(ii) Field at a point on the axis of a charged circular ring.

Let us consider a circular turn of wire of radius *a* carrying a charge q. We would like to calculate E at a point P on the axis of the ring a distance x from its centre.

The arrangement is shown in Fig. 1.4. Let us consider a differential element of the ring of length ds at the top of the ring. Then the charge contained in this element is given by

$$dq = \frac{ds}{2\pi a} \cdot q$$

where $2\pi a$ is the circumference of the ring. This element produces a field dE at the point P.

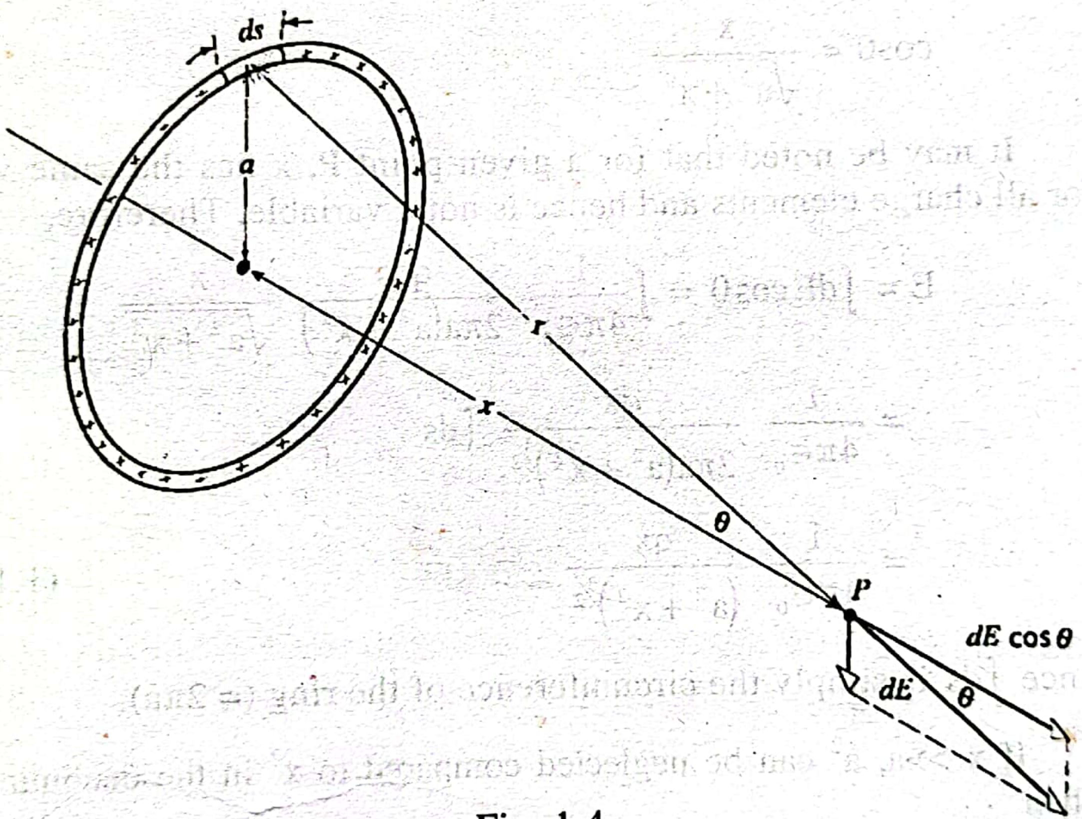

Fig. 1.4

The resultant field **E** at P is found by integrating the effects of all the elements that make up the ring.

or, $$\mathbf{E} = \int d\mathbf{E}$$

The component of dE perpendicular to the axis is cancelled out by an equal opposite component established by the charge element on the opposite side of the ring. Thus only the component of dE parallel to the axis of the ring contributes to the resultant field.

The general vector integral

$$E = \int dE$$

becomes a scalar integral $E = \int dE\cos\theta$

Now dE is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{q\,ds}{2\pi a}\right)\cdot\frac{1}{a^2+x^2}$$

where $r^2 = a^2 + x^2$

Also from the figure we have

$$\cos\theta = \frac{x}{\sqrt{a^2+x^2}}$$

It may be noted that for a given point P, x has the same value for all charge elements and hence is not a variable. Therefore,

$$E = \int dE\cos\theta = \int\frac{1}{4\pi\epsilon_0}\frac{q\,ds}{2\pi a\left(a^2+x^2\right)}\frac{x}{\sqrt{a^2+x^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qx}{2\pi a\left(a^2+x^2\right)^{3/2}}\int ds$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qx}{\left(a^2+x^2\right)^{3/2}} \qquad (1.13)$$

since $\int ds$ is simply the circumference of the ring ($= 2\pi a$).

If $x >> a$, a^2 can be neglected compared to x^2 in the denominator. Then

$$E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{x^2} \qquad (1.14)$$

and is the same as that produced by a point charge.

Thus greater the distance of the point from the charged ring, the more nearly does the field produced by the ring approach that produced by a point charge.

$$\text{or, } y_1 = \frac{eEx_1^2}{4k_0}$$

$$= \frac{(1.6\times10^{-19}\,C)(1.2\times10^4\,N/C)(1.5\times10^{-2}\,m)^2}{(4)(3.2\times10^{-16}\,\text{joule})}$$

$$= 3.4 \times 10^{-4}\text{m} = 0.34 \text{ mm.}$$

the deflection measured, not at the deflecting plates but at the fluorescent screen, is much larger.

1.5 Field due to an electric dipole

An electric dipole consists of two equal but opposite charges separated by a small distance. Fig. 1.8 shows an electric dipole. The charges are +q and –q and they are separated by a distance 2a.

We would like to calculate the field **E** at a point P due to the dipole. P is located at a distance x along the perpendicular bisector of the line joining the charges. We shall assume that x >> a.

