# A multi-age structured SIRVSD model

Implementation of a SIRVSD model with different age groups and heterogeneous contacs in Python

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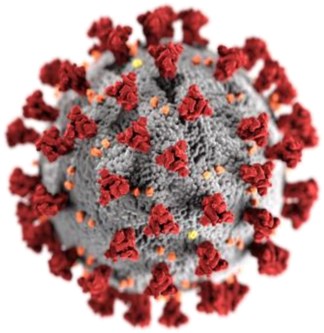
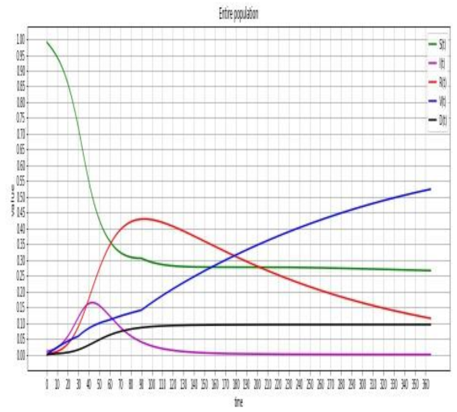
# Introduction to the problem

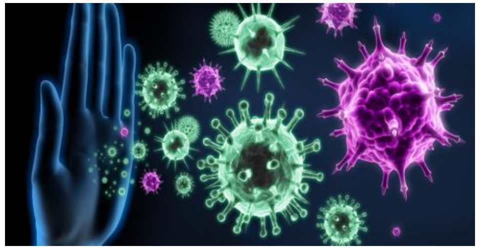


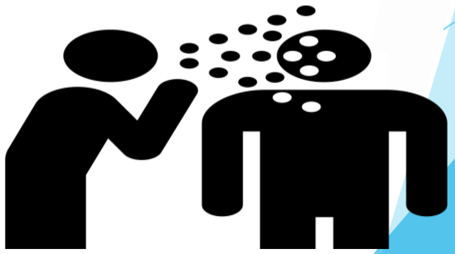
- Covid-19 has focused on forecasting models to be able to monitor and control the number of infections and deaths within a country
- Need to evaluate different vaccination strategies to defeat the infection
- Need to predict the effect of vaccinations to organize a plan to ease restrictions, especially for the country's economy
- Compartmental models are often applied to the mathematical modelling of infectious disease using ODEs
- Population partitioned into different age groups, considering the impact of heterogeneity in susceptibility, mortality and infectivity within the population on the disease trasmission



# Model Assumptions

- S+I+R+V+D = 1
- Some coefficients are the same for all age group (only trasmission, vaccination and mortality coefficients are different)
- We don't consider vital dynamics (reproduction, 'natural' death, migration)
- Heterogeneous contacts and symmetric contact matrix
- Just one injection of the vaccine is considered
- Both recovery and vaccination immunity are not forever, but they ensure 100% protection from infection
- Contacts between individuals are random, the number of infections is proportional to both I and S
- Horizontal transmission





# Model and Parameters

$$\begin{cases} \frac{dS_j}{dt} = \phi R_j - \eta_j(t) S_j + \rho V_j - S_j \sum_{k=1}^{M} \beta_{j,k} I_k \\ \frac{dI_j}{dt} = -\gamma I_j - \mu_j I_j + S_j \sum_{k=1}^{M} \beta_{j,k} I_k \\ \frac{dR_j}{dt} = \gamma I_j - \phi R_j \\ \frac{dV_j}{dt} = \eta_j(t) S_j - \rho V_j \\ \frac{dD_j}{dt} = \mu_j I_j \end{cases} \qquad \forall j \in (0, M]$$

with $M$ = 4 age groups:

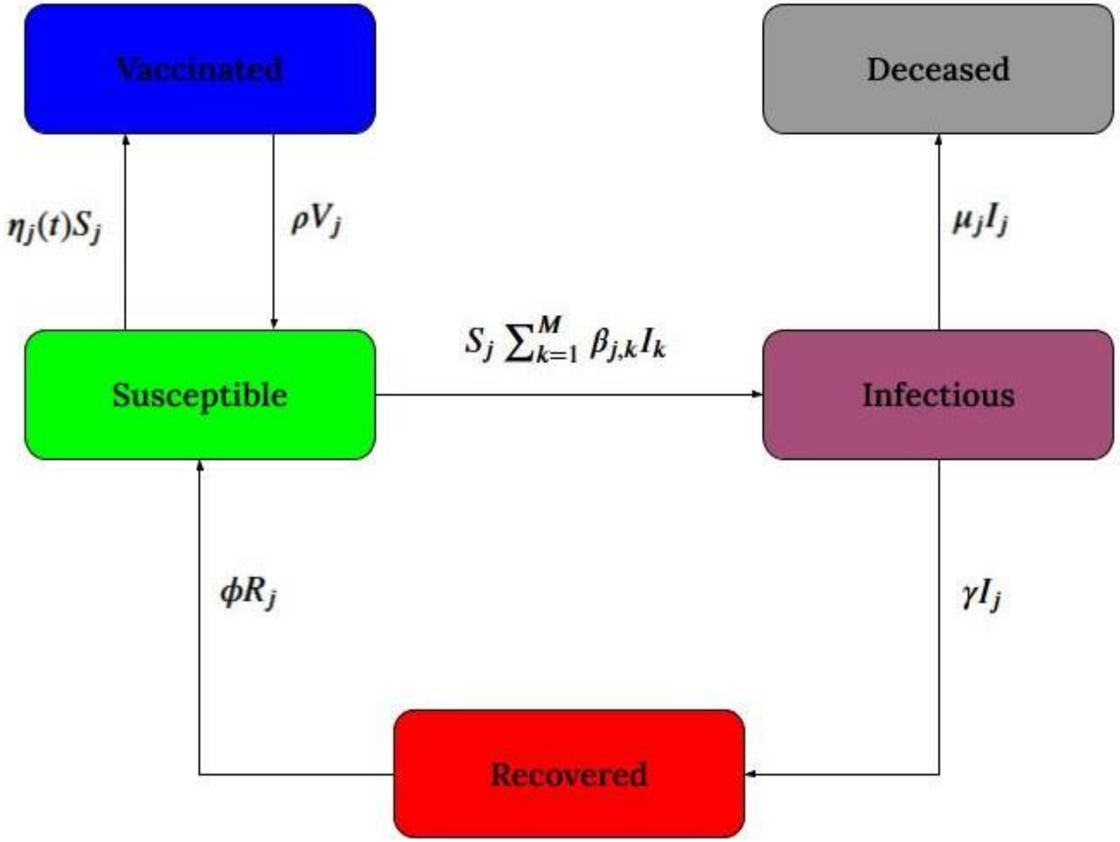
- Children (0-9 years)
- Teenagers (10-19 years)
- Adults (20-69 years)
- Senior Citizens (70+ years)

- $\phi$ is the *transfer coefficient* for loss of immunity from Recovered
- $\rho$ is the *transfer coefficient* for loss of immunity from Vaccinated
- $\beta_{j,k}$ is the *infection coefficient*. We define also the entire *contact matrix:* $\beta = \begin{bmatrix} \beta_{1,1} & \cdots & \beta_{1,M} \\ \vdots & \ddots & \vdots \\ \beta_{M,1} & \cdots & \beta_{M,M} \end{bmatrix}$
- $\gamma$ is the *recovery coefficient* of each infected subject
- $\mu_j$ is the *mortality coefficient* , different for each age group
- $\eta_j(t)$ is a time-dependent *vaccination coefficient*, defined as follows:

$$\eta_j(t) = \begin{cases} 0 & \text{if } t < t_{vacc_j} \\ \eta_j & \text{otherwise} \end{cases}$$

where $t_{vacc_j}$ defines the starting day of the vaccination period

# Transition schema



# Qualitative Analysis

- Case Study: COVID-19 pandemic with different vaccination strategies
  - Initial Configuration

| | Children | Teenagers | Adults | Senior |
|---|---|---|---|---|
| *S(0)* | 99% | 99% | 99% | 99% |
| *I(0)* | 1% | 1% | 1% | 1% |
| *R(0)* | 0% | 0% | 0% | 0% |
| *V(0)* | 0% | 0% | 0% | 0% |
| *D(0)* | 0% | 0% | 0% | 0% |
| $\gamma$ | 1/15 | 1/15 | 1/15 | 1/15 |
| $\mu$ | 0.00009 | 0.00005 | 0.00688 | 0.15987 |
| $\phi$ | 1/180 | 1/180 | 1/180 | 1/180 |
| $\rho$ | 1/270 | 1/270 | 1/270 | 1/270 |

