Project Euler #109: Darts



Problem Statement

This problem is a programming version of Problem 109 from projecteuler.net

In the game of darts a player throws three darts at a target board which is split into twenty equal sized sections numbered one to twenty.



The score of a dart is determined by the number of the region that the dart lands in. A dart landing outside the red/green outer ring scores zero. The black and cream regions inside this ring represent single scores. However, the red/green outer ring and middle ring score double and treble scores respectively.

At the centre of the board are two concentric circles called the bull region, or bulls-eye. The outer bull is worth 25 points and the inner bull is a double, worth 50 points.

There are many variations of rules but in the most popular game the players will begin with a score 301 or 501 and the first player to reduce their running total to zero is a winner. However, it is normal to play a "doubles out" system, which means that the player must land a double (including the double bulls-eye at the centre of the board) on their final dart to win; any other dart that would reduce their running total to one or lower means the score for that set of three darts is "bust".

When a player is able to finish on their current score it is called a "checkout" and the highest checkout is $170: T20\ T20\ D25$ (two treble 20s and double bull).

There are exactly 14 distinct ways to checkout on a score of 6:

D3		
D1	D2	
S2	D2	
D2	D1	
S4	D1	
S1	S1	D2
S1	T1	D1
T1	S1	D1
S1	S3	D1
S3	S1	D1
D1	D1	D1
D1	S2	D1
S2	D1	D1
S2	S2	D1

Note that $D1\ D2$ is considered **different** to $D2\ D1$ as they finish on different doubles. Moreover, the combination $S1\ T1\ D1$ is also considered **different** to $T1\ S1\ D1$.

In addition we shall not include misses in considering combinations; for example, D3 is the **same** as $0\ D3$ and $0\ D3$.

Now imagine you have an infinite number of darts. Now you can stop on every double you get. How many ways you have to checkout with score $\leq N$?

Input Format

A single natural number $N \leq 2^{60}$ - maximum score you need to investigate.

Output Format

The only number - the answer to the problem modulo $10^9 + 9$.

Sample Input

Sample Output

6

Explanation

There are six ways:

- 1) D1: score=2
- 2) S1 D1: score=3
- 3) D2: score=4
- 4) D1 D1: score=4
- 5) S2 D1: score=4
- 6) S1 S1 D1: score=4