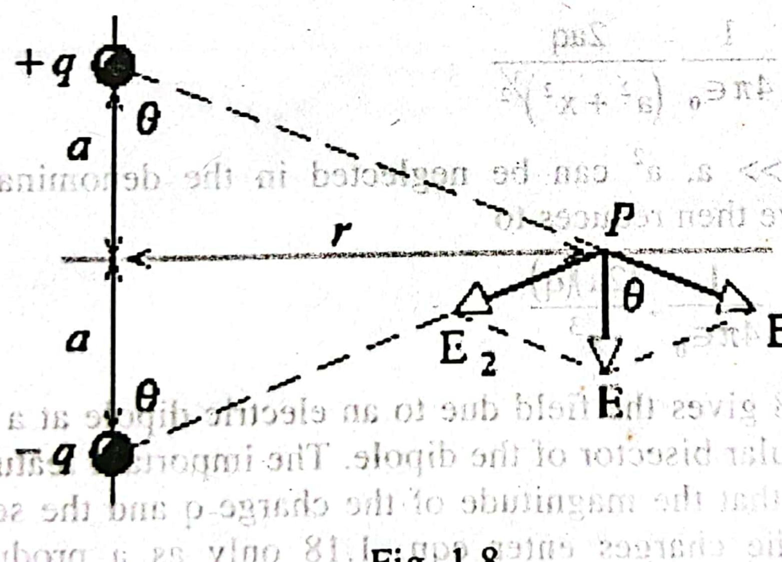

Fig. 1.8

Let the distance of the point P be y from both the charges. Let E_1 be the electric field at the point P due to the charge +q and E_2 be that due to –q. The total field at P due to the dipole is obtained by vector addition of

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

Since the charges have the same magnitude and the distance of the point P from the charges is also same, the magnitudes of the fields E_1 and E_2 are equal. Or,

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{y^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2+x^2}$$

The directions of E_1 and E_2 are indicated by arrows in Fig. 1.8. The horizontal components of E_1 and E_2 cancel each other. Hence the vector sum of E_1 and E_2 points vertically downwards and has the magnitude

$$E = 2E_1 \cos\theta$$

From the figure

$$\cos\theta = \frac{a}{y} = \frac{a}{\sqrt{a^2+x^2}}$$

Substituting the values of E_1 and $\cos\theta$ in eqn. 1.17 we obtain

$$E = \frac{2}{4\pi\epsilon_0}\frac{q}{(a^2+x^2)}\cdot\frac{a}{\sqrt{a^2+x^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2aq}{(a^2+x^2)^{3/2}}$$

since $x >> a$, a^2 can be neglected in the denominator. The equation above then reduces to

$$E \cong \frac{1}{4\pi\epsilon_0}\cdot\frac{(2a)(q)}{x^3} \tag{1.18}$$

Eqn. 1.18 gives the field due to an electric dipole at a point on the perpendicular bisector of the dipole. The important feature of the expression is that the magnitude of the charge q and the separation 2a between the charges enter eqn. 1.18 only as a product. This means that if we measure **E** at various distances from the electric dipole (assuming $x >> a$), we can never deduce '*q*' and '*a*' separately but only the product 2aq; if q were doubled and 'a' simultaneously cut in half, the electric field at large distance from the dipole would not change.

The product 2aq is called the *electric dipole moment* and is denoted by *p*. It is a vector having the direction along the axis of the dipole from the negative to the positive charge.

Another important feature of eqn. 1.18 hat the field E varies with distance x as $1/x^3$. This implies that when two equal and opposite charges are placed close to each other, their separate fields at distant points almost, *but not quite*, cancel each other; whereas for a point charge, E(x) drops off more slowly, namely as $1/x^2$ (eqn. 1.7).

1.6 A dipole in an external electric field

An electric dipole is placed in a uniform external electric field **E**; its dipole moment **p** making an angle θ with this field [Fig. 1.9(a)]. The two forces (**F** and – **F**) acting on the charges are equal and opposite where

$$F = qE$$

The net force on the dipole is clearly zero. But since the forces do not act along the same line, there is a net torque on the dipole about an axis passing through the centre O of the dipole given by

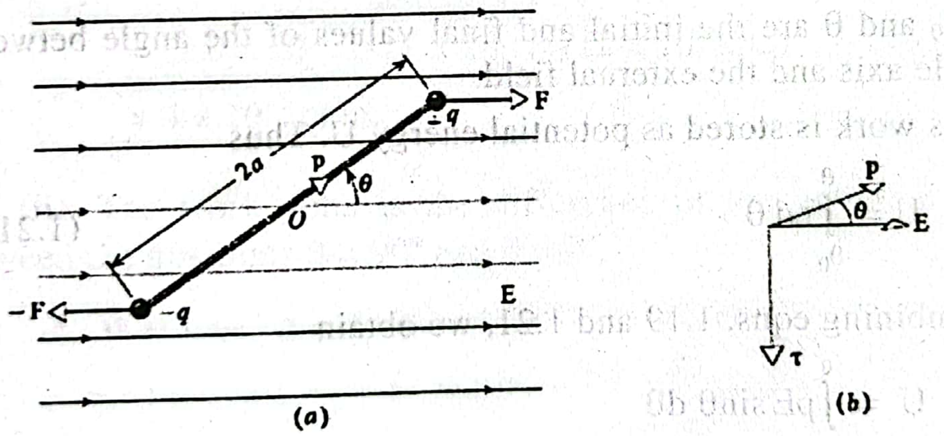

Fig. 1.9

$$\tau = \text{magnitude of a force} \times \text{perpendicular distance between the forces.}$$

$$= F \times 2a \sin\theta$$

$$= qE \times 2a \sin\theta$$

$$= 2aq\, E \sin\theta$$

$$= p\, E \sin\theta \qquad (p = 2aq) \qquad (1.19)$$

eqn. 1.19 can be written in vector form as

$$\tau = p \times E \tag{1.20}$$

the appropriate vectors being shown in Fig. 1.9(b).

Thus, when an electric dipole is placed in an external electric field, it experiences a torque tending to align it with this field. Hence, work (positive or negative) must be done by an external agent to change the orientation of an electric dipole in an external electric field. This work is stored as potential energy U in the system consisting of the dipole and the arrangement used to set up the external field. The work done to change the orientation of the dipole through a small angle $d\theta$ is given by

$$dW = \tau.dq$$

where τ is the torque exerted by the agent and does the work.