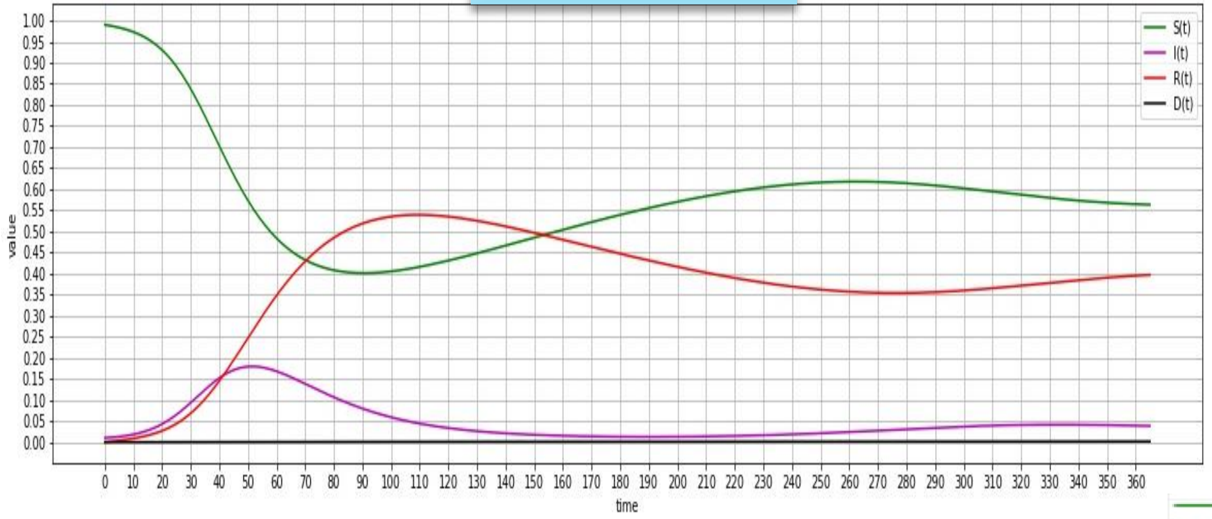
| $\beta$ | Children | Teenagers | Adults | Senior |
|---|---|---|---|---|
| **Children** | 0.05 | 0.01 | 0.04 | 0.008 |
| **Teenagers** | 0.01 | 0.09 | 0.08 | 0.008 |
| **Adults** | 0.04 | 0.08 | 0.1 | 0.02 |
| **Senior** | 0.008 | 0.008 | 0.02 | 0.03 |

Some clarifications
- fifteen days for recovering
- six months of immunity for recovered
- nine months of immunity for vaccinated
- mortality coefficient computed from [ISS covid-19 report]

