



**UNITED
INTERNATIONAL
UNIVERSITY**

ELECTRIC POTENTIAL

SAGAR DUTTA

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ELECTRIC POTENTIALS

3

TODAY'S TOPICS

- Electric potential and its calculation
- Electric potential energy
- Relationship between Field and Potential
- Potential due to a point charge, dipole, continuous charge distribution
- Electric field calculation from electric potential
- Equipotential surface
- Potential gradient.

ELECTRIC POTENTIAL

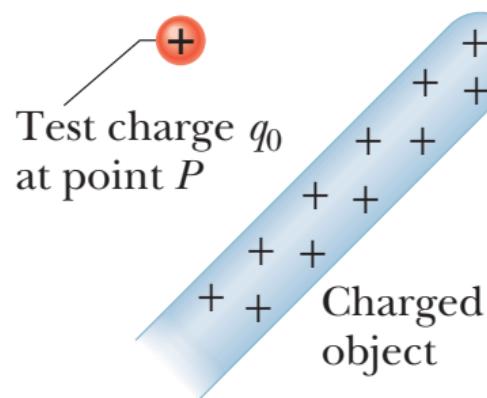
$$E_k = \frac{1}{2}mv^2$$



Where does this kinetic energy comes from?

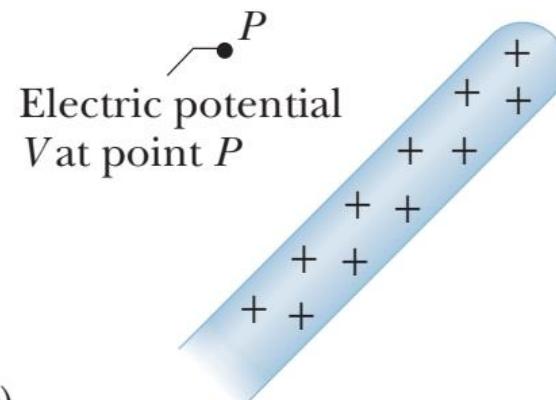
Potential Energy

Electric Potential and Electric Potential Energy



(a)

The rod sets up an electric potential, which determines the potential energy.



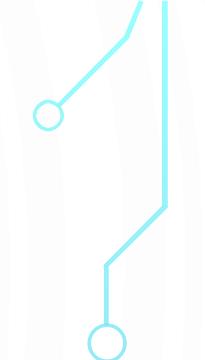
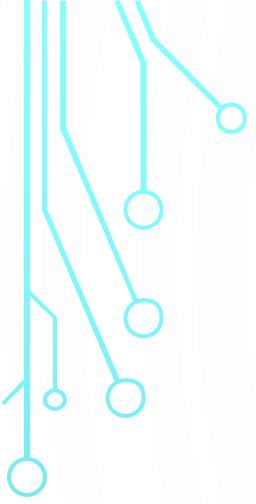
(b)

We are going to define the electric potential (or *potential* for short) in terms of electric potential energy, so our first job is to figure out how to measure that potential energy. Back in Chapter 8, we measured gravitational potential energy U of an object by (1) assigning $U = 0$ for a reference configuration (such as the object at table level) and (2) then calculating the work W the gravitational force does if the object is moved up or down from that level. We then defined the potential energy as being

$$U = -W \quad (\text{potential energy}). \quad (24-1)$$

Next, we define the electric potential V at P in terms of the work done by the electric force and the resulting potential energy:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0} \quad (\text{electric potential}). \quad (24-2)$$

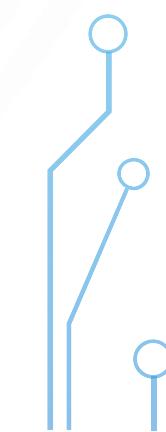


(electric potential energy) = (particle's charge) $\left(\frac{\text{electric potential energy}}{\text{unit charge}} \right)$,

or

$$U = qV,$$

(24-3)



Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector \vec{E} all along any path connecting those points. To make the calculation, we find the work done on a positive test charge by the field as the charge moves from i to f , and then use Eq. 24-6.

Consider an arbitrary electric field, represented by the field lines in Fig. 24-6, and a positive test charge q_0 that moves along the path shown from point i to point f . At any point on the path, an electric force $q_0\vec{E}$ acts on the charge as it moves through a differential displacement $d\vec{s}$. From Chapter 7, we know that the differential work dW done on a particle by a force \vec{F} during a displacement $d\vec{s}$ is given by the dot product of the force and the displacement:

$$dW = \vec{F} \cdot d\vec{s}. \quad (24-15)$$

For the situation of Fig. 24-6, $\vec{F} = q_0\vec{E}$ and Eq. 24-15 becomes

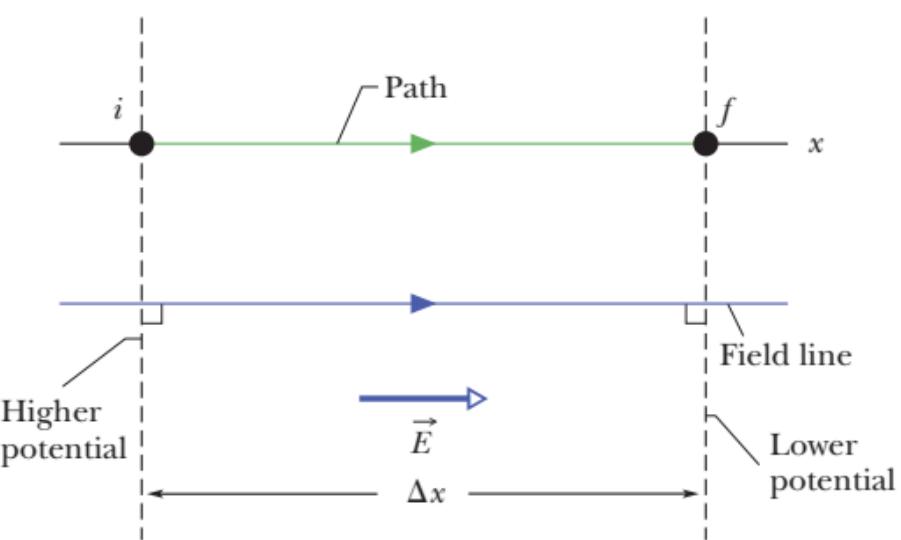
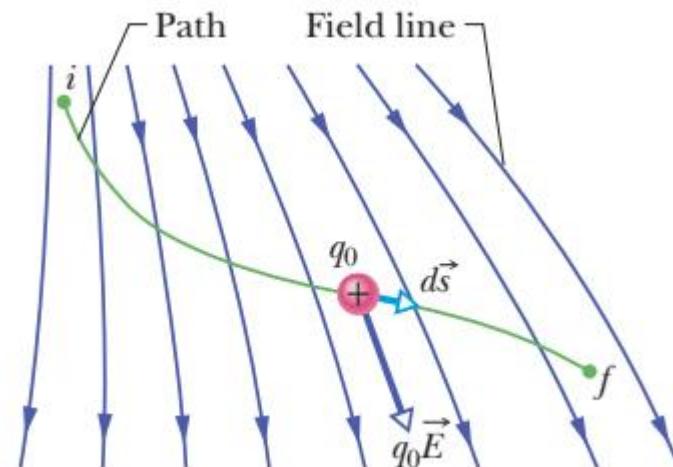
$$dW = q_0\vec{E} \cdot d\vec{s}. \quad (24-16)$$