Then the work done to turn the dipole from an initial orientation θ_0 to a final orientation θ is given by

$$W = \int_{\theta_0}^{\theta} dW = \int_{\theta_0}^{\theta} \tau.d\,\theta$$

where θ_0 and θ are the initial and final values of the angle between the dipole axis and the external field.

This work is stored as potential energy U. Thus

$$U = \int_{\theta_0}^{\theta} \tau.d\,\theta \tag{1.21}$$

Combining eqns. 1.19 and 1.21, we obtain

$$U = \int_{\theta_0}^{\theta} pE\sin\theta\, d\theta$$

$$= pE\int_{\theta_0}^{\theta} \sin\theta\, d\theta$$

$$= pE \left|-\cos\theta\right|_{\theta_0}^{\theta}$$

$$= -\,pE\,(\cos\theta - \cos\theta_0)$$

Since we are interested only in *changes* in potential energy, the reference orientation θ_0 can be chosen to have any convenient value, say 90º. This gives,

$$U = -pE\cos\theta$$

This, again, can be put in the vector from

$$U = -\mathbf{p}\cdot\mathbf{E}$$

***

Example 1.19 An electric dipole consists of two opposite charges of magnitude 2.0×10^{-6}C separated by a distance $l = 1.0$cm. It is placed in an external electric field of 2.0×10^5 N/C.

(i) What maximum torque does the field exert on the dipole?

(ii) How much work must an external agent do to turn the dipole from its initial alignment given by $\theta = 0^\circ$ to final alignment $\theta = 90^\circ$?

Soln.

(i) The maximum torque is found by putting $\theta = 90^\circ$ in eqn. 1.19

$\tau = pE\sin\theta$ $\qquad$ $q = 2.0 \times 10^{-6}$C

$\qquad\qquad\qquad\qquad$ $l = 1.0 \text{ cm} = 1 \times 10^{-2}$m

$= qlE\sin 90^\circ$ $\qquad$ $E = 2.0 \times 10^5$ N/C

$= (2.0 \times 10^{-6}\text{C})(1 \times 10^{-2}\text{m})(2 \times 10^5\text{N/C})$

$= 4 \times 10^{-3}$ N.m.

(ii) The work done is the difference in the potential energy between the positions $\theta = 90^\circ$ and $\theta = 0^\circ$.

$$W = U_{90^\circ} - U_{0^\circ} = -pE\cos 90^\circ - (-pE\cos 0^\circ)$$

$$= pE = qlE = 4 \times 10^{-3} \text{ joules.}$$

Example 1.20 A molecule of water vapour (H_2O) has an electric dipole moment of magnitude $p = 6.2 \times 10^{-30}$C.m. The dipole moment arises because the effective centre of positive charge does not coincide with the effective centre of negative charge.

(i) How far apart are the effective centres of positive and negative charge in a molecule of H_2O?

(ii) What is the maximum torque on a molecule of H_2O in an electric field of magnitude 1.5×10^4 N/C?

(iii) Suppose the dipole moment of a molecule of H_2O is initially pointing in a direction opposite to the field. How much work is done by the electric field in rotating the molecule into alignment with the field?

Soln.

(i) There are 10 electrons and, correspondingly, 10 positive charges in this molecule. We can write, for the magnitude of the dipole moment,

$$p = qd = (10e)\,d$$

where d is the separation between the centres of the positive and negative charges. e is the elementary charge. Thus

$$d = \frac{p}{10e} = \frac{6.2\times10^{-30}\,\text{C.m}}{(10)(1.6\times10^{-19}\,\text{C})}$$

$$= 3.9 \times 10^{-12}\,\text{m} = 3.9\ \text{pm}$$

(ii) As in eqn. 1.19, the torque is maximum when $\theta = 90°$. Substituting this value in that equation yields,

$$\tau = pE\sin\theta$$

$$= (6.2 \times 10^{-30}\,\text{C.m})\,(1.5 \times 10^4\,\text{N/C})\,(\sin 90°)$$

$$= 9.3 \times 10^{-26}\,\text{N.m.}$$

(iii) The work done in rotating the dipole from $\theta_0 = 180°$ to $\theta = 0°$ is given by eqn.

$$W = \int_{\theta_0}^{\theta} pE\sin\theta . d\theta$$

$$= \left|-pE\cos\theta\right|_{180°}^{0°}$$

$$= -pE\cos 180° - (-pE\cos 0°)$$

$$= pE + pE$$

$$= 2pE = (2)\,(6.2 \times 10^{-30}\,\text{C.m})\,(1.5 \times 10^4\,\text{N/C})$$

$$= 1.9 \times 10^{-25}\,\text{J.}$$

Application of Gauss' law

(i) *Deduction of Coulomb's law*

Coulomb's law can be deduced from Gauss' law and symmetry considerations. To do so, let us apply Gauss' law to an isolated point charge q and consider that the charge is surrounded by a spherical surface as in Fig. 2.5. Although Gauss' law holds for any surface whatever, information can be extracted more readily for a spherical surface of radius r with the charge q at the centre. The advantage of this surface is that, from symmetry consideration the field **E** is normal to the surface and is constant in magnitude for all points on it. *A closed surface so imagined under symmetry consideration is called a Gaussian surface* – a term which we shall often use.

In Fig. 2.5 **E** and **dS** at any point on the Gaussian surface are directed radially outward. The angle between them is zero, and the quantity **E . dS** = E dS cosθ becomes EdS. Gauss' law then becomes

$$\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = \epsilon_0 \oint EdS = q \tag{2.8}$$

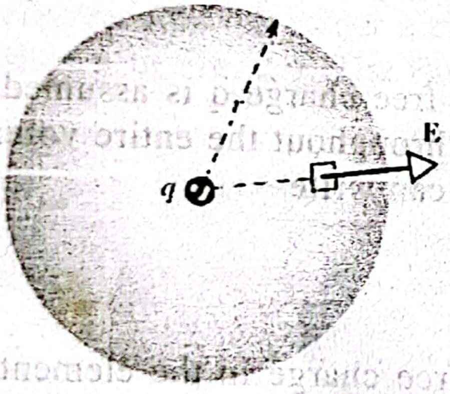

Fig. 2.5

Since E is constant for all points on the surface of the sphere, it can be taken outside the integral, and consequently eqn. 2.8 becomes

$$\epsilon_0 E \oint dS = q \tag{2.9}$$

But $\oint dS$ is simply the area $4\pi r^2$ of the sphere. Eqn. (2.9) therefore reduces to

$$\epsilon_0 E 4\pi r^2 = q$$

$$\text{or,} \quad E = \frac{q}{4\pi\epsilon_0 r^2} \tag{2.10}$$

Eqn. (2.10) gives the magnitude of electric field strength **E** at any point a distance r from an isolated point charge q.

