Nestings and Intersections between Partitional Complexes

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Abstract: *The formalization of musical texture is the main objective of* Partitional Analysis*. Each integer partition corresponds to a specific textural configuration and is used as a tool to organize and systematize the work with textures through the compositional process.* Partitional complexes*, on the other hand, are sets of partitions, observed in the analysis of musical excerpts, that work in tune to create stable temporal domains where a referential partition projects, extends or presents itself as dominant. The number of partitions and complexes for a certain instrumental, vocal or electronic medium is finite and implies nestings and intersections that can provide important information about textural possibilities available to the composer. In the present work, the relationships established between distinct partitional complexes are discussed, as well as the characterization of an hierarchy related to the number of total choices that each complex offers to the composer.*

Keywords: *Partitional Analysis. Partitional Complexes. Musical Texture. Musical Analysis. Theory of Integer Partitions.*

Musical texture is a diffuse concept and is often interpreted in distinct and even contradictory ways in its various uses ([8]). Two major interpretations have overcome since the term was coined in the early twentieth century, and today the term has acquired many meanings, due to an extreme diversification of sonorous and instrumental resources that occurred from the 1950s to the present day, with the popularization of recording techniques and electronic and digital instruments.

The first interpretation (*texture-plot*), inherited from the traditional classification of texture in generalized categories of *monophony*, *homophony*, *polyphony* and *heterophony*, refers to the combination of vocal or instrumental parts, based mainly on compositional production (*poiesis*) and covering music based on notes.

The second one (*texture-sonority*), developed from the instrumentation and orchestration manuals, is mainly concerned with perception (*esthesis*) and the sound quality of the timbral combinations, as well as sonological researches.

In any case, the texture is still a sub-formalized field at the present and needs further investigation and systematization

One of the pioneering and most influential authors in the work of formalizing the texture is Wallace Berry ([3], pp. 184-199). Berry proposed a coding of the textured configurations taking into account two forces, which he calls *quantitative* and *qualitative* curves. The quantitative curve is determined by the number of sounding components at each time point (called by him as *density-number*), while the qualitative curve is the result of the evaluation of dependence and independence relations between vocal or instrumental parts, determined by the congruence or contrast between them.

The representation of the qualitative curves is given by stacked numbers, corresponding to the various lines and blocks involved in a given point of time (Figure 1). It is curious that, as a pioneer, Berry did not concern with the investigation of the set of these configurations *per se*, independent of the analytical applications that are presented in his book. Berry's representations are, in fact, finite, and can be read through the *Theory of Integer Partitions* simply as *partitions*, that is, representations of integers through sums of integers (called, then, *parts*).

Figure 1: *Textural configurations in Milhaud, first sonnet for choir* A Peine si le coeur, images et figures *([3], p. 187).*

Partitional Analysis (*PA* for short), on the other hand, is an original contribution that can be viewed as a radical expansion of Berry's work, which, in addition to offering the exhaustive taxonomy of the field of textural configurations, as well as its topological and metric mapping, presents formal structures that can be applied to various fields of texture, such as melodic texture, orchestration, form, among many others. In this sense, these formal structures gain as much or even greater importance than the original application as creative tools, because they end up describing a deep level of the organization of musical flows and the possibilities of discourses based on simultaneous, even non-musical, temporal transformations. A much broader field to explore.

In the present work, the textural possibilities available to the composer (as he makes his choices about textural configurations) will be approached. This interaction between the composer's choices and the possibilities that open up at each stage of his actions is referred inside PA as a *compositional game*.

It will be shown later that each textural configuration has a specific number of possible realizations, which can significantly impact compositional thinking. In addition, since partitions always work in sets, also specific, the intersections between different *Partitional Complexes* (that is, sets of partitions that work in tune to make up a global partition) also turn important, by defining how successive partitional domains interact.

Partitional Analysis

The main feature of *Partitional Analysis* ([16], [13]) and its main distinction from Berry's *Textural Analysis* lies in the understanding of how partitions are established, that is, through *binary relations* ([13], p. 33-38).

Every composer of concert music has gone through the experience of searching for a musical interval within a harmonic structure, either in choral exercises, or in the process of analysis of the harmonic content of an instrumental chord, for instance. This exhaustive search can eventually be mandatory. For instance, in a choir, comparing the voices, (e. g., SA, ST, SB, AT, AB, TB) to be assured of all intervals. In this case, the number of assessments is provided by the two-by-two combination of the number of voices, a very well known mathematical function.

In the domain of textural configurations, it is not just the intervals that are evaluated, but the quality of *collaboration* or *counterposition*, which will produce, after all, the individualization of textural elements, like lines and blocks, and, at the same time, the vertical differentiation from one to another element. In the present work, we will consider just the rhythmic congruence or counterposition, determined by the combination of point of attack (or onset) and duration - what is called in PA by *Rhythmic Partitioning* ([13], p. 35 *et seqs*, [7]) .

For each textural configuration, there is a pair of indices corresponding to the total number of found *collaborations* and *counterpositions* between the sounding components (in the case of the string quartet, these components could be each musician, playing in ordinary mode, as an example). This counting reflects how much there is homogeneity and agreement between the sound sources, on one hand; on the other, how much there is diversity, disagreement. These two indices are called in PA by *agglomeration* and *dispersion* indices, respectively, or, for short, *(a, d)*.

