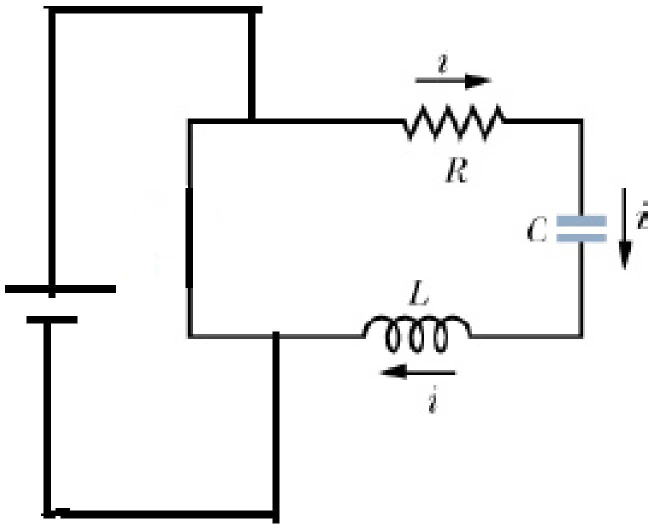
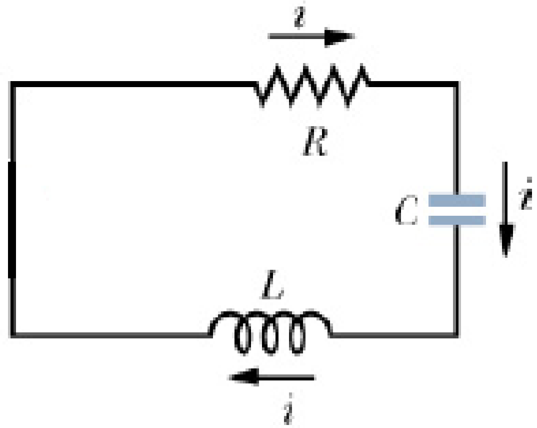
RLC circuit





- Voltage across resistor R

$$V_R = iR$$

- Voltage across capacitor C

$$V_C = \frac{Q}{C}$$

- Voltage across inductor L

$$V_L = L\frac{di}{dt}$$

- According to Kirchhoff's voltage law

$$iR + \frac{Q}{C} + L\frac{di}{dt} = 0$$

Rewrite the equation

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

Comparing with the equation

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0{}^2 x = 0$$

Where

$$\gamma = \frac{R}{L} \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

Three distinguish cases are

i) $$\frac{1}{LC} > \frac{R^2}{4L^2}$$ Oscillatory behavior

ii) $$\frac{1}{LC} = \frac{R^2}{4L^2}$$ Critical damping

iii) $$\frac{1}{LC} < \frac{R^2}{4L^2}$$ Over damping

Case i) $\frac{1}{LC} > \frac{R^2}{4L^2}$

Solution of the differential equation

$$Q(t) = Ae^{-\frac{R}{2L}t}\cos(\omega_1 t + \phi)$$

Where in damping,

Angular Frequency of oscillation, $\omega_1 = \sqrt{(\frac{1}{LC} - \frac{R^2}{4L^2})}$

Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{(\frac{1}{LC} - \frac{R^2}{4L^2})}$$

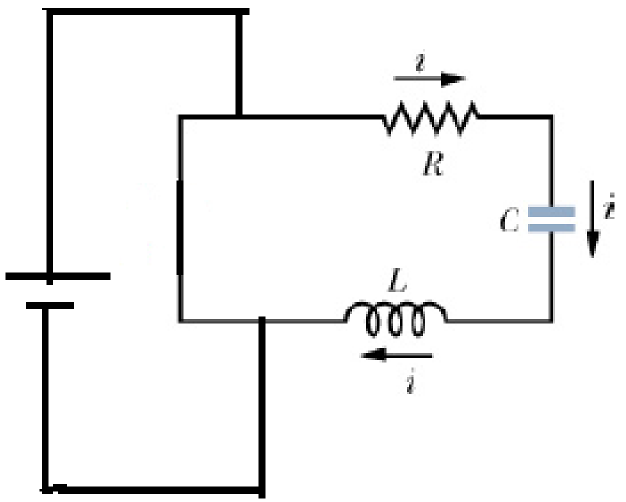
When resonance occurs, the driven frequency w_d from an external emf source is equal to the natural frequency w_0.

Simply we can calculate,

$$\omega_d = \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{(resonance).}$$

Angular Resonant Frequency,

$$\omega_0 = \sqrt{\frac{1}{LC}}$$



Linear Resonant Frequency,

$$f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)}$$

Here both of them are Resonant Frequency, if you are trying to calculate Linear resonant frequency you will calculate f_0 and if you want to calculate resonant angular frequency you will calculate w_0