If now a second point charge q_0 is placed at the point where **E** has been calculated, the force acting on q_0 is

$$F = Eq_0$$

Combining this with eqn. (2.10) we obtain

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

which is nothing but the Coulomb's law of electrostatic force. Coulomb's law can, therefore, be deduced from Gauss' law and symmetry considerations.

(ii) *Electric field due to a charged sphere*

Let us consider a solid conducting sphere of radius R. if an amount of charge q is placed on this sphere, it will distribute itself uniformly over the surface of the sphere. *No charge can reside in the interior of the region*, because it will disturb the normal distribution of charges in this region of the conductor and hence will give rise to an unbalanced electric field causing an current to flow. Since there cannot be any such current in the case of electrostatic problems, *a conducting sphere must carry all the added charge on its surface*. The distribution of charge over the surface must also be uniform, otherwise there will exist a component of electric field tangential to the surface thus causing a current to flow on the surface. Again, this is not allowed under the assumed electrostatic condition. The distribution of change should be such that it would not create a tangential component of the field, so that the electric field **E** must be normal to the surface at all points on it. Since no tangential component of the field exists on the surface of the conductor, the surface will be an equipotential surface. The above discussion is true for any conductor of arbitrary shape. Consider a point P near but outside a uniform charged sphere of radius R with a charge q. Let us imagine a spherical Gaussian surface of radius r (r > R) passing through the point P. The charge q is now surrounded by the Gaussian surface. By the spherical symmetry of the problem, if the electric field strength at P be **E**, then it must have the same magnitude at all points over the Gaussian surface and everywhere normal to it.

Further, since both **E** and dS at any point on the Gaussian surface are directed radially outwards, the angle between them is zero. So from Gauss' law, we have

$$\oint \mathbf{E} \cdot d\mathbf{S} = \oint E \cos 0° dS = \oint E\, dS = \frac{q}{\epsilon_0}$$

Since E is uniform for all points on the surface, it can be brought outside the integral. Or,

$$E\oint dS = E.\,4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\text{or, } E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad (2.11)$$

Eqn. 2.11 is equivalent to the field at a point which is at a distance r from a point charge q. Thus *while finding the electric field outside a charged sphere, the charge can be considered to be concentrated at the centre of the sphere* and the formula for the point charge can be used.

Let us now consider a point inside the charged sphere and imagine a Gaussian surface of radius r (r < R) through the point. Applying Gauss' law we have

$$\oint \mathbf{E}.\,d\mathbf{S} = E\oint dS = E.4\pi r^2 = \frac{q'}{\epsilon_0}$$

where q' is the charge inside the Gaussian surface. But since all the charge resides on the surface of the charged sphere, there is no charge inside the sphere, *i.e.*, the charge inside the Gaussian surface is zero. Or, q' = 0. Hence the above relation reduces to

$$E\,.\,4\pi r^2 = 0$$

$$\text{or, } E = 0$$

Michael Faraday carried out an experiment designed to show that excess charge resides on the outside surface of a conductor. He built a large metal-covered box, which he mounted on insulating supports and charged it with a powerful electrostatic generator. In Faraday's words:

I went into the cube and lived in it, and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them ...... though all the time the outside of the cube was very powerfully charged, and large sparks and brushes were darting off from every part of its outer surface.

(iii) *Field due to a spherically symmetric charge distribution (volume distribution of charge)*

Fig. 2.6 shows a spherical region of radius R having a uniform volume distribution of charge. The charge density ρ, *i.e.*, the charge per unit volume (measured in coul/metre^3), at any point depends only on the distance of the point from the centre. This condition is

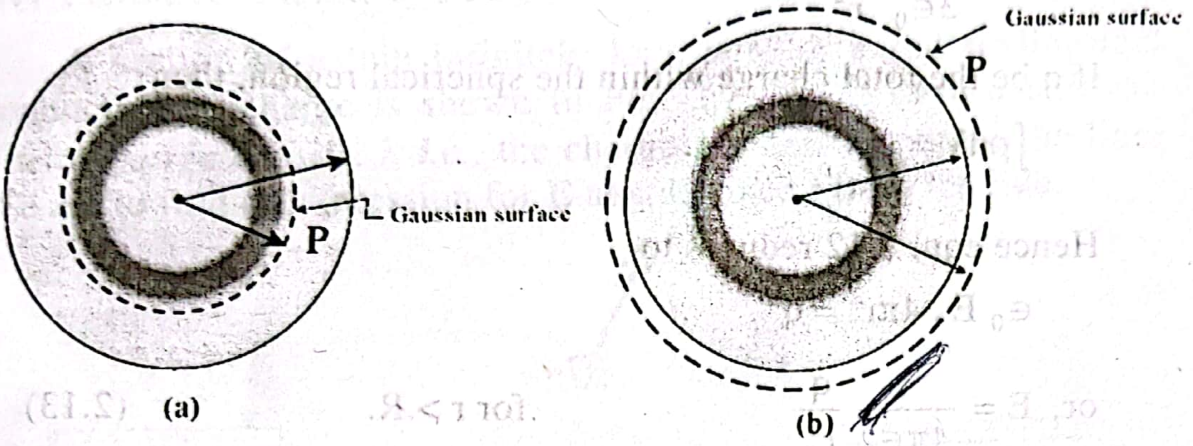

Fig. 2.6

referred to as spherical symmetry. Let us find an expression for E for points (b) outside and (a) inside the charge distribution. *The sphere of Fig. 2.6 cannot be a conductor because in that case all the charges given to the sphere i.e., the excess charges will reside on its surface.*