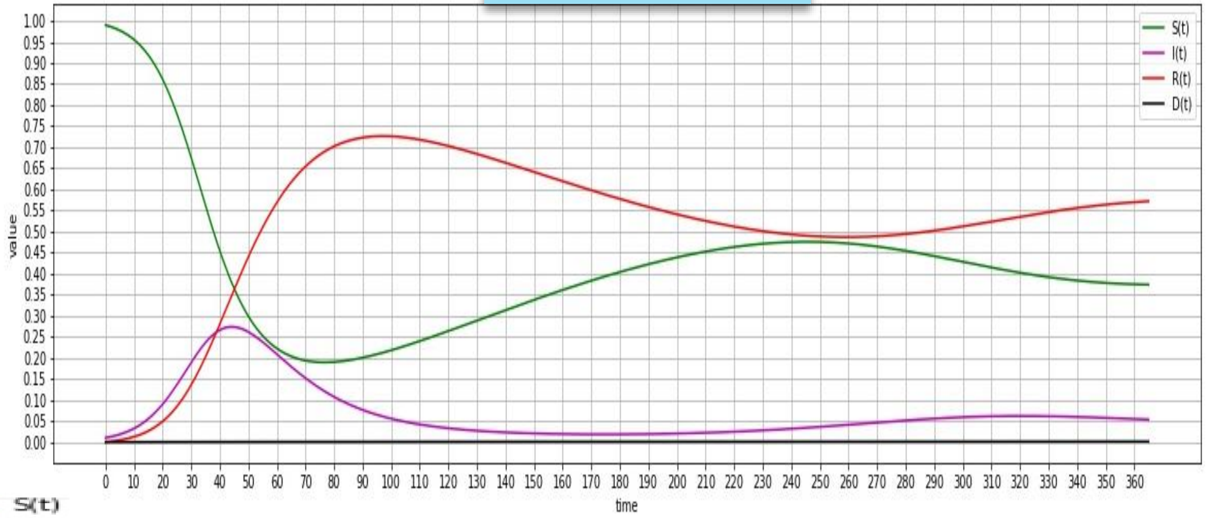
Fine-grained analysis

# No Vaccination

## Children

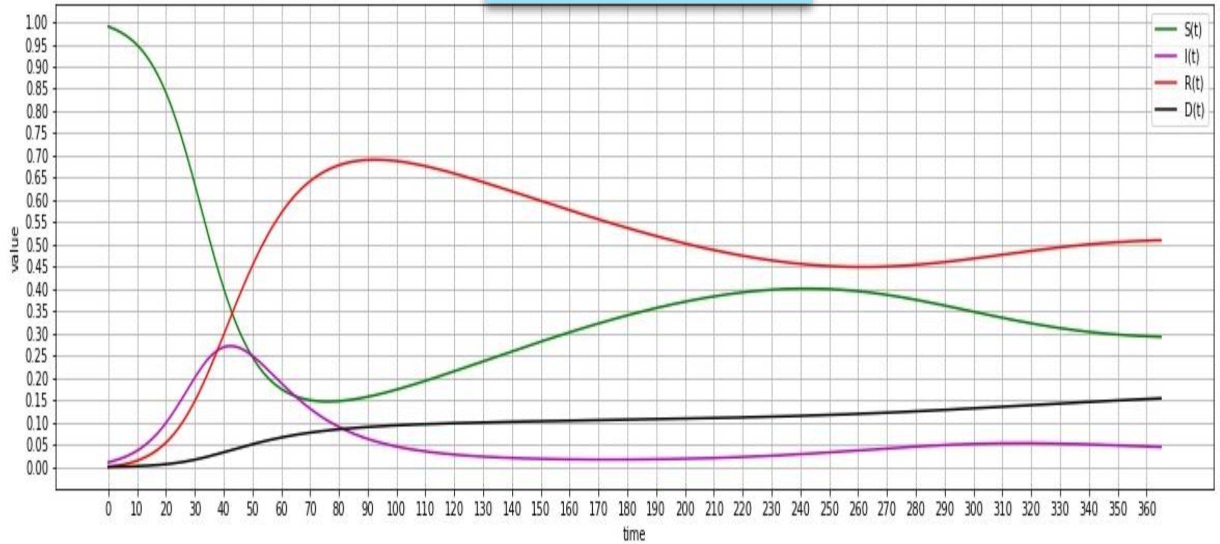



## Teenagers

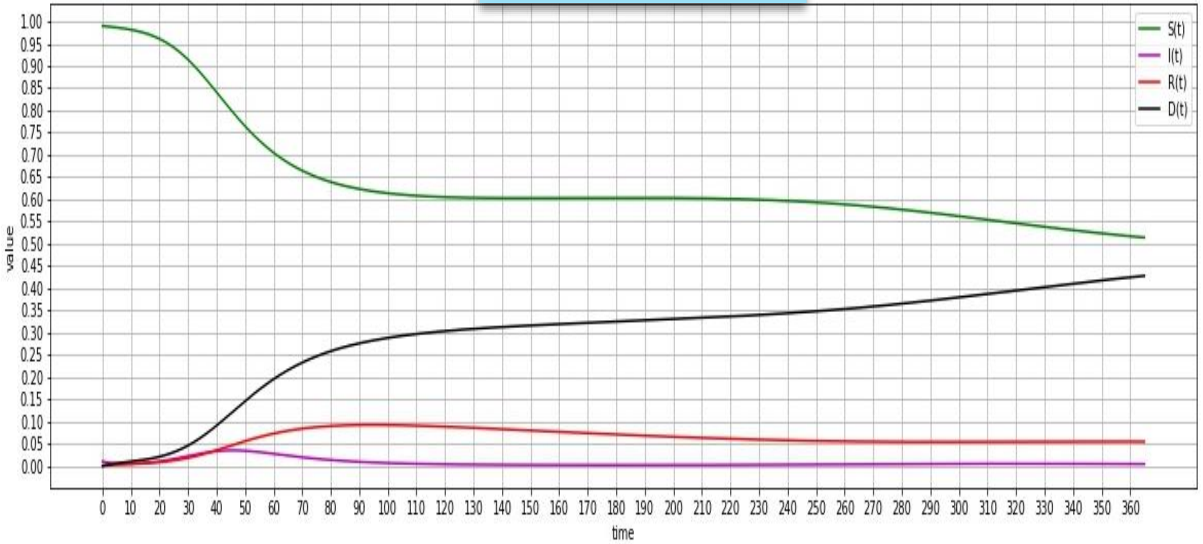




## Adults

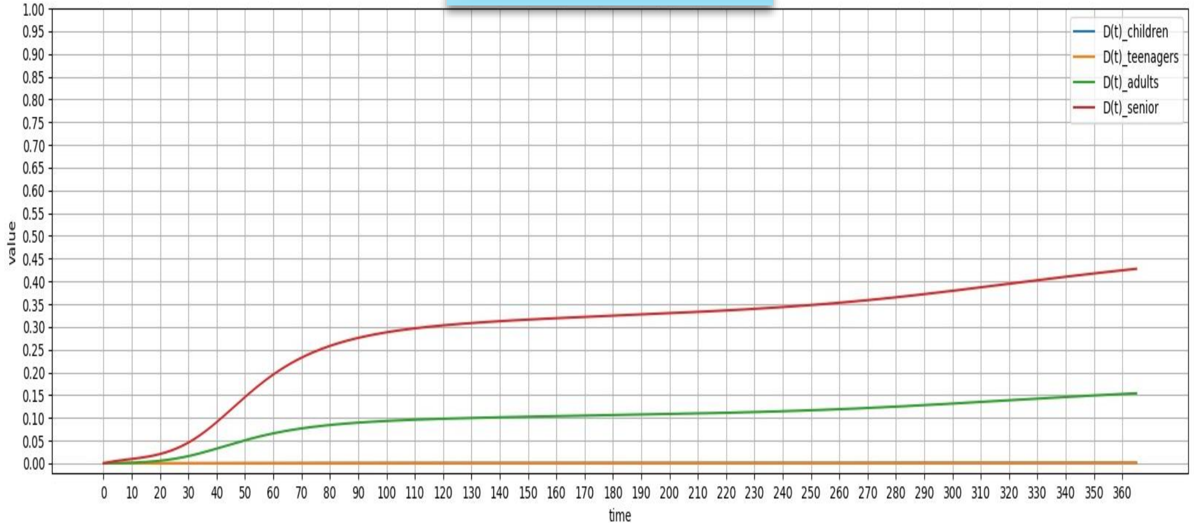



## Senior

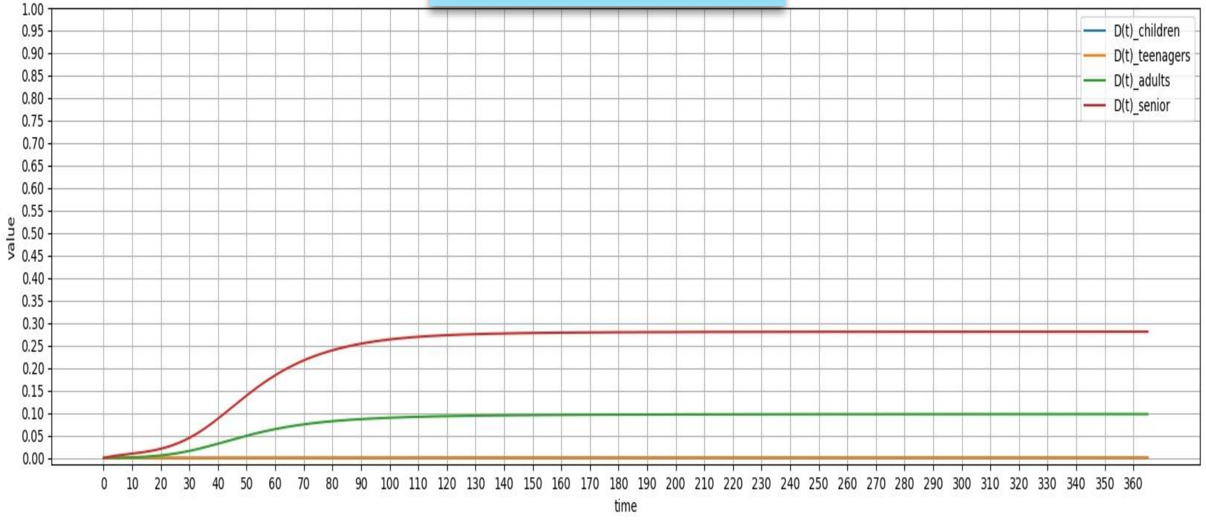


# Mortality comparison for each vaccination strategy

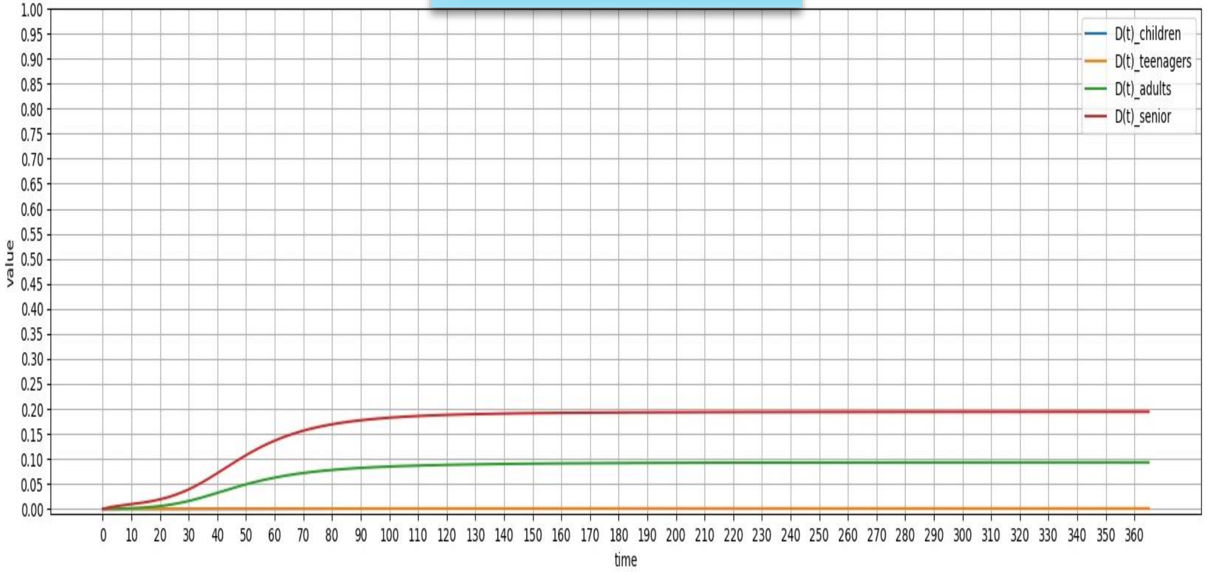
