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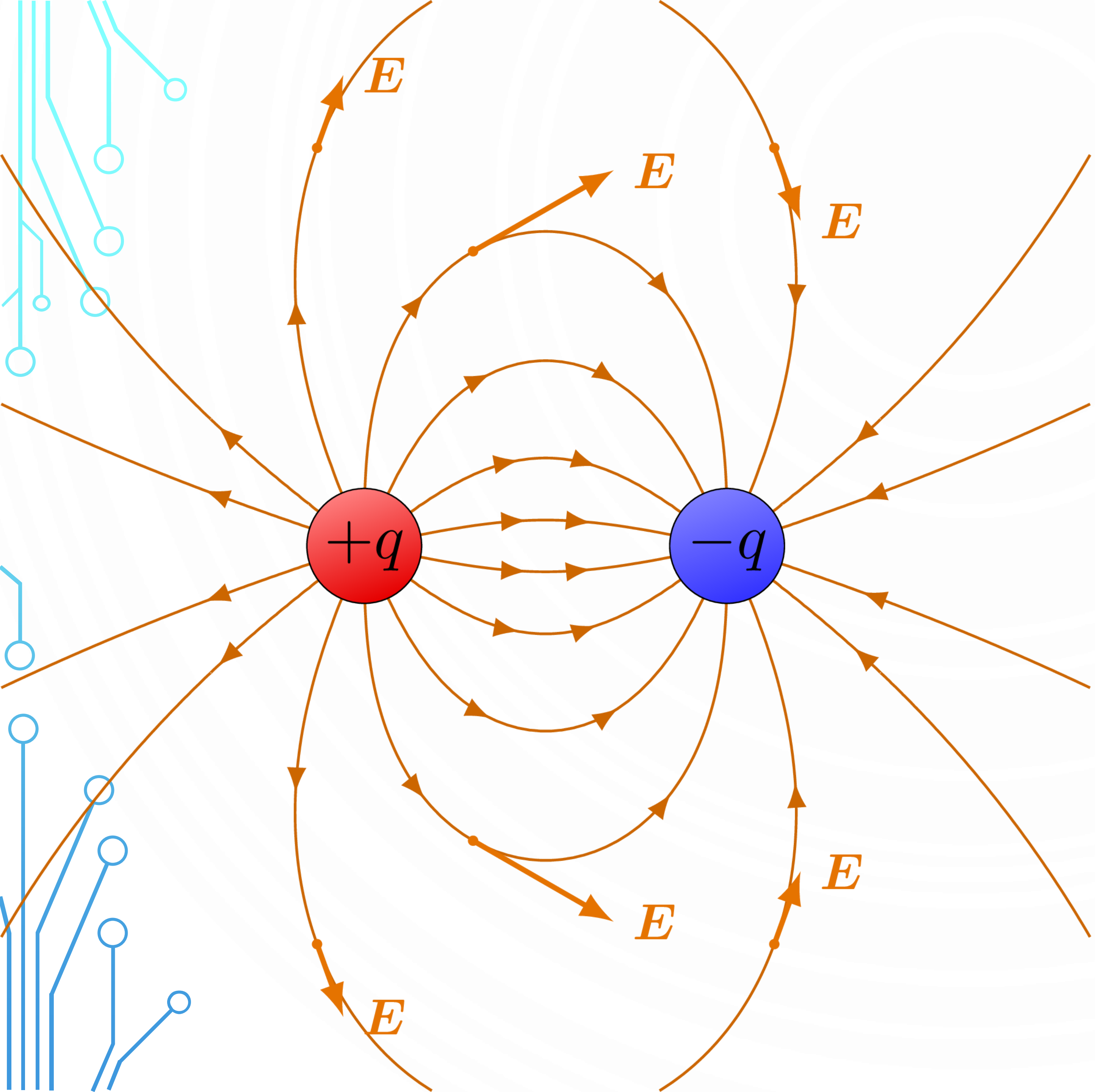
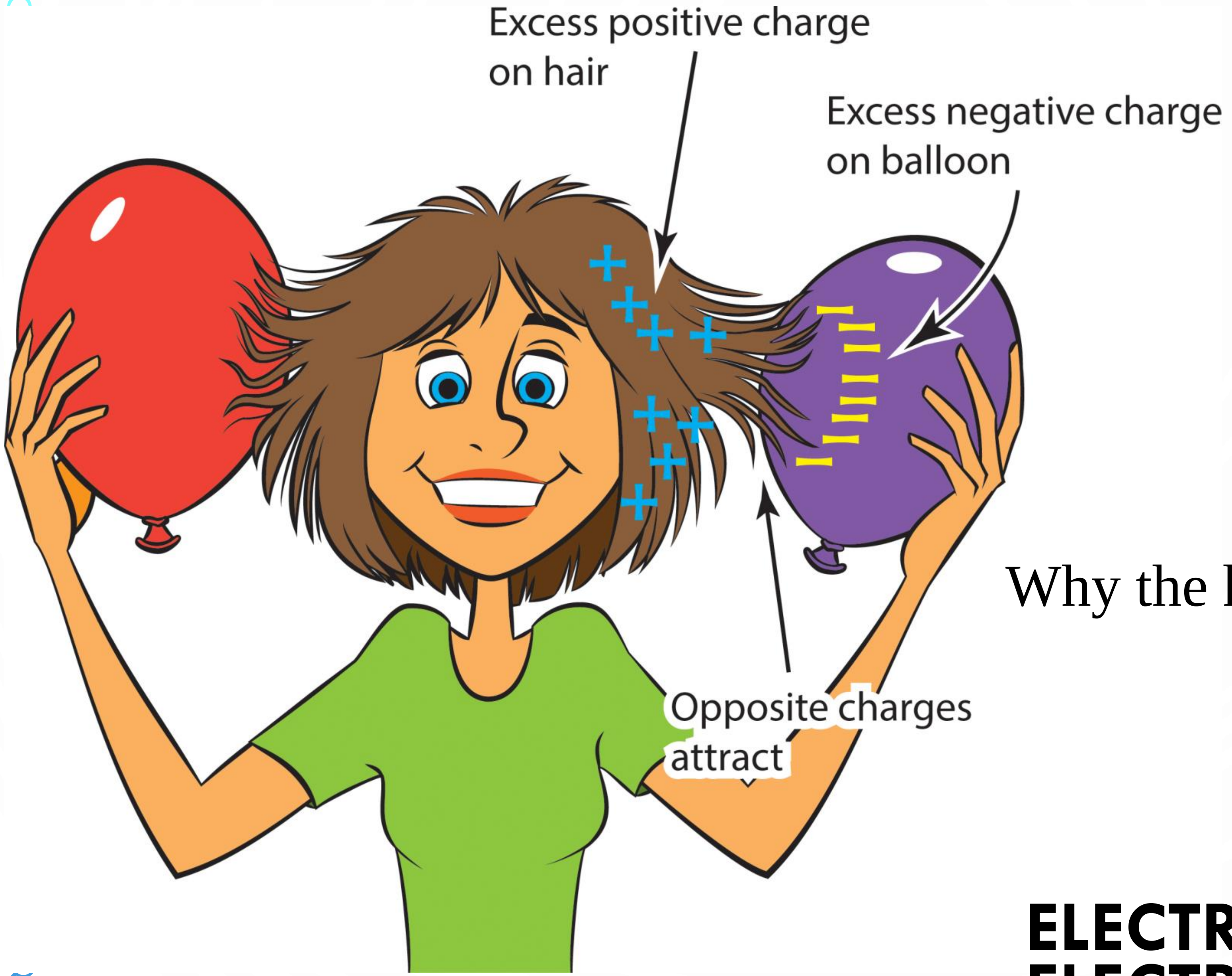



