Homework 2

Instructions

The assignment contains three exercises, but more may be added later in the week. Solutions should be turned in LaTeX. Collaboration with other people in the class is allowed, as long as it is clearly acknowledged, but the final write-up and understanding **must** be your own. **Due date: Wednesday, September 25**

Exercise 1: Reading

Carefully read sections 1.3 (**Regular Expressions**) and 1.4 (**Nonregular Languages**) of our textbook. Turn in a signed pledge that you have read and studied the aforementioned sections.

Exercise 2: Regular Expressions

Turn in your solutions to exercises 1.19 and 1.20 in our textbook (p. 86). **In addition turn in exercises** 1.29 **part b) and** 1.30 **(p. 88).**

Exercise 3: Proving Lower Bounds

For $n \in \mathbb{N}$, define $K_n = \{s \in \{0,1\}^* \mid \text{the n-th from last symbol in s is } 1\}$. Denote by α a lower bound on the number of states for any DFA recognizing K_n. Prove the best α that you are able to. The argument should be as formal as possible.

Exercise 4: Regular Languages

For an integer $k \geq 2$, let $\Sigma_k \triangleq \{0, \dots, k-1\}$. For $w \in \Sigma_k^* \setminus \{\epsilon\}$, let $w_1 \in \Sigma_k$ denote the first symbol in w. For $S \subseteq \mathbb{N}$, define the language

$$Rep_k(S) \triangleq \{w \in \Sigma_k^* \mid w \text{ is the representation in base } k \text{ of a number in } S \text{ s.t. } w_1 \neq 0\}$$

Given any two distinct prime numbers p, p', can you find a set $S \subseteq \mathbb{N}$ such that the language $Rep_p(S)$ is regular, but for the same set S, the language $Rep_{p'}(S)$ is not regular? The argument should be as formal as possible.

Exercise 5: Regular Languages

Let $L \subseteq \Sigma^*$ be a regular, infinite language for some alphabet Σ. Prove that L can be partitioned into two infinite, disjoint, regular languages. The argument should be as formal as possible.

Good luck, and remember that this homework is a pledged activity.