## No vaccination



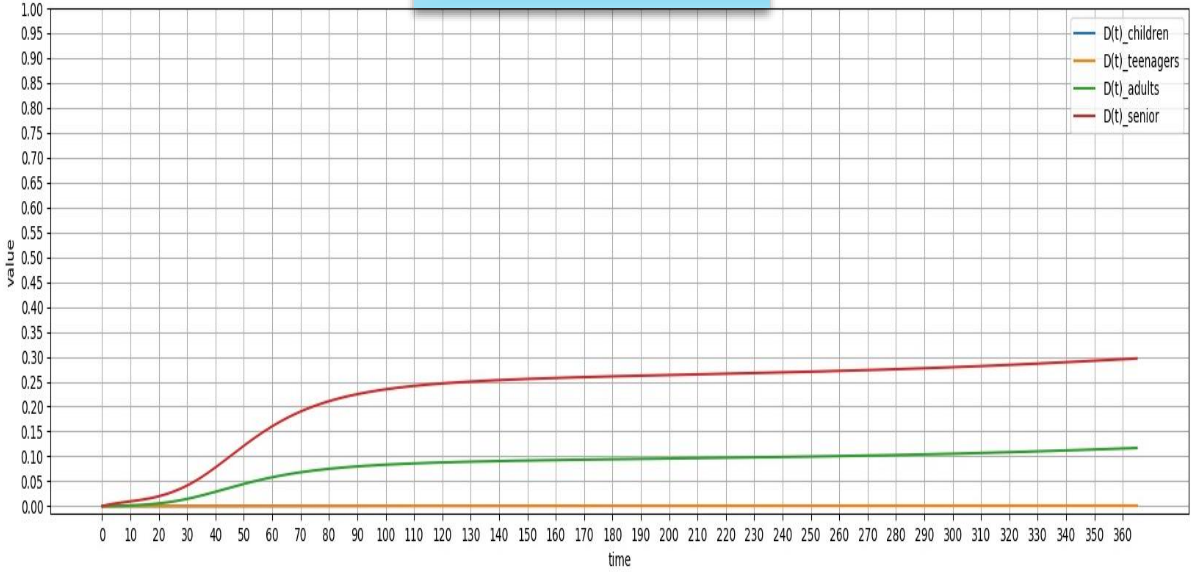

## Ascending order




## Descending order

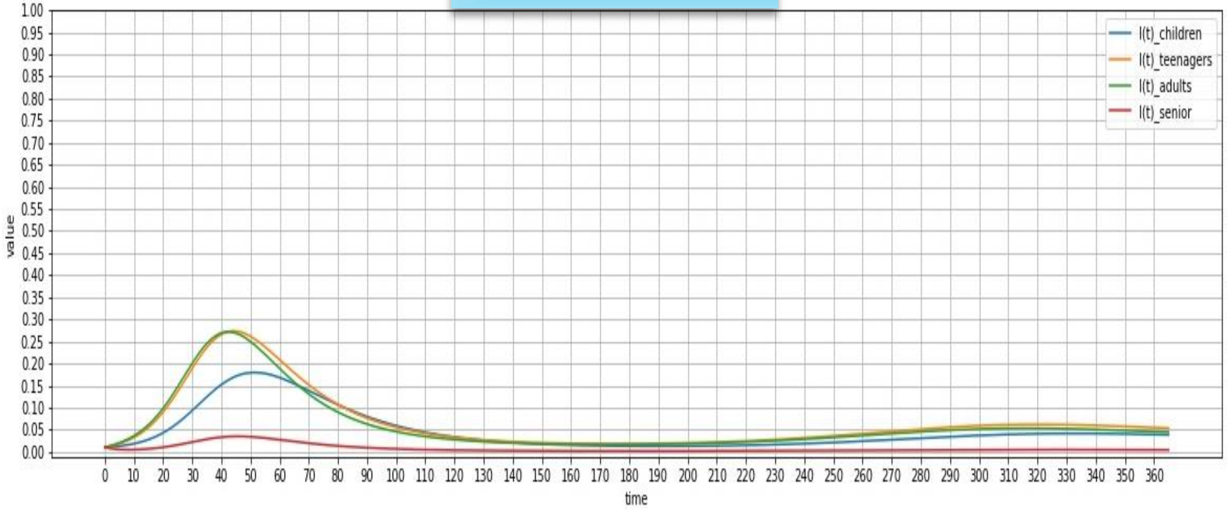



## Same Time

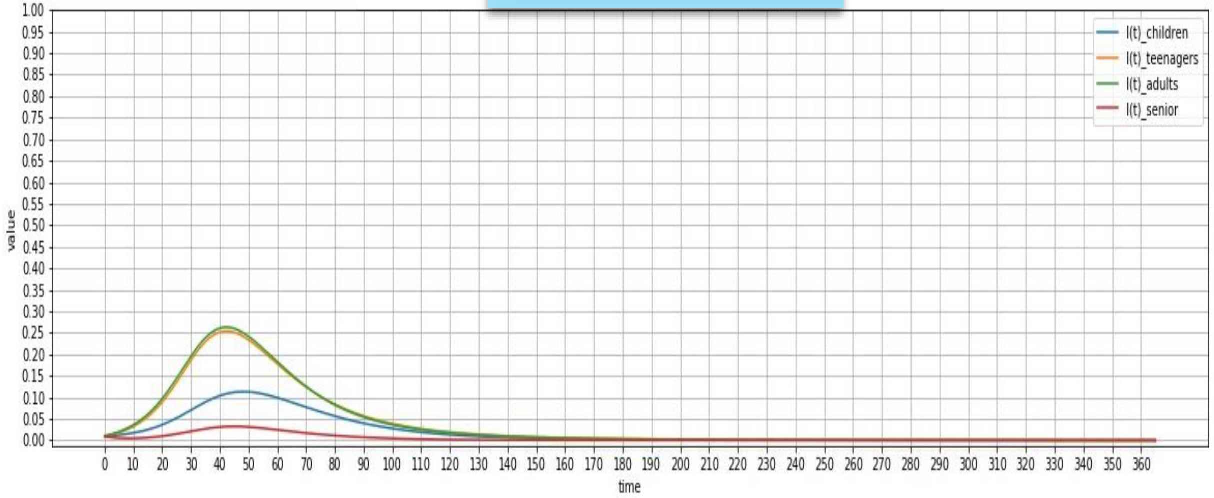


# Infections comparison for each vaccination strategy

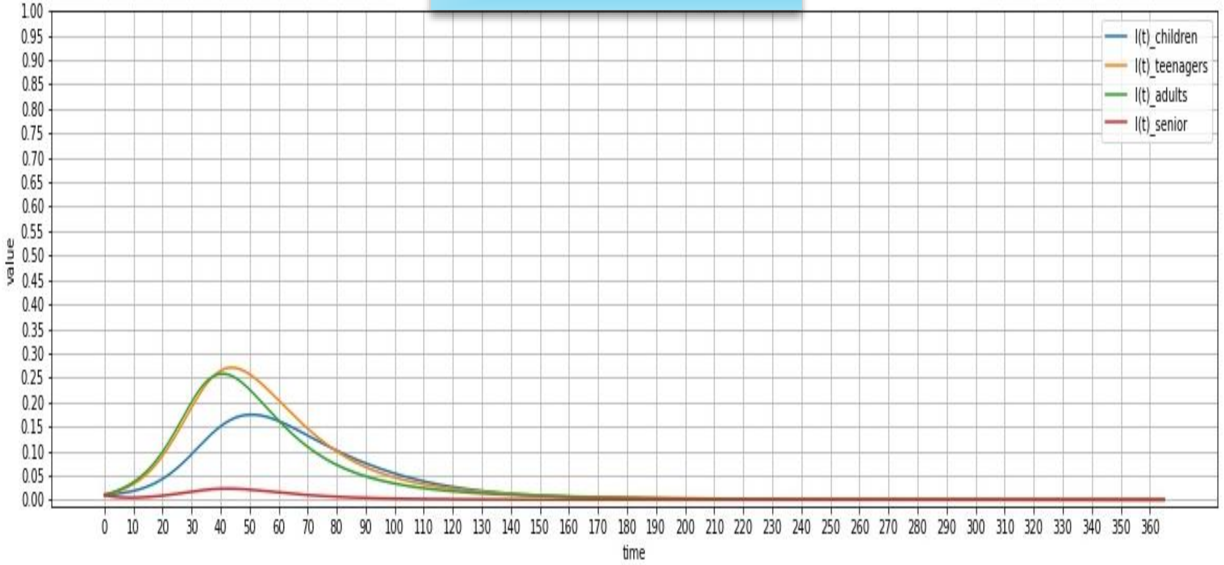
## No vaccination



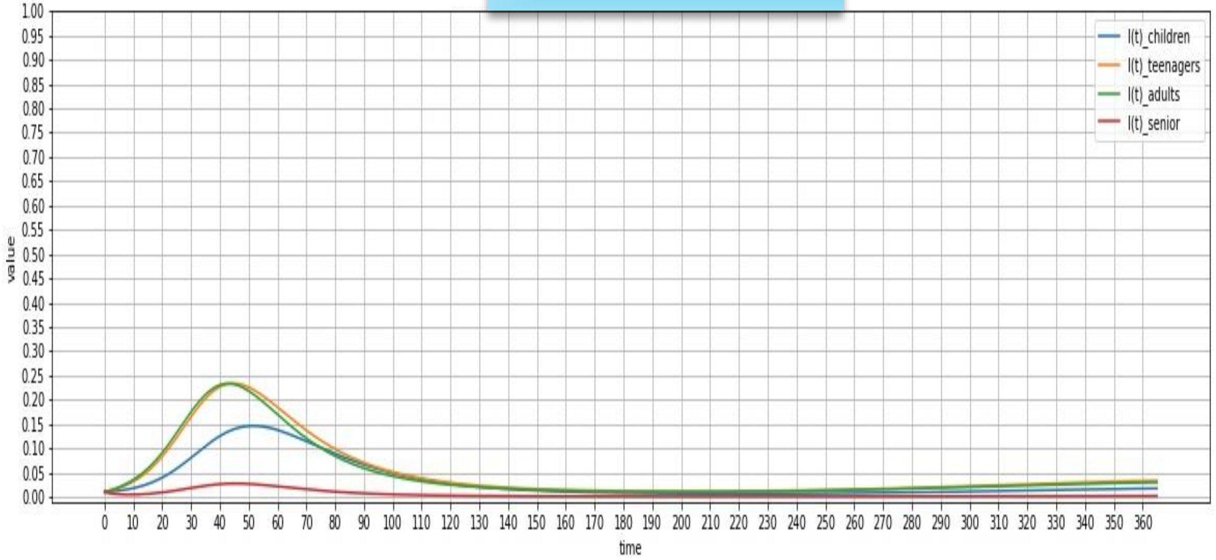