# ELECTRIC FIELD

SAGAR DUTTA




Why the hair attracts the balloons?

# ELECTRIC CHARGE, ELECTRIC FIELD

# TODAY'S TOPICS

- History of Electric Field
- What is Electric Field?
- Electric Field Lines
- Electric Field due to a point charge
- Electric Field due to an electric dipole

# DISCOVERY

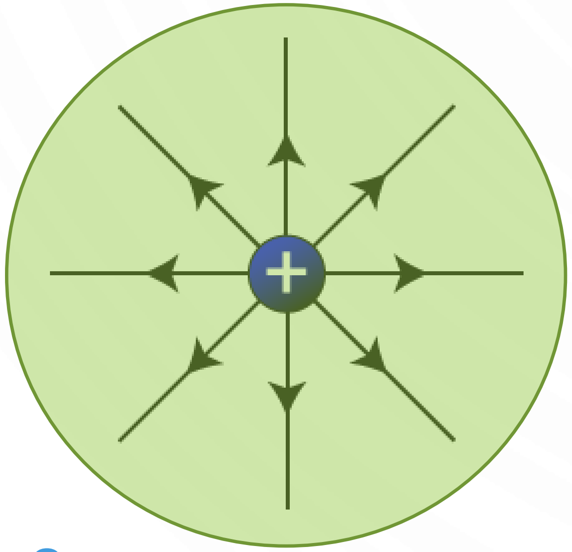
- The concept of the electric field was introduced by Michael Faraday.

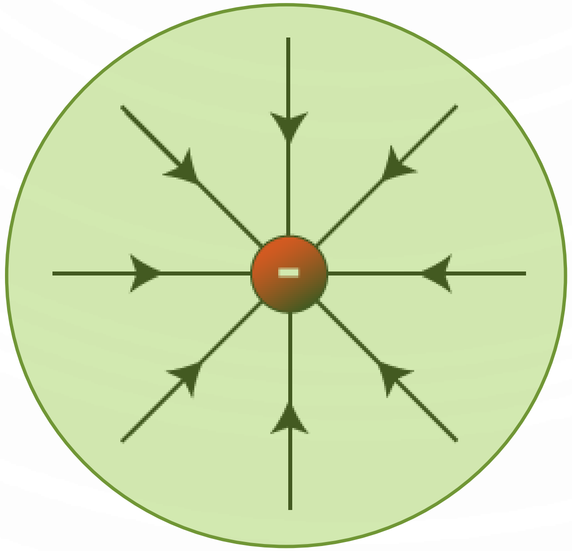
- An electric field is created by a charged body in the space that surrounds it, and results in a force exerted on any other charges placed within the field.

# ELECTRIC FIELD

An electric field (sometimes E-field) is the physical field that surrounds electrically charged particles and exerts force on all other charged particles in the field, either attracting or repelling them.

The S. I . Unit of electric field is N/C



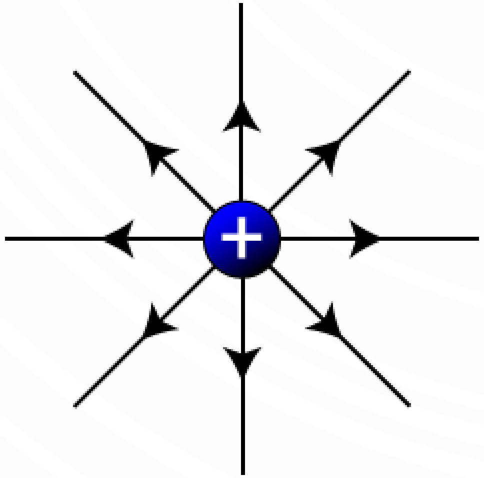


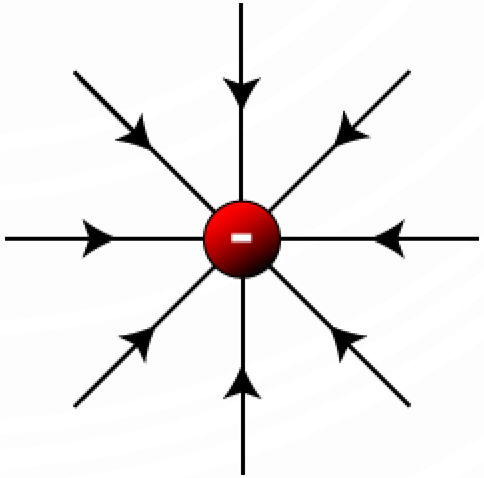
If **F** is the electrostatic force experienced by test charge $q_0$ with electric field **E**, then