(a) *Field for points outside the sphere (r > R)*

Let us consider a point P outside the sphere at a distance r from the centre of the sphere. Consider a spherical Gaussian surface of radius r passing through the point. From symmetry and uniformity of the density, the field is radial and uniform over the Gaussian surface. Hence the angle between E and dS will be zero. Applying Gauss' law, we have

$$\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = \epsilon_0 \oint E\cos 0^\circ dS$$

$$= \epsilon_0 E 4\pi r^2 = \int_V \rho dV \qquad (2.12)$$

Since there is no charge outside the spherical region of charges, *i.e.*, $\rho = 0$ for $r > R$, the right side of eqn. 2.12 becomes

$$\int_V \rho dV = \rho . \frac{4}{3}\pi R^3$$

Hence eqn. 2.12 gives

$$E.4\pi r^2 = \frac{1}{\epsilon_0}\rho.\frac{4\pi}{3}R^3$$

$$\text{or, } E = \frac{\rho}{3\epsilon_0}\frac{R^3}{r^2} \quad \text{for } r > R.$$

If q be the total charge within the spherical region, then

$$\int_v \rho dV = q$$

Hence eqn. 2.12 reduces to

$$\epsilon_0 E.4\pi r^2 = q$$

$$\text{or, } E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{for } r > R. \qquad (2.13)$$

Thus for r > R, the total distribution of charges may be considered to be concentrated at the centre of the spherical region.

(b) *E for points inside the sphere (r < R)*

Now consider a point P at a distance r from the centre of the sphere of charge. Imagine a spherical Gaussian surface of radius r passing through the point. Again from symmetry consideration, $\vec{E}$ and $d\vec{S}$ will point in the same direction. Applying Gauss' law we obtain

$$\epsilon_0 \oint \vec{E}.d\vec{S} = \epsilon_0 \oint E\cos 0^\circ dS$$

$$= \epsilon_0 E\oint dS = \epsilon_0 E.4\pi r^2 = \int_v \rho dV = q' \text{ (say)}$$

where q' is the portion of q contained within the sphere of radius r.

$$\therefore\ E = \frac{1}{\epsilon_0}\frac{q'}{4\pi r^2} = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}$$

$$\text{Now } q = \rho.\frac{4}{3}\pi R^3 \text{ and } q' = \rho.\frac{4}{3}\pi r^3$$

$$\therefore\ q' = q\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = q\left(\frac{r}{R}\right)^3$$

Hence the expression for E for points inside the sphere (r < R) becomes

$$E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3} \quad \text{for} \quad r < R. \qquad (2.14)$$

(iv) *Field due to a line of charge*

A section of a thin infinitely long wire or a rod (cylindrical conductor) of charge is shown in Fig. 2.7. Let it have a uniform *linear charge density,* λ *i.e.,* the charge per unit length on the line. We are to find an expression for **E** at a distance r from the line.

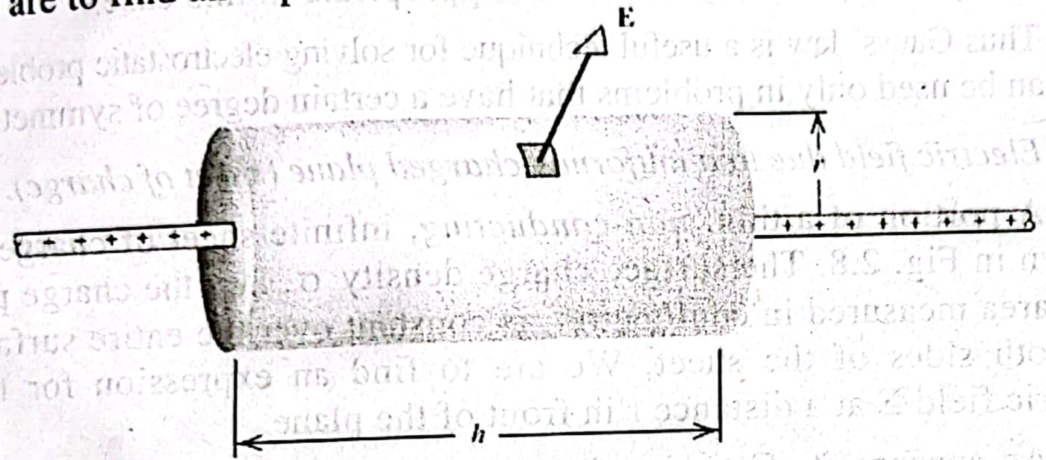

Fig. 2.7

As a Gaussian surface, let us choose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis. From symmetry, the field is everywhere constant on the Gaussian surface and for a positive charge directed radially away from the axis so that **E** and dS are in the same direction on the curved surface. There will be no flux through the circular caps at two ends because **E** here lies in the surface at every point; therefore the angle between **E** and dS is 90° at every point.

Applying Gauss' law, we therefore have,

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = \epsilon_0 \oint E\cos 0° dS$$

$$= \epsilon_0 E\, 2\pi rh = q \qquad (2.15)$$

where q is the charge enclosed by the cylinder and 2πrh is the volume of the cylinder. But q = λh. So eqn. 2.15 reduces to

$$\epsilon_0 E\, 2\pi rh = \lambda h$$

$$\text{or,}\quad E = \frac{\lambda}{2\pi\epsilon_0 r} \qquad (2.16)$$

The same problem was also solved in Art. 1.3 by using the method of integration. It may be noticed that how much simpler the solution becomes when Gauss' law is applied. It may also be noticed that the solution using Gauss' law is possible only if the Gaussian surface is choosen in such a way that full advantage of the radial symmetry of the electric field set up by a long line of charge can be taken. One is free to choose any surface for a Gaussian surface such as a cube or a sphere. However, they are not all useful for the problem at hand, only the cylindrical surface is appropriate in this case.

Thus Gauss' law is a useful technique for solving electrostatic problems but can be used only in problems that have a certain degree of symmetry.