Once the pair of indices *(a, d)* is established, two graphical tools are elaborated to visualize the textural progressions in a particular piece or musical work: the *indexogram* and *partitiogram*.

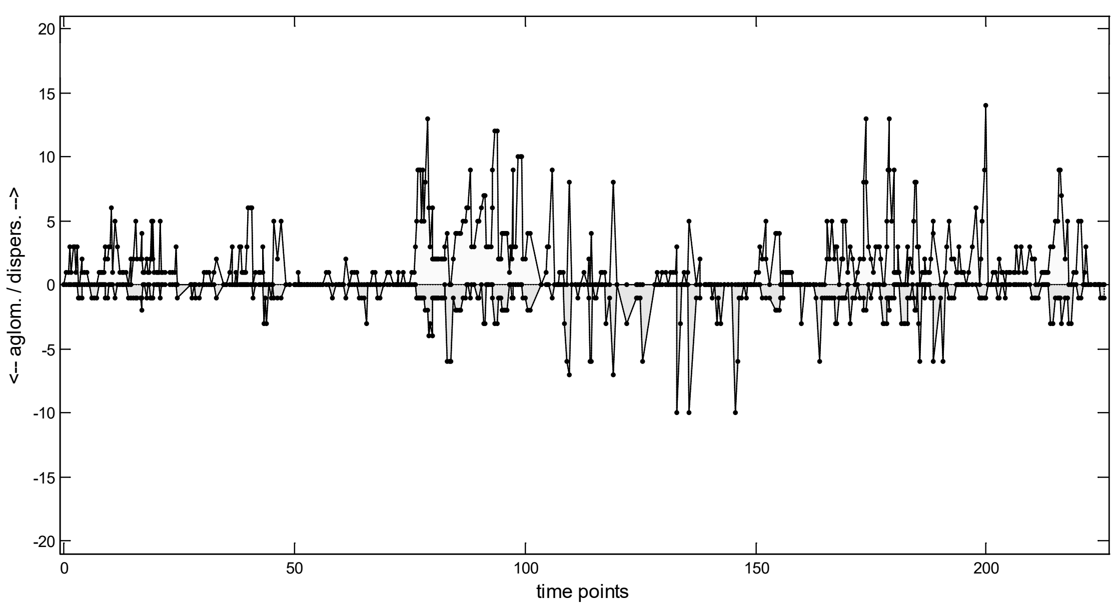

Figure 2: *Indexogram of* Le Marteau sans Maître – I - avant "L'Artisanat furieux" *([4], [12], p. 3). The peaks in dispersion area indicate points of maximum polyphony, while the peaks in agglomeration area mark the more massive blocks. The two indices are partially independent, as variety and mass can assume many combinations. Graph produced by Parsemat® ([15]).*

The Indexogram shows the individual progression of each index over time, updated in each new attack or time point where any event occurs (usually, the onset of a single note or chord). The graph is splitted horizontally in its median portion, so that agglomeration index is presented with negative signal. The intention is to allow the visualization of the interval between indices, in a wave chart-like design.

Partitiogram presents all the partitions referring to a certain density-number, plotted in a plane according to their indices of agglomeration (x axis) and dispersion (y axis). Besides being an exhaustive taxonomy of all available textural configurations (called in PA as *lexset*), it is also a topological representation of its metrics (which reveal adjacencies, proximity, and degrees of similarity or kinship between partitions).

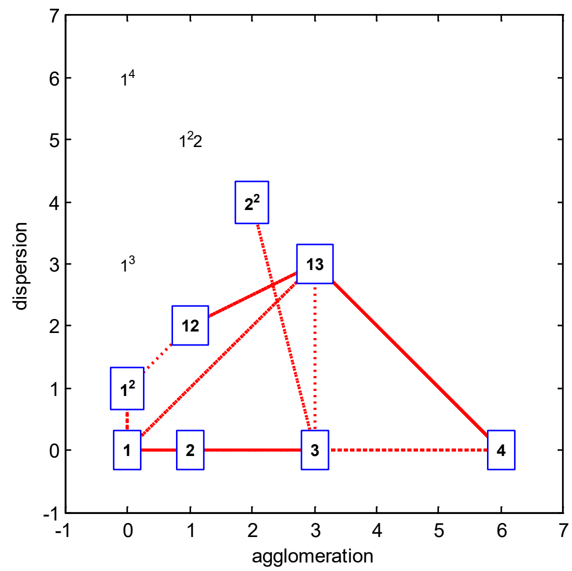

Figure 3: *Partitiogram of Leo Brower's* Estudio 20 *from* Estudios Sencillos *for guitar ([6]). Partitions are mapped by agglomeration and dispersion indices. Used partitions from lexset of integer 4 are marked in blue squares and connections are established between successive configurations presented in the piece. In this case, there are three unused partitions (1^3), ($1^2 2$) and (1^4) – precisely the most polyphonic ones. Graph produced by Parsemat® ([15]).*

Since it represents all the available vocabulary for a particular instrumental, vocal or sound environment, it is possible to read the texture in specific excerpts or works as a sequence of partitions, considering that they are finite and therefore treatable from a compositional point of view.

The adjacency relationships found in the partitiogram can be classified according to their intrinsic qualities, thus forming networks of operators. divided in two general categories: simple and compound.

