1. $t = 0.25\ s$ one point to the next such point time.

Distance between two point is 36cm.

(a) period $T = 2t$

$= 2 \times 0.25$

$= 0.50\ s$

(b) frequency $f = \frac{1}{T}$

$= \frac{1}{0.50}$

$= 2\ Hz$

(c) amplitude $A = \frac{x}{2}$

$= \frac{36}{2}$

$= 18\ cm$

2. $m = 680\ gm$

$= 0.68\ kg$

$k = 65\ Nm^{-1}$

$x_{max} = A = 11\ cm$

$= 0.11\ m$

(i) $T = 2\pi\sqrt{m/k}$

$= 2\pi\sqrt{\frac{0.68}{65}}$

$= 0.643\ s$

time period

(i) angulare frequency $\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{65}{0.68}}$

$= 9.77$ r

(iii) phase constant

$x = A\cos(\omega t + \delta)$

$t=0,\ 11 = 11\cos\delta$

$\Rightarrow \cos\delta = 1$

$\Rightarrow \delta = \cos^{-1}(1)$

$= 0°$

(iv) $x = A\cos(\omega t + \delta)$

$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \delta)$

$t = 4\text{sec} \quad = -0.11 \times 9.77 \sin(9.77 \times 4 + 0)$

$= -1.055\ ms^{-1}$

$v_{max} = \omega A$

$= 9.77 \times 0.11$

$= 1.0747\ ms^{-1}$

(v) $v = -A\omega \sin(\omega t + \delta)$

$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t + \delta)$

$t = 4\text{sec} = -0.11 \times (9.77)^2 \cos(9.77 \times 4 + 0)$

$= -1.982 \ ms^{-2}$

$a_{max} = \omega^2 A$

$= (9.77)^2 \times 0.11$

$= 10.5 \ ms^{-2}$

(vi) displacement $x = A\cos(\omega t + \delta)$

$t = 0 \text{ sec} \quad x = 0.11 \cos(9.77 \times 0 + 0)$

$= 0.11 \ m$

$t = 7 \text{ sec} \quad x = 0.11 \cos(9.77 \times 7 + 0)$

$= 0.0823 \ m$

(vii) $v = \omega\sqrt{A^2 - x^2}$

$x = 0.11, \quad v = \sqrt{0.11 - 0.11} \times 9.77 = 0 \ ms^{-1}$

$x = 0.04 \quad v = 9.77\sqrt{0.11 - 0.04} = 1.0011 \ ms^{-1}$

[3] $m = 0.12\ kg$

$A = 8.5\ cm = 0.085\ m$

$T = 0.20\ s$

(a) Magnitude of the maximum force

$F_{max} = m a_{max}$

$= m\omega^2 A$

$= 0.12 \times (31.41)^2 \times 0.085$

$= 10.06\ N$

but $\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{0.20}$

$= 31.41\ rad/s$

(b) $\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow \frac{k}{m} = \omega^2$

$\Rightarrow k = \omega^2 m$

$= (31.41)^2 \times 0.12$

$= 118.4\ N/m$

[4] back and forth distance $2x = 2\ mm$

$\therefore x_{max} = \frac{2}{2}\ mm$

$= 1 \times 10^{-3}\ m$

$f = 120\ Hz$

(a) Amplitude $A = x_{max}$

$= 1 \times 10^{-3}\ m$

(b) maximum blade speed v_{max}

$v_{max} = \omega A$

$= 2\pi f \times A$

$= 2\pi \times 120 \times 1 \times 10^{-3}$

$= 0.754\ ms^{-1}$

(c)