To find the total work W done on the particle by the field as the particle moves from point i to point f , we sum—via integration—the differential works done on the charge as it moves through all the displacements $d\vec{s}$ along the path:

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-17)$$

If we substitute the total work W from Eq. 24-17 into Eq. 24-6, we find

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-18)$$



Motion Through an Electric Field

Change in Electric Potential. If we move from an initial point i to a second point f in the electric field of a charged object, the electric potential changes by

$$\Delta V = V_f - V_i.$$

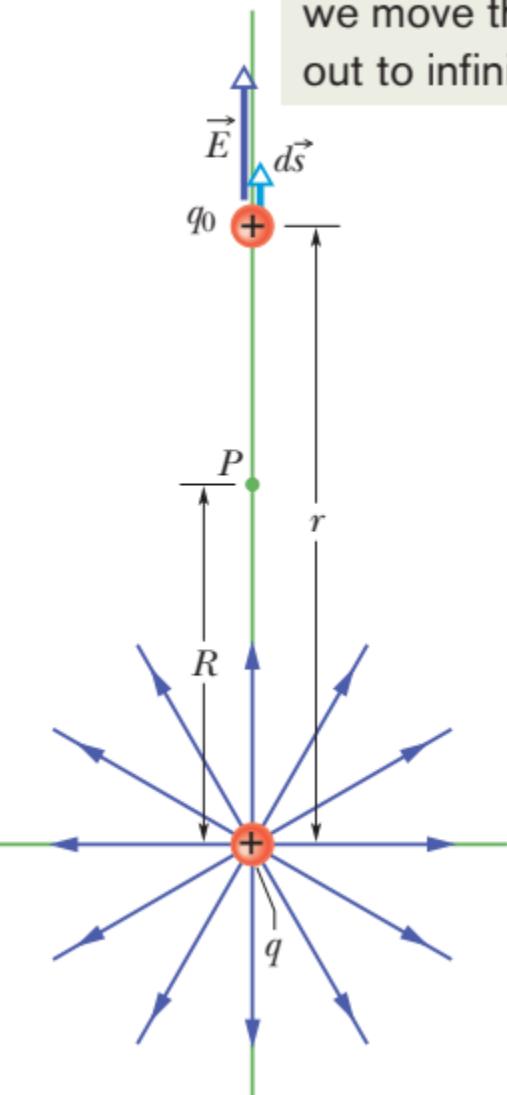
If we move a particle with charge q from i to f , then, from $U = qV$, potential energy of the system changes by

$$\Delta U = q \Delta V = q(V_f - V_i). \quad (24-4)$$

The change can be positive or negative, depending on the signs of q and ΔV . It can also be zero, if there is no change in potential from i to f (the points have the same value of potential). Because the electric force is conservative, the change in potential energy ΔU between i and f is the same for all paths between those points (it is *path independent*).

Potential Due to a Charged Particle

To find the potential of the charged particle, we move this test charge out to infinity.



Consider a point P at distance R from a fixed particle of positive charge q (Fig. 24-9). To use Eq. 24-18, we imagine that we move a positive test charge q_0 from point P to infinity. Because the path we take does not matter, let us choose the simplest one—a line that extends radially from the fixed particle through P to infinity.

To use Eq. 24-18, we must evaluate the dot product

$$\vec{E} \cdot d\vec{s} = E \cos \theta ds. \quad (24-22)$$

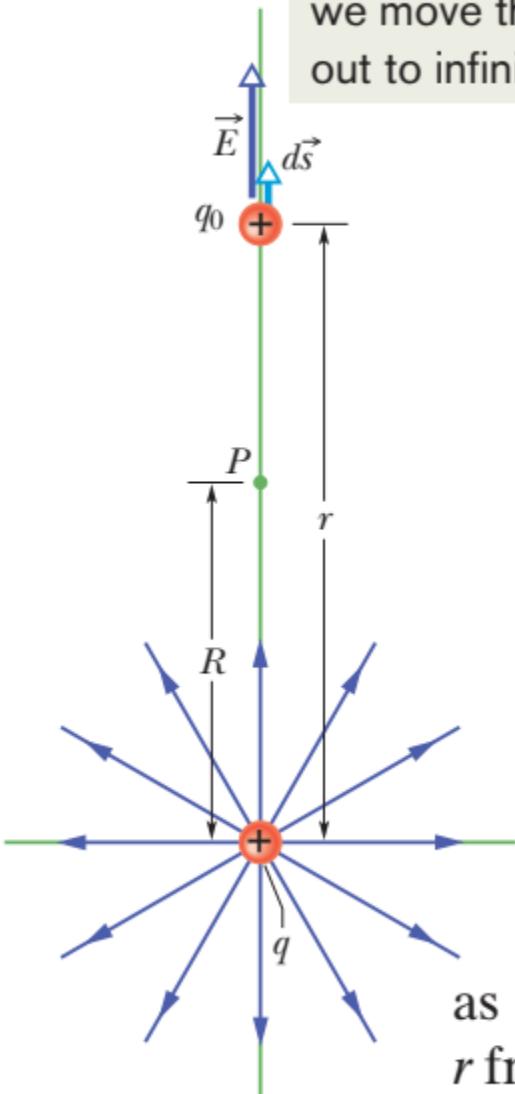
The electric field \vec{E} in Fig. 24-9 is directed radially outward from the fixed particle. Thus, the differential displacement $d\vec{s}$ of the test particle along its path has the same direction as \vec{E} . That means that in Eq. 24-22, angle $\theta = 0$ and $\cos \theta = 1$. Because the path is radial, let us write ds as dr . Then, substituting the limits R and ∞ , we can write Eq. 24-18 as

$$V_f - V_i = - \int_R^\infty E dr. \quad (24-23)$$

we set $V_f = 0$ (at ∞) and $V_i = V$ (at R)