Simple operators (*resizing* and *revariance*) are the very basis of the construction of texture itself. Compound ones (*transfer* and *concurrence*) are combinations of simple operators and are necessary to explain some adjacencies in PA diagrams. For each one, it is assigned a letter for easier reference.

Resizing (m) a part means to change its thickness. The positive resizing implies the inclusion of more sounding components to a block, making it "fatter"; the negative resizing is, on the contrary, the thickening of a part, subtracting a sounding component from a block.

Revariance (v) is the changing of variety (number of parts) inside a textural configuration.

Positive revariance implies adding an unitary part to the partition and negative revariance means subtracting an unitary part from it.

Transference (t) arises when resizing and revariance are applied together, but with opposite signals (positive resizing with negative revariance, and vice-versa). The consequence is that one sounding component is displaced from a part to another, without affecting the overall density-number. This kind of operation is very common in traditional concert music.

Concurrence (c), on the other side, is the consequence of articulation of resizing and negative revariance, with the same signals (positive resizing and revariance, or the opposite). This causes increment of the distinctions between parts (blocks become more massive and lines are multiplied) and the change in the global density-number. In this way, concurrence is different of the former operators, in the sense that it is not and relation of adjacency. The concept is included here, anyway, because it is relevant to describe some musical situations, where the contrast between successive textural configurations are the rule (for instance, in some *avant-garde* styles).

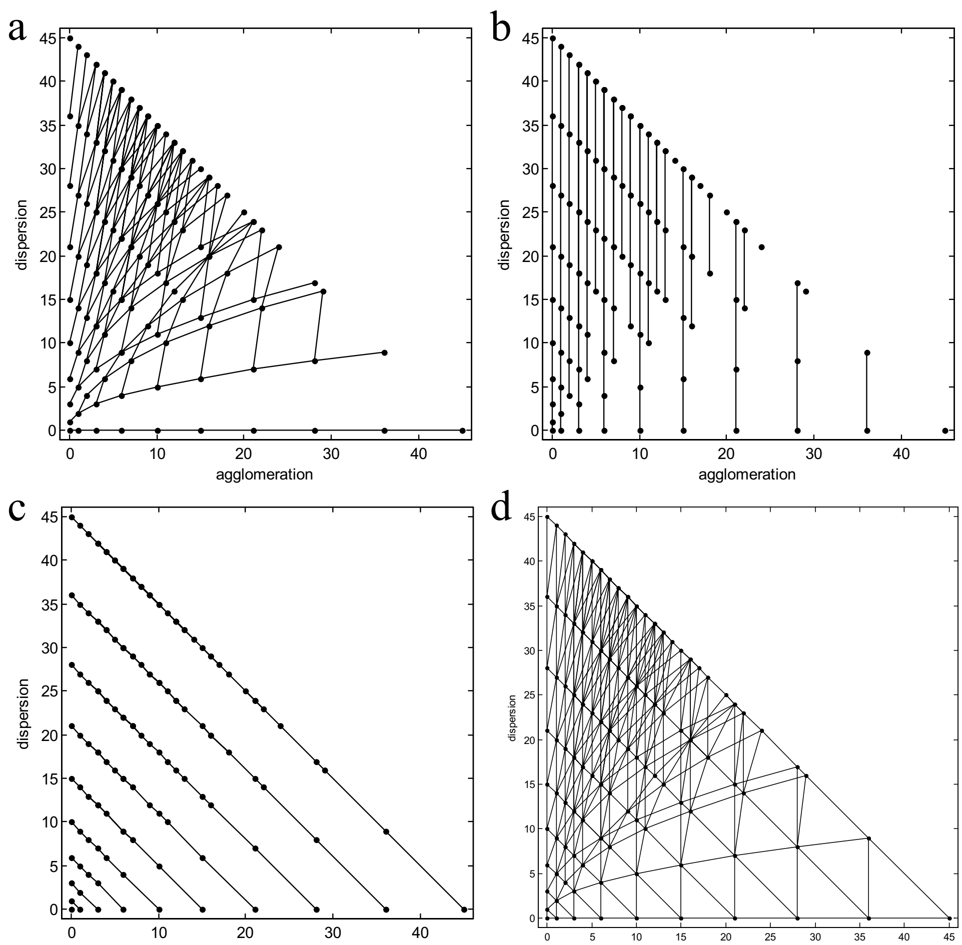

Figure 4: *Networks of operators for* $dn = 10$: mnet *(a),* vnet *(b),* tnet *(c), and the overall combination of the three basic nets (d). Each point corresponds to a partition or group of h-related partitions ([19], p. 70). Graph produced by Parsemat® ([15]).*

According to the used operators, some networks are formed, each with a different type of connection, transformation or syntax ([8]).

The three basic networks, constructed from the resizing *(m)*, revariance *(v)* and transfer *(t)* operators respectively, are presented.