$a_{max} = \omega^2 A$

$= (2\pi f)^2 \times 0.1 \times 10^{-3}$

$= 568.5\ ms^{-2}$

5. $k = 400\ N/m$; $x = 0.100\ m$

$v = -13.6\ ms^{-1}$; $a = -123\ ms^{-2}$

(a) $a = -\omega^2 x$

$\Rightarrow -123 = -\omega^2 \times 0.100$

$\Rightarrow \omega = \sqrt{\frac{123}{0.100}}$

$= 35.07\ rad/s$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{35.07}{2\pi}$$

$$= 5.58 \text{ Hz}$$

(b) Mass of the block m

$$\omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \frac{k}{m} = \omega^2$$

$$\Rightarrow m = \frac{k}{\omega^2} = \frac{400}{(35.07)^2}$$

$$= 0.325 \text{ kg}$$

(c)

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow 13.6 = 35.07\sqrt{A^2 - (0.100)^2}$$

$$\Rightarrow A = \sqrt{(0.100)^2 + \left(\frac{13.6}{35.07}\right)^2}$$

$$= 0.4004 \text{ m}$$

6. $m = 25 \text{ gm} = 0.025 \text{ kg}$

$k = 400 \text{ dyns/cm}$ $\quad v_{max} = A = 10 \text{ cm}$

$$= \frac{400 \times 10^{-5}}{10^{-2}} \text{ N/m} \qquad = 0.1 \text{ m}$$

$$= 0.4 \text{ N/m}$$

(i) Time period $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{0.025}{0.4}}$

$= 1.57\ s$

(ii) frequency $f = \frac{1}{T}$

$= \frac{1}{1.57}$

$= 0.637\ Hz$

(iii) angular frequency $\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{0.4}{0.025}}$

$= 4\ rad/s$

(iv) Maximum Velocity v_{max}

$v_{max} = \omega A$

$= 4 \times 0.1$

$= 0.4\ ms^{-1}$

7

$m = 1.68 \times 10^{-27}\ kg$

$f = 1 \times 10^{14}\ Hz$

$A = 1 \times 10^{-10}\ m =$ ✗

$F = kx$

$F = (2\pi f)^2 \times m \times x$

$= (2\pi \times 10^{14})^2 \times 1.68 \times 10^{-27} \times 1 \times 10^{-10}$

$= 6.63 \times 10^{-10}$ N

$\omega^2 = \frac{k}{m}$

$\Rightarrow (2\pi f)^2 = \frac{k}{m}$

$\Rightarrow k = (2\pi f)^2 \times m$

8

$v_{max} = \omega A$ —— (i)

$a_{max} = \omega^2 A$ —— (ii)

(ii) ÷ (i) $\frac{\omega^2 A}{\omega A} = \frac{1.57}{1}$

$\Rightarrow \omega = 1.57$ rad/s

Time period $T = \frac{2\pi}{\omega}$

$= \frac{2\pi}{1.57}$

$= 4$ s

9

$A = 5$ m

$x = 3$ m

$a = 48$ ms^{-2}

$$a = \omega^2 x$$

$$\Rightarrow \omega^2 = \frac{a}{x}$$

$$\Rightarrow \omega = \sqrt{a/x}$$

$$= \sqrt{\frac{48}{3}}$$

$$= 4 \text{ rad/s}$$

(i) velocity $v = \omega\sqrt{A^2 - x^2}$

$$= 4\sqrt{5^2 - 3^2}$$

$$= 16 \text{ ms}^{-1}$$

(ii) Time period $T = \frac{2\pi}{\omega}$

$$= \frac{2\pi}{4}$$

$$= 1.57 \text{ s}$$

(iii) Maximum velocity v_{max}

$$v_{max} = \omega A$$

$$= 4 \times 5$$

$$= 20 \text{ ms}^{-1}$$

10

$x_{max} = 0.37\ cm$

$f = 0.25\ Hz$

(i) Time period $T = \frac{1}{f} = \frac{1}{0.25} = 4\ s$

(ii) Angular frequency $\omega = 2\pi f$

$= 2\pi \times 0.25$

$= 1.57\ rad/s$

$= \pi/2$

(iii) Amplitude $A = x_{max}$

$= 0.37\ cm$

$= 0.0037\ m$

10 (iv) displacement, Calculator in Radian Mode

$$x = A\cos(\omega t + \delta)$$

$$= 0.0037 \cos\left(\frac{\pi}{2} \times 3 + 0\right)$$

$A = 0.0037$ rad/s
$t = 3s$

$$\therefore x = 0$$

(v) $$v = \frac{dx}{dt}$$

$$= -A\omega \sin(\omega t + \delta)$$

$$= -0.0037 \times \frac{\pi}{2} \sin\left(\frac{\pi}{2} \times 3 + 0\right)$$

$$= 5.809 \times 10^{-3} \, ms^{-1}.$$

[11]