Potential Due to a Charged Particle

To find the potential of the charged particle, we move this test charge out to infinity.



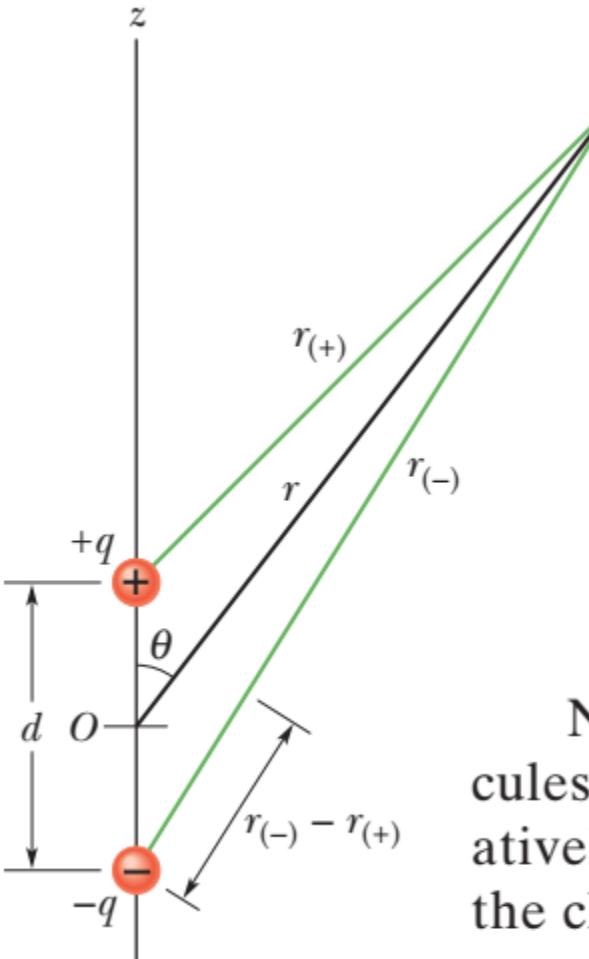
as the electric potential V due to a particle of charge q at any radial distance r from the particle.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\begin{aligned} 0 - V &= -\frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{1}{r^2} dr = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_R^\infty \\ &= -\frac{1}{4\pi\epsilon_0} \frac{q}{R}. \end{aligned}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

POTENTIAL DUE TO AN ELECTRIC DIPOLE

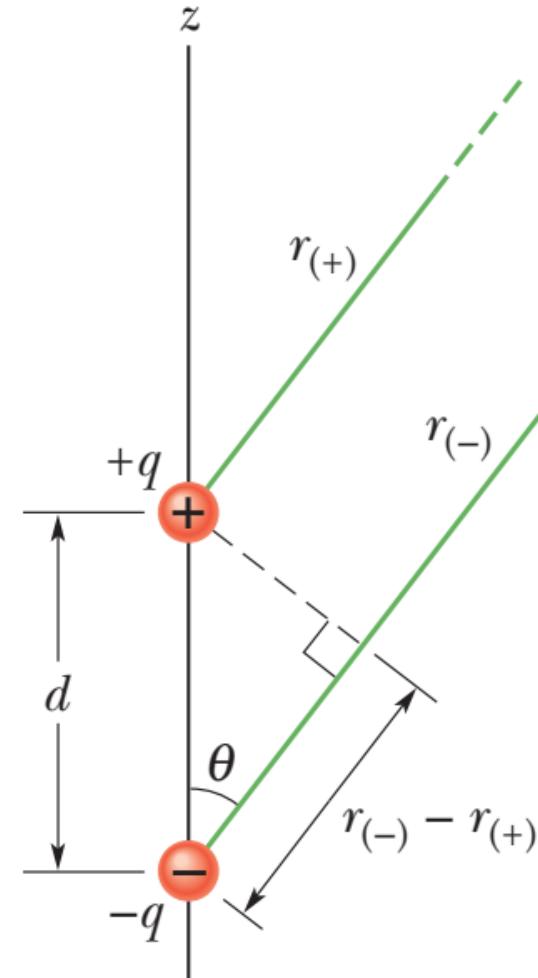


$$\begin{aligned} V &= \sum_{i=1}^2 V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right) \\ &= \frac{q}{4\pi\epsilon_0} \frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}. \end{aligned}$$

Naturally occurring dipoles—such as those possessed by many molecules—are quite small; so we are usually interested only in points that are relatively far from the dipole, such that $r \gg d$, where d is the distance between the charges and r is the distance from the dipole's midpoint to P . In that case, we can approximate the two lines to P as being parallel and their length difference as being the leg of a right triangle with hypotenuse d (Fig. 24-13b). Also, that difference is so small that the product of the lengths is approximately r^2 . Thus,

$$r_{(-)} - r_{(+)} \approx d \cos \theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

POTENTIAL DUE TO AN ELECTRIC DIPOLE



$$V = \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2},$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} \quad (\text{electric dipole})$$

$p (= qd)$ is the magnitude of the electric dipole moment \vec{p}
The vector \vec{p} is directed along the dipole axis, from the negative to the positive charge

What is the electric potential at point P , located at the center of the square of charged particles shown in Fig. 24-11a? The distance d is 1.3 m, and the charges are

$$\begin{aligned} q_1 &= +12 \text{ nC}, & q_3 &= +31 \text{ nC}, \\ q_2 &= -24 \text{ nC}, & q_4 &= +17 \text{ nC}. \end{aligned}$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four particles.

