons:

- Logic is the foundation for computer operation
- Logical conditions are common in programs

  Example:

  Selection: if (score <= max) { ... }

  Iteration: while (i<limit && list[i]!=sentinel) ...
- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).

  Example: Trees, Graphs, Recursive Algorithms, . . .
- Programs can be proven correct.
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

PROPOSITION

A **statement** or a **proposition**, is a declarative sentence that is **either TRUE or FALSE, but not both.**

Example:

- 4 is less than 3.
- 7 is an even integer.
- Washington, DC, is the capital of United State.

Example

i) Why do we study mathematics?
ii) Study logic.
iii) What is your name?
iv) Quiet, please.

The above sentences are not propositions. Why ?

(i) & (iii) : is question, not a statement.
(ii) & (iv) : is a command.

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Example

i) The temperature on the surface of the planet Venus is 800 F.

ii) The sun will come out tomorrow.

Propositions? Why?

- Is a statement since it is either true or false, but not both.
- However, we do not know at this time to determine whether it is true or false.

CONJUNCTIONS

- Compound propositions formed in English with the word "**and**"
- Formed in logic with the caret symbol **(" ∧ ")**
- **True** only when **both participating propositions are true**.

TRUTH TABLE: This tables aid in the evaluation of **compound propositions**.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

True (T), False (F)

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Example

p : 2 is an even integer
q : 3 is an odd number

propositions

p ∧ q

symbols

: 2 is an even integer and 3 is an odd number

statements

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p : today is Monday
q : it is hot

p ∧ q: today is Monday and it is hot

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Example

Proposition

p : 2 divides 4

q : 2 divides 6

Symbol & Statement

$p \wedge q$: 2 divides 4 and 2 divides 6.

or,

$p \wedge q$: 2 divides both 4 and 6.

Example

Proposition

p : 5 is an integer
q : 5 is not an odd integer

boleh guna
and / but

Symbol & Statement

$p \wedge q$: 5 is an integer and 5 is not an odd integer.

or,

$p \wedge q$: 5 is an integer but 5 is not an odd integer.

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DISJUNCTION

- Compound propositions formed in English with the word "**or**",
- Formed in logic with the caret symbol ("$\vee$")
- **True** when **one or both** participating propositions are true.

The truth table for $p \vee q$

p	q	p Vq
T	T	T
T	F	T
F	T	T
F	F	F

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Example

p: 2 is an integer
q: 3 is greater than 5

p ∨ ***q*** : 2 is an integer or 3 is greater than 5

p : 1+1=3
q *:* A decade is 10 years

p ∨ ***q*** **:** 1+1=3 or a decade is 10 years

Example

p : 3 is an even integer
q : 3 is an odd integer

p $\vee$ **q**
3 is an even integer or 3 is an odd integer
or
3 is an even integer or an odd integer

NEGATION

Negating a proposition simply flips its value. Symbols representing negation include:

$$\neg x \,,\ \overline{x}\,,\ {\sim}x,\ x' \quad \text{(NOT)}$$

Let *p* be a proposition.
The negation of *p, written* ¬ *p* is the statement obtained by negating statement *p.*

The truth table of ¬ *p*

p	¬*p*
T	F
F	T

Example

p : 2 is positive

¬ *p* : 2 is not positive.

p : 4 is less than 3

¬ *p* : 4 is not less than 3.

CONDITIONAL PROPOSITIONS

Let ***p*** *and* ***q*** be propositions.

"if ***p***, *then* ***q***"

is a statement called a **conditional proposition**, written as

$$p \to q$$

CONDITIONAL PROPOSITIONS

The truth table of $p \rightarrow q$

=> Cause and effect relationship

FALSE if p = True and q =false

p	q	p→ q
T	T	T
T	F	F
F	T	T
F	F	T

TRUE if both true or p=false for any value of q

Example

p : today is Sunday

q : I will go for a walk

p → q : If today is Sunday, then I will go for a walk.

p : I get a bonus

q : I will buy a new car

p → ***q***: If I get a bonus, then I will buy a new car

Example

p : x/2 is an integer.

q : x is an even integer.

p → q : if x/2 is an integer, then x is an even integer.

