Part I

Electricity and Magnetism

1. Concept of Charge and Coulomb's Law
2. Electric Fields
3. Gauss' Law
4. Electric potential
5. Capacitance
6. Current and Resistance
7. Circuit
8. Magnetic Fields

Lecture on Electricity: Coulomb’s Law

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Reference Books: 1. Fundamentals of Physics-
By Halliday-Resnick-Walker (10^{th} edition)

2. Physics for Engineers (Par-II)-By Dr. Giasuddin Ahmed

Concept of charge

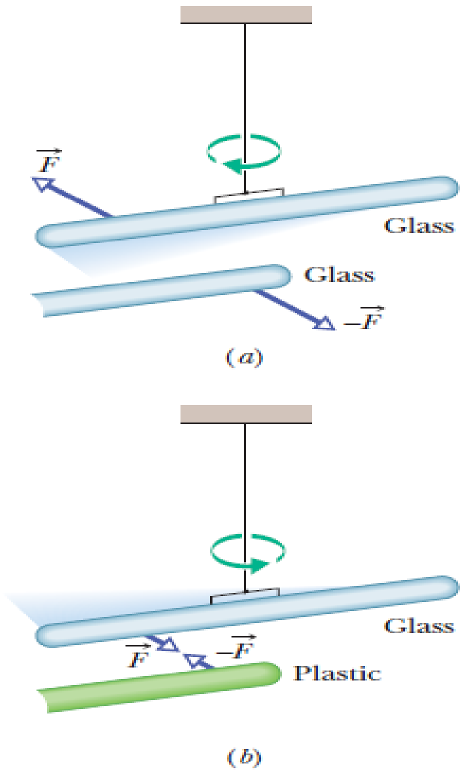


Figure 21-1 (*a*) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (*b*) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

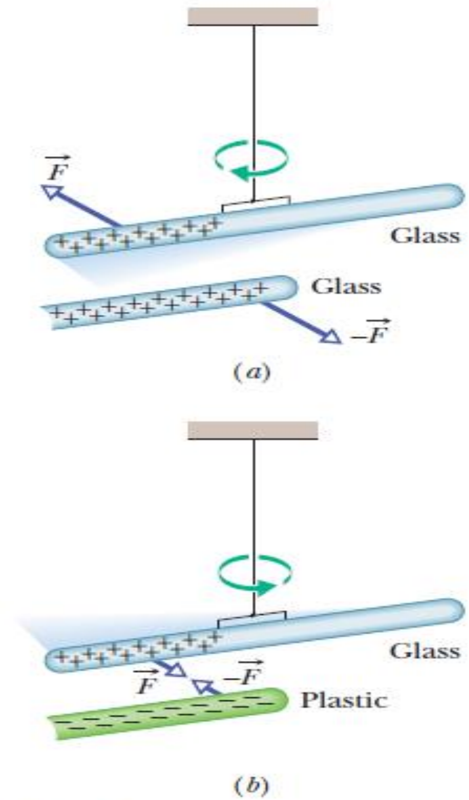


Figure 21-2 (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

Introduction Continued

- What is charge?
- How do we visualize it.
- What is the model.
- We only know charge exists because in experiments electric forces cause objects to move.

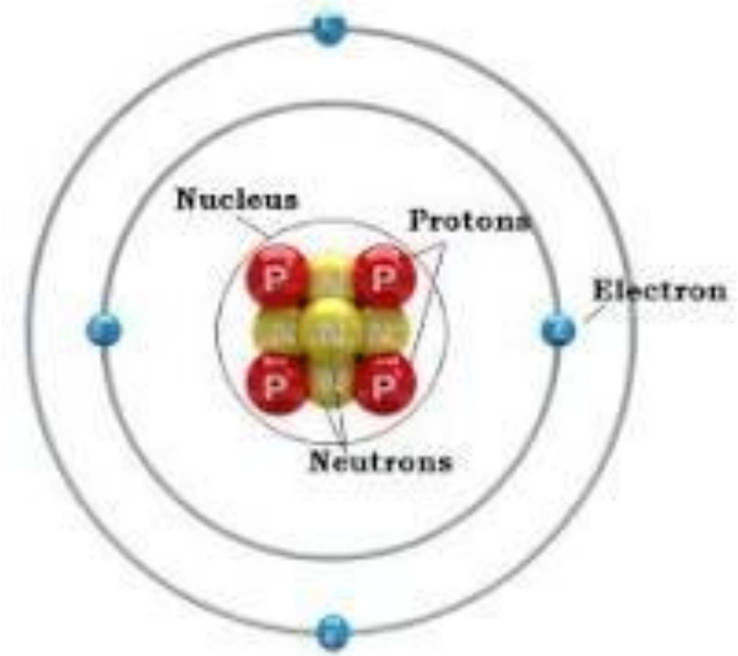
- Electrostatics: study of electricity when the charges are not in motion. Good place to start studying E&M because there are lots of demonstrations.

Some preliminaries

- **Electron**: Considered a point object with radius less than 10^{-18} meters with electric charge e= -1.6 x 10^{-19} Coulombs (SI units) and mass m_e= 9.11 x 10^{-31} kg

- **Proton**: It has a finite size with charge +e, mass m_p= 1.67 x 10^{-27} kg and with radius
  - 0.805 +/-0.011 x 10^{-15} m scattering experiment
  - 0.890 +/-0.014 x 10^{-15} m Lamb shift experiment

- **Neutron**: Similar size as proton, but with total charge = 0 and mass m_n=
  - Positive and negative charges exists inside the neutron



In dry weather, you can produce a spark by walking across certain types of carpet and then bringing one of your fingers near a metal doorknob, metal faucet, or even a friend. You can also produce multiple sparks when you pull, say, a sweater from your body or clothes from a dryer. Sparks and the “static cling” of clothing (similar to what is seen in Fig. 21-1) are usually just annoying. However, if you happen to pull off a sweater and then spark to a computer, the results are more than just annoying.

These examples reveal that we have electric charge in our bodies, sweaters, carpets, doorknobs, faucets, and computers. In fact, every object contains a vast amount of electric charge. **Electric charge** is an intrinsic characteristic of the fundamental particles making up those objects; that is, it is a property that comes automatically with those particles wherever they exist.