$$x = 10\cos(3\pi t + \pi/3)$$

(i) displacement at $t = 2.5\,s$

$$x = 10\cos(3\pi \times 2.5 + \pi/3)$$

$$= 3.66\ m$$

(ii) Velocity (calculator in Radian Mode)

$$v = \frac{dx}{dx} = -10 \times 3\pi \sin(3\pi t + \pi/3)$$

$$t = 3\,s = -10 \times 3\pi \sin(3\pi \times 3 + \pi/3)$$

$$= 81.62\ ms^{-1}$$

(iii) $$a = \frac{dv}{dt} = -10 \times (3\pi)^2 \cos(3\pi t + \pi/3)$$

$$t = 2\ sec = -10 \times (3\pi)^2 \cos(3\pi \times 2 + \pi/3)$$

$$= -444.13\ ms^{-2}$$

(iv) $$V_{max} = \omega A$$

$$= 3\pi \times 10$$

$$= 94.24\ ms^{-1}$$

12

$$x = 10\sin(10t - \pi/6)$$

$A = 10, \quad \omega = 10$

(i) $f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.592 \text{ Hz}$

(ii) $T = \frac{1}{f} = \frac{1}{1.592} = 0.6285 \text{ s}$

(iii) maximum displacement x_{max}

$$x_{max} = A$$
$$= 10$$

(iv) maximum velocity v_{max}

$$v_{max} = \omega A$$
$$= 10 \times 10$$
$$= 100 \text{ ms}^{-1}$$

(v) maximum acceleration a_{max}

$$a_{max} = \omega^2 A$$
$$= 10^2 \times 10$$
$$= 1000 \text{ ms}^{-2}$$

13

$A = 4\,m$

$f = 0.5\,Hz$ $\quad \omega = 2\pi f = 2\pi \times 0.5 = \pi$

$\delta = \pi/4$

(i) $T = \frac{1}{f} = \frac{1}{0.5} = 2\,s$

(ii) $x = A\sin(\omega t + \delta)$

$= 4\sin(\pi t + \pi/4)$

(iii) $x = 4\sin(\pi t + \pi/4)$

$v = \frac{dx}{dt} = 4 \times \pi \cos(\pi t + \pi/4)$

$t = 5sec = 4 \times \pi \cos(\pi \times 5 + \pi/4)$

$= -8.8857\ ms^{-1}$

$a = \frac{dv}{dt} = -4\pi^2 \sin(\pi t + \pi/4)$

$t = 5sec = -4\pi^2 \sin(\pi \times 5 + \pi/4)$

$= 27.91\ ms^{-2}$

Problem 14: Given, $m = 2\,kg$

$k = 196\ N/m$

$A = 5\,cm = 0.05\,m$

$t = 0$

a) ω, f, T

Soln: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{196}{2}} = 9.9\ rad/s$

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{9.9}{2\pi} = 1.58\ Hz$

$\therefore T = \frac{1}{f} = \frac{1}{1.58} = 0.635$

b) x vs t eqn.