BICONDITIONAL

Let ***p*** *and* ***q*** *be propositions.*

*"**p** if and only if **q**"*

is a statement called a **biconditional proposition**, written as

$$p \leftrightarrow q$$

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BICONDITIONAL

true kalau
sama TT / FF

The truth table of $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example

p *:* my program will compile
q *:* it has no syntax error.

p $\leftrightarrow$ ***q*** : My program will compile if and only if it has no syntax error.

p : x is divisible by 3
q : x is divisible by 9

p $\leftrightarrow$ ***q*** : x is divisible by 3 if and only if x is divisible by 9.

LOGICAL EQUIVALENCE

- The compound propositions ***Q*** and ***R*** are made up of the propositions $p_1, \ldots, p_n$.

- ***Q*** and ***R*** are logically equivalent and write,

  $Q \equiv R$

  provided that given any truth values of $p_1, \ldots, p_n$, either ***Q*** and ***R*** are **both true** or ***Q*** and ***R*** are **both false**.

Example

$\boldsymbol{Q} = p \to q$　　$\boldsymbol{R} = \neg q \to \neg p$

Show that, $\boldsymbol{Q} \equiv \boldsymbol{R}$

The truth table shows that, $Q \equiv R$

p	q	$p \to q$	$\neg q \to \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

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Example

Show that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

The truth table shows that, $\neg (p \rightarrow q) \equiv p \wedge \neg q$

p	q	$\neg(p \rightarrow q)$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

PRECEDENCE OF LOGICAL CONNECTIVES

Precedence of logical connectives is as follows:

not	$\neg$	↑	Highest
and	$\wedge$		
or	$\vee$		
If…then	$\rightarrow$		
If and only if	$\leftrightarrow$		Lowest

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Example

Construct the truth table for, **A** = $\neg(p \vee q) \rightarrow (q \wedge p)$

Solution

p	q	$(p \vee q)$	$\neg(p \vee q)$	$(q \wedge p)$	A
T	T	T	F	T	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

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LOGIC & SET THEORY

Logic and set theory go very well togather. The previous definitions can be made very succinct:

$$x \notin A \text{ if and only if } \neg(x \in A)$$

$$A \subseteq B \text{ if and only if } (x \in A \rightarrow x \in B) \text{ is True}$$

$$x \in (A \cap B) \text{ if and only if } (x \in A \wedge x \in B)$$

$$x \in (A \cup B) \text{ if and only if } (x \in A \vee x \in B)$$

$$x \in A - B \text{ if and only if } (x \in A \wedge x \notin B)$$

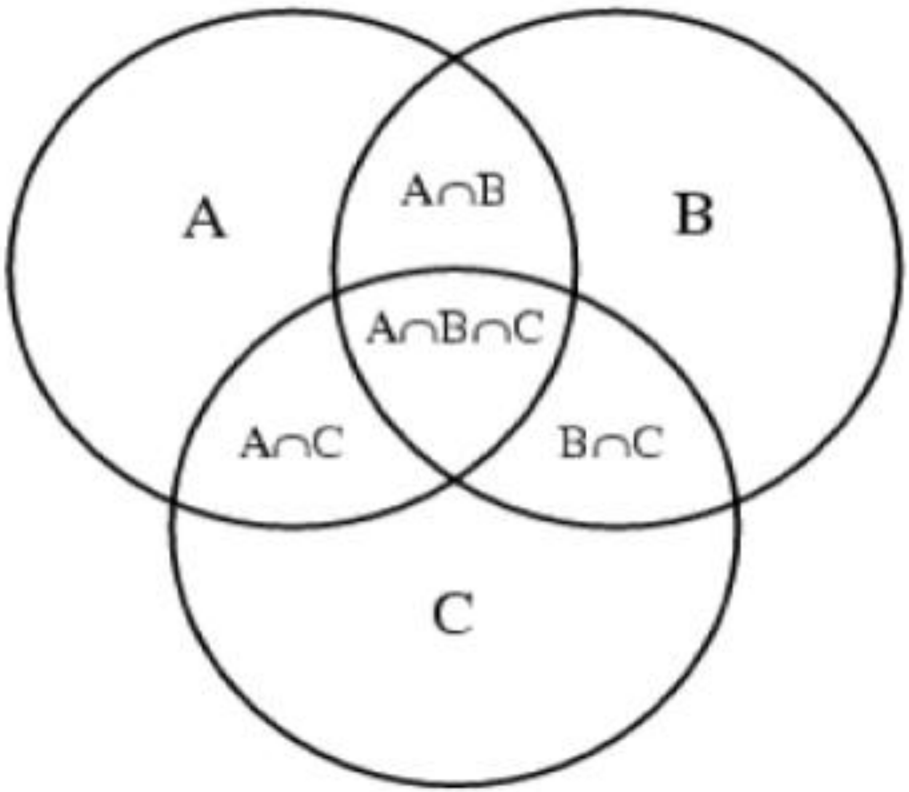
$$x \in A \,\Delta\, B \text{ if and only if } (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)$$