The vast amount of charge in an everyday object is usually hidden because the object contains equal amounts of the two kinds of charge: *positive charge* and *negative charge*. With such an equality—or *balance*—of charge, the object is said to be *electrically neutral;* that is, it contains no *net* charge. If the two types of charge are not in balance, then there *is* a net charge. We say that an object is *charged* to indicate that it has a charge imbalance, or net charge. The imbalance is always much smaller than the total amounts of positive charge and negative charge contained in the object.

We can understand these two demonstrations in terms of positive and negative charges. When a glass rod is rubbed with silk, the glass loses some of its negative charge and then has a small unbalanced positive charge (represented by the plus signs in Fig. 21-2*a*). When the plastic rod is rubbed with fur, the plastic gains a small unbalanced negative charge (represented by the minus signs in Fig. 21-2*b*). Our two demonstrations reveal the following:

Charges with the same electrical sign repel each other, and charges with opposite electrical signs attract each other.

The “positive” and “negative” labels and signs for electric charge were chosen arbitrarily by Benjamin Franklin. He could easily have interchanged the labels or used some other pair of opposites to distinguish the two kinds of charge. (Franklin was a scientist of international reputation. It has even been said that Franklin’s triumphs in diplomacy in France during the American War of Independence were facilitated, and perhaps even made possible, because he was so highly regarded as a scientist.)

Methods of Charging Objects: Friction, Contact, and Induction

- Normally atoms are in the lowest energy state. This means that the material is electrically neutral. You have the same number of electrons as protons in the material.
- How do we change this?
- How do we add more electrons than protons or remove electrons?

Summary Comments

- Silk(+) on teflon(-)
- Silk (-) on acrylic (+)
- Wood doesn’t charge
- Charged objects always attract neutral objects

- Show Triboelectric series
- Not only chemical composition important, structure of surface is important - monolayer of molecules involved, quantum effect.
(nanotechnology)

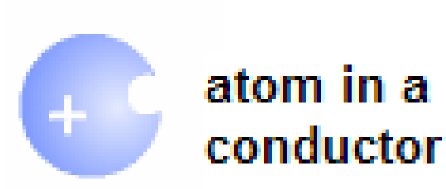
Triboelectric series

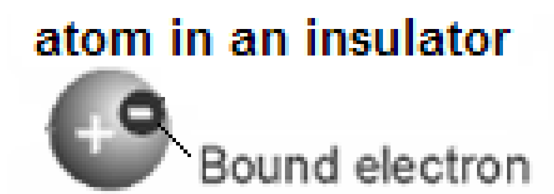
http://www.sciencejoywagon.com/physicszone/lesson/07elecst/static/triboele.htm

Positive (Lose electrons easily)
Air
Human Hands
Asbestos
Rabbit Fur
Glass
Mica
Acrylic
Human Hair
Nylon
Wool
Fur
Lead
Silk
Aluminum
Paper
Cotton
Steel
Wood
Amber
Sealing Wax
Hard Rubber
Nickel, Copper
Brass, Silver
Gold, Platinum
Sulfur
Acetate, Rayon
Polyester
Styrene
Orlon
Saran
Balloon
Polyurethane
Polypropylene
Vinyl (PVC)
Silicon
Teflon
Negative (Gains electrons easily)

Conductors and insulators

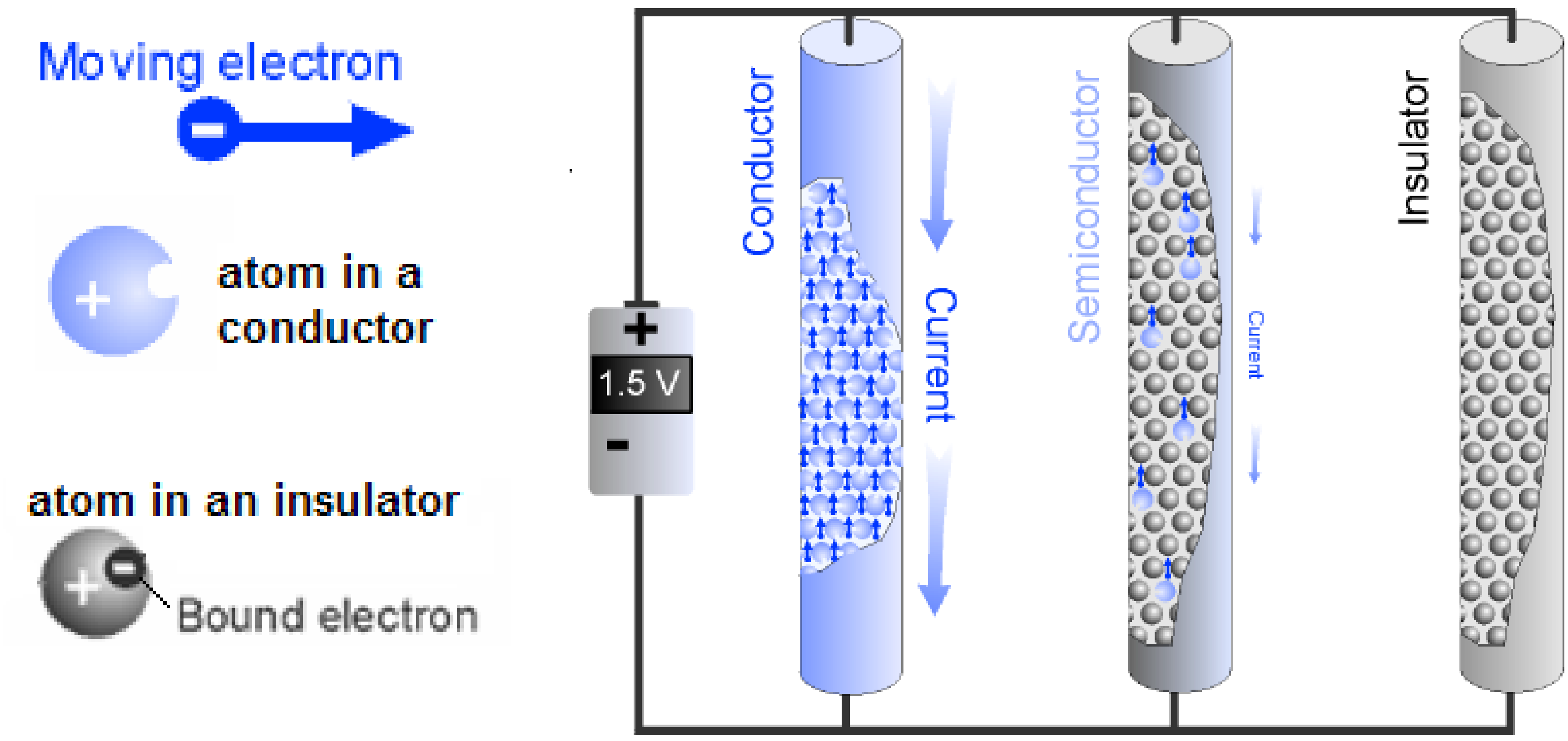
- All materials contain electrons.
- The electrons are what carry the current in a conductor.
- The electrons in insulators are not free to move—they are tightly bound inside atoms.