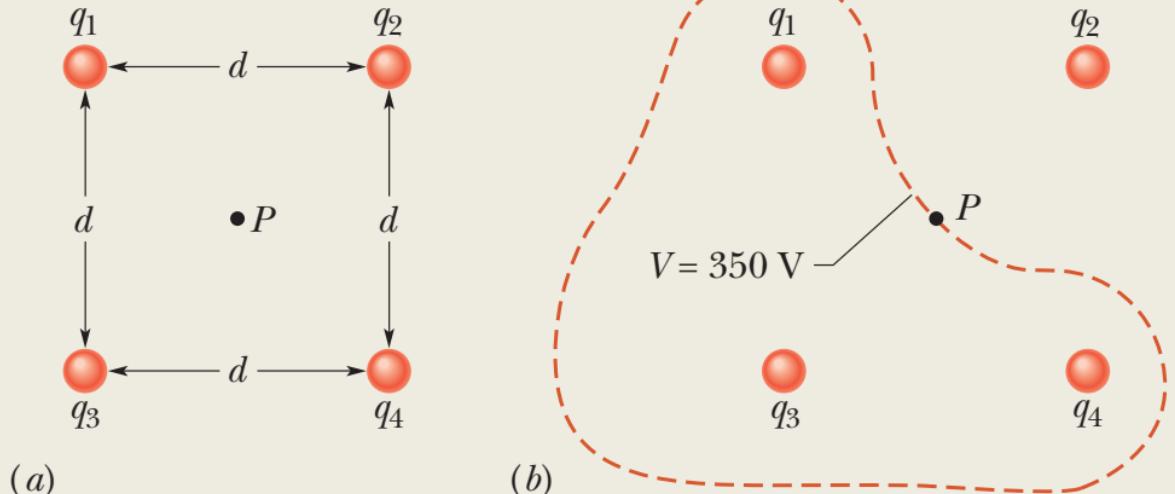


Figure 24-11 (a) Four charged particles. (b) The closed curve is a (roughly drawn) cross section of the equipotential surface that contains point P .

(Because electric potential is a scalar, the orientations of the particles do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^4 V_i = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r} \right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$\begin{aligned} q_1 + q_2 + q_3 + q_4 &= (12 - 24 + 31 + 17) \times 10^{-9} \text{ C} \\ &= 36 \times 10^{-9} \text{ C}. \end{aligned}$$

$$\begin{aligned} \text{Thus, } V &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(36 \times 10^{-9} \text{ C})}{0.919 \text{ m}} \\ &\approx 350 \text{ V}. \end{aligned} \quad (\text{Answer})$$

Close to any of the three positively charged particles in Fig. 24-11a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P . The curve in Fig. 24-11b shows the intersection of the plane of the figure with the equipotential surface that contains point P .



(a) In Fig. 24-12a, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R . Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. Because

electric potential is a scalar, the orientations of the electrons do not matter. (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C , Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\epsilon_0} \frac{e}{R}. \quad (\text{Answer}) \quad (24-28)$$

Because of the symmetry of the arrangement in Fig. 24-12a, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C ,

$$\vec{E} = 0. \quad (\text{Answer})$$

(b) The electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-12b). At C , find the electric potential and describe the electric field.

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

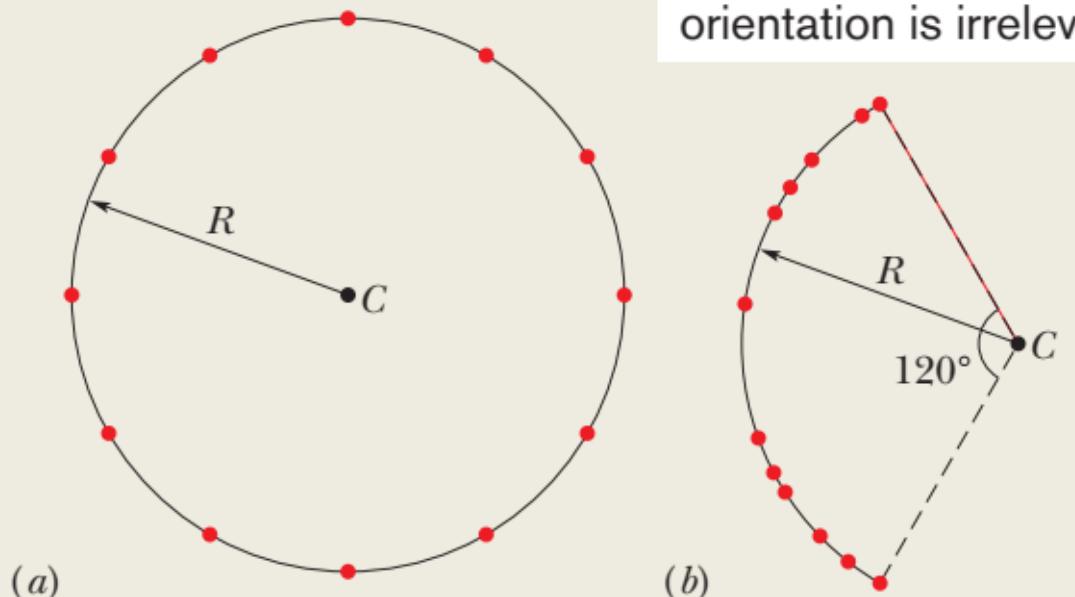
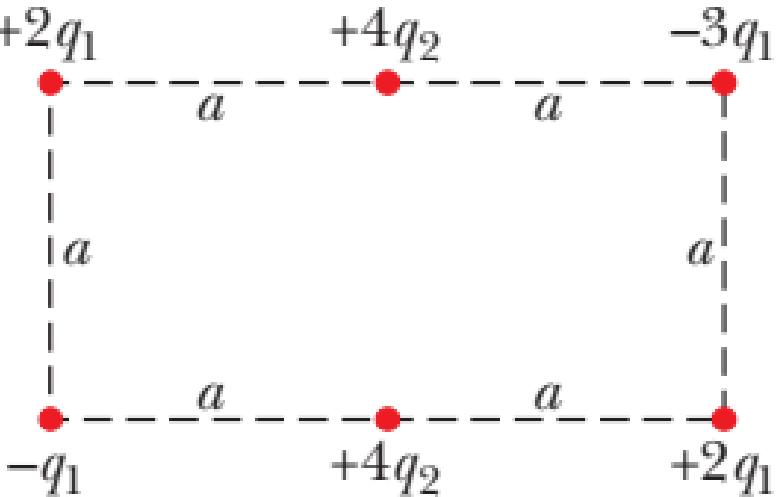


Figure 24-12 (a) Twelve electrons uniformly spaced around a circle.
(b) The electrons nonuniformly spaced along an arc of the original circle.



