CHAPTER – I

COULOMB'S LAW & ELECTRIC FIELD

1.1 Concept of charge

Let a glass rod, rubbed with silk, be hung from a long silk thread as in Fig. 1.1. If another glass rod, again rubbed with silk, is held near the rubbed end of the first rod, the rods will *repel* each other. On the other hand, a hard-rubber rod, rubbed with fur, will *attract* the glass rod. But two rubber rods rubbed with fur will repel each other. This phenomenon serves us to introduce the concept of charge. We say that as a result of rubbing, the rods acquire a new property

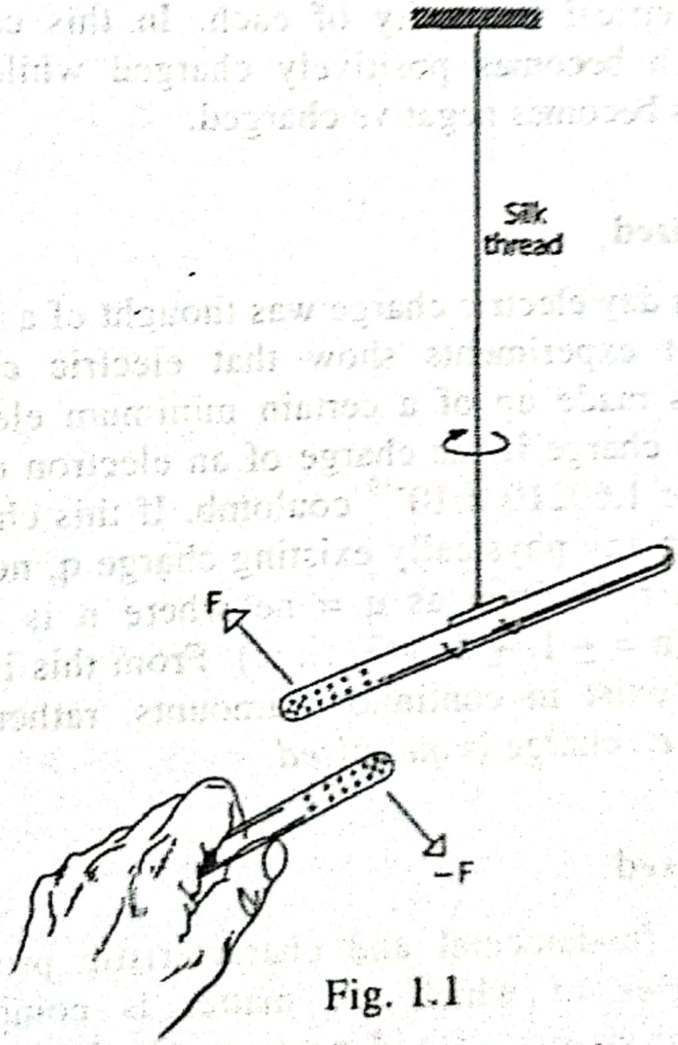


Fig. 1.1

– they become electrically charged and the charges on the two rods exert forces on one another. Clearly the charges on the glass the hard rubber rods must be different in nature. Benjamin Franklin who, among many other accomplishments, is regarded as the first

American physicist, named the kind of electric charge that appears on the glass rod as *positive* and the kind that appears on hard-rubber *negative*. The positive and negative labels and signs for electric charge were chosen arbitrarily by Franklin – but the names have remained to this day. Thus the simple experiments described above can be summed up by saying that

like charges repel each other and unlike charges attract each other.

Charge itself, however, is not created during the process of rubbing. In the light of modern view of bulk matter, an object in its normal state is electrically neutral *i.e.*, it contains equal amounts of positive and negative charge. If the two bodies like glass and silk are rubbed together, electrons are transferred from one to the other, upsetting the electrical neutrality of each. In this case electrons leave glass which becomes positively charged while silk which receives electrons becomes negative charged.

Charge is quantized

In Franklin's day electric charge was thought of as a continuous fluid. Subsequent experiments show that electric charge is not continuous but is made up of a certain minimum electric charge. This fundamental charge is the charge of an electron or proton and has the magnitude 1.60210×10^{-19} coulomb. If this charge is given the symbol e, then any physically existing charge q, no matter what its origin is, can be written as $q = ne$ where n is a positive or negative integer ($n = \pm 1, \pm 2, \pm 3, \ldots\ldots$). From this it follows that charge does not exist in continuous amounts, rather it exists in discrete *packets i.e.*, charge is *quantized*.

Charge is conserved

Charge is a fundamental and characteristic property of the elementary particles of which the matter is composed. These particles are electron, proton and neutron. Of these electrons are negatively charged, protons positively charged and neutrons are neutral. Although the mass of a proton is 1840 times heavier than the mass of an electron, the magnitude of its charge is the same as that of electron. The masses of proton and neutron are approximately equal.