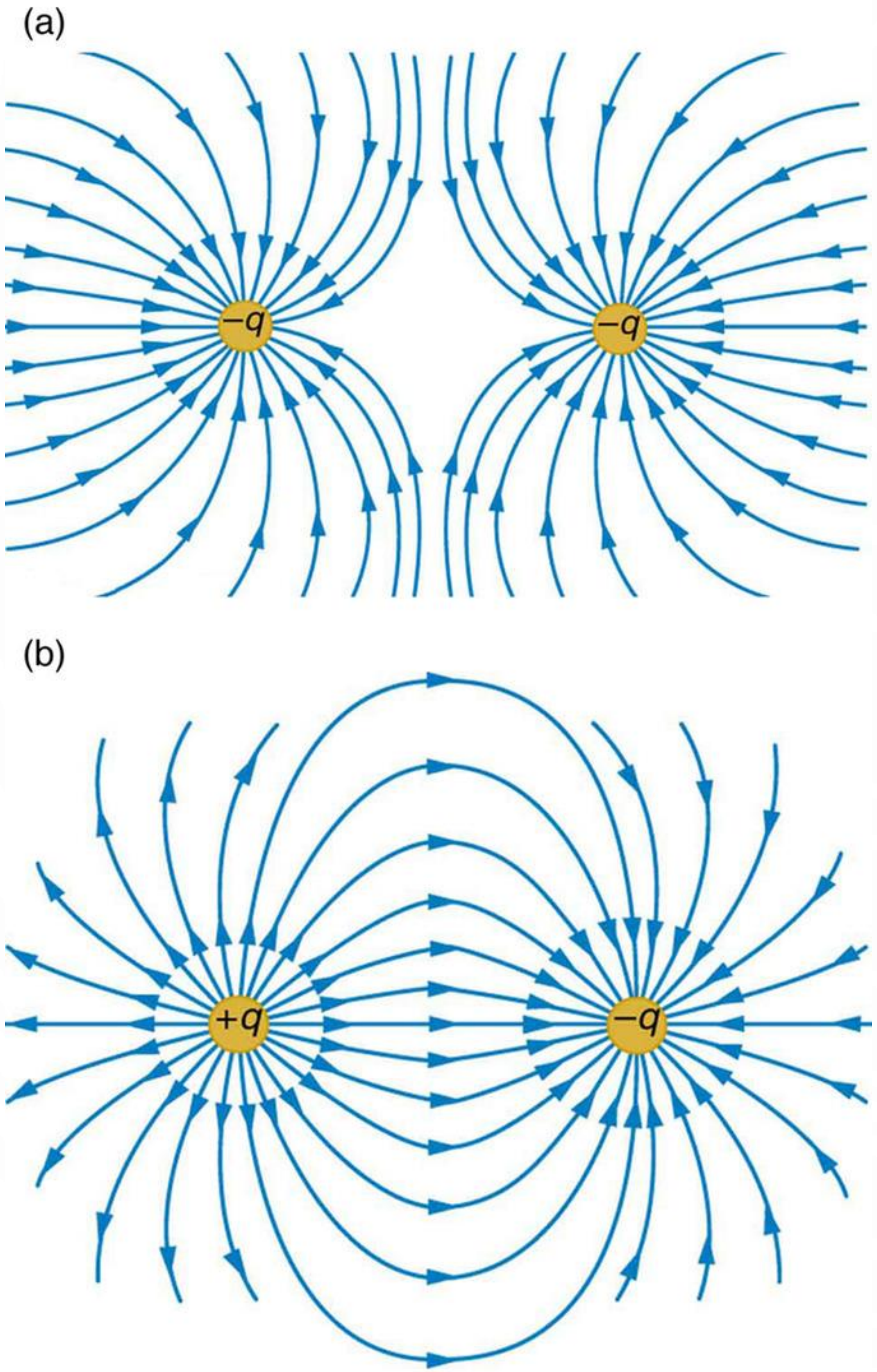
$$\vec{E} = \frac{\vec{F}}{q_0}$$

# ELECTRIC FIELD LINES

An *electric field line* is an imaginary line or curve drawn through a region of empty space so that its tangent at any point is in the direction of the electric field vector at that point.



