Lecture: Standing Waves

Ref book: Physics for Engineers - Giasuddin Ahmad (Part-1)

University Physics - Sears, Zemansky, Young & Freedman

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Web ref provided on slides

Simple Harmonic Motion: Review

The position x of an object moving in simple harmonic motion as a function of time has the following form:

$$x = A \cos(\omega t + \phi)$$

i.e. the object periodically moves back and forth between the amplitudes $x=+A$ and $x=-A$.

The time it takes for the object to make one full cycle is the period $T=2\pi/\omega=1/f$, where f is the frequency of the motion.

Thus, the angular speed in terms of T and f reads

$$\boldsymbol{\omega = 2\pi/T} \quad \textbf{and} \quad \boldsymbol{\omega = 2\pi f}$$

Ref: toppr.com

Wave Nature

- Nature of waves:
  - A wave is a traveling disturbance that transports energy from place to place.
  - There are two basic types of waves: transverse and longitudinal.
  - Transverse: the disturbance travels perpendicular to the direction of travel of the wave.
  - Longitudinal: the disturbance occurs parallel to the line of travel of the wave.

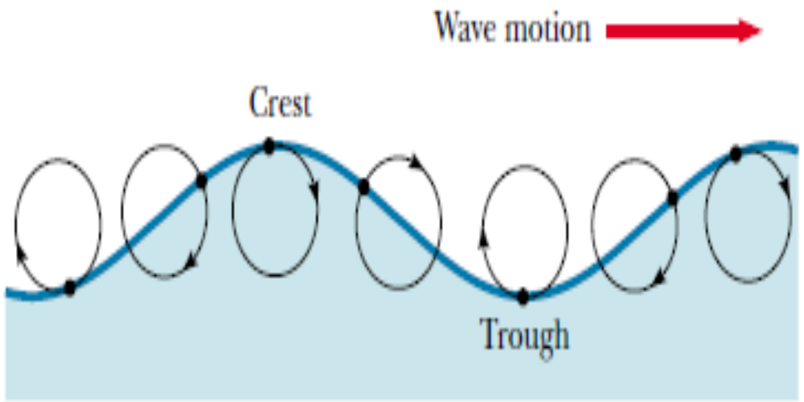
Ref: google

Wave Nature

- Examples:
  - Longitudinal: Sound waves (e.g. air moves back & forth)
  - Transverse: Light waves (electromagnetic waves, i.e. electric and magnetic disturbances)

  The source of the wave, i.e. the disturbance, moves continuously in

  simple harmonic motion, generating an entire wave, where each part of the wave also performs a simple harmonic motion.



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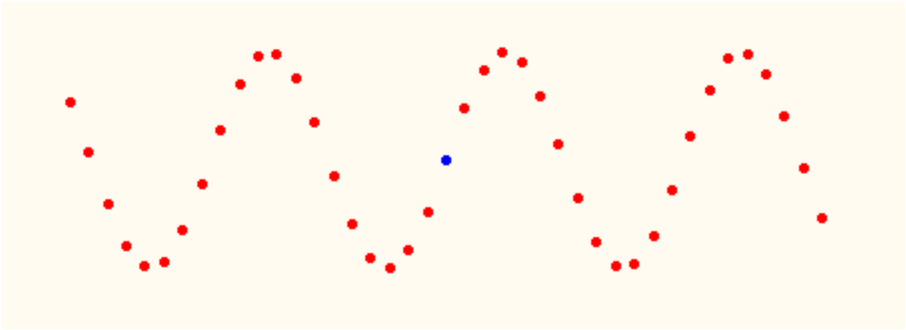
Wave Nature

- **Transverse:** The medium oscillates perpendicular to the direction the wave is moving.
  - ➡ Water waves