There are then two kinds of charge, positive and negative; and an ordinary piece of matter contains equal amount of each kind. Thus ordinary matter is electrically neutral. When we say that an object is charged, we mean that either it has an excess of electrons (in which case the object is negatively charged) or an excess of protons (in which case the object is positively charged). The charge of a body, then, refers to net charge or excess charge. It is the algebraic sum of the charges present in the body and may be positive, negative or zero.

Like many other physical entities charge cannot be created or destroyed. When a glass rod is rubbed with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that charge is not created by rubbing but is merely transferred from one object to another, slightly disturbing the electrical neutrality of each object in the process. Thus the total charge of an isolated (or closed) system cannot change – the individual charges can be combined or regrouped in different ways. This is known as the *principle of conservation of charges*.

Conductors and insulators

A metal rod held in hand and rubbed with fur will not seem to develop a charge. If the same rod is provided with a glass or hard rubber handle and then rubbed with fur without touching it with hands, it will be charged. The explanation is that metals, the human body and the earth are *conductors* of electricity while glass, hard rubber, plastics, etc. are *insulators* (also called *dielectrics*).

In conductors electric charges are free to move through the material, whereas in insulators they are not. Although there are no perfect insulators, the insulating ability of fused quartz is 10^{25} times as great as that of copper; so that for many practical purposes fused quartz as well as many other materials behave as if they were perfect insulators.

There is yet another class of materials called *semiconductors* whose ability to conduct electricity is intermediate between conductors and insulators. Among the elements, silicon and germanium are well known examples. The electrical conductivity of semiconductors can often be greatly increased by adding very small amounts of other elements. For example, the conducting ability of silicon or germanium can be immensely enhanced by adding traces of arsenic or boron to these elements. This property of semiconductors has many practical applications.

1.2 Coulomb's law

The *electrostatic force* of attraction or repulsion between two charges (strictly speaking point charges) was first measured quantitatively by Charles Augustine de Coulomb in 1875. The apparatus used by him is shown in Fig. 1.2.

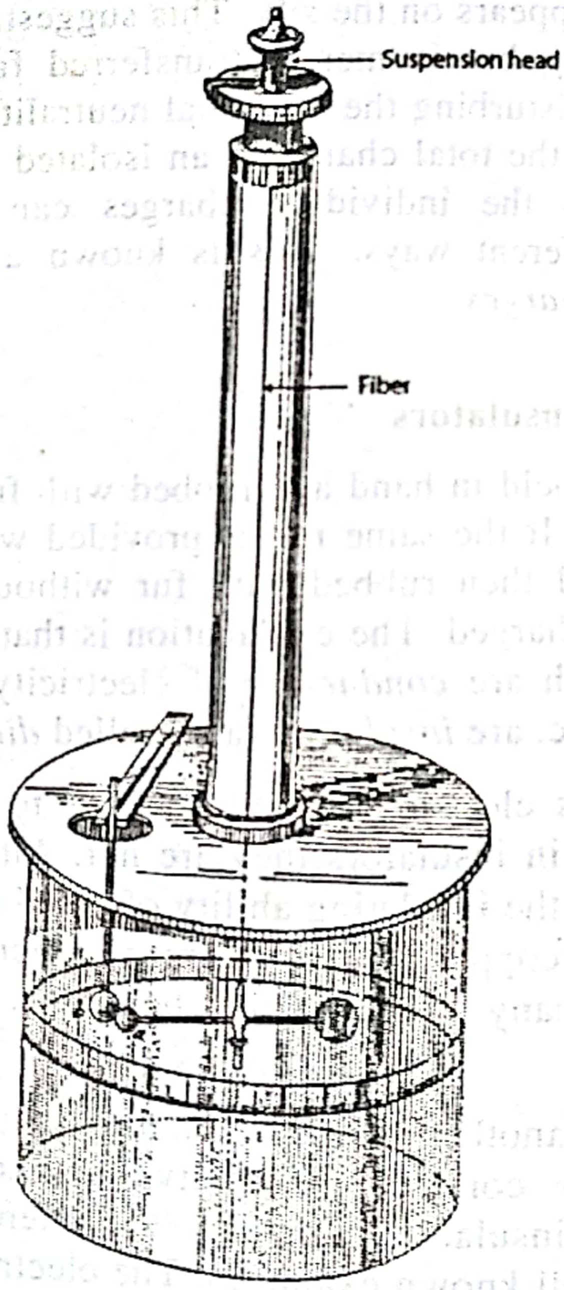


Fig. 1.2

The charges are confined to small spheres a and b. Of the two spheres b is fixed while a is suspended by a fibre. As b is fixed, the electric force on a will tend to twist the suspension fibre. Coulomb cancelled out this twisting effect by turning the suspension head through the angle θ needed to keep the two charges at a particular separation. The angle θ is then a relative measure of the electric force acting on a. The device of Fig. 1.2 is called a *torsion balance.*