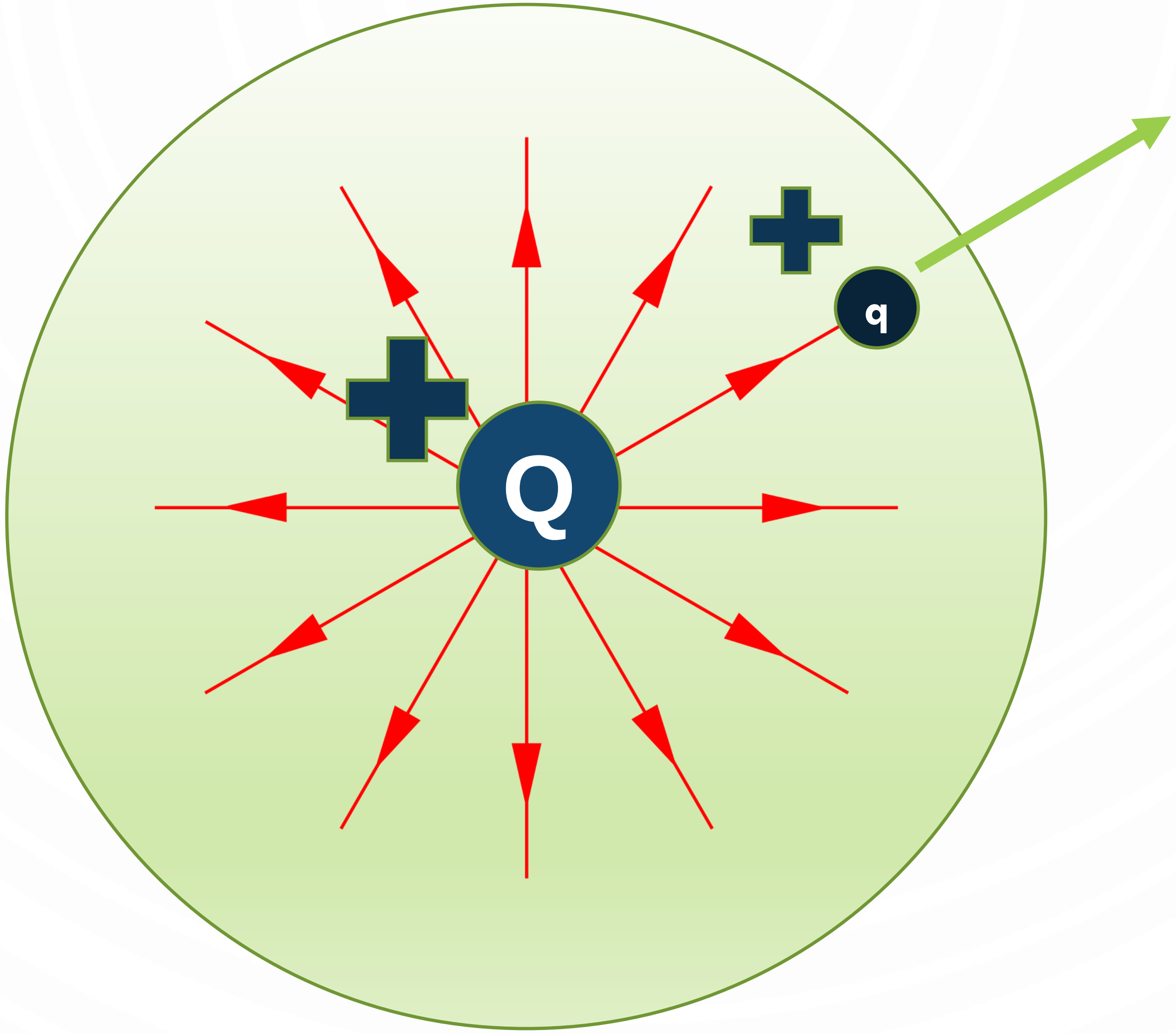





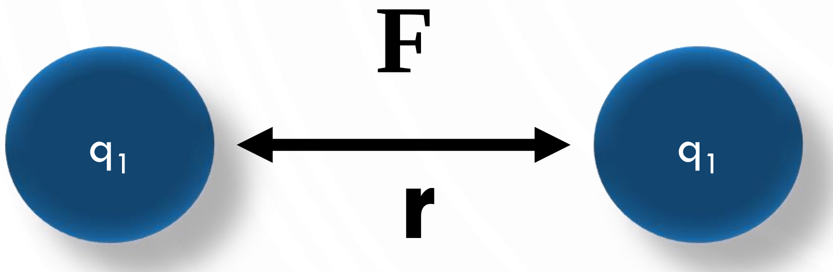

# PROPERTIES OF ELECTRIC FIELD LINES

1. Electric field lines always begin on a positive charge and end on a negative charge
2. Never form closed curves.
3. The number of electric field lines ∝ magnitude of the charge.
4. Electric field lines never intersect.
5. The field lines are straight, parallel and uniformly spaced In an uniform electric field.
6. The electric field lines can never form closed loops, as line can never start and end on the same charge.
7. These field lines always flow from higher potential to lower potential.
8. If the electric field in a given region of space is zero, electric field lines do not exist.
9. The tangent to a line at any point gives the direction of the electric field at the point.

# ELECTRIC FIELD DUE TO POINT CHARGE



# COULOMB'S LAW



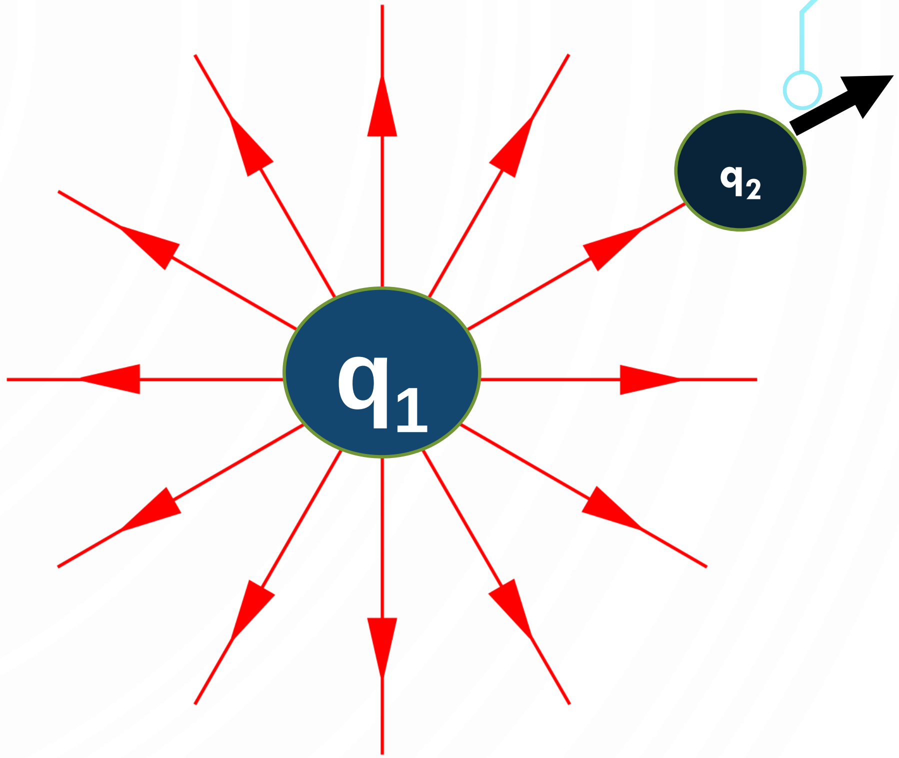

Let ,
The amount of two charges that acts here = $q_1$ and $q_2$
Electrostatic force between the two charges = **F**
Here r **=** distance between the two charges
$k = \frac{1}{4\pi\varepsilon o}$ = Coulombs Constant
$k = 8.987752 \times 10^{9}\ Nm^2/C^2$

$$\vec{F} = k\,\frac{q_1 q_2}{r^2}\,\hat{r} \qquad \text{(Coulomb's law),}$$

$\hat{r}$ is a unit vector along an axis extending through the two particles

# ELECTRIC FIELD DUE TO POINT CHARGE

Let ,
the strength of an electric field = $E$
The charge that creates the electric field = Q
Charge that has been placed in the electric field = q
Coulombs force exerted on the charge q placed in the electric field,
Here $\underline{r}$ = direction of electric field
$k = \frac{1}{4\pi\varepsilon o}$ = Coulombs Constant = $8.987752\times10^{9}\ Nm^2/C^2$




$$\mathbf{E} = \frac{F}{q2}\,\underline{\mathbf{r}}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