(v) *Electric field due to a uniformly charged plane (sheet of charge).*

A portion of a thin, *non-conducting*, infinite sheet of charge is shown in Fig. 2.8. The surface charge density σ, *i.e.*, the charge per unit area measured in coul/metre^2, is constant over the entire surface on both sides of the sheet. We are to find an expression for the electric field **E** at a distance r in front of the plane.

An appropriate Gaussian for the problem is a pill-box of cross-sectional area S and height 2r, arranged to pierce the plane (r on each side) as shown in the figure. From symmetry **E** is uniform at every point and points at right angles to the end caps away from the plane. This means E and dS are in the same direction on the end caps. Since **E** does not pierce the cylindrical surface, *i.e.*, **E** and dS are perpendicular to each other at every point on the surface, there is no contribution to the flux from this source. Hence, applying Gauss' law, we obtain

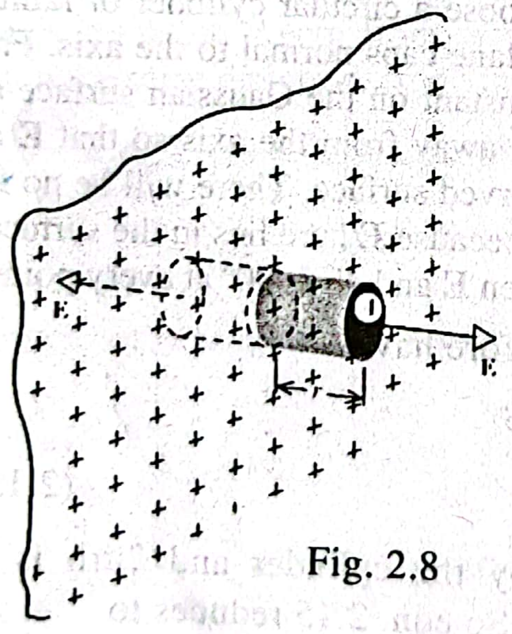

Fig. 2.8

$$\epsilon_0 \oint \mathbf{E} . d\mathbf{S} = \epsilon_0 \oint E \cos 0° dS = q \tag{2.17}$$

where q is the charge enclosed by the area of cross-section S.

Now $\oint E\cos 0°dS = (ES + ES)$

And $q = \int_s \sigma . dS = \sigma.S$

So eqn. 2.17 becomes

$$\epsilon_0 (ES + ES) = \sigma S$$

$$\text{or, } 2ES = \frac{\sigma S}{\epsilon_0}$$

$$\text{or, } E = \frac{\sigma}{2\epsilon_0}$$

Eqn. 2.18 shows that the field due to an infinite sheet of charge is independent of the distance from the plane, i.e., E is same for all points on each side of the plane.

(vi) *Electric field due to infinite charged conducting plate*

When a charge is given to a conducting plate, the same is distributed over the entire surface of the plate. If the plate is of uniform *size and thickness*, the surface density (σ) of the plate is uniform and is same on both sides of the surfaces. We are to find an expression for the electric field at a point P in front of the plate.

An appropriate Gaussian surface for the problem is a cylinder through P, which is normal to the surface of the plate. Let S be the cross-sectional area of the cylinder. One end of the cylinder is *inside the conductor* (Fig. 2.9) and so does not contribute to the electric flux as no charge exists inside the conductor. Also no contribution of flux is given by the curved surface of the cylinder as E and dS are perpendicular to each other at every point on this surface. So it is only end cap around P which contributes towards electric flux. So we have

$$\oint \mathbf{E}.d\mathbf{S} = \oint E\cos 0°dS$$

$$= E\oint dS = ES$$

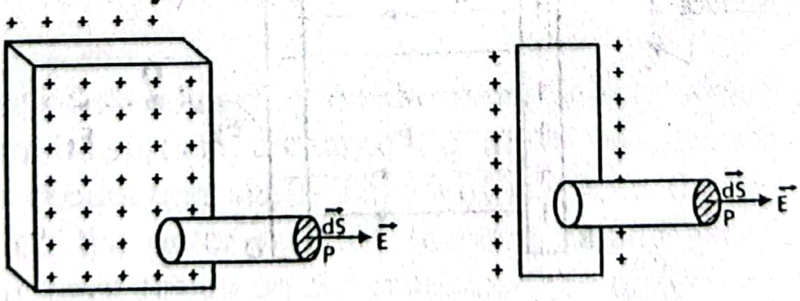

Fig. 2.9

as **E** and **dS** point in the same direction on the end cap.

Charge enclosed by the Gaussian surface of cross-sectional area S is

$q = \sigma.S.$

So from Gauss' theorem

$$\therefore \oint \mathbf{E} \cdot d\mathbf{S} = ES = \frac{q}{\epsilon_0} = \frac{\sigma S}{\epsilon_0}$$

$$\text{or, } E = \frac{\sigma}{\epsilon_0} \tag{2.19}$$

It may be noted that the electric field due to a charged conducting plate is twice the field due to a plane sheet of charge having the same surface charge density. It is due to the fact that the same charge is present on its both sides.

(vii) *Electric field due to two parallel charged plates*

Let AB and CD be two charged parallel plates of very great extent. Of the two plates AB has positive charge while CD negative, the charge density σ being same for both the plates.