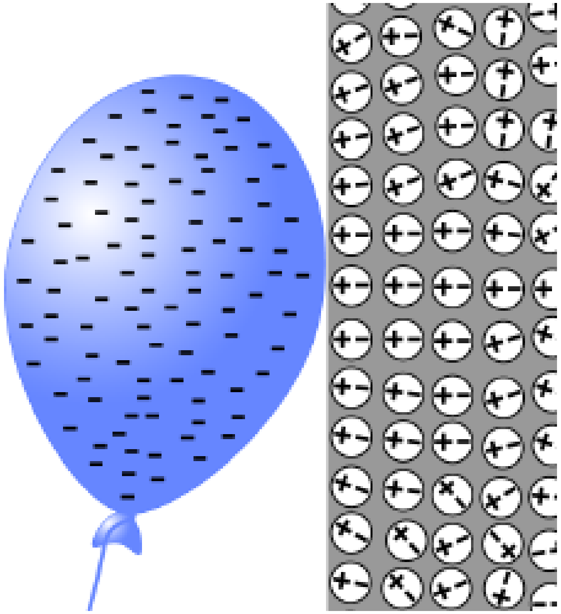
Conductors and insulators

- A **semiconductor** has a few free electrons and atoms with bound electrons that act as insulators.



Conductors and insulators

- When two neutral objects are rubbed together, charge is transferred from one to the other and the objects become oppositely charged.
- This is called charging by friction.
- Objects charged by this method will attract each other.



Conductors and Insulators

Conductors	Insulators
Electrons flow easily between atoms	Electron flow is difficult between atoms
1-3 valence electrons in outer orbit	5-8 valence electrons in outer orbit
Examples: Silver, Copper, Gold, Aluminum	Examples: Mica, Glass, Quartz

Summary:
Electrostatics is based on 4 four empirical facts

- Conservation of charge
- Quantization of charge
- Coulomb’s Law
- The principle of superposition

Quantization of charge

In Benjamin Franklin's day, electric charge was thought to be a continuous fluid—an idea that was useful for many purposes. However, we now know that fluids themselves, such as air and water, are not continuous but are made up of atoms and molecules; matter is discrete. Experiment shows that "electrical fluid" is also not continuous but is made up of multiples of a certain elementary charge. Any positive or negative charge q that can be detected can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots, \tag{21-11}$$

in which e, the **elementary charge,** has the approximate value

$$e = 1.602 \times 10^{-19}\ \mathrm{C}. \tag{21-12}$$

Conservation of charge

If you rub a glass rod with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but only transfers it from one body to another, upsetting the electrical neutrality of each body during the process. This hypothesis of **conservation of charge,** first put forward by Benjamin Franklin, has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles. No exceptions have ever been found. Thus, we add electric charge to our list of quantities—including energy and both linear and angular momentum—that obey a conservation law.

Examples of Conservation of Charge

$$e^- + e^+ \rightarrow \gamma + \gamma \qquad \text{(annihilation).}$$

$$\gamma \rightarrow e^- + e^+ \qquad \text{(pair production).}$$

Conservation of charge

- Rubbing does not create charge, it is transferred from one object to another

- Teflon negative - silk positive

- Acrylic positive - silk negative

- Nuclear reactions $\gamma^0 = e^+ + e^-$

- Radioactive decay ${}^{238}U_{92} = {}^{234}Th_{90} + {}^{4}He_{2}$

- High energy particle reactions $e^- + p^+ = e^- + \pi^+ + n^0$

What is meant by quantization of charge?

- Discovered in 1911 by Robert A. Millikan in the oil drop experiment
- The unit of charge is so tiny that we will never notice it comes in indivisible lumps.

Example-1: Suppose in a typical experiment we charge an object up with a nanoCoulomb of charge (10^{-9} C). How many elementary units of charge is this?

Solution: We know,

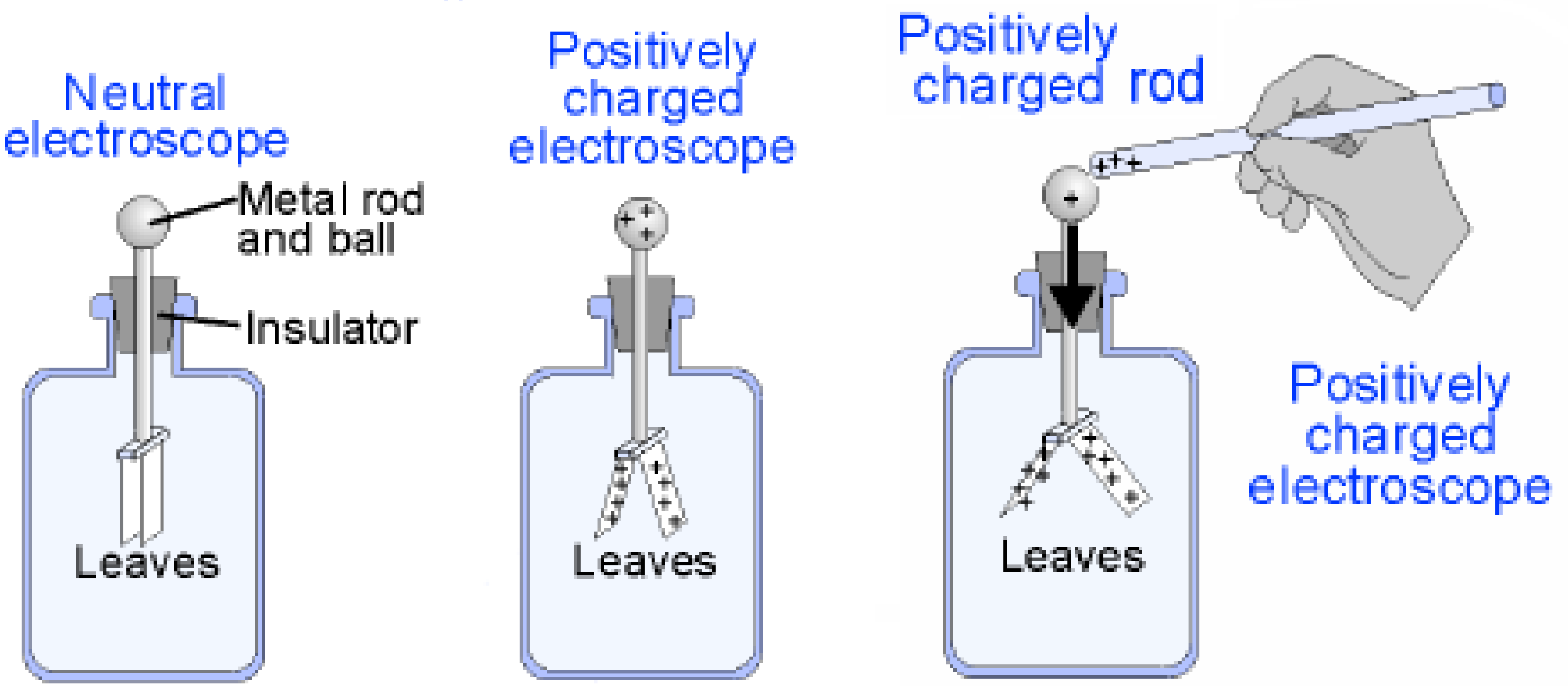
$$Q = N \times e \quad \textbf{so} \quad N = \frac{Q}{e} = \frac{10^{-9}\,C}{1.6 \times 10^{-19}\,C} = 6 \times 10^{9}$$