## Ascending order




## Descending order




## Same Time



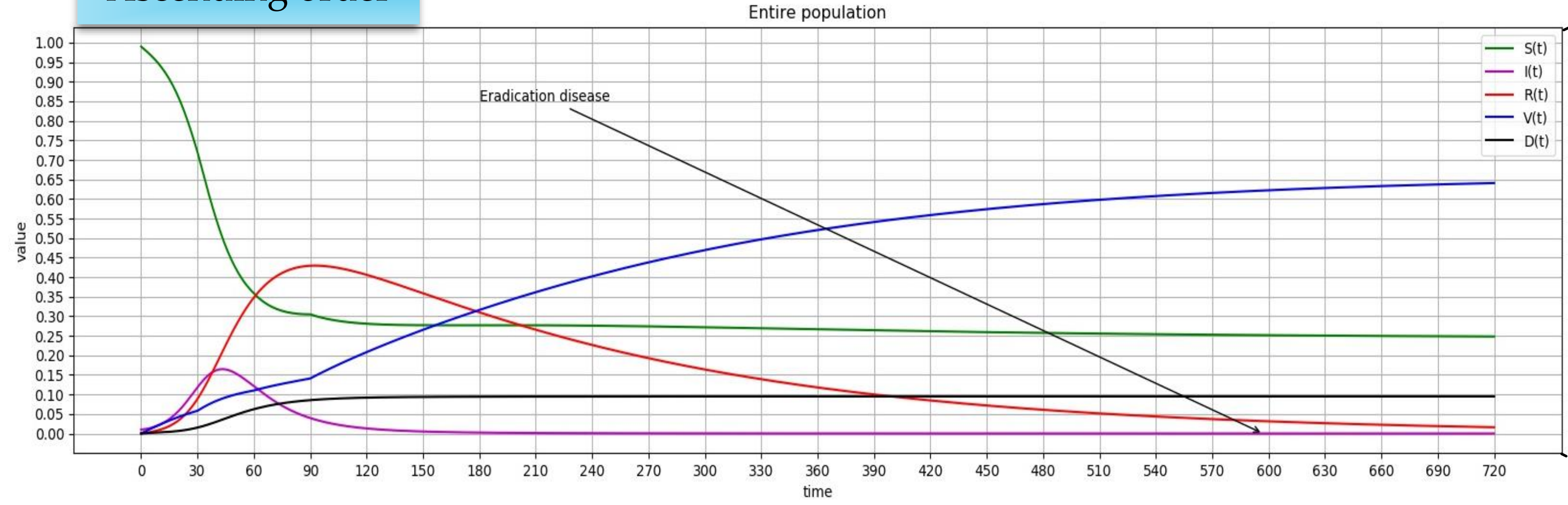
# Eradication disease

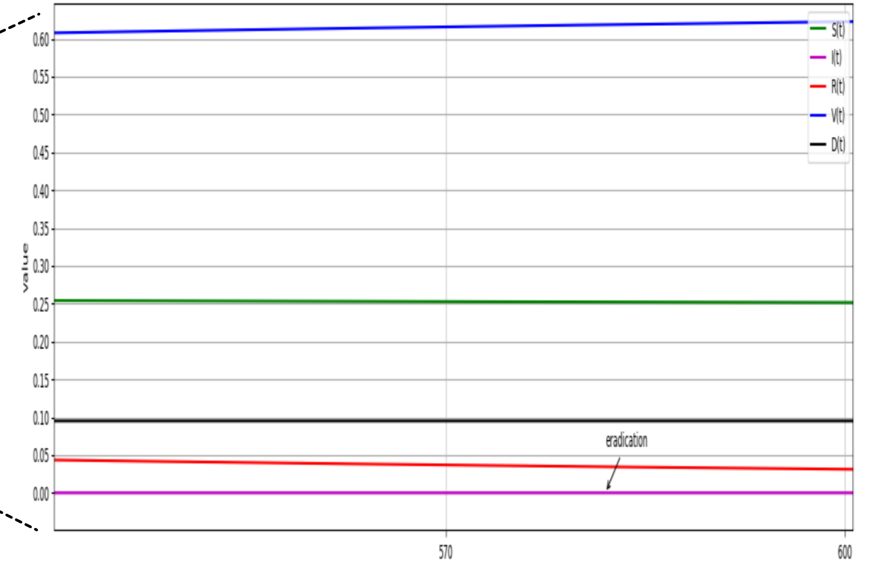
Coarse-grained analysis

| Ascending | Children | Teenagers | Adults | Senior |
|---|---|---|---|---|
| $\eta$ | 0.01 | 0.01 | 0.01 | 0.01 |
| *VACC-DAY* | 0 | 30 | 60 | 90 |

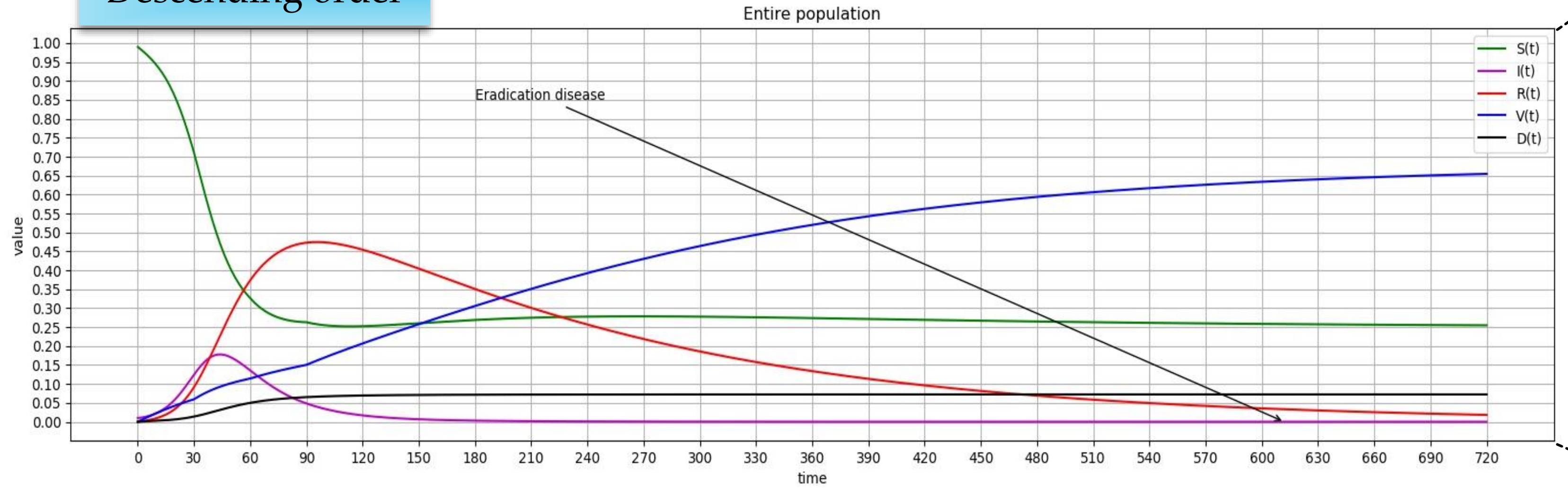
Descending equal only with reverse order for *VACC_DAY*