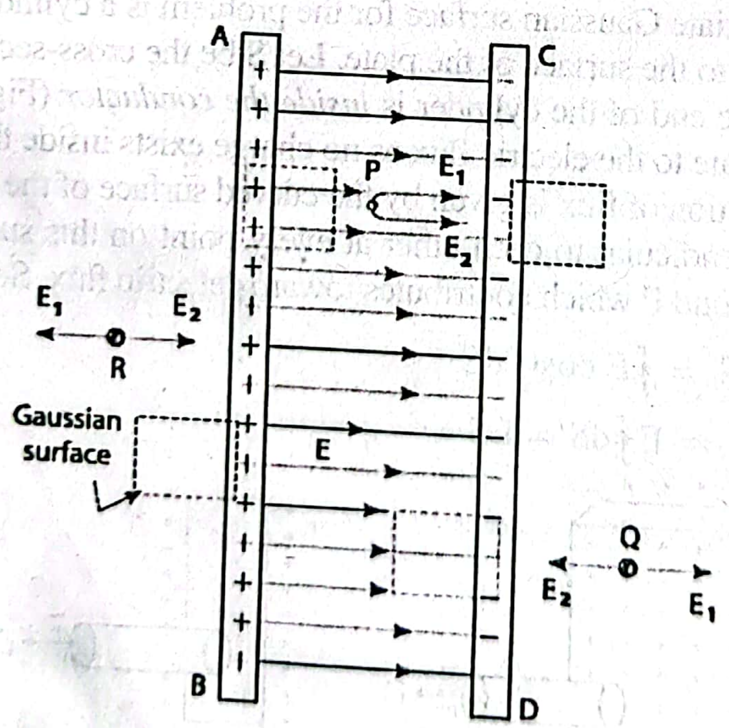

Fig. 2.10

At the point P inside the plates (Fig. 2.10), the field due to AB is $E_1 = \dfrac{\sigma}{2\epsilon_0}$ (eqn. 2.18) and points to the right. The field due to CD is $E_2 = \dfrac{\sigma}{2\epsilon_0}$ which also points to the right. [the electric field points away from the positive charge and points towards the negative charge]. Since electric fields superpose, the total field at P is

$$E = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

the direction being from plate AB to the plate CD, *i.e.*, from the positive charge to the negative charge.

Let us now consider a point Q outside the plates. The field due to AB is $\dfrac{\sigma}{2\epsilon_0}$ towards the right while the field due to CD is also $\dfrac{\sigma}{2\epsilon_0}$ but points towards the left. Thus the total field at Q is zero. This proves that the field due to parallel charged plates is constant at any point between the plates but vanishes at points outside the plates.

***Example 2.2** A spherical conducting shell of inner radius b and outer radius a carries a charge q. A second spherical conductor of radius c and charge q' is introduced inside it through a hole. Determine the electric fields in the regions (i) outside the outer conductor; (ii) inside the outer spherical shell; (iii) the space between the two spheres; and (iv) within the inner sphere.*

Soln.

Because of the charge q' on the inner sphere which is uniformly distributed on its surface, a charge – q' must appear on the inner walls of the outer conducting shell. This will result in the appearance of a charge +q' on the outer wall of the conducting shell. Therefore a charge q + q' must reside on the outer surface of the outer conducting shell. The arrangement of charges is shown in Fig. 2.11. The electric fields can be obtained by the application of Gauss' law.

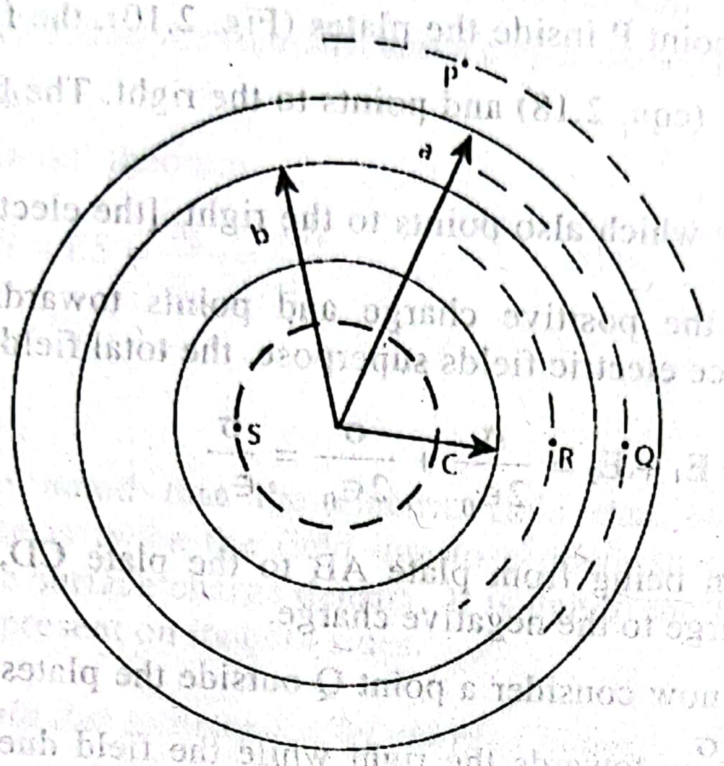

Fig. 2.11

(i) In the region outside the outer conductor, say at point P, we obtain $\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = \sum q$.

Now the total charge enclosed by the Gaussian surface passing through the point P is q + q' + (– q') + q' = q + q'.

$$\therefore \; \epsilon_0 E . 4\pi r^2 = q + q' ; \quad \text{or,} \quad E = \frac{1}{4\pi\epsilon_0} \frac{(q+q')}{r^2}$$

(ii) At a point inside the spherical shell, say at point Q. The total charge enclosed by the Gaussian surface passing through Q is – q' + q' = 0. Thus, applying Gauss' law we obtain

$$\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = \sum q = 0 ; \quad \text{or,} \quad \epsilon_0 E\, 4\pi r^2 = 0 ; \quad \text{or,} \quad E = 0.$$

(iii) At a point in the space between the two spheres, say at point R. The total charge enclosed by the Gaussian surface passing through R is q'. Hence

$$\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = q' ; \quad \text{or,} \quad \epsilon_0 E\, 4\pi r^2 = q' ; \quad \text{or,} \quad E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2}.$$

(iv) At a point within the inner sphere, say at a point S. Charge enclosed by the Gaussian surface passing through S is zero.

$$\epsilon_0 \oint \mathbf{E}.d\mathbf{S} = 0 ; \quad \text{or,} \quad \epsilon_0 E\, 4\pi r^2 = 0 ; \quad \text{or,} \quad E = 0.$$

***Example 2.3** The magnitude of the average electric field normally present in the Earth's atmosphere just above the surface of the Earth is above 150 N/C, directed downward. What is the total net surface charge carried by the Earth? Assume the earth to be a conductor.*

Soln.