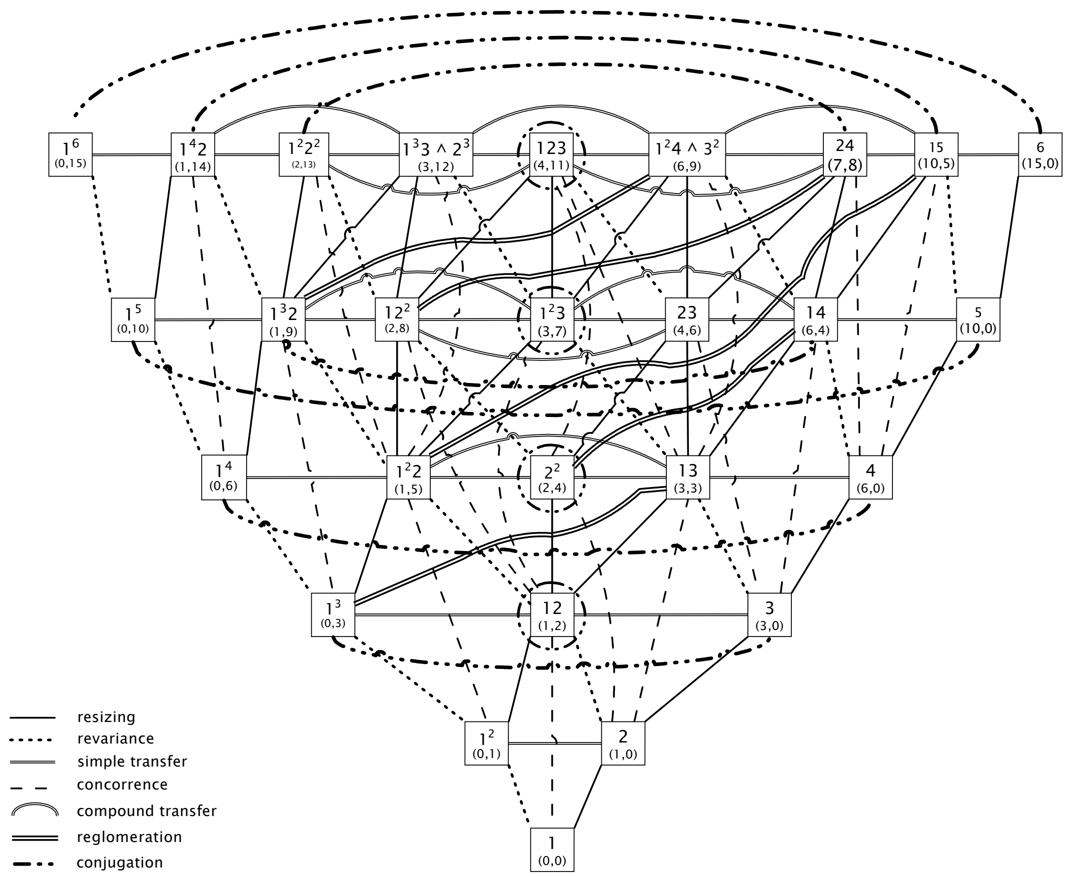

Figure 5: *Partition Young Lattice for* $dn = 6$. *Basic operators (resizing, revariance, [simple] transfer and concurrence)are presented along with compound and auxiliary ones. The overall resultant net can be used by the composer as a board game, where each type of move means a kind of textural transformation ([19], p. 69)*

Mnet is a fractal structure, whose lines always start with polyphonic configurations, which have their parts gradually resized, and which allows the bifurcation in some points, when there is more than one part available for the resizing. This operator is the one that brings for the partitiogram the greatest irregularity and unpredictability, especially in densities greater than 6.

Vnet is a more predictable network, where each row maintains the fixed agglomeration index, only with more parts being added. For this reason, vertical lines are formed, which start whenever there is a new configuration imbricated by the resizing.

Tnet is a network delimited by density-numbers, that is, all elements components of the same line have the same number of sounding components. Up to density-number 6, tnet presents itself in a linear, simple way. From there, bifurcations make the network increasingly complex, as some partitions start to establish multiple transfers. This unfortunately is not visible in the tnet graph, as the lines are superimposed.

Partitiogram can be read also as a *Hasse Diagram*. This graph is constructed with the basic elements of a list or taxonomy, connected by the relation of inclusion, in a bottom-up arrangement.

Each element can be docked in its superior connected neighbour (and in all the connected superior elements) as a subgroup. This graph can show all the relations of a set with maximum economy of information. The representation of a lexset of a number, for example, the integer 4, in a Hasse diagram, shows 11 elements ([2], p. 108) .

Partitional Young Lattice (*PYL*), on the other side, is an adaptated visualization of the original Hasse Diagram for integer partitions. It includes, beyond the partitions themselves, the subscription of agglomeration and dispersion indices and the qualification of the relations of inclusion as operators ([13], [19]).

In PA, concepts are generally observed as tools within compositional games (even though they may be used in many other ways, for example in musical analysis or hermeneutics). In this sense, both the partitiogram and PYL are seen as a phase space or a board game, respectively, where trajectories are traced, as the composer progresses in his creative work.

There are currently some important theoretical expansions and applications of PA in musical analysis and composition. They just fall outside the scope of this paper.

Partitional Complexes

Partitional Complexes can be defined as a bunch of partitions that cooperate to set a referential partition domain.

The main ideia is that the independence between parts transcends the simple contraposition. In a texturally diverse environment, independence is built when there is complete autonomy between parts. This situation presupposes a certain detachment among parts, which can cause eventual congruences, as a result of fortuitous movements that come to occur without detriment of the global textural conception.