Soln:

$x = 0.05\cos(9.9t)$ | $\omega = 9.9\ rad/s$, $\delta = 0$

15 $m = 500\ gm$

$= 0.5\ kg$

$x = 7\ cm$

$= 0.07\ m$

When $x = 3\ cm$ velocity $v = 40\ cm/s$

$= 0.4\ m/s$

(i) $F = mg$

$F = kx$

$mg = kx$

$\Rightarrow k = \frac{mg}{x} = \frac{0.5 \times 9.8}{0.07}$

$= 70\ N/m$

(ii) $\omega = \sqrt{k/m} = \sqrt{\frac{70}{0.5}}$

$= 11.83\ rad/s$

(iii) $T = \frac{2\pi}{\omega} = \frac{2\pi}{11.83} = 0.53\ s$

(iv) Initial potential energy $U = \frac{1}{2}kx^2$

$= \frac{1}{2} \times 70 \times (0.07)^2$

$= 0.1715\ j$

(v) The initial kinetic energy

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \times 0.5 \times (0.4)^2$$

$$= 0.04 \text{ J}$$

(vi) $E_{total} = E_K + E_P$

$$= 0.04 + 0.1715$$

$$= 0.2115 \text{ J}$$

$$E = \frac{1}{2} k A^2$$

$$\Rightarrow A^2 = \frac{2E}{k}$$

$$\Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \times 0.2115}{70}}$$

$$= 0.077 \text{ m}$$

$= 0.025 \text{ kg}$

16

$k = 400 \text{ dyns/cm}$

$= \frac{400 \times 10^{-5}}{10^{-2}}$

$= 0.4 \text{ N/m}$

(a) Time period $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{0.025}{0.4}}$

$= 1.57 \text{ s}$

(b) frequency $f = \frac{1}{T} = \frac{1}{1.57}$

$= 0.636 \text{ Hz}$

(c) angular frequency $\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{0.4}{0.025}}$

$= 4 \text{ rad/s}$

(e) Amplitude $A = \sqrt{\frac{v^2}{\omega^2} + x^2}$

$= \sqrt{\frac{(40\times10^{-2})^2}{4^2} + (10\times10^{-2})^2}$

$= 0.02 \text{ m}$

(f) The angle $\theta = \tan^{-1}\left(\frac{x\omega}{v}\right)$

$= \tan^{-1}\left(\frac{10\times10^{-2}\times 4}{40\times10^{-2}}\right)$

$= 45^\circ \text{ or } \frac{\pi}{4}$

(d) Total energy $= \frac{1}{2} k A^2$

$= \frac{1}{2} \times 0.4 \times (0.02)^2$

$= 8 \times 10^{-5} \text{ J}$

(g) $v_{max} = \omega A$

$= 4 \times 0.02$

$= 0.08 \ ms^{-1}$

(h) $a_{max} = \omega^2 A$

$= 4^2 \times 0.02$

$= 0.32 \ ms^{-2}$

(i) $x = A \sin(\omega t + \delta)$

$v = \frac{dx}{dt} = A\omega \cos(\omega t + \delta)$

$= 0.02 \times 4 \cos(4t + \pi/4)$

$= 0.08 \cos(4t + \pi/4)$

$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \delta)$

$= -.32 \sin(4t + \pi/4)$

(j) $x = 0.02 \sin(4 \times \pi/8 + \pi/4)$

$t = \pi/8 \quad = 0.0141 \ m$

$v = 0.08 \cos(4 \times \pi/8 + \pi/4)$

$= -0.0565 \ ms^{-1}$

$$x_{max} = A = 20\ cm$$
$$= 0.2\ m$$

(17) $\omega^2 = \frac{k}{m}$

$\Rightarrow k = \omega^2 m$

$= (2\pi f)^2 \times 2.72 \times 10^5$

$= (2\pi \times 10)^2 \times 2.72 \times 10^5$

$= 1.07 \times 10^9\ N/m$

(a) Mechanical energy $= \frac{1}{2} k A^2$

$= \frac{1}{2} \times 1.07 \times 10^9 \times$

$= 21.47 \times 10^6$

(b) In equilibrium point $x = 0$

$\therefore v = \sqrt{A^2 - x^2}\,\omega$

$= \sqrt{A^2}\,\omega = \omega A$

$= 2\pi f \times 0.2$

$= 12.56\ ms^{-1}$