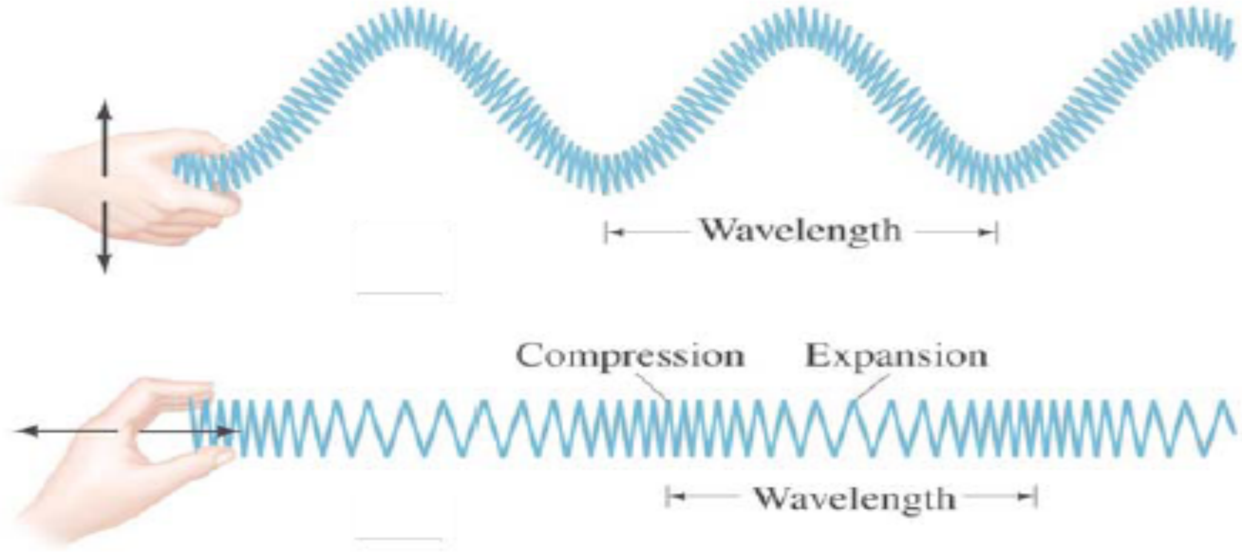


- **Longitudinal:** The medium oscillates in the same direction as the wave is moving
  - ➡ Sound

Ref: google

Wave Nature

Types of Waves: Transverse and Longitudinal



The motion of particles in a wave can be either **perpendicular** to the wave direction (**transverse**) or **parallel** to it (**longitudinal**).

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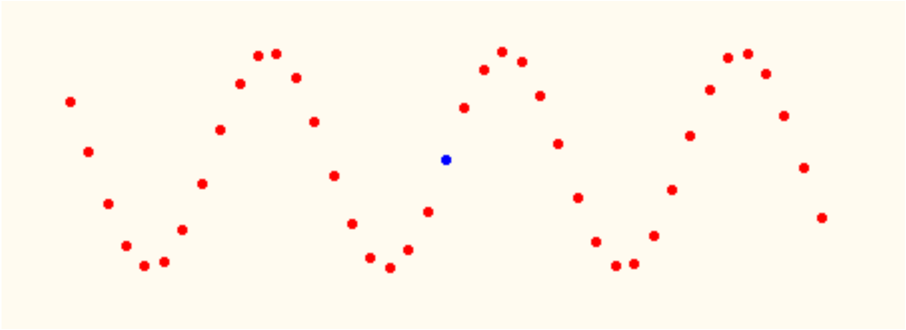
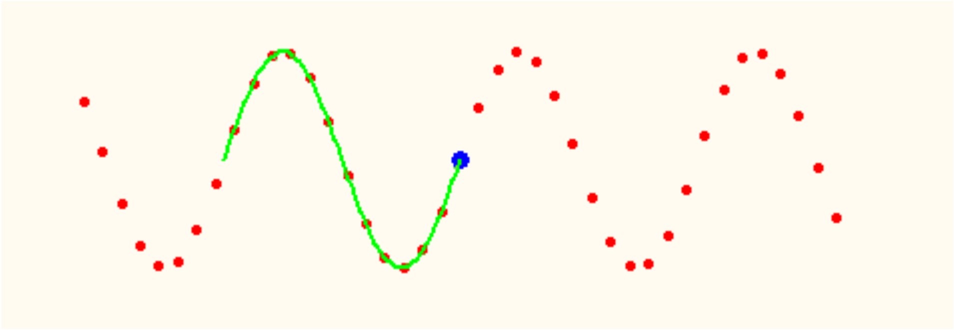
Wave Nature

- Period: The time *T* for a point on the wave to undergo one complete oscillation.