For an organic realization of a given textural configuration, it is necessary to take this dynamics into account, organizing the textural thinking into hierarchical layers.

One of the possibilities of organizing textural configurations comes from the consideration of:

- *Subpartitions*: those textures that are revealed in the partial or incomplete presentations of a referential partition;
- *Subsums*: eventual congruences that their parts offer, constituted by all the sums of the parts of the referential partition.
- *Subsums of subpartitions*: when both processes can be applied concurrently.

As an example, the partition (1^3) or (1.1.1) have, as incomplete presentations of its parts, the subpartitions (1) and (1.1). The subsums are (2) and (3), resulting from the sums of (1.1) and (1.1.1), respectively. There are a subsum of subpartitions, (1.2), where two parts are summed and one preserved. Finally, the partitional complex of (1^3) has six elements: $\{(1^3),(1),(1.1),(2),(3),(1.2)\}$. This is equivalent to say that constructing a three voice polyphony implies in the articulation of some or all the partitions of the complex.

We then have a distinct development of each ingredient for each partition, thus defining a differentiated number of choices for the composer. Massive partitions are the most limited, including only themselves in its partitional complex, while polyphonic ones offer the most numerous alternatives, encompassing all the partitions for the correspondent density-number. Among the massive and polyphonic partitions, there are those partitions that mix blocks and lines together (Table 1).

The information presented in Table 1 can be arranged in a Hasse Diagram through the combination of two relationships: "is subpartition" and "is subsum" (Figure 6). Complexes can be constituted by following a top-down direction from the chosen referential partition through all

Table 1: *Partitional complexes for quartets: referential partitions, with their cardinalities, density-number, subpartitions, subsums, subpartitions of subsums, partitional complex, and the cardinality of the partitional complex [20], p. 123, [9], p. 35-36.*

referential partition	**card.**	**DN**	**subpartitions (Sp)**	**subsums (Ss)**	**subpartitions of subsums**	**partitional complex**	**complex card.**
(1)	1	1	-	-	-	(1)	1
(2)	1	2	-	-	-	(2)	1
(1^2)	2	2	(1)	(2)	-	(1^2), (1), (2)	3
(3)	1	3	-	-	-	(3)	1
(1.2)	2	3	(1), (2)	(3)	-	(1.2), (1), (2), (3)	4
(1^3)	3	3	(1), (1^2)	(2), (3)	(1.2)	(1^3), (1), (1^2), (2), (3), (4), (1.2)	6
(4)	1	4	-	-	-	(4)	1
(1.3)	2	4	(1), (3)	(4)	-	(1.3), (1), (3), (4)	4
(2^2)	2	4	(2)	(4)	-	(2^2), (2), (4)	3
$(1^2 2)$	3	4	(1), (1^2), (1.2), (2)	(3), (4)	(1.3), (2^2)	$(1^2 2)$, (1), (1^2), (1.2) (2), (3), (4), (2^2), (1.3)	9
(1^4)	4	4	(1), (1^2), (1^3)	(2), (3), (4)	(1.2), (1.3), (2^2), $(1^2 2)$	(the whole lexset)	11

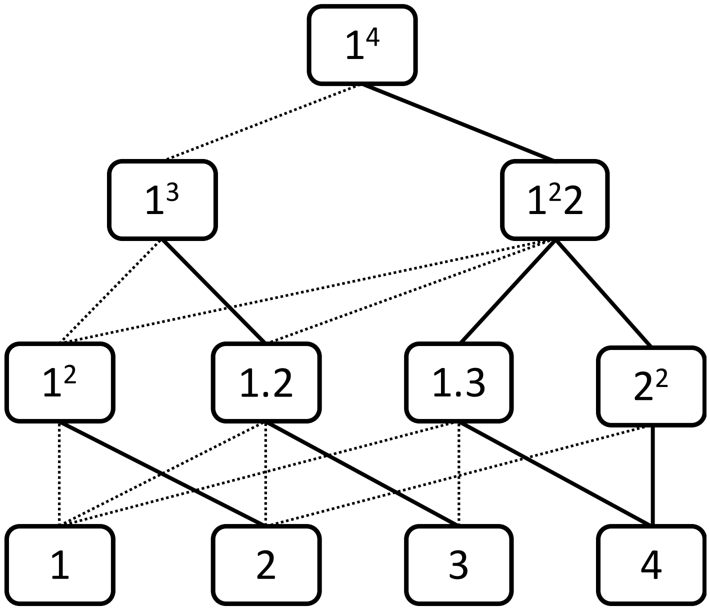

Figure 6: *Partitions for quartet, arranged in partitional complexes. Each partition constitutes its own complex by gathering all top-down connections departing from it. Lines are read bottom-up: dotted lines indicate* subpartitions *and full lines indicate* subsums*. Four levels of crescent textural complexity are constituted.*

connections below. From this point of view, it turns clear that for a quartet (density-number 4) there are four levels of gradual complexity, read bottom-top. Once more, massive partitions – (1), (2), (3) and (4) – stay alone inside its own complexes, as stated before. Complexes of polyphonic partitions – (1^2), (1^3) and (1^4), on the other hand, embrace all partitions from integer 1 to its own density number. The arrangement of Figure 6 shows also some imbalanced distribution, as the partition (1^22) have four immediate bottom connections (and 13 in total), while (1^3) has only two (7 in total). Each connection corresponds to an available path for more or less (depending on the number of moves in the graph) parsimonious transformation between textural configurations. In this specific case (partitional complexes), the transformations occurs with the ommision or addition of a part (line or block), or the merging of existent parts without any subtraction.