Lines of force terminate on negative charges so that the Earth's electric field points downward. Hence the average surface charge density σ must be negative. Thus from the relation $E = \sigma/\epsilon_0$ we obtain

$$\sigma = \epsilon_0 E = (8.85 \times 10^{-12}\ C^2/N.m^2)(-150\ N/C)$$

$$= -1.33 \times 10^{-9}\ C/m^2.$$

The Earth's total charge q is given by the product of its surface charge density (σ) and its surface area $4\pi R^2$. Thus

$$q = \sigma.4\pi R^2 = (-1.33 \times 10^{-9}\ C/m^2)(4\pi)(6.37 \times 10^6 m)^2$$

$$= -6.8 \times 10^5\ C$$

$$= -680\ KC.$$

***Example 2.4** A plastic rod, whose length L is 220 cm and whose radius R is 3.6 mm, carries a negative charge q of magnitude 3.8 x 10^{-7}C, spread uniformly over its surface. What is the electric field near the midpoint of the rod, at a point on its surface?*

Soln.

Although the rod is not infinitely long, for a point on its surface and near its midpoint, it may be considered to be effectively infinitely long.

The linear charge density for the rod is

$$\lambda = \frac{q}{L} = \frac{-3.8\times10^{-7}\ C}{2.2m} = -1.73 \times 10^{-7} C/m$$

We then have from eqn. 2.16

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{-1.73\times10^{-7}\ C/m}{(2\pi)(8.85\times10^{-12} C^2/N.m^2)(0.0036m)}$$

$$= -8.6 \times 10^5\ N/C.$$

Example 2.5 *Portions of two large sheets of charge with uniform surface charge densities $s_+ = +6.8\ mC/m^2$ and $s_- = -4.3 mC/m^2$ are shown in Fig. 2.12. Find the electric field E (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.*

Soln.

The electric field due to each sheet separately will be first determined. The resultant field will then be determined by using the principle of superposition.

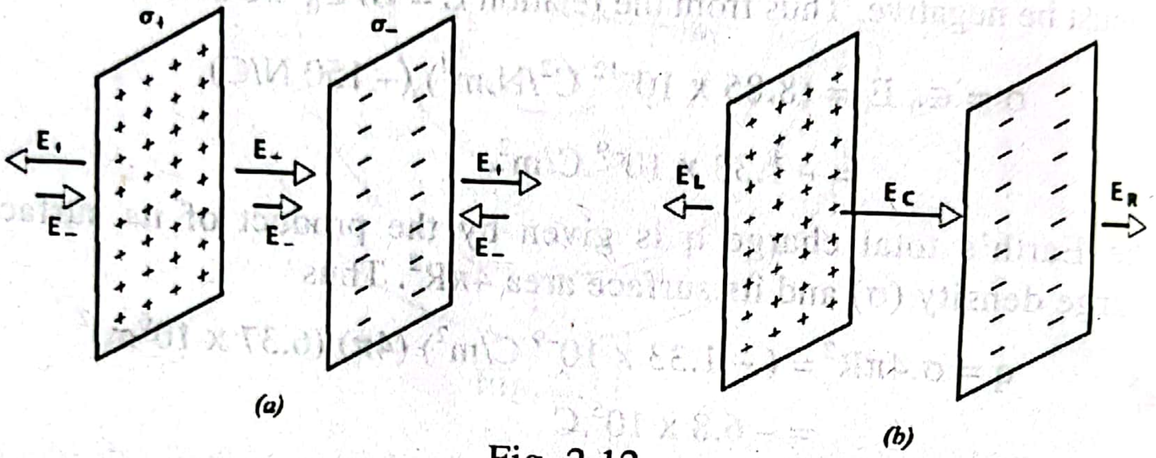

Fig. 2.12

For the electric field due to the positive sheet we have,

$$E_+ = \frac{\sigma_+}{2\epsilon_0} = \frac{6.8\times10^{-6}C/m^2}{(2)(8.85\times10^{-12}C^2/N.m^2)} = 3.84 \times 10^5 N/c.$$

Similarly for the negative sheet,

$$E_- = \frac{\sigma_-}{2\epsilon_0} = \frac{4.3\times10^{-6}C/m^2}{(2)(8.85\times10^{-12}C^2/N.m^2)} = 2.43 \times 10^5 N/c.$$

The direction of the fields are shown in the figure for left of the sheets, between the sheets and right of the sheets. The resultant fields in these regions are obtained following the principle of superposition. For a point on the left of the sheets we have,

$$E_L = -E_+ + E_- = -3.8 \times 10^5\ N/C + 2.43 \times 10^5 N/C$$
$$= -1.4 \times 10^5\ N/C.$$

The resultant field therefore points towards the left.

For a point on the right, the electric field E_R has the same magnitude but points towards the right.

Between the sheets, the two fields point to the same direction. They, therefore, add to give

$$E_C = 3.84 \times 10^5 \text{ N/C} + 2.43 \times 10^5 \text{ N/C} = 6.3 \times 10^5 \text{ N/C}.$$

***Example 2.6** Two parallel plates of area 20 cm^2 each are separated by a distance of 10 mm. If one plate carries a charge $+2 \times 10^{-9}$C, and the other carries a charge -2×10^{9}C, calculate the electric field in the space between them.*

Soln.

Surface charge density for each of the plate is given by

$$\sigma = \frac{2\times10^{-9}}{20\times10^{-4}} \text{Cm}^{-2} = 10^{-6} \text{ Cm}^{-2}.$$

Since one of the plate carries positive charge while the other carries negative charge, the electric field in the space between them

$$E = \frac{\sigma}{\epsilon_0} = \frac{10^{-6}\text{Cm}^{-2}}{8.85\times10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 1.13 \times 10^5 \text{ N/C}.$$

***Example 2.7** The electric strength of air is about 3.0×10^6 N/C which means that if the electric field exceeds this value, sparking will occur. What is the largest charge a 0.50 cm. radius sphere can hold if sparking is not to occur in the air surrounding it?*

Soln.

For a point outside a sphere, all the charge on the surface can be considered to be located at the centre of the sphere. Thus the value of q can be obtained from the relation.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\text{or, } 3.0 \times 10^6 \text{ N/C} = \frac{1}{(4\pi)(8.85\times10^{-12}\text{C}^2/\text{N.m}^2)}\frac{q}{(5.0\times10^{-3}\text{m})^2}$$

$$\text{or, } q = 8.34 \times 10^{-9}\text{C} = 8.34\text{nC}.$$