••16 GO Figure 24-37 shows a rectangular array of charged particles fixed in place, with distance $a = 39.0$ cm and the charges shown as integer multiples of $q_1 = 3.40 \text{ pC}$ and $q_2 = 6.00 \text{ pC}$. With $V = 0$ at infinity, what is the net electric potential at the rectangle's center? (*Hint:* Thoughtful examination of the arrangement can reduce the calculation.)

16. In applying Eq. 24-27, we are assuming $V \rightarrow 0$ as $r \rightarrow \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

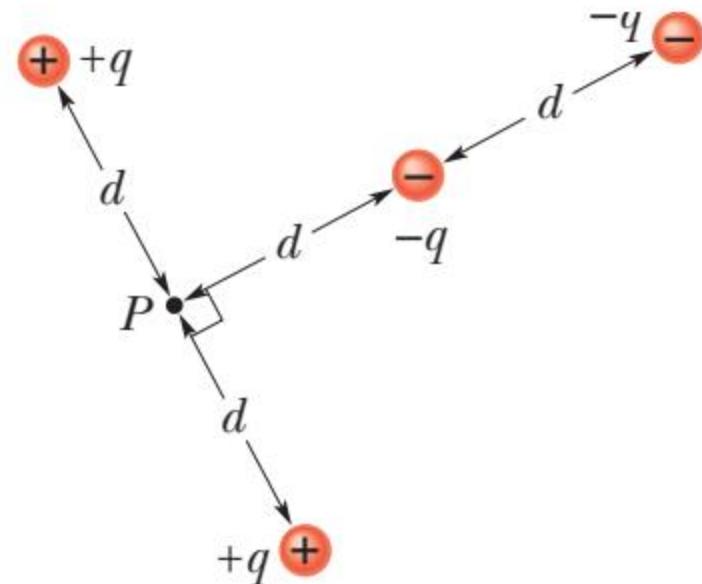
$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \frac{4q_2}{a/2} + \frac{1}{4\pi\epsilon_0} \frac{4q_2}{a/2} = \frac{16q_2}{4\pi\epsilon_0 a} = \frac{16(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(6.00 \times 10^{-12} \text{ C})}{0.39 \text{ m}} \\ &= 2.21 \text{ V}. \end{aligned}$$

••17 GO In Fig. 24-38, what is the net electric potential at point P due to the four particles if $V = 0$ at infinity, $q = 5.00 \text{ fC}$, and $d = 4.00 \text{ cm}$?

17. A charge $-5q$ is a distance $2d$ from P , a charge $-5q$ is a distance d from P , and two charges $+5q$ are each a distance d from P , so the electric potential at P is

$$V = \frac{q}{4\pi\epsilon_0} \left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d} \right] = \frac{q}{8\pi\epsilon_0 d} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(5.00 \times 10^{-15} \text{ C})}{2(4.00 \times 10^{-2} \text{ m})}$$
$$= 5.62 \times 10^{-4} \text{ V.}$$

The zero of the electric potential was taken to be at infinity.



••19 In Fig. 24-40, particles with the charges $q_1 = +5e$ and $q_2 = -15e$ are fixed in place with a separation of $d = 24.0$ cm. With electric potential defined to be $V = 0$ at infinity, what are the finite (a) positive and (b) negative values of x at which the net electric potential on the x axis is zero?

19. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k \left(\frac{q_1}{d_1} + \frac{q_2}{d_2} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{x} + \frac{-3}{d-x} \right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k \left(\frac{q_1}{d_1} + \frac{q_2}{d_2} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{-x} + \frac{-3}{d-x} \right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

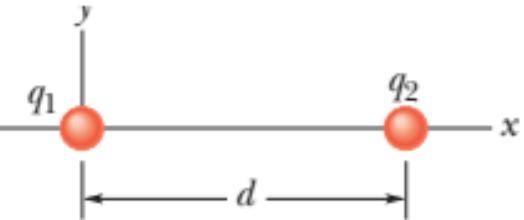


Figure 24-40 Problems 19 and 20.

(a) Figure 24-8a shows two points i and f in a uniform electric field \vec{E} . The points lie on the same electric field line (not shown) and are separated by a distance d . Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from i to f along the path shown, which is parallel to the field direction.

KEY IDEA

We can find the potential difference between any two points in an electric field by integrating $\vec{E} \cdot d\vec{s}$ along a path connecting those two points according to Eq. 24-18.

Calculations: We have actually already done the calculation for such a path in the direction of an electric field line in a uniform field when we derived Eq. 24-21. With slight changes in notation, Eq. 24-21 gives us

$$V_f - V_i = -Ed. \quad (\text{Answer})$$

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-8b.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf . At all points along line ic , the displace-

ment $d\vec{s}$ of the test charge is perpendicular to \vec{E} . Thus, the angle θ between \vec{E} and $d\vec{s}$ is 90° , and the dot product $\vec{E} \cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$. Ah, we should have seen this coming. The points are on the same equipotential surface, which is perpendicular to the electric field lines.