- Speed: The wave moves one wavelength λ in one period *T* so its speed is $v = \lambda / T$.



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Wave Nature

- The speed of a wave is a ***constant*** that depends only on the medium, not on the amplitude, wavelength or period:

  λ and T are related !

- $\lambda = v\,T$ or $\lambda = 2\pi\, v / \omega$ (since $T = 2\pi / \omega$)

  or $\lambda = v / f$ (since $T = 1/ f$)

- Recall f = cycles/sec or revolutions/sec
  - $\omega = 2\pi f$

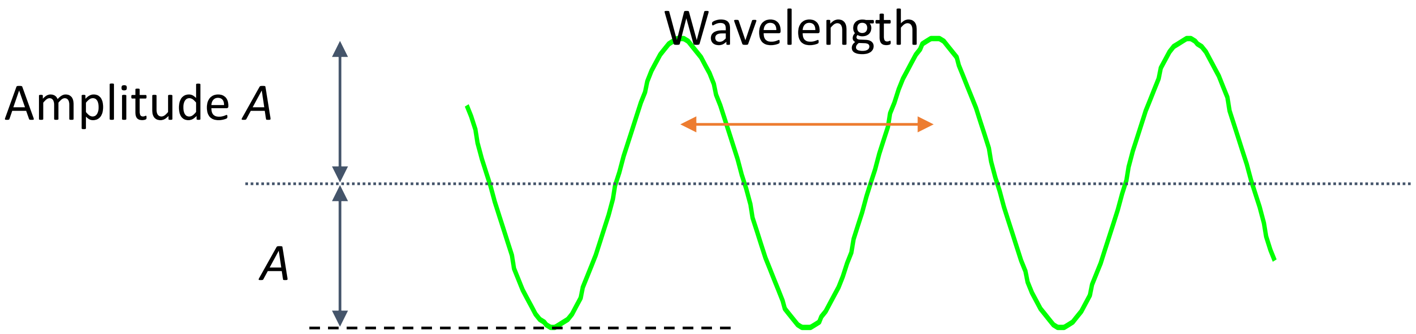
Is the speed of a wave particle the same as the speed of the wave ?
No. Wave particle performs simple harmonic motion: $v = A\,\omega \sin \omega t$.

Ref: google

Wave Nature

- Amplitude: The maximum displacement *A* of a point on the
- wave.

- Wavelength: The distance λ between identical points
- on the wave.



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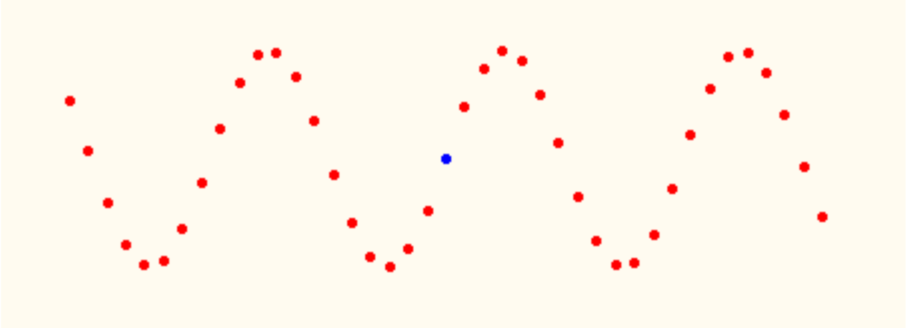
Equation for a Plane Progressive Wave

- The simplest type of wave is the one in which
- the particles of the medium are set into simple
- harmonic vibrations as the wave passes
- through it. The wave is then called a simple
- harmonic wave.