= six billion units of charge or 6 billion electrons.

Ans: 6×10^{9} **unit charge**

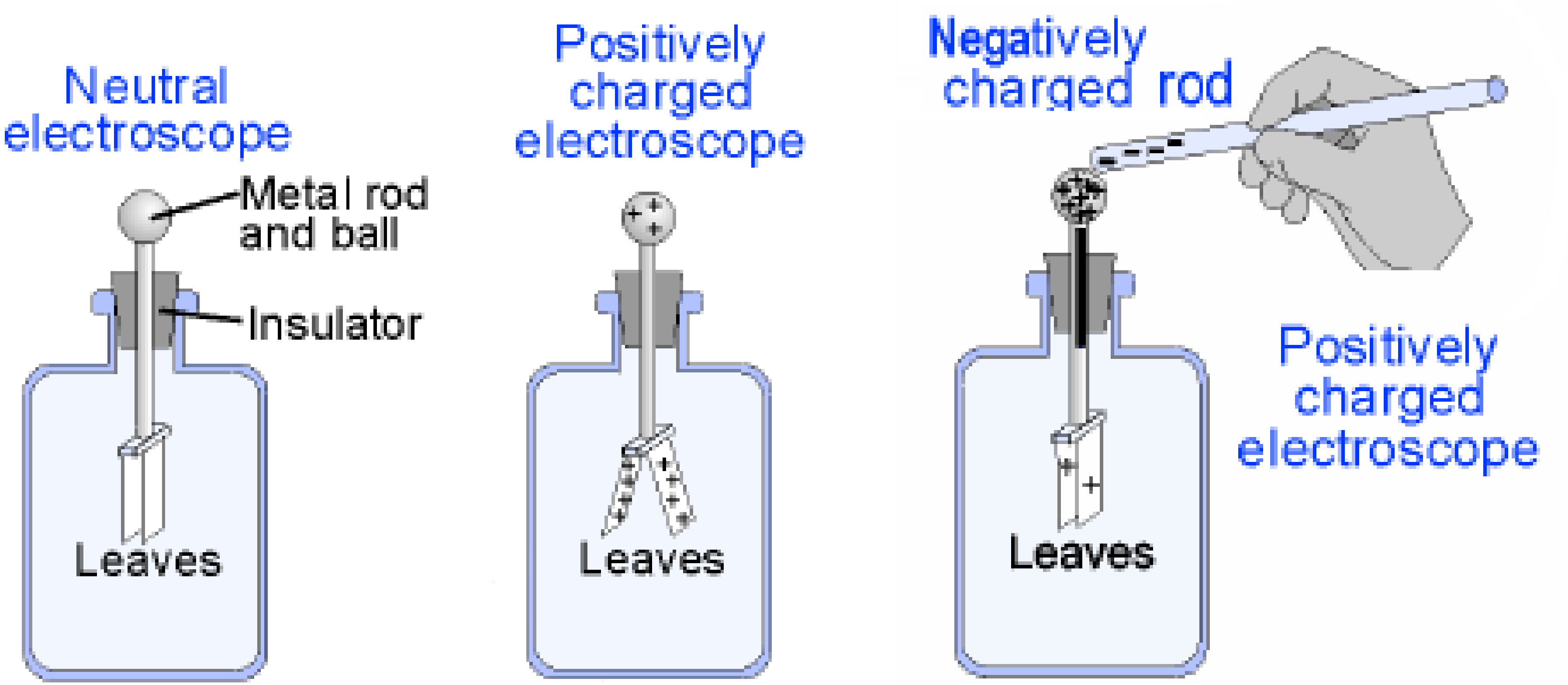
Electric forces

- The forces between the two kinds of charge can be observed with an electroscope.



Electric forces

- Charge can be transferred by conduction.



Electric current

- Current is the movement of electric charge through a substance.