For line cf we have $\theta = 45^\circ$ and, from Eq. 24-18,

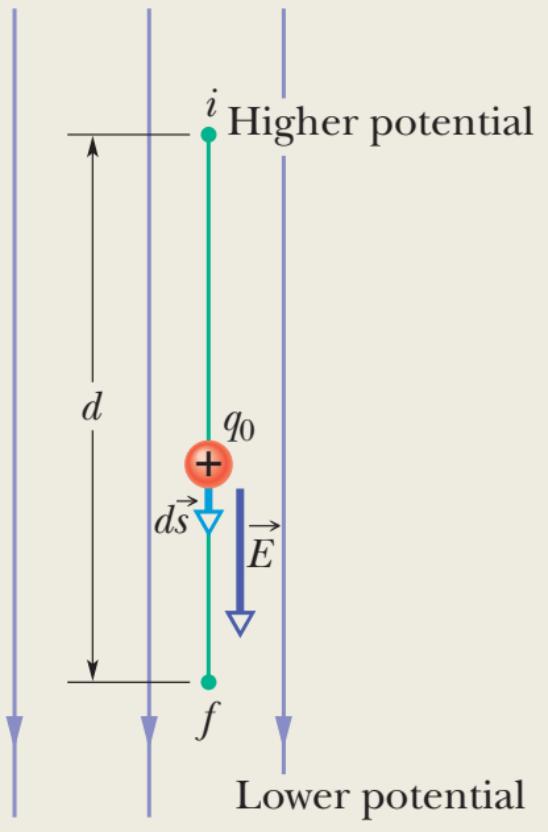
$$\begin{aligned} V_f - V_i &= - \int_c^f \vec{E} \cdot d\vec{s} = - \int_c^f E(\cos 45^\circ) ds \\ &= -E(\cos 45^\circ) \int_c^f ds. \end{aligned}$$

The integral in this equation is just the length of line cf ; from Fig. 24-8b, that length is $d/\cos 45^\circ$. Thus,

$$V_f - V_i = -E(\cos 45^\circ) \frac{d}{\cos 45^\circ} = -Ed. \quad (\text{Answer})$$

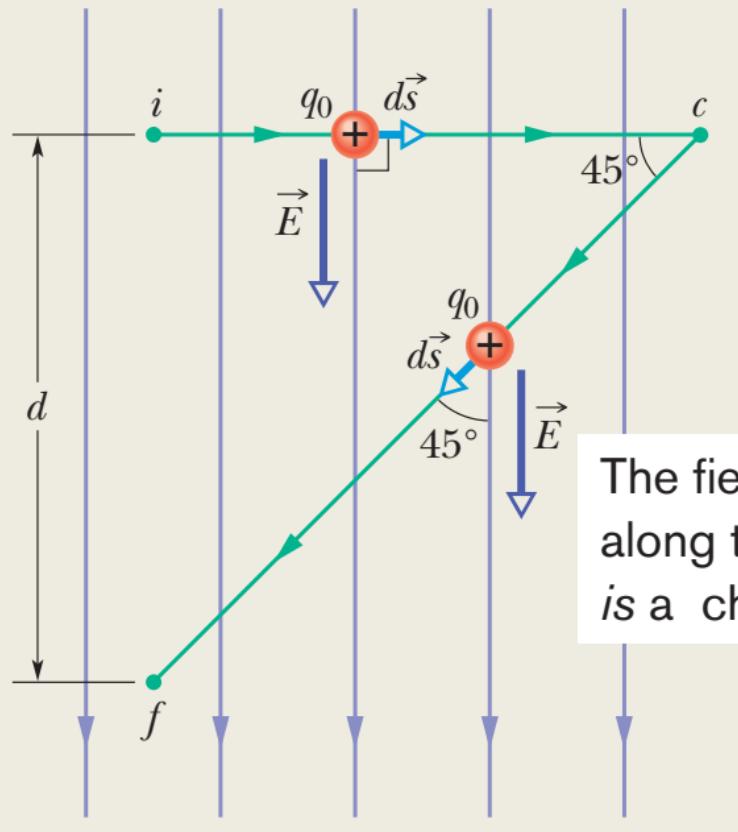
This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 24-18.

The electric field points *from* higher potential *to* lower potential.



(a)

The field is perpendicular to this *ic* path, so there is no change in the potential.



(b)

The field has a component along this *cf* path, so there is a change in the potential.

Figure 24-8 (a) A test charge q_0 moves in a straight line from point *i* to point *f*, along the direction of a uniform external electric field. (b) Charge q_0 moves along path *icf* in the same electric field.

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electric force \vec{F} due to the electric field \vec{E} that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150 \text{ N/C}$ and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electric force causes it to move vertically upward through a distance $d = 520 \text{ m}$ (Fig. 24-3)? Through what potential change does the electron move?

KEY IDEAS

- (1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-5 ($W = -\Delta U$) gives the relation.
- (2) The work done by a constant force \vec{F} on a particle undergoing a displacement \vec{d} is

$$W = \vec{F} \cdot \vec{d}.$$

- (3) The electric force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19} \text{ C}$).

Calculations: Substituting the force equation into the work equation and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta,$$

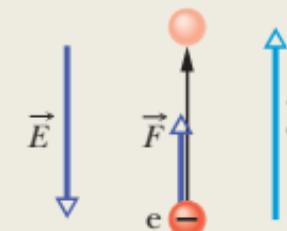


Figure 24-3 An electron in the atmosphere is moved upward through displacement \vec{d} by an electric force \vec{F} due to an electric field \vec{E} .

where θ is the angle between the directions of \vec{E} and \vec{d} . The field \vec{E} is directed downward and the displacement \vec{d} is directed upward; so $\theta = 180^\circ$. We can now evaluate the work as

$$\begin{aligned} W &= (-1.6 \times 10^{-19} \text{ C})(150 \text{ N/C})(520 \text{ m}) \cos 180^\circ \\ &= 1.2 \times 10^{-14} \text{ J}. \end{aligned}$$

Equation 24-5 then yields

$$\Delta U = -W = -1.2 \times 10^{-14} \text{ J}. \quad (\text{Answer})$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by $1.2 \times 10^{-14} \text{ J}$. To find the change in electric potential, we apply Eq. 24-4:

$$\begin{aligned} \Delta V &= \frac{\Delta U}{-q} = \frac{-1.2 \times 10^{-14} \text{ J}}{-1.6 \times 10^{-19} \text{ C}} \\ &= 4.5 \times 10^4 \text{ V} = 45 \text{ kV}. \quad (\text{Answer}) \end{aligned}$$

This tells us that the electric force does work to move the electron to a *higher* potential.