Intersection, nesting and partitional complexes

The introit of the tenth piece from Schoenberg's *Pierrot Lunaire* (*Raub*) is a remarkable example of an organization of textural language through a hierarchical arrangement. There are two distinct regions, the first (mm. 1-2.3), with more rarefaction and discontinuity, with loose notes in staccato; the second (mm.2.2.2-3), more continuous and repetitive, with coordination between the majority of the attacks of all instruments.

Change from one region to the other occurrs smoothly, as the flute begins the second region while the strings are still finishing the first one, drawing a quite diagonal division line between the two domains.

Inside the atomic level of partitions, there are a sudden change of behaviour as well. The sequence of partitions of the first region orbits around a limited set of configurations, all belonging to the partition complex of (1.1.4), yet the referential partition in fact does not appear at all:

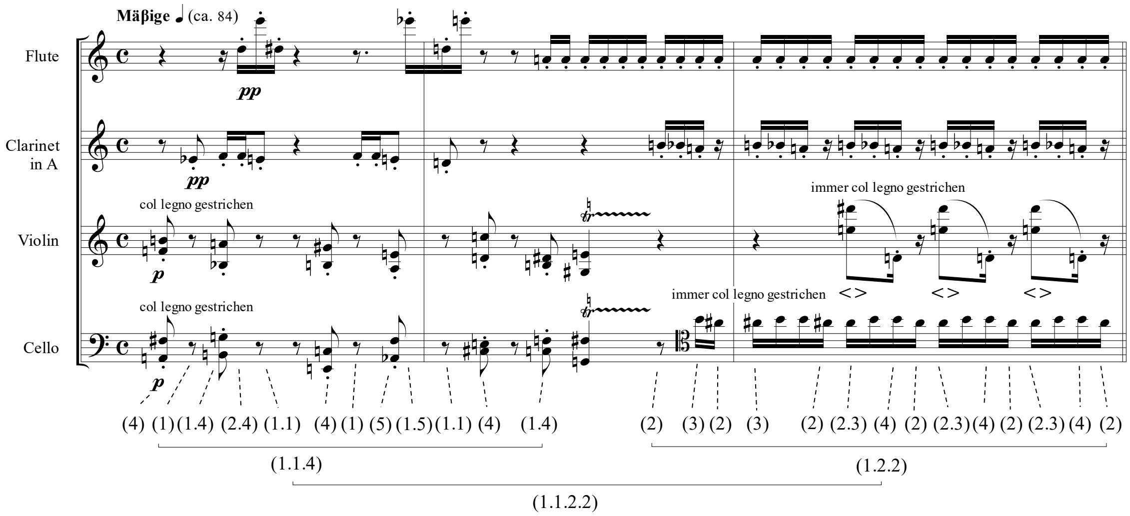

Figure 7: *Partition complexes in Schoenberg's* Raub, *from* Pierrot Lunaire *([21], p. 89, mm. 1-3)*

subpartitions (1), (1.1), (1.4), (4); subsums (1.5), (5) and (2.4).

The emergence of the referential partition (1.1.4) is not surprising at all, considering the motivic role of the wind instruments against the massive blocks of the strings on the initial measures of the piece. What is more striking here is that it was not necessary, at any point of the region, to present this relations literally, through a real partition. That is, an concrete realization of a polyphony with extense lines with attacks and prolongations sustained by blocks configured to working all together. On the contrary, the referential partition is, in this little region, quite virtual.

The second region brings partitions which would not be compatible with the previous complex - for instance, (3) and (2.3). But all the presented partitions can be ascribed to another referential partition - in this case, (1.2.2), with subpartition (2) and subsums (3), (2.3) and (4). The elements that induce this result are identifiable as well - the clarinet, assuming a more independent role, due to a periodic interruption, distinguishing itself from the flute and cello, which are at this point filling in a layer of continuous successive attacks. The violin has a more independent role, with the isolated articulation of two notes in a more distant register.

The constitution of the complex can be observed in more detail by the way the partitions are vertically structured. For instance, the alternation between (2) and (3) at the beginning of second region is due to the gesture of clarinet, sometimes participating in the block, sometimes absent. This indicates a textural structure that tends to (1.2), which would be a subpartition of (1.2.2), but in fact is not explicitly stated as such. Similarly, the partition (2.3) is the result of the interval of the violin (2), opposed to the simultaneity formed by the winds and cello (3), in the moments where the clarinet is mixed in the block.

Looking further, the two complexes – (1.1.4) and (1.2.2) – are, in its turn, a subpartition and a subsum, respectively, of (1.1.2.2). This super-complex is also never stated literally, but it can be seen in the overall instrumental partition, in the first region – flute (1), clarinet (1), violin in double stops (2) and cello in double stops (2). In the second region, we have the clarinet motive (1), the insistent notes of flute and cello (2), the double stop of violin in treble register (2) and, finally, the open string of the violin, that can be thought as an independent layer (1), in a lower register.