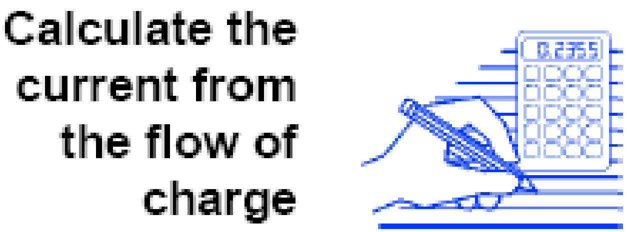
Current (amps)

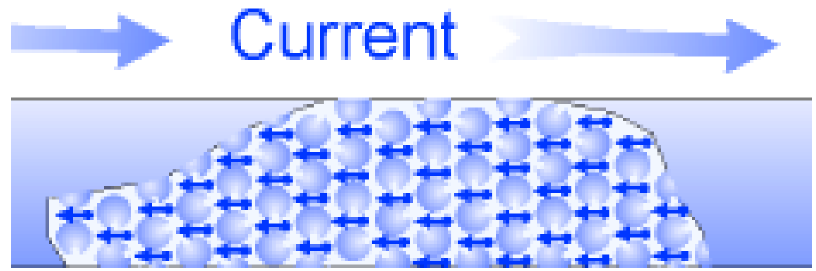
$$I = \frac{q}{t}$$

Charge that flows (coulombs)

Time (sec)

Calculate current

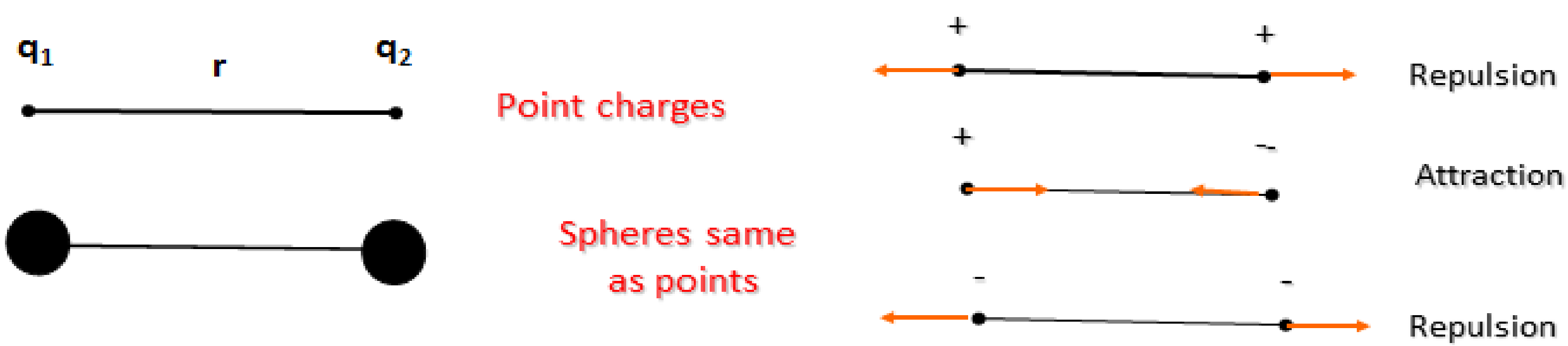




- Two coulombs of charge pass through a wire in five seconds.
- Calculate the current in the wire.

Coulombs Law

If two charged particles are brought near each other, they each exert a force on the other. If the particles have the same sign of charge, they repel each other (Figs. 21-6*a* and *b*). That is, the force on each particle is directed away from the other particle, and if the particles can move, they move away from each other. If, instead, the particles have opposite signs of charge, they attract each other (Fig. 21-6*c*) and, if free to move, they move closer to each other.



This force of repulsion or attraction due to the charge properties of objects is called an **electrostatic force.** The equation giving the force for charged *particles* is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it. In terms of the particles in Fig. 21-7, where particle 1 has charge q_1 and particle 2 has charge q_2, the force on particle 1 is

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{\mathrm{r}} \qquad \text{(Coulomb's law)}, \tag{21-1}$$

Curiously, the form of Eq. 21-1 is the same as that of Newton's equation (Eq. 13-3) for the gravitational force between two particles with masses m_1 and m_2 that are separated by a distance r:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{\mathrm{r}} \qquad \text{(Newton's law)}, \tag{21-2}$$

in which G is the gravitational constant.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \qquad \text{(Coulomb's law).} \tag{21-4}$$

The constants in Eqs. 21-1 and 21-4 have the value

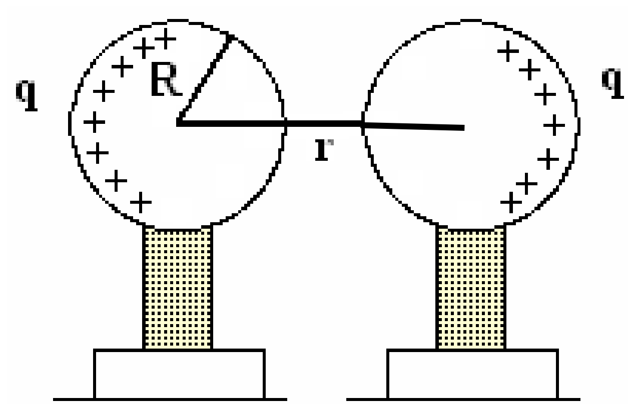
$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}. \tag{21-5}$$

The quantity ε_0, called the **permittivity constant**, sometimes appears separately in equations and is

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}. \tag{21-6}$$

Uniformly charged metal spheres of Radius R

$$F=\frac{kq^2}{(r)^2}$$



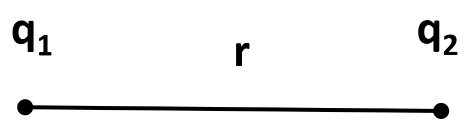
$$F- \frac{kq^2}{(r+2R)^2}$$

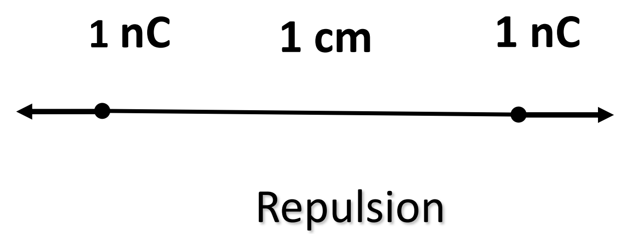
Demo: Show uniformity of charge around sphere using electrometer.
Demo: Show charging spheres by induction using electrometer

Coulombs Law
Two Positive Charges

Example-2: What is the force between two positive charges each 1 nano Coulomb, 1cm apart in a typical demo? Why is the force so weak here?

Solution:





$$F = \frac{kq_1q_2}{r^2}$$

$$F = \frac{\left(10^{10}\,\frac{Nm^2}{C^2}\right)\left(10^{-9}\,c\right)^2}{\left(10^{-2}\,m\right)^2} = 10^{-4}\,N$$

(equivalent to a weight of something with a mass of 10^{-5} kg = 10^{-2} gm or 10 mg - long strand of hair)

Coulombs Law
Two Pennies without electrons

Example-3: (i) What is the force between two 3 gm pennies one meter apart if we remove all the electrons from the copper atoms? [Modeling] (ii) What is their acceleration as they separate?