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Equation for a Plane Progressive Wave

Consider a particle O in the medium.
The displacement at any instant of time is given by

$$y = A \sin \omega t \ldots\ldots.(1)$$

Where A is the amplitude, ω is the angular frequency of the wave. Consider a particle P at a distance x from the particle O on its right. Let the wave travel with a velocity v from left to right. Since it takes some time for the disturbance to reach P, its displacement can be written as

Ref: google

Equation for a Plane Progressive Wave

$y = A \sin(\omega t - \phi)$………(2)

Where ϕ is the phase difference between the particles O and P.

We know that a path difference of λ corresponds to a phase difference of 2π radians. Hence a path difference of x corresponds to a phase difference of

$$\frac{2\pi}{\lambda} \cdot x$$

Equation for a Plane Progressive Wave

$$\phi = \frac{2\pi x}{\lambda}$$

Displacement of particle P is

$$y = A\sin(\omega t - \frac{2\pi x}{\lambda}) \ldots\ldots\ldots.(3)$$

But $\omega = \frac{2\pi}{T}$

$$y = A\sin(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}) \ldots\ldots\ldots.(5)$$

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Equation for a Plane Progressive Wave

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

But $v = \frac{\lambda}{T}$ or $T = \frac{\lambda}{v}$

$$y = A\sin 2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)$$

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)\text{...........}(6)$$

Similarly, for a particle at a distance x to the left of 0, the equation for the displacement is given by

$$y = A\sin\frac{2\pi}{\lambda}(vt + x)\text{.............}(7)$$

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Differential Equation for Wave Motion

We have wave equation

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)\text{.........}(1)$$

Differentiating equation with respect to x,
We get,

$$\frac{dy}{dx} = A\frac{2\pi}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)\text{..........}(2)$$

Ref: google

Differential Equation for Wave Motion

$\frac{dy}{dx}$ represents the strain or the compression. When $\frac{dy}{dx}$ is positive, a rarefaction takes place and when $\frac{dy}{dx}$ is negative, a compression takes place.

The velocity of the particle whose displacement y is represented by equation, is obtained by differentiating it with respect to t, since velocity is the rate of change of displacement with respect to time.

$$\frac{dy}{dt} = A\frac{2\pi v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)..........(3)$$

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Differential Equation for Wave Motion

Comparing equations (2) and (3) we get,

$$\frac{dy}{dt} = v\frac{dy}{dx} \text{.........}(4)$$

Particle velocity = wave velocity x slope of the displacement curve or strain.

Differentiating equation (2)

$$\frac{d^2y}{dx^2} = -A\frac{4\pi^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt-x)\text{.........}(5)$$

Differential Equation for Wave Motion

Differentiating equation (3)

$$\frac{d^2y}{dt^2} = -A\frac{4\pi^2v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt-x)..........(6)$$

Comparing eqs (5) and (6)

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}.........(7)$$

Equation (7) represents the differential equation of wave velocity.

Ref: google

Wave Nature: Sample Problems

Ex. The equation of a traveling wave is

$$y = 4.0 \sin \pi (0.10x - 2t)$$

Find (i) wavelength, (ii) speed and
(iii) frequency of oscillating particle of the wave

Wave Nature: Sample Problems

Ex. When a simple harmonic wave is propagated through a medium, the displacement of a particle in cm at any instant is

$$y = 10\sin\frac{2\pi}{100}(36000t - 20)$$

Calculate the amplitude, wave velocity, wavelength, frequency and period of the oscillating particle.

Wave Nature: Sample Problems

When a simple harmonic wave is propagated through a medium, the displacement of the particle at any instant of time is given by

$$y = 5.0 \sin \pi(360t - 0.15x)$$

calculate
(i)the amplitude of the vibrating particle,
(ii)wave velocity,
(ii)wave length,
(iv)frequency and
(v) time period.

Ref: google

Wave Nature: Sample Problems

Ex. A simple harmonic wave of amplitude 8 units travels a line of particles in the direction of positive X axis. At any instant for a particle at a distance of 10cm from the origin, the displacement is +6units and at a distance a particle from the origin is 25units, the displacement is +4units. Calculate the wavelength.