All these relations are presented then in a partial hierarchical graph of the complex (1.1.2.2), where the two branches of referential partitions (1.1.4) and (1.2.2) generate the remaining ones

(Figure 8).

The branches are also not equally balanced, with the first region, represented by the branch (1.1.4), more populated than the second one, correspondent to partition (1.2.2). This implies that the involved transformation from first to last region is descendent in terms of information – there are more redundancy and confination inside the second branch.

This inference is also supported by formal features. In the first region, blocks articulated by the strings are superimposed and followed by a brief polyphony played by the winds, and that structure is repeated once with little variation in the temporal interval between its elements, at a time distance of nearly five eighths relative to the first presentation.

The second region, in turn, presents more repetitions of shorter modules. The pattern of winds and cello is constructed gradually, lasting three beats and, after presented in its complete form, is stated four times, three of them with a repeated pattern of the violin. This foreground repetition is reflected too in the sequence of partitions, that exhibits for three times the pattern $< (2.3)(4)(2) >$. In the first region, due to the displacement of the elements in the repetition, there are no recognizable patterns in the surface flux of partitions, which grants a bigger amount of information.

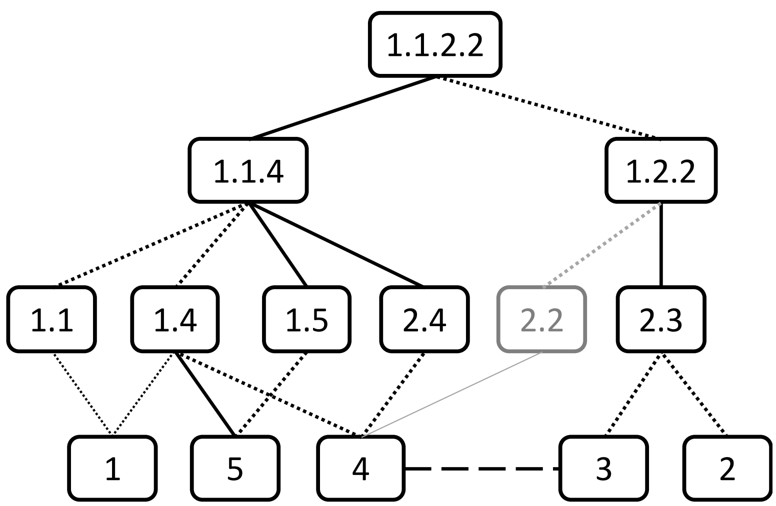

Figure 8: *Partial presentation of partitional complex (1.1.2.2) in Pierrot Lunaire, X - Raub ([21], p. 89).*

The only relation that is not motivated by internal operations of the complex occurs between partitions (3) and (4). Partition (4), specifically, do not fit very well in the frame of second region (1.2.2), because the intermediary partition, (2.2), a subpartition of (1.2.2), is absent, and this gap between (4) and (1.2.2) creates a disconnection between the two configurations. The partition (4) arises when an extraneous pitch is added (just the open string of the violin) to the sounding block (3). This operation is, in fact, not a relation inside a complex, but a real transformation – a simple *resizing* from (3) to (4).

A return to the Berry's example ([3], p. 187) can bring some insights about the deep interaction of partitions and complexes. Berry used an observation window corresponding to the measure, which arises some analytical questions – for example, the binary metric structure of the *fugato*, that leads the author to disregard some combinations that occurs in shorter durations, like, for instance, the convergence between S and A in measure 4, or the beginning of the *tutti* in measure 6, which is registered by Berry only in measure 7 (Figure 1). Some papers was addressed on this subject, trying to explain the cognitive process of defining intuitively what partitions are more important or valid, covering, without expressive results, some hyphotesis like "the most prominent ones" (in terms of peaks of dispersion or agglomeration), or "the more extensive in terms of temporal durations" ([10], [14])

Assessing the configurations with a more refined window (as PA re-evaluate the configuration for each attack, detecting, exhaustively, all used partitions) leads to a more complex and counter-intuitive result. On the other way, it brings some information that is not accessible through an intuitive appreciation.

Applying the concept of complexes can bring some enlightment, even in a very know structure as the *fugato*, where each partial complex gradually blooms from the previous one.

Figure 9: *Partitions and complexes in Milhaud,* A Peine si le coeur, images et figures. *Every complex is encompassed by the coming one, in a nested structure. Referential partition (1^3) is not explicitly articulated.*

The interaction between the complexes lead to a embracing structure (1^4), which is compatible with the polyphonic language proposed by the composer. One of the intriguing questions that arouse from the comparation of Berry's analysis and the PA results was the complete absence of the partition (1^3) in the micro-surface of the texture. Even if we consider the traditional gesture of gradual accrual of voices in *fugato* style, a cognitive basis to sustain this so naturally placed conclusion was considered as necessary. The concept of partitional complex can answer this question with ease.

The main point, in this case, is that the construction of partition (1^3) cannot be confined just to the observation of measure 3, where in fact it is not present. It is just the accumulation of all partitions that were articulated until this point that constitutes the complex. The partitions (3) and (1.2) are not sufficient to characterize the complex (1^3). If the piece were summarized by a discourse based on the third measure, certainly it would not be possible to understand the complex as such, but as a complex (1.2), instead.