Solution: (i) We know,

$$F=\frac{kq_1q_2}{r^2}=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)q^2}{(1m)^2}$$

The force is

$$F=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)\left(1.4\ \times 10^5 C\right)^2}{1m^2}=2\times 10^{20}N$$

The atom Cu has 29 protons and a 3 gm penny has

$$=\left(\frac{3gm}{63.5gm}\right)\times 6\times 10^{23}\,atoms = 3\times 10^{22}\,atoms$$

The total charge is $q = 29\times 3\times 10^{22}\,atoms\times 1.6\times 10^{-19}C = 1.4\times 10^{5}C$

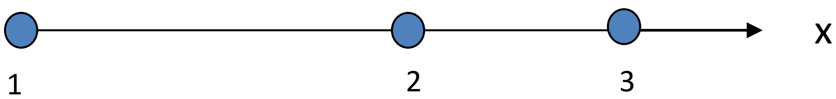
(ii) What is their acceleration as they separate?

$$a=\frac{F}{m}=\frac{2x10^{20}N}{3x10^{-3}kg}=7x10^{22}\ \frac{m}{s^2}$$

Principle of Superposition
Three charges In a line

- In the previous example we tacitly assumed that the forces between nuclei simply added and did not interfere with each other. That is the force between two nuclei in each penny is the same as if all the others were not there. This idea is correct and is referred to as the Principle of Superposition.

Example-4: Example of charges in a line.



Three charges lie on the x axis: q_1=+25 nC at the origin, q_2= -12 nC at x =2m, q_3=+18 nC at x=3 m. What is the net force on q_1?

Solution: We simply add the two forces keeping track of their directions. Let a positive force be one in the + x direction.

$$F=-kq_1\left(\frac{q_2}{(2m)^2}+\frac{q_3}{(3m)^2}\right)$$

$$=-\left(10^{10}\,\tfrac{Nm^2}{C^2}\right)(25\times10^{-9}\,C)\left(\frac{-12\times10^{-9}\,C}{(2m)^2}+\frac{18\times10^{-9}\,C}{(3m)^2}\right)$$

$$=2.5\times10^{-7}\,N$$

Finding the net force due to two other particles

(a) Figure 21-8*a* shows two positively charged particles fixed in place on an *x* axis. The charges are $q1=1.60\times10^{-19}$ C and $q2 = 3.20\times10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force on particule 1 from particle 2?

(b) Figure 21-8*c* is identical to Fig. 21-8*a* except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge $q3 = 3.20 \times 10^{-19}$ C and is at a 3/4R distance from particle 1.What is the net electrostatic force on particle 1 due to particles 2 and 3?

(c) Figure 21-8*e* is identical to Fig. 21-8*a* except that particle 4 is now included. It has charge $q4 = 3.20 \times 10^{-19}$ C, is at a distance from particle 1, and lies on a line that makes an angle u 60° with the *x* axis. What is the net electrostatic force on particle 1 due to particles 2 and 4?

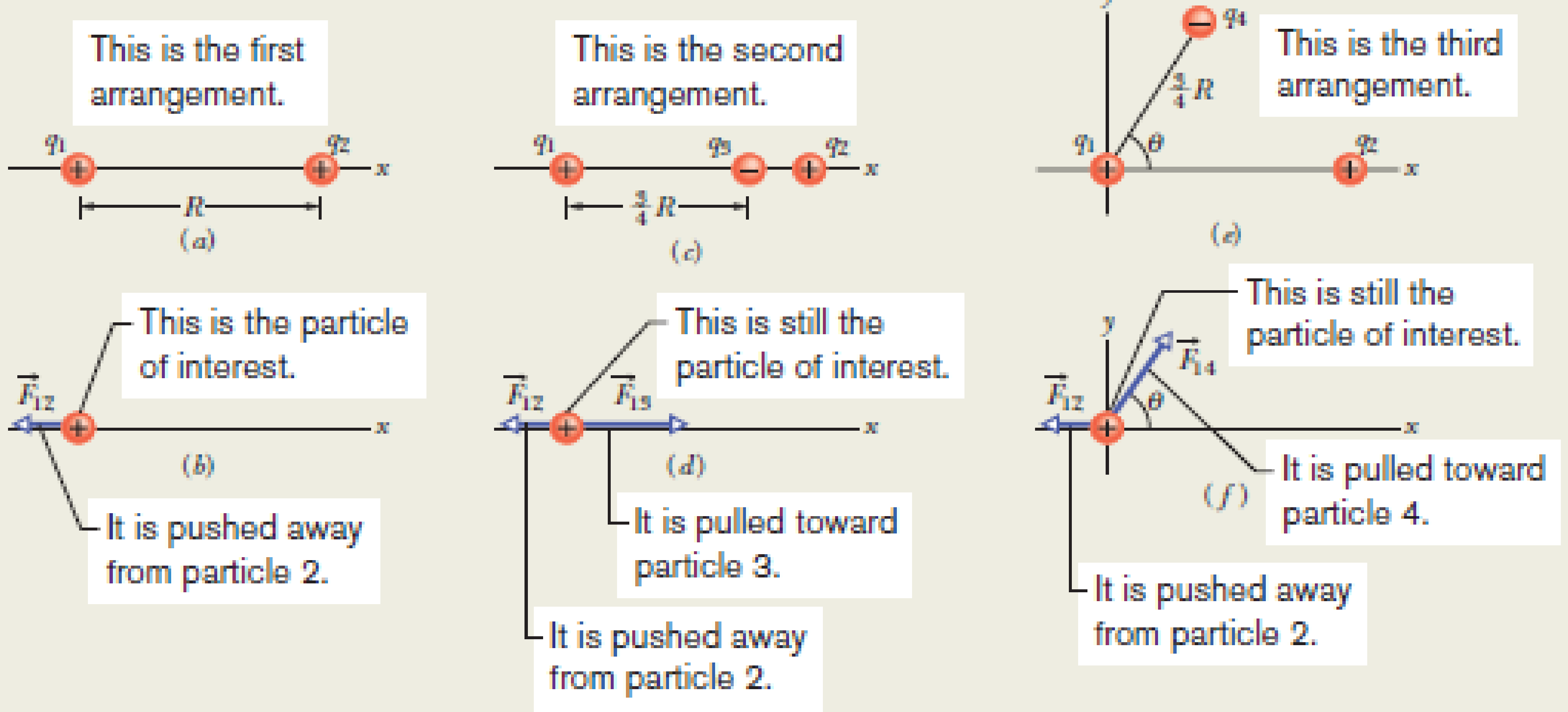


Figure 21-7 (*a*) Two charged particles of charges q_1 and q_2 are fixed in place on an x axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21-7 is available in *WileyPLUS* as an animation with voiceover.)