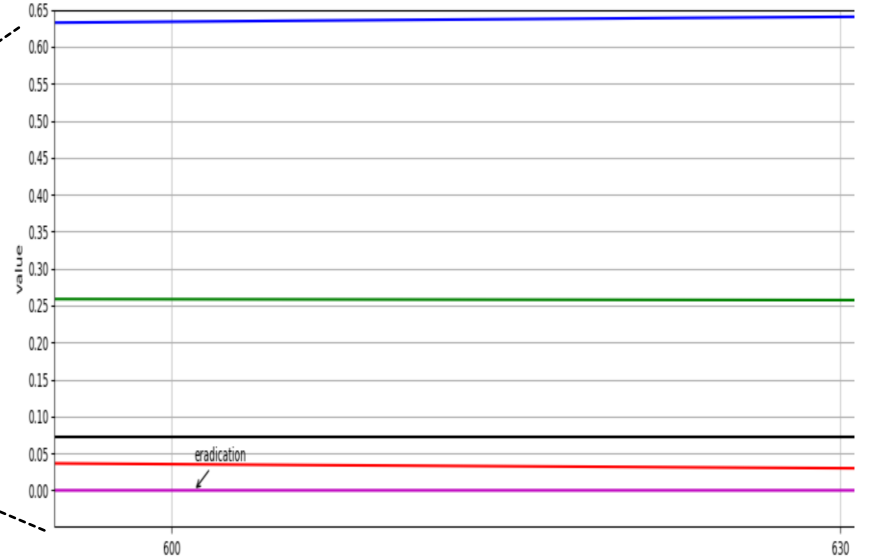
## Ascending order




## Descending order



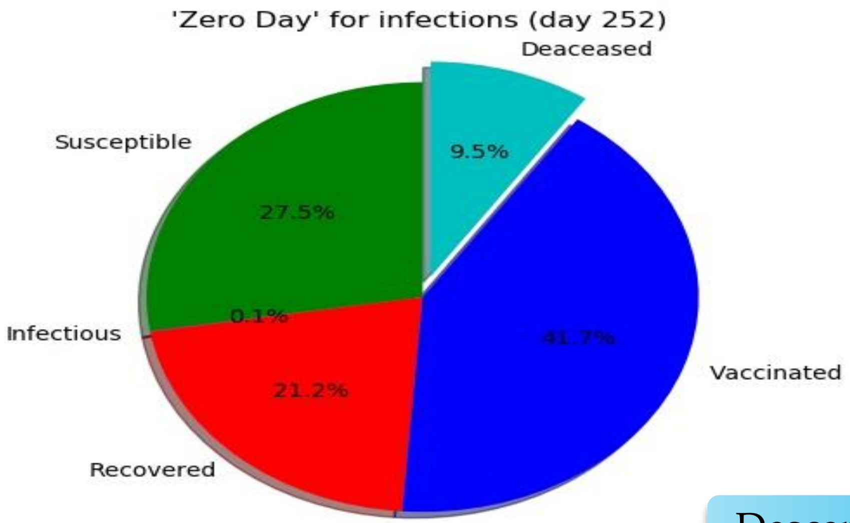



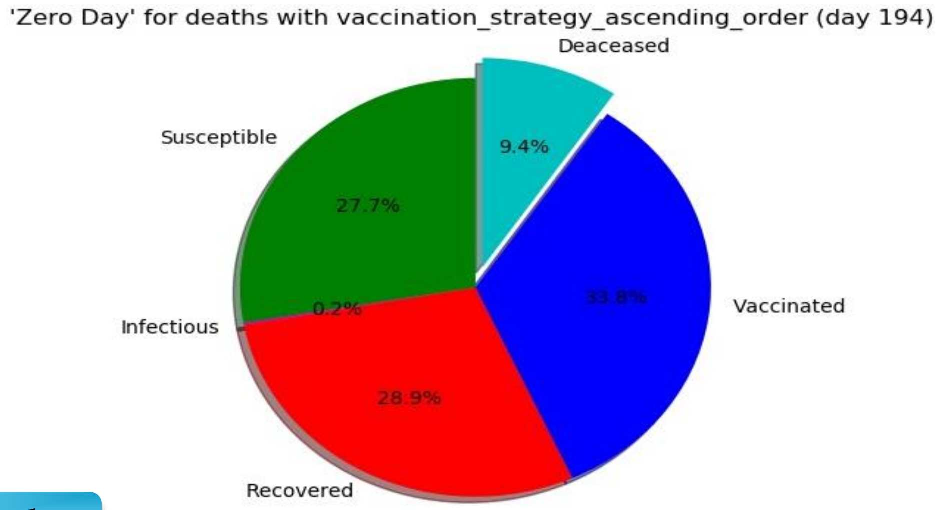
Coarse-grained analysis

# ‘Zero Days’

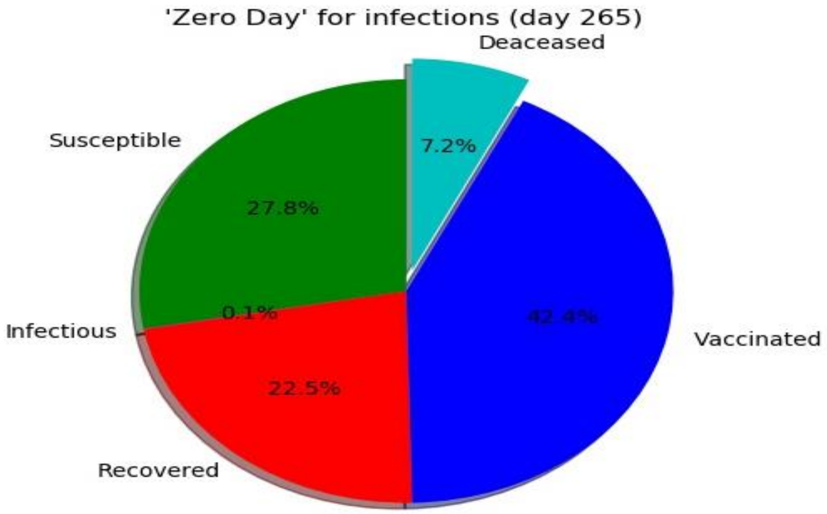
i.e. first day without new infections (or deaths)