From Coulombs Law the force F between charge Q and q

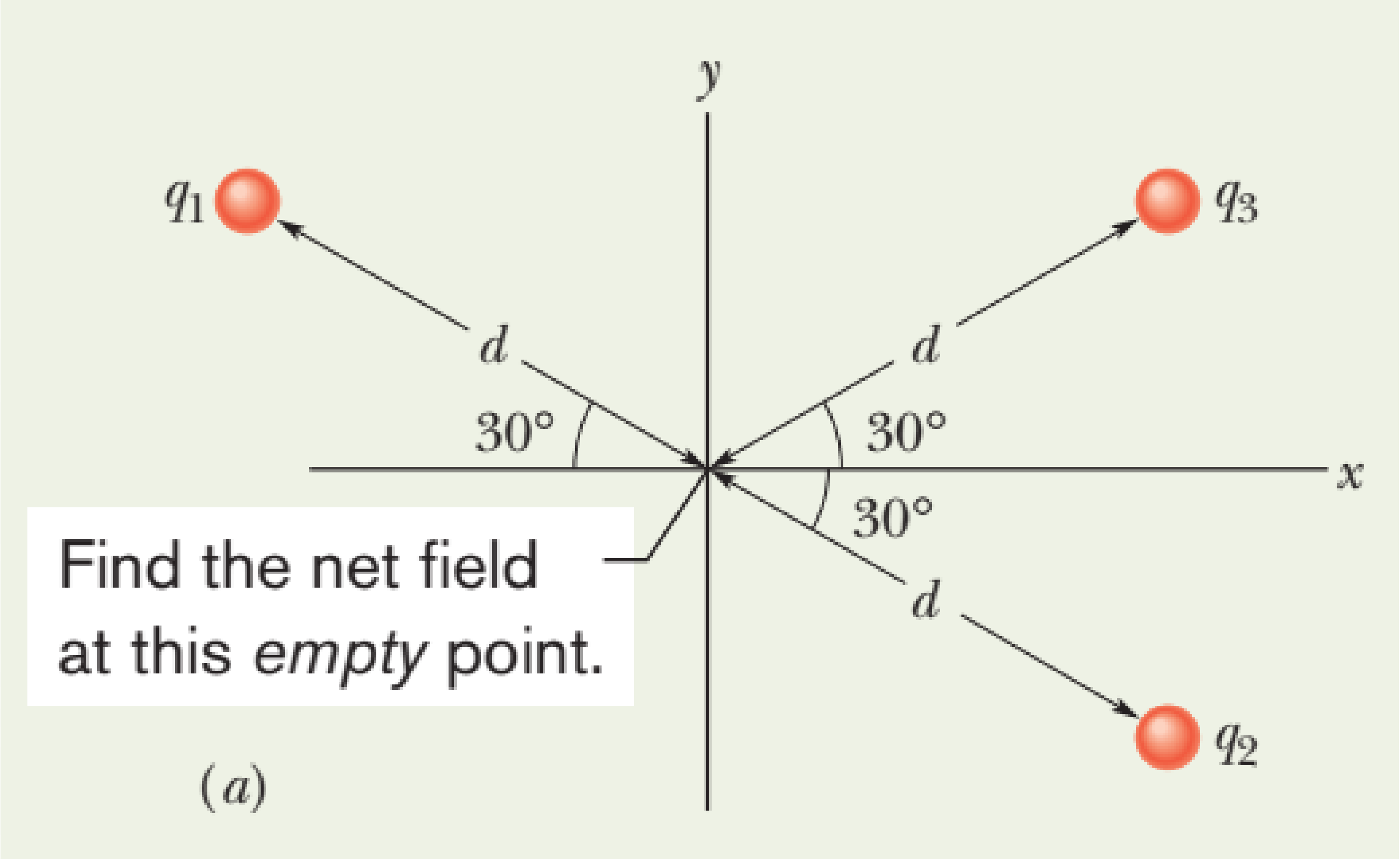
$$\mathbf{F} = \frac{kq1\,q2}{r^2}\,\underline{\mathbf{r}}\ \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

So the electric field generated by a charge Q

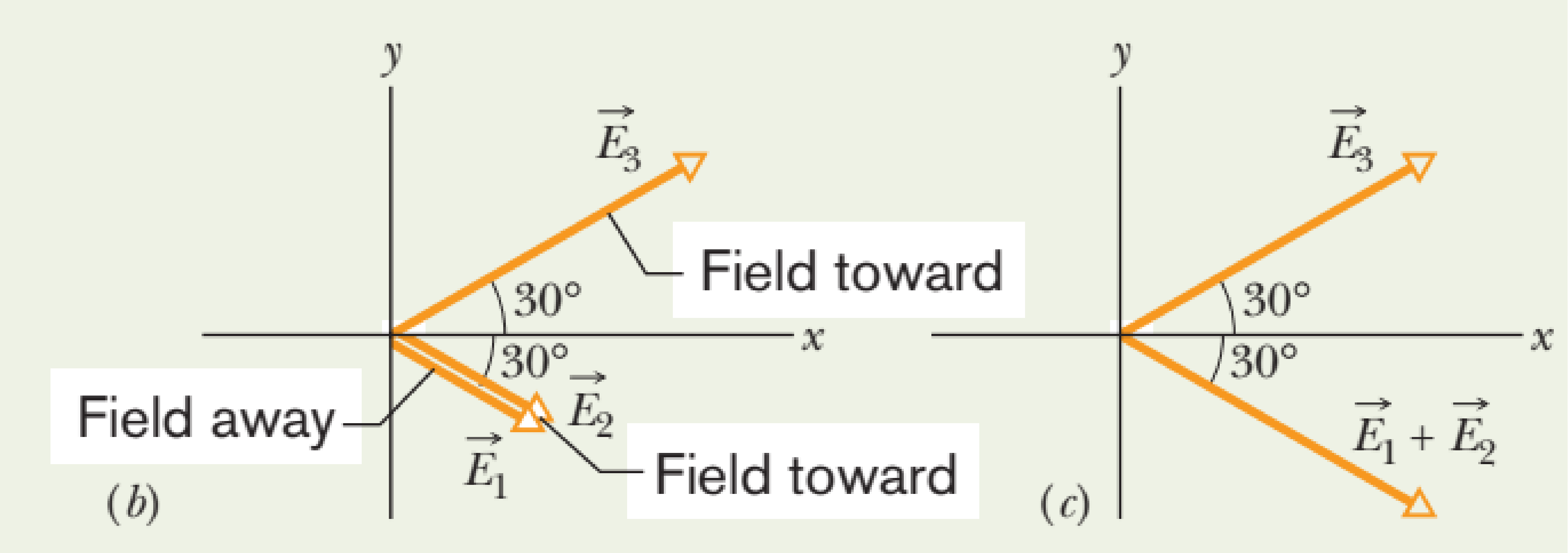
$$\mathbf{E} = \frac{kq1}{r^2}\,\underline{\mathbf{r}}\ \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

# PROBLEM

Figure 22-7$a$ shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance $d$ from the origin. What net electric field $\vec{E}$ is produced at the origin?



# SOLUTION


y
$\vec{E}_3$
Field toward
30°
x
30°
$\vec{E}_2$
Field away
$\vec{E}_1$
Field toward
(b)
y
$\vec{E}_3$
30°
x
30°
$\vec{E}_1 + \vec{E}_2$
(c)

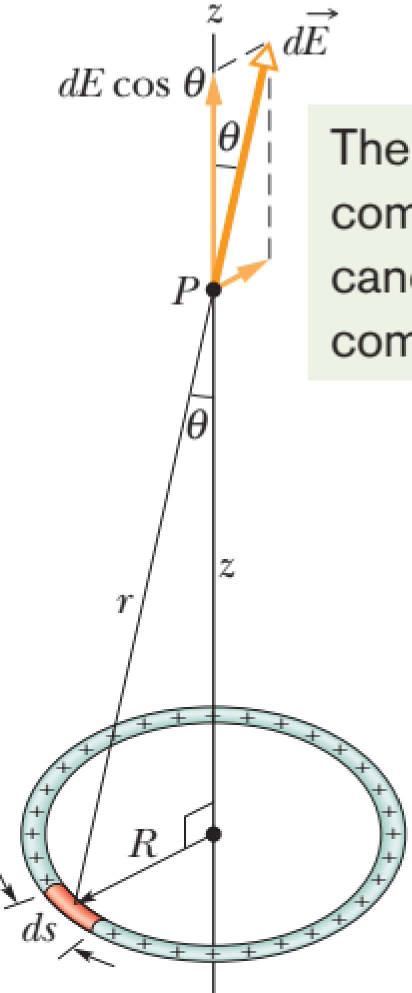
$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

$$E = 2E_{3x} = 2E_3 \cos 30^\circ$$
$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