$$x \in A' \text{ if and only if } \neg(x \in A)$$

$$X \in P(A) \text{ if and only if } X \subseteq A$$

Venn Diagrams

Venn Diagrams are used to depict the various unions, subsets, complements, intersections etc. of sets.



Logic and Sets

are closely related

Tautology	Set Operation Identity
$p \vee q \leftrightarrow q \vee p$	$A \cup B = B \cup A$
$p \wedge q \leftrightarrow q \wedge p$	$A \cap B = B \cap A$
$p \vee (q \vee r) \leftrightarrow (p \vee q) \vee r$	$A \cup (B \cup C) = (A \cup B) \cup C$
$p \wedge (q \wedge r) \leftrightarrow (p \wedge q) \wedge r$	$A \cap (B \cap C) = (A \cap B) \cap C$
$p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$p \wedge \neg q \leftrightarrow p \wedge \neg (p \wedge q)$	$A - B = A - (A \cap B)$
$p \wedge \neg (q \vee r) \leftrightarrow (p \wedge \neg q) \wedge (p \wedge \neg r)$	$A - (B \cap C) = (A - B) \cup (A - C)$
$p \wedge \neg (q \wedge r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$	$A - (B \cup C) = (A - B) \cap (A - C)$
$p \wedge (q \wedge \neg r) \leftrightarrow (p \wedge q) \wedge \neg (p \wedge \neg r)$	$A \cap (B - C) = (A \cap B) - (A \cap C)$
$p \vee (q \wedge \neg r) \leftrightarrow (p \vee q) \wedge \neg (r \wedge \neg p)$	$A \cup (B - C) = (A \cup B) - (C - A)$
$p \wedge \neg \vee (q \wedge \neg r) \leftrightarrow (p \wedge \neg q) \vee (p \wedge r)$	$A - (B - C) = (A - B) \cup (A \cap C)$

The above identities serve as the basis for an "algebra of sets".

Logic and Sets

are closely related

Tautology

$$p \wedge p \leftrightarrow p$$

$$p \vee p \leftrightarrow p$$

$$p \wedge \neg(q \wedge \neg q) \leftrightarrow p$$

$$p \vee \neg(q \wedge \neg q) \leftrightarrow p$$

Set Operation Identity

$$A \cap A = A$$

$$A \cup A = A$$

$$A - \varnothing = A$$

$$A \cup \varnothing = A$$

Contradiction

$$p \wedge \neg p$$

$$p \wedge (q \wedge \neg q)$$

$$p \wedge \neg p$$

Set Operation Identity

$$A - A = \varnothing$$

$$A \cap \varnothing = \varnothing$$

$$A - A = \varnothing$$

The above identities serve as the basis for an "algebra of sets".

Theorem for Logic

Let ***p*, *q*** and ***r*** be propositions.

Idempotent laws:

$p \wedge p \equiv p$

$p \vee p \equiv p$

Truth table

p	$p \wedge p$	$p \vee p$
T	T	T
F	F	F

Theorem for Logic

Double negation law:

$$\neg \neg p \equiv p$$

Commutative laws:

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Associative laws:

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$
$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

Theorem for Logic

Distributive laws:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

PROVE

Theorem for Logic

Absorption laws:

$$p \land (p \lor q) \equiv p$$
$$p \lor (p \land q) \equiv p$$

PROVE

p	q	$p \land (p \lor q)$	$p \lor (p \land q)$
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	F

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Theorem for Logic

De Morgan's laws:

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$
$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$

The truth table for $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

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Thank You