POTENTIAL DUE TO A CONTINUOUS CHARGE DISTRIBUTION

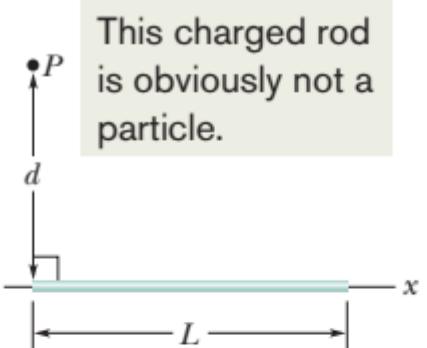
In Fig. 24-15a, a thin nonconducting rod of length L has a positive charge of uniform linear density λ . Let us determine the electric potential V due to the rod at point P , a perpendicular distance d from the left end of the rod.

We consider a differential element dx of the rod as shown in Fig. 24-15b. This (or any other) element of the rod has a differential charge of

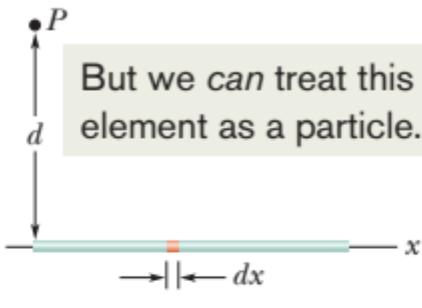
$$dq = \lambda dx. \quad (24-33)$$

This element produces an electric potential dV at point P , which is a distance $r = (x^2 + d^2)^{1/2}$ from the element (Fig. 24-15c). Treating the element as a point charge, we can use Eq. 24-31 to write the potential dV as

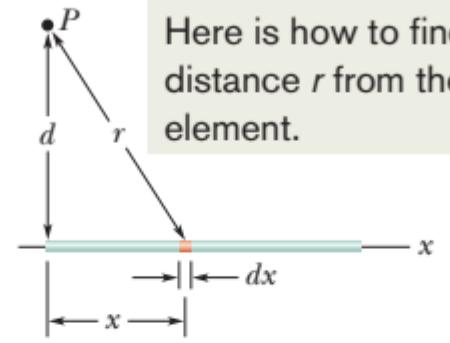
$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}. \quad (24-34)$$



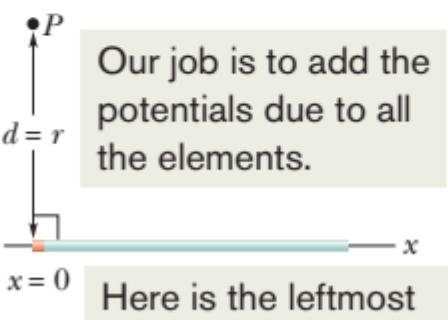
(a)



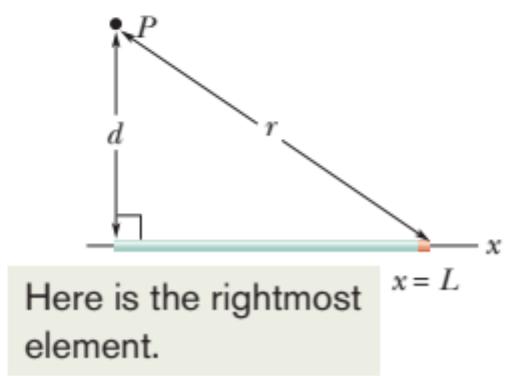
(b)



(c)



(d)



(e)

Since the charge on the rod is positive and we have taken $V = 0$ at infinity, we know from Module 24-3 that dV in Eq. 24-34 must be positive.

We now find the total potential V produced by the rod at point P by integrating Eq. 24-34 along the length of the rod, from $x = 0$ to $x = L$ (Figs. 24-15d and e), using integral 17 in Appendix E. We find

$$\begin{aligned} V &= \int dV = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx \\ &= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}} \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln(x + (x^2 + d^2)^{1/2}) \right]_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln(L + (L^2 + d^2)^{1/2}) - \ln d \right]. \end{aligned}$$

We can simplify this result by using the general relation $\ln A - \ln B = \ln(A/B)$. We then find

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]. \quad (24-35)$$

Because V is the sum of positive values of dV , it too is positive, consistent with the logarithm being positive for an argument greater than 1.

Charged Disk

In Module 22-5, we calculated the magnitude of the electric field at points on the central axis of a plastic disk of radius R that has a uniform charge density σ on one surface. Here we derive an expression for $V(z)$, the electric potential at any point on the central axis. Because we have a circular distribution of charge on the disk, we could start with a differential element that occupies angle $d\theta$ and radial distance dr . We would then need to set up a two-dimensional integration. However, let's do something easier.

In Fig. 24-16, consider a differential element consisting of a flat ring of radius R' and radial width dR' . Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

in which $(2\pi R')(dR')$ is the upper surface area of the ring. All parts of this charged element are the same distance r from point P on the disk's axis. With the aid of Fig. 24-16, we can use Eq. 24-31 to write the contribution of this ring to the electric potential at P as

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}. \quad (24-36)$$

We find the net potential at P by adding (via integration) the contributions of all the rings from $R' = 0$ to $R' = R$:

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} (\sqrt{z^2 + R^2} - z). \quad (24-37)$$

Note that the variable in the second integral of Eq. 24-37 is R' and not z , which remains constant while the integration over the surface of the disk is carried out. (Note also that, in evaluating the integral, we have assumed that $z \geq 0$.)

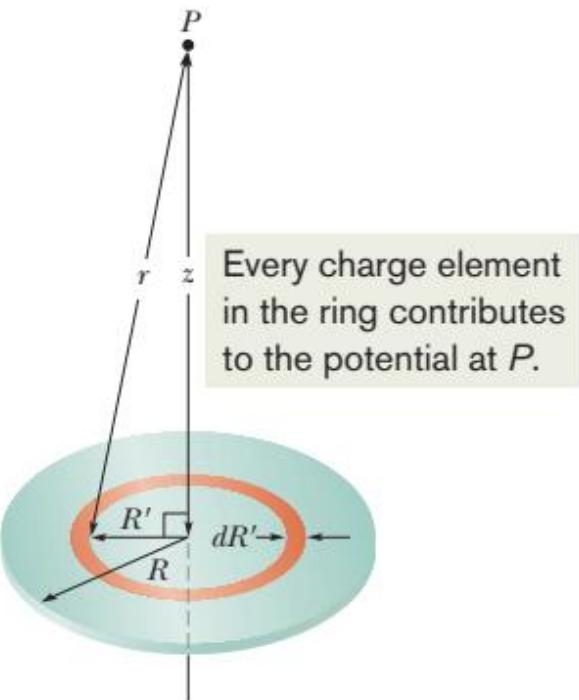


Figure 24-16 A plastic disk of radius R , charged on its top surface to a uniform surface charge density σ . We wish to find the potential V at point P on the central axis of the disk.