# The Electric Field Due to a Line of Charge

Let $ds$ be the (arc) length of any differential element of the ring. Since $\lambda$ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds. \qquad (22\text{-}10)$$




The perpendicular components just cancel but the parallel components add.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

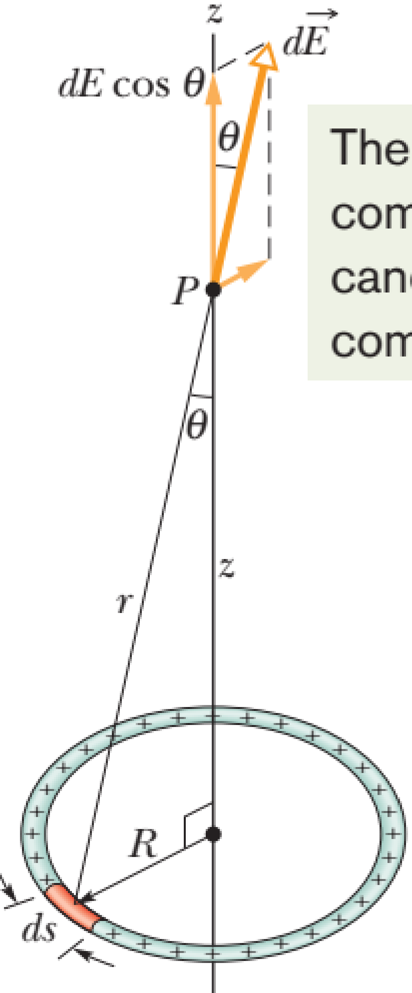
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}. \qquad \cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}.$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

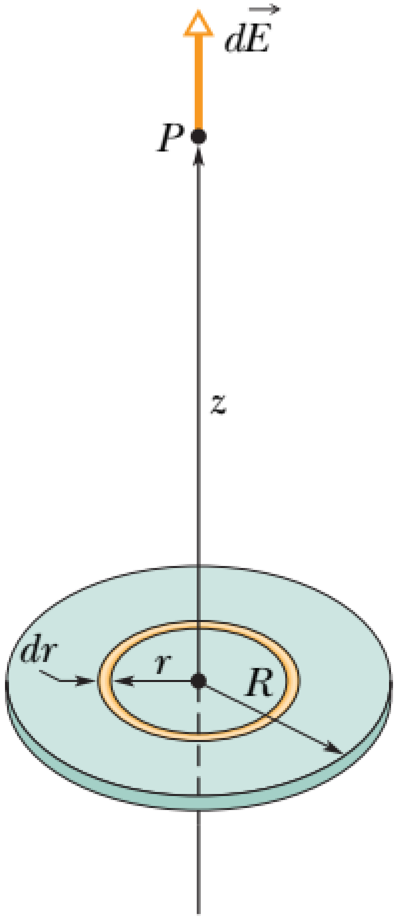
# The Electric Field Due to a Line of Charge




The perpendicular components just cancel but the parallel components add.

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$$

# The Electric Field Due to a Charged Disk




$$dq = \sigma\, dA = \sigma\,(2\pi r\, dr)$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0 (z^2 + r^2)^{3/2}}$$

$$dE = \frac{\sigma z}{4\varepsilon_0} \frac{2r\, dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr.$$

To solve this integral, we cast it in the form $\int X^m\, dX$ by setting $X = (z^2 + r^2)$, $m = -\frac{3}{2}$, and $dX = (2r)\, dr$. For the recast integral we have

$$\int X^m\, dX = \frac{X^{m+1}}{m+1},$$

and so Eq. 22-24 becomes

$$E = \frac{\sigma z}{4\varepsilon_0} \left[ \frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R. \tag{22-25}$$

# The Electric Field Due to a Charged Disk

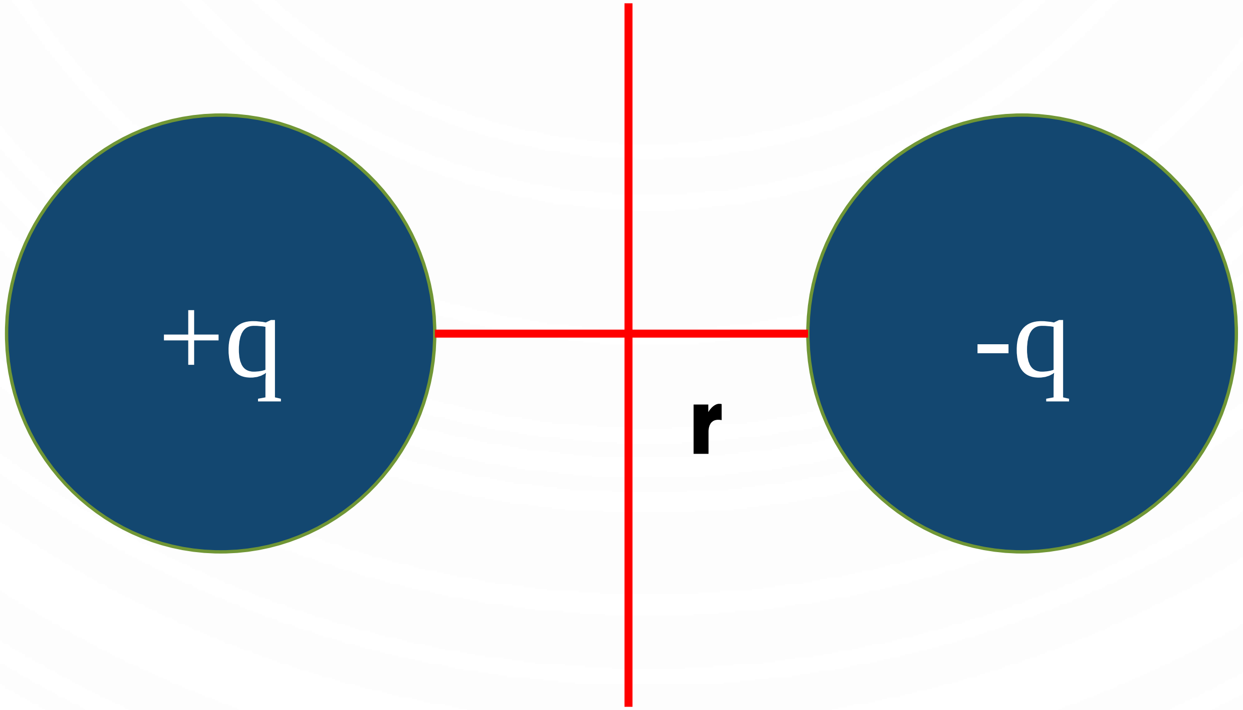
$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad \text{(charged disk)}$$