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$
$$= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(0.0200\ \mathrm{m})^2}$$
$$= 1.15 \times 10^{-24}\ \mathrm{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24}\ \mathrm{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)}$$

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(\frac{3}{4})^2(0.0200\ \mathrm{m})^2}$$
$$= 2.05 \times 10^{-24}\ \mathrm{N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}.$$

The net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{13}$$
$$= -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (2.05 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}}$$
$$= (9.00 \times 10^{-25}\ \mathrm{N})\hat{\mathrm{i}}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\mathrm{net}}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25}\ \mathrm{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

Four particles: We can rewrite Eq. 21-4 as

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$
$$= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2}) \times \frac{(1.60 \times 10^{-19}\ \mathrm{C})(3.20 \times 10^{-19}\ \mathrm{C})}{(\frac{3}{4})^2(0.0200\ \mathrm{m})^2}$$
$$= 2.05 \times 10^{-24}\ \mathrm{N}.$$

Then from Eq. 21-7, we can write the net force $\vec{F}_{1,\mathrm{net}}$ on particle 1 as

$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

Method 1. *Summing directly on a vector-capable calculator.* For $\vec{F}_{12}$, we enter the magnitude 1.15×10^{-24} and the angle 180°. For $\vec{F}_{14}$, we enter the magnitude 2.05×10^{-24} and the angle 60°. Then we add the vectors.

Method 2. *Summing in unit-vector notation.* First we rewrite $\vec{F}_{14}$ as

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{\mathrm{i}} + (F_{14}\sin\theta)\hat{\mathrm{j}}.$$

Substituting 2.05×10^{-24} N for F_{14} and 60° for θ, this becomes

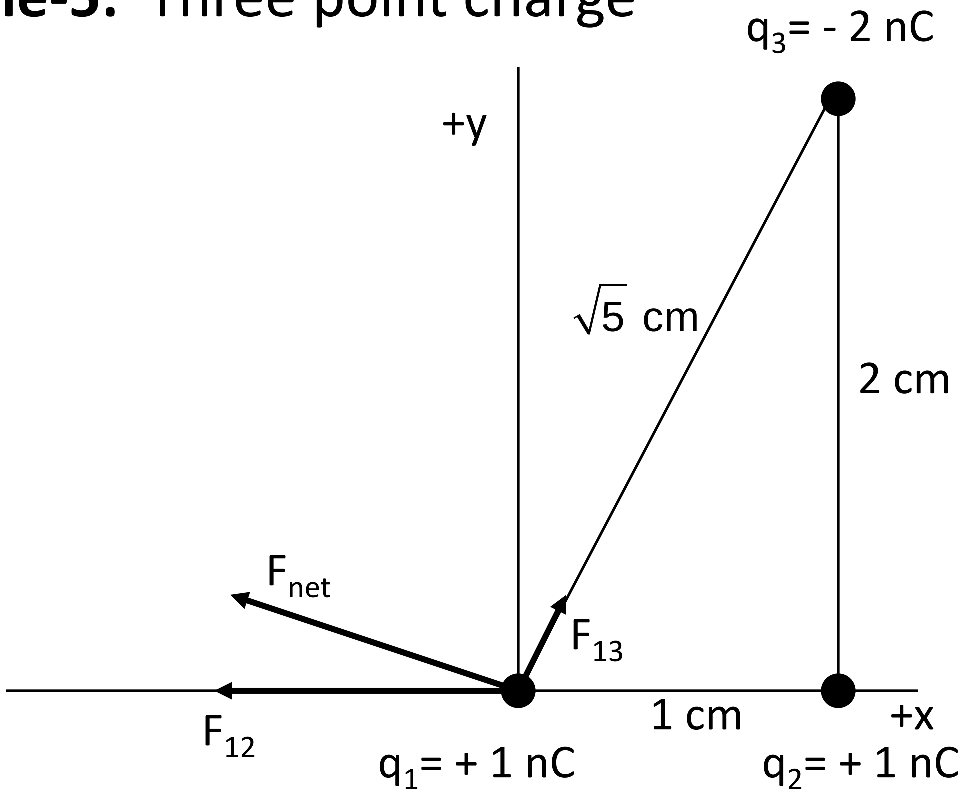
$$\vec{F}_{14} = (1.025 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (1.775 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{j}}.$$

Then we sum:

$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{14}$$
$$= -(1.15 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (1.025 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{i}} + (1.775 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{j}}$$
$$\approx (-1.25 \times 10^{-25}\ \mathrm{N})\hat{\mathrm{i}} + (1.78 \times 10^{-24}\ \mathrm{N})\hat{\mathrm{j}}.$$

(Answer)

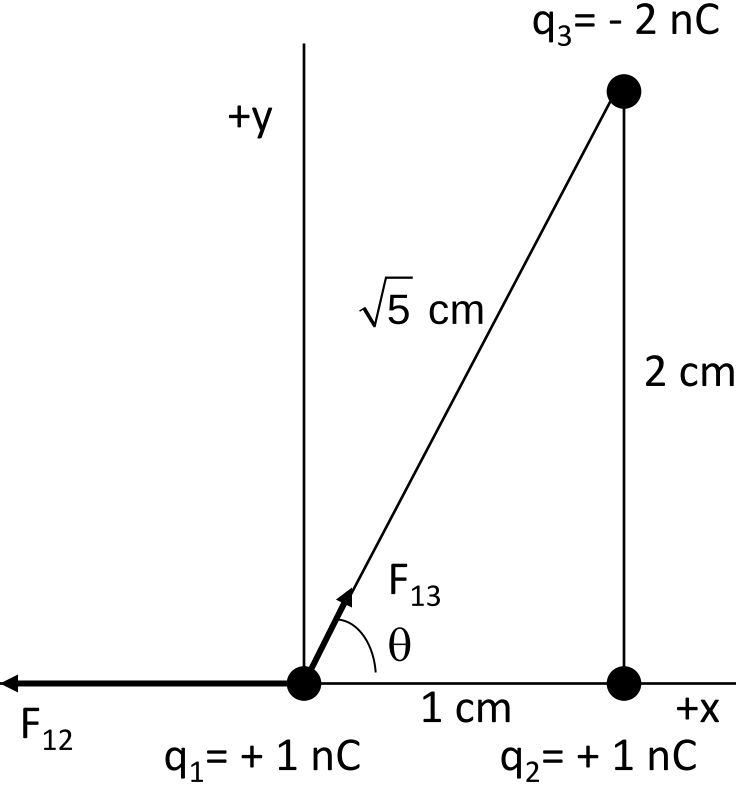
Example-5: Three point charge

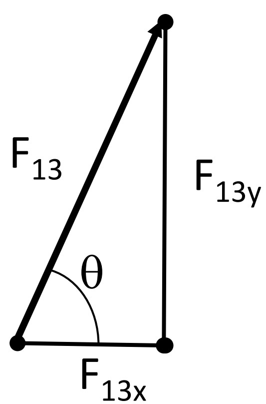


Question: What is the net force on q_1 and in what direction?