Calculating the Field from the Potential

Figure 24-17 shows cross sections of a family of closely spaced equipotential surfaces, the potential difference between each pair of adjacent surfaces being dV . As the figure suggests, the field \vec{E} at any point P is perpendicular to the equipotential surface through P .

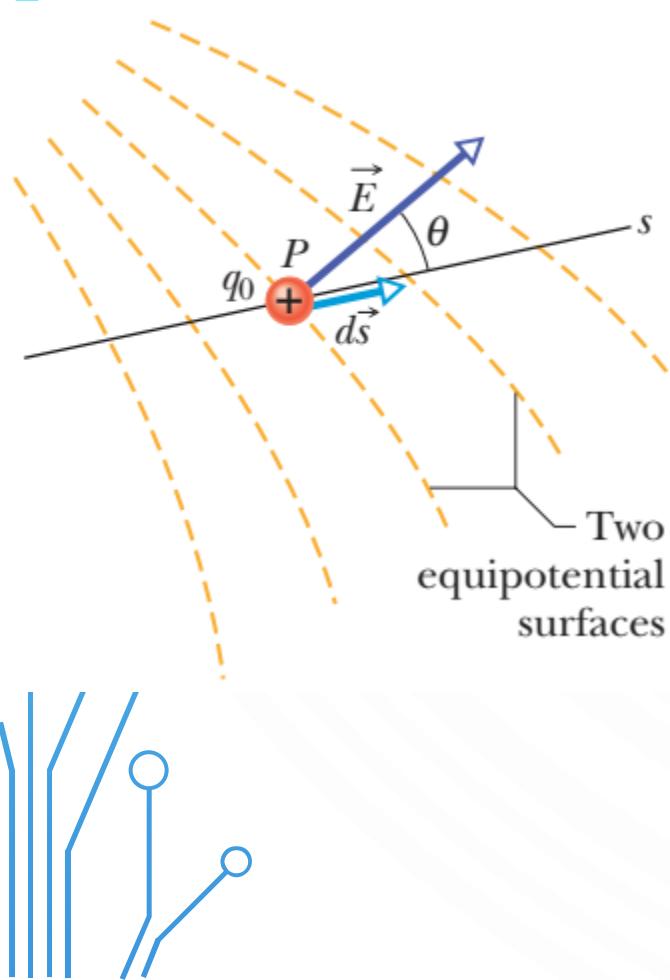
Suppose that a positive test charge q_0 moves through a displacement $d\vec{s}$ from one equipotential surface to the adjacent surface. From Eq. 24-6, we see that the work the electric field does on the test charge during the move is $-q_0 dV$. From Eq. 24-16 and Fig. 24-17, we see that the work done by the electric field may also be written as the scalar product $(q_0 \vec{E}) \cdot d\vec{s}$, or $q_0 E(\cos \theta) ds$. Equating these two expressions for the work yields

$$-q_0 dV = q_0 E(\cos \theta) ds$$

$$E \cos \theta = -\frac{dV}{ds}.$$

Since $E \cos \theta$ is the component of \vec{E} in the direction of $d\vec{s}$,

$$E_s = -\frac{\partial V}{\partial s}$$



ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF CHARGED PARTICLES

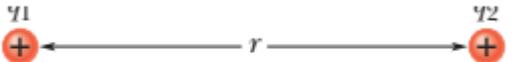


Figure 24-18 Two charges held a fixed distance r apart.

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (\text{two-particle system}).$$

- The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other. For two particles at separation r ,

$$U = W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}.$$

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12 \text{ cm}$ and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150 \text{ nC}$.

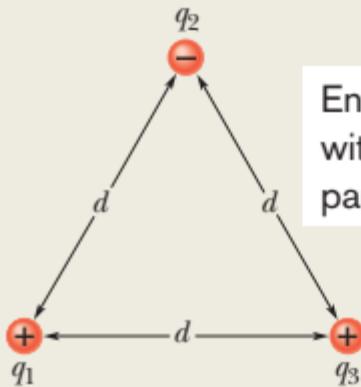
KEY IDEA

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Calculations: Let's mentally build the system of Fig. 24-19, starting with one of the charges, say q_1 , in place and the others at infinity. Then we bring another one, say q_2 , in from infinity and put it in place. From Eq. 24-46 with d substituted for r , the potential energy U_{12} associated with the pair of charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}.$$

We then bring the last charge q_3 in from infinity and put it in



Energy is associated with each pair of particles.

Figure 24-19 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2 . From Eq. 24-46, with d substituted for r , that sum is

$$W_{13} + W_{23} = U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{d} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{d}.$$

The total potential energy U of the three-charge system is the sum of the potential energies associated with the three pairs of charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$\begin{aligned}
 U &= U_{12} + U_{13} + U_{23} \\
 &= \frac{1}{4\pi\epsilon_0} \left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d} \right) \\
 &= -\frac{10q^2}{4\pi\epsilon_0 d} \\
 &= -\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9} \text{ C})^2}{0.12 \text{ m}} \\
 &= -1.7 \times 10^{-2} \text{ J} = -17 \text{ mJ}.
 \end{aligned}$$

(Answer)

The negative potential energy means that negative work would have to be done to assemble this structure, starting with the three charges infinitely separated and at rest. Put another way, an external agent would have to do 17 mJ of positive work to disassemble the structure completely, ending with the three charges infinitely far apart.

The lesson here is this: If you are given an assembly of charged particles, you can find the potential energy of the assembly by finding the potential of every possible pair of the particles and then summing the results.