If we let $R \to \infty$ while keeping $z$ finite, the second term in the parentheses in Eq. 22-26 approaches zero, and this equation reduces to

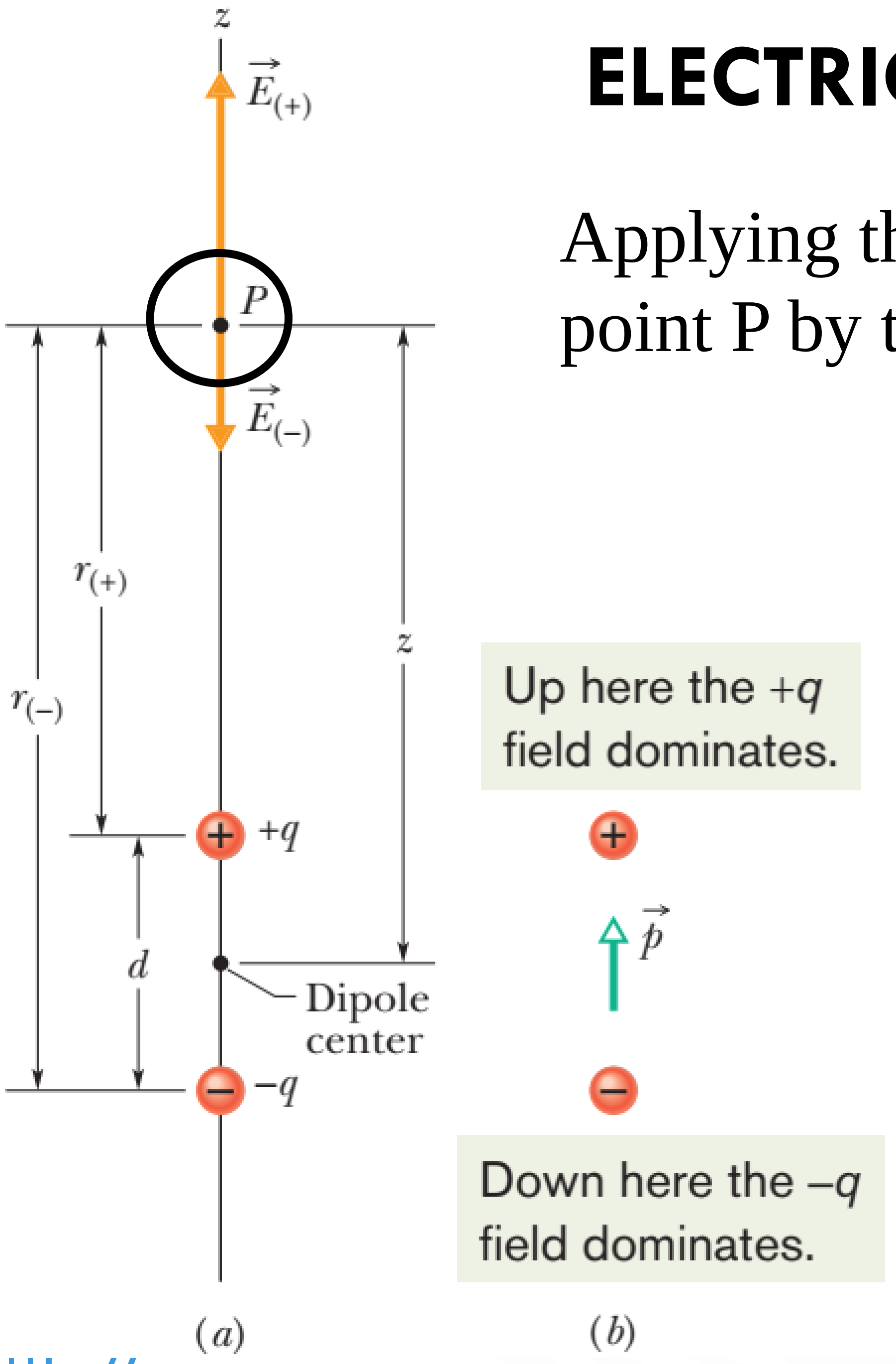
$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(infinite sheet).} \tag{22-27}$$

# ELECTRIC DIPOLE

An electric dipole is defined as a couple of opposite charges q and –q separated by a distance d.



# ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE




Applying the superposition theorem the electric field at point P by the dipoles , $E = E_{(+)} - E_{(-)}$