Hint : Find x and y components of force on q_1 due to q_2 and q_3 and add them up.

Solution:





x and y Components of force due to q_2

$$F_{12x} = -10^{10}\,\tfrac{\mathrm{Nm}^2}{\mathrm{C}^2}\,\frac{(10^{-9}\,\mathrm{C})^2}{(10^{-2}\,\mathrm{m})^2} = -1\times10^{-4}\,\mathrm{N}$$

$$F_{12y} = 0$$

x and y Components of force due to q_3

Magnitude of Force due to q_3

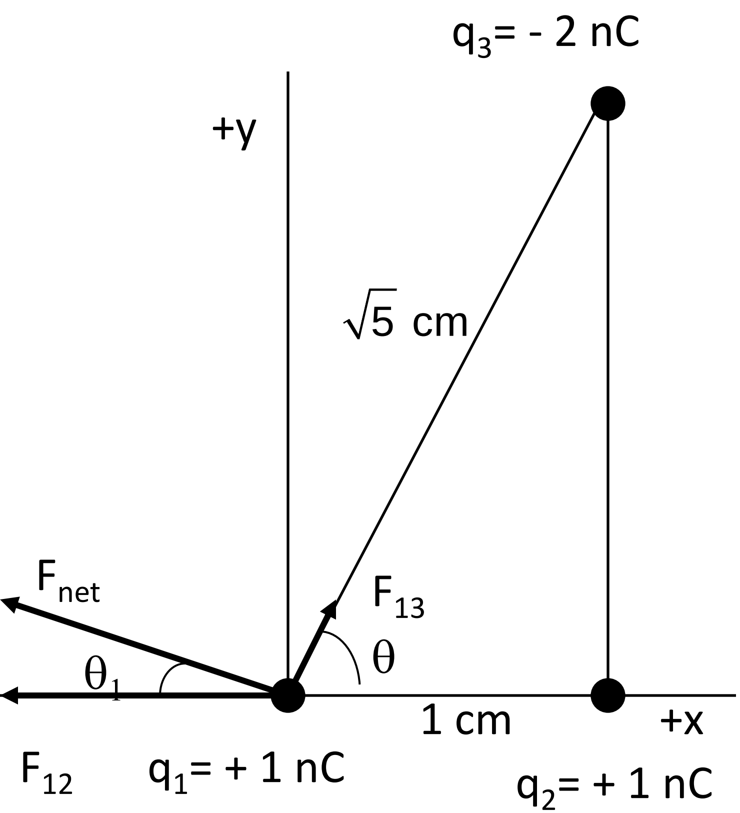
$$|F_{13}| = 10^{10}\,\tfrac{\mathrm{Nm}^2}{\mathrm{C}^2}\,\frac{(2\times10^{-9}\,\mathrm{C})(1\times10^{-9}\,\mathrm{C})}{(\sqrt{5}\times10^{-2}\,\mathrm{m})^2} = 0.40\times10^{-4}\,\mathrm{N}$$

$$\tan\theta = \frac{\text{y - axis value}}{\text{x - axis value}} \quad \Rightarrow \theta = \tan^{-1}(\tfrac{2}{1}) = 63.43\,\mathrm{deg}$$

$$F_{13x} = F\cos\theta = (0.40\mathrm{N})(\cos 63.43) = 0.179\times10^{-4}\,\mathrm{N}$$

$$F_{13y} = F\sin\theta = (0.40\mathrm{N})(\sin 63.43) = 0.358\times10^{-4}\,\mathrm{N}$$

Example Cont.



Total force along the x-axis

$$F_{x_{net}} = F_{12x} + F_{13x} = (-1\times10^{-4} + 0.179\times10^{-4})N = -0.821\times10^{-4}N$$

Total force along the y-axis

$$F_{y_{net}} = F_{12y} + F_{13y} = (0 + 0.358\times10^{-4})N = 0.358\times10^{-4}N$$

$$F_{net} = \sqrt{F_{x_{net}}^2 + F_{y_{net}}^2} = \sqrt{((0.821)^2 + (0.358)^2)\times(10^{-4})^2}N$$

$$F_{net} = +0.802\ 10^{-4}N$$

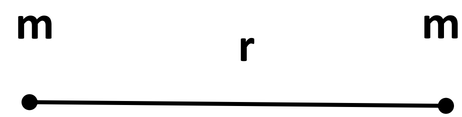
$$\tan\theta_1 = \frac{F_y}{F_x}$$

$$\theta_1 = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad \theta_1 = \tan^{-1}\left(\frac{0.358\times10^{-4}N}{-0.821\times10^{-4}N}\right)$$

$$\theta_1 = 23.6^\circ \text{ from the } -x \text{ axis}$$

Example-6: In an atom can we neglect the gravitational force between the electrons and protons? What is the ratio of Coulomb's electric force to Newton's gravity force for 2 electrons separated by a distance r ?

Solution:



$$\mathbf{F_c = \frac{kee}{r^2}}$$

$$F_g = \frac{Gmm}{r^2}$$

$$\frac{F_c}{F_g} = \frac{ke^2}{Gm^2}$$

$$\frac{F_c}{F_g} = \frac{\left(10^{10}\, Nm^2/C^2\right)\left(1.6\times10^{-19}\, C\right)^2}{\left(6.67\times10^{-11}\, Nm^2/kg^2\right)\left(9.1\times10^{-31}\, kg\right)^2}$$

$$= 4.6\times10^{42}$$

Huge number, pure ratio