After the progression reaches its apex, the gradual agglomeration that follows occurs within the complex (1^4), being just an possibility of the writing for voice quartet or choir. In fact, this simplification, as Berry states, is very common in endings and leads to closure.

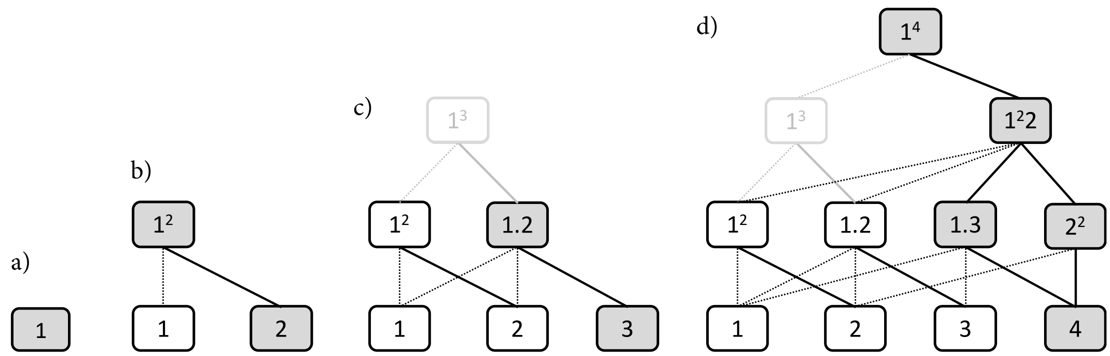

Figure 10: *Sucession of partitional complexes in Milhaud,* A Peine si le coeur, images et figures*: (1), m. 1 (a); (1^2), mm. 1 through 1.3.2 (b); (1^3), mm. 1 through 3.3.1 (c); (1^4), all the excerpt (d). Referential partition (1^3) is never explicitly articulated.*

The middle section of the second movement (mm. 12-28) of Gyorgy Ligeti's *Bagatelles* for woodwind quintet ([17]) is an example of construction of a dramatic curve based mainly in textural development. This time, all regions are very well defined and all referential partitions are stated very clearly (FIgure 11).

There are four domains, referring to partitions (1^2), (1^32), (1^22) and (1^5). They are grouped in two large segments with progressive accrual, each one with two domains.

First segment lasts for 10 measures. It begins with an sparse texture, with only two voices in a calm polyphony, that is suddenly filled with long notes in the fifth to seventh measures, until all the instruments are presented. After that, the initial texture returns, with a profile slightly more prominent.

The second segment lasts for eight measures, where the overall density is noticeably greater than previous one. It begins with a more dramatic tensioning, articulated through the gradual narrowing of the recursive motives, creating superpositions that make the texture also more weighted. The arrival of the massive partition (5) is the apex, from which the following configurations constitute a dissolution.

As the example of Schoenberg, we have also, in the second segment, greater redundancy caused by cyclic return of a partitional segment, in this case the sequence $<(3)(1.2)(1^22)>$. That repetition is clearly used to articulate saturation and to value the arrival of the climax.

Referential partition of first region is (1^2) and, in fact, it stays as the only significant one, as the (1) is only a departure to start the structure. In this sense, the region is very static.

The second region has (1^32), denser partition, found in the local apex. Flute and clarinet are working together, with some sintony with the horn, because of dynamics (whose consideration would cause a trigger of partition (1^23), belonging also to the local complex)

Third region (1^22) seems as a recession compared to the previous complex, but this occurs, as stated before, for balancing further crescendo of the fourth region. A sequence is repeated, just to prepare the arrival of fourth region. In this sense, this complex is a prefix for the last one.

The last region is far more complex, as it articulates the most disperse partition, (1^5), and begins the descent to the more agglomerated one, (5). This contrast is just the main resource to give to this region its dramatic character. The dissolution just uses two partitions kin to (5) in terms of agglomeration. The imbalance of the parts, in this case, have an important role to the rest of the music to come, that will be entirely structured in blocks of (2) and (3), being the (2) constituted by unisons, most of the time.

Figure 11: *Sucession of partitional complexes in Ligeti*, Sechs Bagatellen für Bläserquintett *(II, mm. 12-28)*

Conclusions

Different variety of choices provided by each refential partition can be read as degrees of liberty that a composer have in his creative process. Building a section of a piece with a massive partition – for instance, (4) – implies in a restriction that would be lighter with some other partition, like ($1^2$2). Obviously, the composer always have the freedom to explore other qualities or possibilities he has at hand - harmonies, timbres, rhythms, among many others; but within the specific field of rhythmic partitioning, the distinction is substantial and certainly has a considerable impact on other aspects of compositional work.

In this sense, each partition offers a different potential to develop parsimounious relations inside its complex.

Relations of the various levels of nesting and intersections create an hierarchy comparable to the Schenkerian concepts of *foreground*, *middleground* and *fundamental structure*. Here is a rich field to be explored further by analysts and composers.

There are at the present moment some research being made inside MusMat Research Group concerning this type of analysis, and considering also other kinds of partitional organization, drawn from observation of textural repertoire.

Applications of the partitional complexes to diverse partitionings, like *melodic partitioning*, *event partitioning*, *spectral partitioning* and others ([13]) are in course. Each partitioning has its own idiosyncrasy and has to be evaluated from scratch in this respect, since each handled material has its own nature.

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