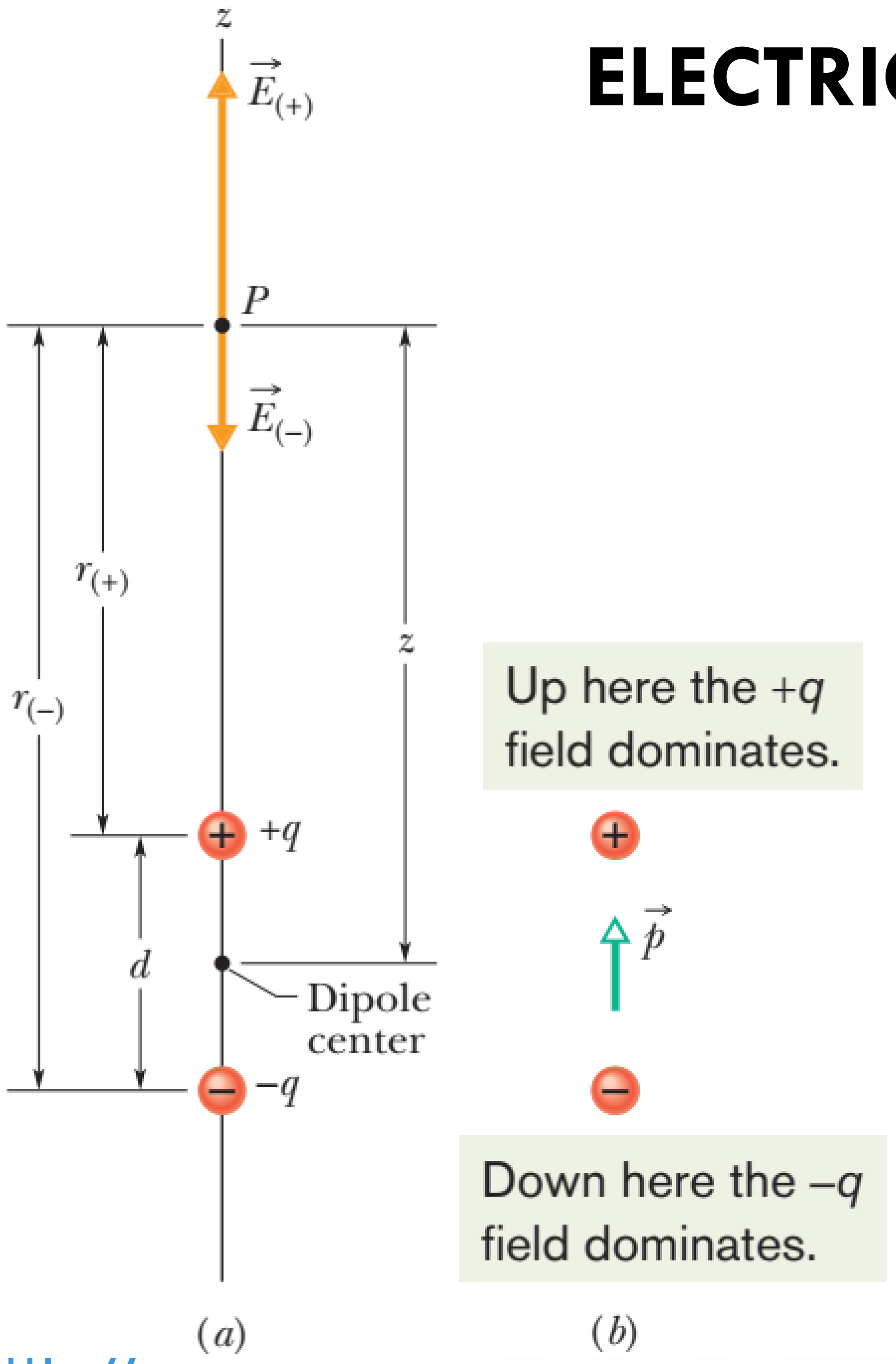
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}$$

Simplifying the equation we get

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

# ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE




$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$

**P** = qd

Here p = magnitude of electric dipole moment

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{(electric dipole)}$$

# PROBLEM

electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge $-q$ at cloud height $h$ and charge $+q$ at below-ground depth $h$ (Fig. 22-9*c*). If $q = 200$ C and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30$ km somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

***Calculations:*** We write that equation as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9*c*. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\text{ C})(2)(6.0\times10^3\text{ m})}{(30\times10^3\text{ m})^3}$$
$$= 1.6\times10^3\text{ N/C.} \qquad \text{(Answer)}$$

# PROBLEM

A neutral water molecule ($H_2O$) in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30}$ C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

***Calculations:*** There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

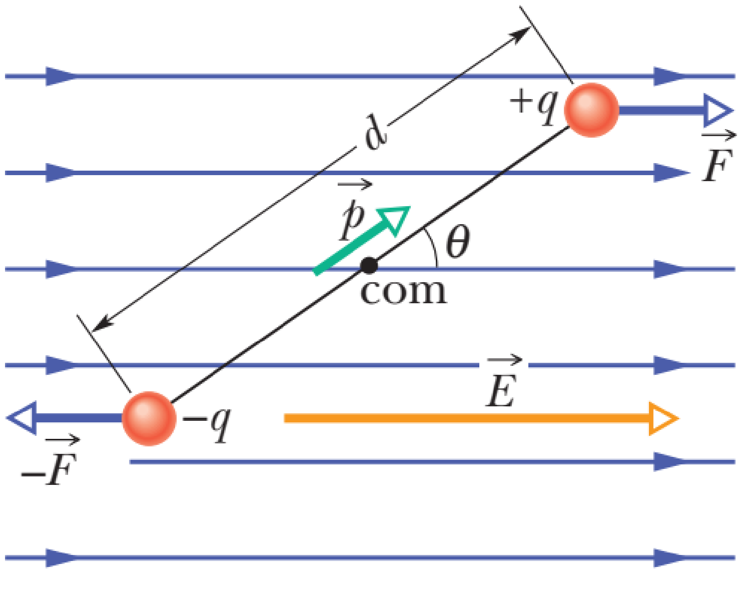
in which $d$ is the separation we are seeking and $e$ is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \mathrm{C\cdot m}}{(10)(1.60 \times 10^{-19}\ \mathrm{C})}$$

$$= 3.9 \times 10^{-12}\ \mathrm{m} = 3.9\ \mathrm{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

# A Dipole in an Electric Field

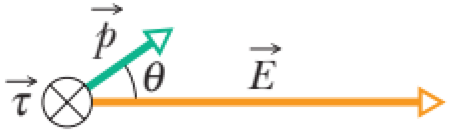



(a)

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. 22-19*a*) and with the same magnitude $F = qE$. Thus, *because the field is uniform,* the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance $x$ from one end and thus a distance $d - x$ from the other end. From Eq. 10-39 ($\tau = rF \sin \phi$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \tag{22-32}$$

The dipole is being torqued into alignment.




(b)

$$\tau = pE \sin \theta.$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

$$\tau = -pE \sin \theta.$$

(b) If the molecule is placed in an electric field of $1.5 \times 10^4$ N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

**Calculation:** Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\begin{aligned}\tau &= pE \sin \theta \\ &= (6.2 \times 10^{-30}\ \mathrm{C \cdot m})(1.5 \times 10^4\ \mathrm{N/C})(\sin 90°) \\ &= 9.3 \times 10^{-26}\ \mathrm{N \cdot m}. \qquad \text{(Answer)}\end{aligned}$$

# Potential Energy of an Electric Dipole

$$U = -W = -\int_{90°}^{\theta} \tau \, d\theta = \int_{90°}^{\theta} pE \sin\theta \, d\theta.$$

$$U = -pE\cos\theta.$$

$$U = -\vec{p} \cdot \vec{E}$$

$$W = -\Delta U = -(U_f - U_i),$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

$$
\begin{aligned}
W_a &= U_{180°} - U_0 \\
&= (-pE\cos 180°) - (-pE\cos 0) \\
&= 2pE = (2)(6.2 \times 10^{-30}\ \mathrm{C\cdot m})(1.5 \times 10^{4}\ \mathrm{N/C}) \\
&= 1.9 \times 10^{-25}\ \mathrm{J}. \qquad \text{(Answer)}
\end{aligned}
$$