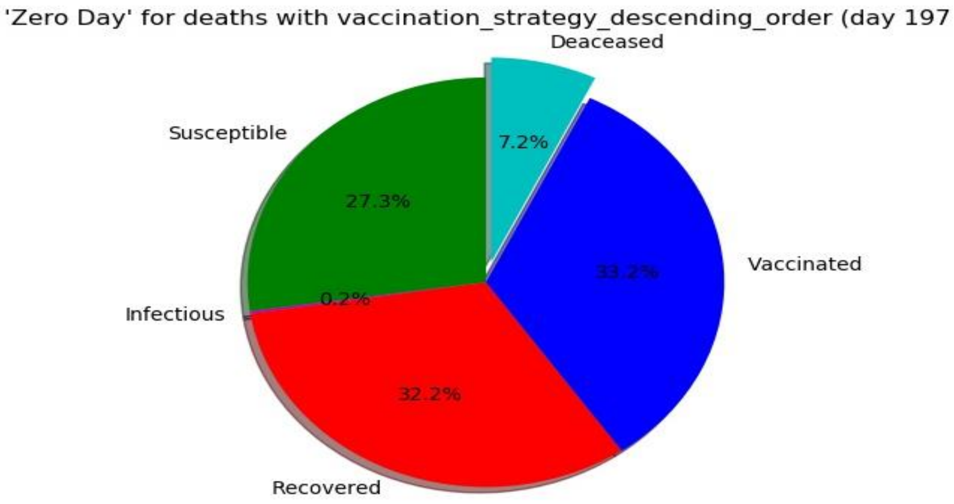
## Ascending order




## Descending order



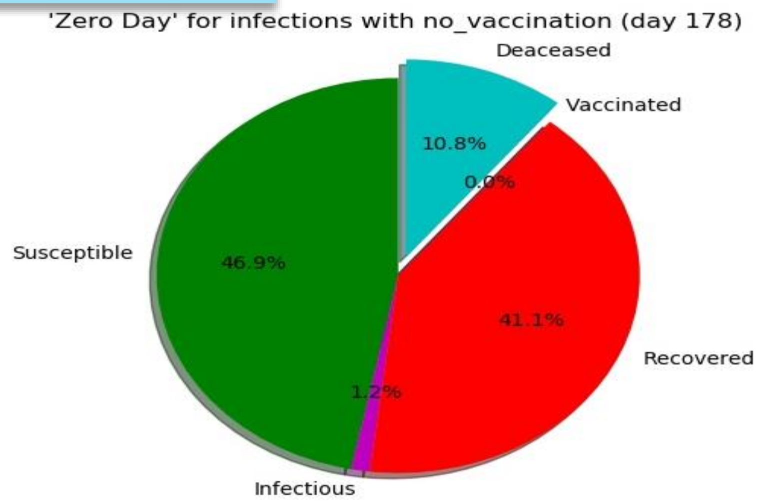



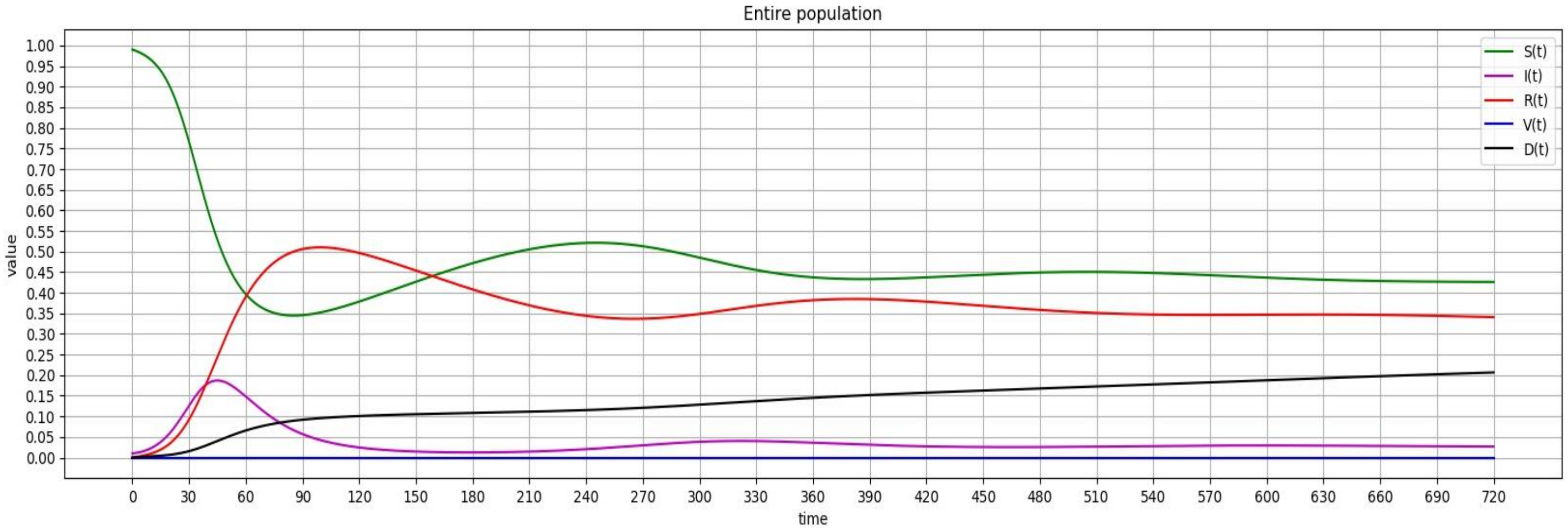
# Endemic infection

## Coarse-grained analysis

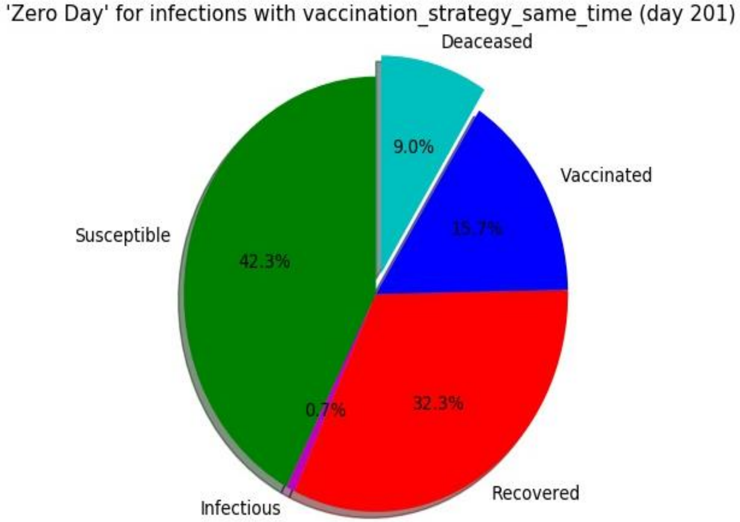
| Same Time | Children | Teenagers | Adults | Senior |
|---|---|---|---|---|
| $\eta$ | 0.0025 | 0.0025 | 0.0025 | 0.0025 |
| *VACC-DAY* | 0 | 0 | 0 | 0 |

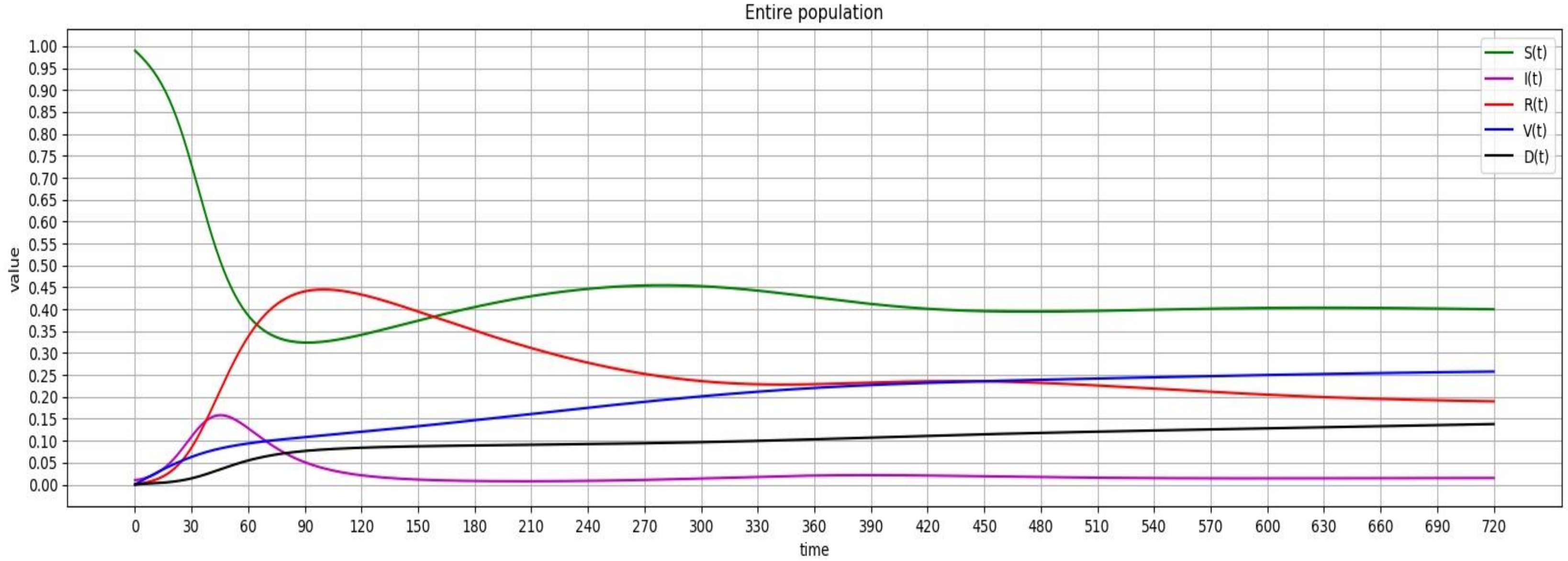
## No vaccination




## Same Time

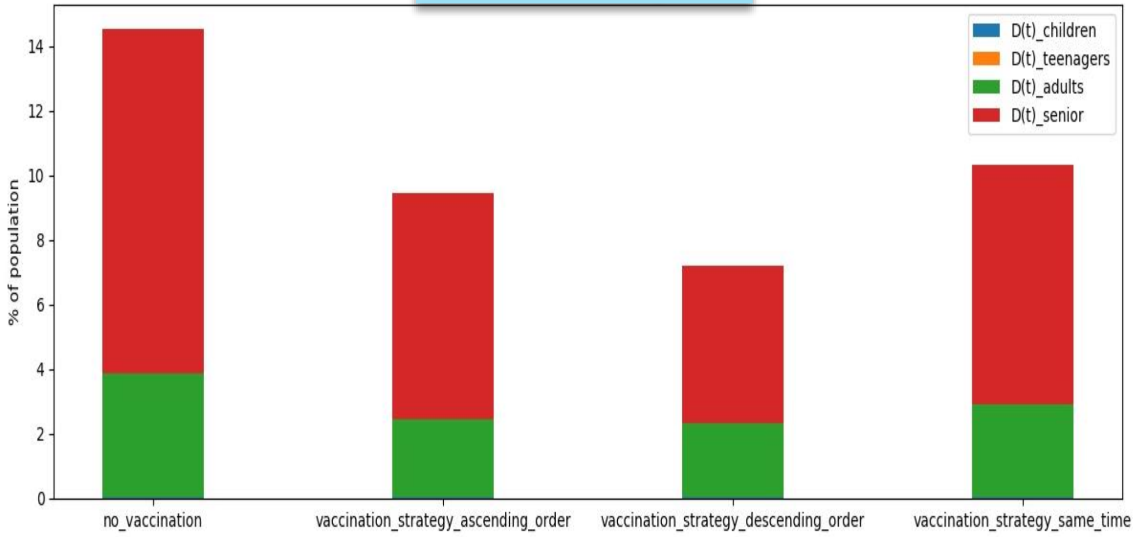


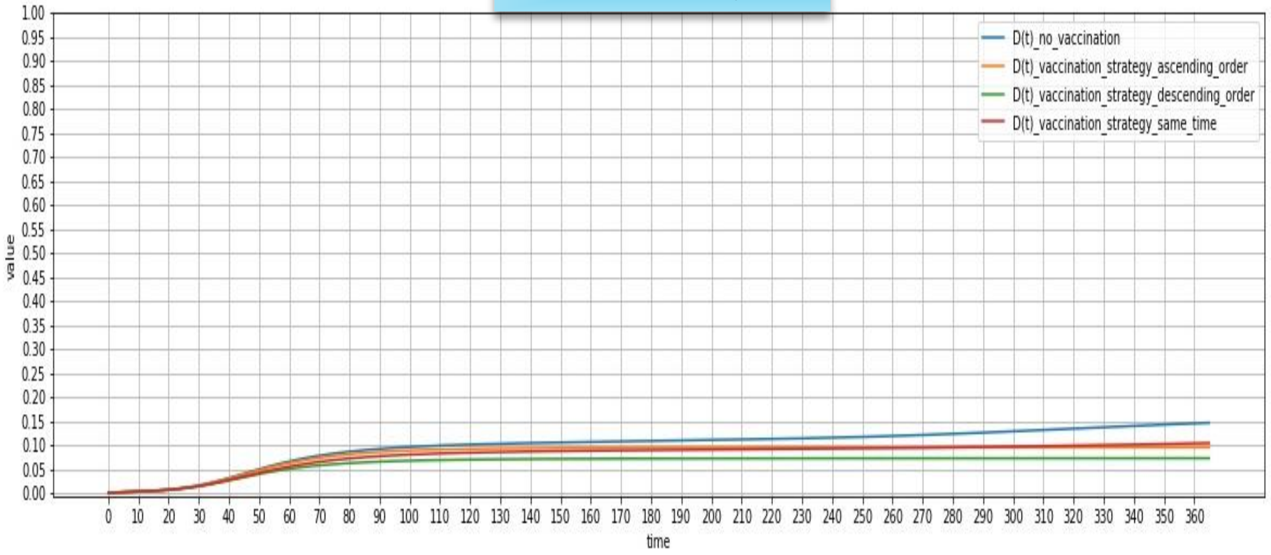
## Coarse-grained analysis

# Final vaccination strategies comparison

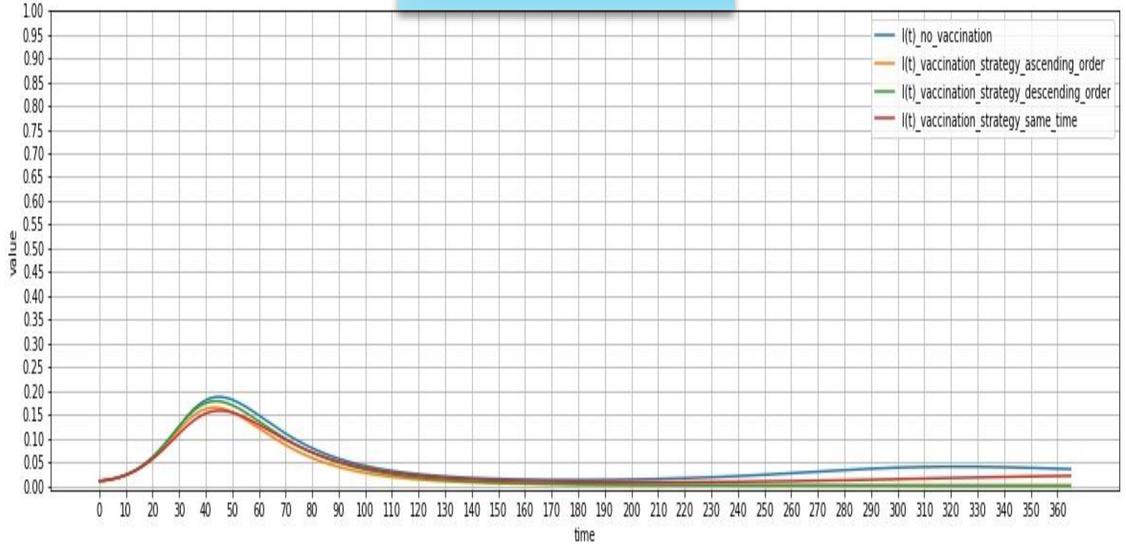
### Mortality



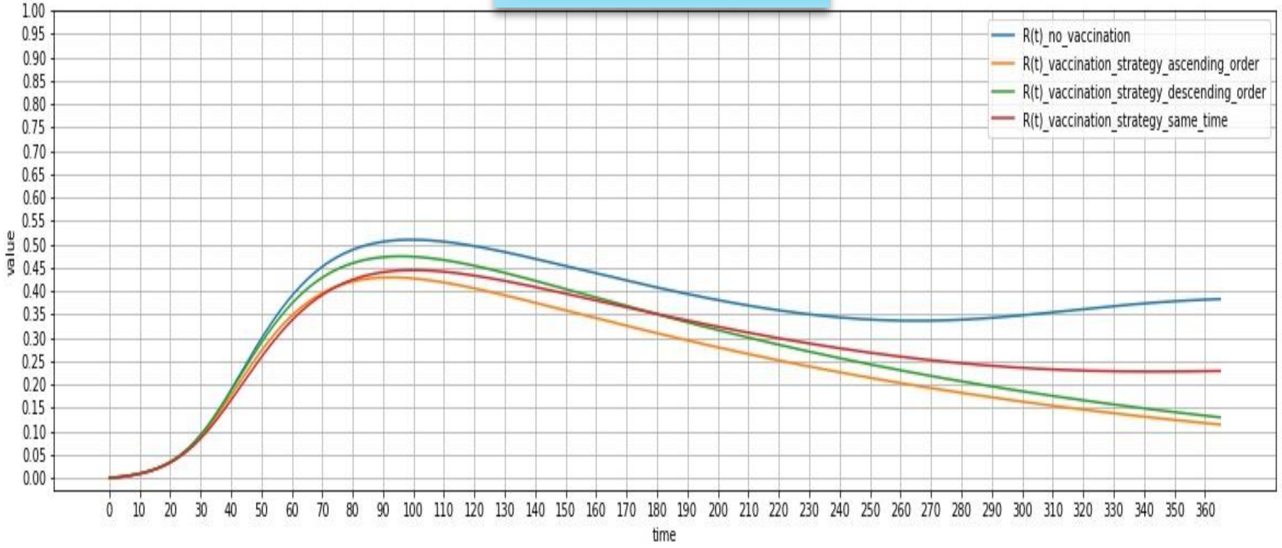

### Mortality




### Infections

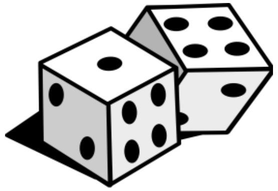


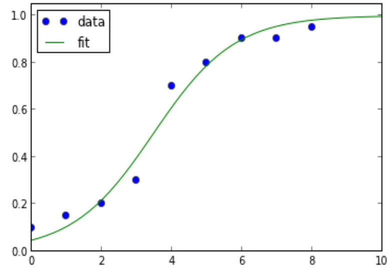

### Recovery



# Final Considerations and Improvements

- This model with ODEs is simple and intuitive and we obtain some expected experimental results
- The model could include compartments like quarantine, intensive care occupation and could take into account the preventive measures used
- Vaccination rate could be described by a logistic function
- Probabilities could be included in the model
- Implement an interactive dashboard with daily measurements
- Parameters estimation and fitting with a real (and accurate) dataset




# References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- A statistical methodology for data-driven partitioning of infectious disease incidence into age-groups
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- Matplotlib Documentation