Coulomb at first kept the magnitude of the charges on the spheres a and b constant and varied the distance of separation between the charges. His first experimental results can be represented by

$$F \propto \frac{1}{r^2}$$

where F is the magnitude of the force that acts on each of the two charges separated by a distance r. As required by Newton's third law of motion, these forces must act along the line joining the charges but point in opposite directions. It may be noted that the magnitude of the force on each charge is the same, even though the charges may be different.

Coulomb then studied how the electrical force varied with the relative size of the charges on the spheres of his torsion balance by keeping the distance fixed. If a charged conducting sphere is touched to an exactly similar but uncharged conducting sphere, the original charge will divide equally between the spheres. Coulomb varied the relative size of the charges by this technique and his experimental result can be represented by

$$F \propto q_1 q_2$$

where q_1 and q_2 are the charges on the two spheres.

Thus the electrostatic force exerted on one charge by another charge depends directly on the product of the magnitudes of the two charges and inversely on the square of their separation. That is

$$F \propto \frac{q_1 q_2}{r^2} \tag{1.1}$$

Turning the above proportionality into an equation by introducing a constant of proportionality k, the force between two charges is given by

$$F = k\frac{q_1 q_2}{r^2} \tag{1.2}$$

Eqn. (1.2) is called *Coulomb's law* and generally holds for *point charges*. Charged objects whose sizes are much smaller than the distance between them are usually referred to as point charges.

The SI unit of charge is the *coulomb* (abbreviation C), which is defined as *the amount of charge* that *flows in one second when there is a steady current of one ampere*. That is

$$dq = idt \tag{1.3}$$

where dq (in coulombs) is the charge transferred by a current i (in amperes) during the interval dt (in seconds). For example, a current of 1A delivers a charge of 1×10^{-6} C in a time of 10^{-6} s.

In the SI system, the constant k is expressed in the following form:

$$k = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0, called the *permittivity constant* of free space (vacuum), must have that value which makes the right hand-side of eqn. (1.2) equal to the left-hand side. This value turns out to be

$$\epsilon_0 = 8.85418 \times 10^{-12} \text{ coul}^2 / \text{nt - m}^2.$$

The constant k has the corresponding value

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N.m}^2/\text{coul}^2.$$

With this choice of k, Coulomb's law can be written as

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \tag{1.4}$$

When k has the above value, expressing q in coulombs and r in metres gives the force in newtons.

Coulomb's law has survived every experimental test; no exceptions to it have ever been found. The significance of Coulomb's law goes far beyond the description of the forces acting between charged balls or rods. This law, when incorporated into the structure of quantum physics, correctly describes (a) the electric forces that bind the electrons of an atom to its nucleus, (b) the forces that bind atoms together to form molecules, and (c) the forces that bind atoms or molecules together to form solids or liquids. Most of the forces of our daily experience that are not gravitational in nature are electrical.

1.3 Gravitational force and electrical force

In eqn. (1.2), F is the magnitude of the force acting on either particle owing to the charge on the other, and q_1 and q_2 are the *magnitudes* (or *absolute values*) of the charges of the two particles. The gravitational force between the two particles is given by

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the masses of the two particles, G being gravitational constant. By analogy with G, the constant k may be called the *electrostatic constant*. Both laws are inverse square laws and both involve a property of the interacting particles – the mass in one case and the charge in the other.

However there are dissimilarities between the laws too. The gravitational forces are always attractive but electrostatic forces may be either attractive or repulsive, depending on the signs of the two charges. This difference arises from the fact that, although there is only one kind of mass, there are two kinds of charges.

Example 1.1 *Determine the force between two free electrons spaced 1 Å (10^{-10}m) apart (a typical atomic dimension).*

Soln.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \qquad q_1 = q_2 = -1.6 \times 10^{-19}\text{C}$$

$$r = 1\,\text{Å} = 10^{-10}\text{m}$$

$$= \frac{(9.0\times10^9)(-1.6\times10^{-19})^2}{(1.0\times10^{-10})^2} \qquad \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ \text{N.m}^2/\text{coul}^2$$

$$= 2.3 \times 10^{-8}\ \text{N}$$

$$= 23\ \text{nN}.$$

The force is repulsive.

***Example 1.2** The uranium nucleus contains a charge 92 times that of a proton. If a proton is shot at the nucleus, how large a repulsive force does the proton experience due to the nucleus when it is 1×10^{-11}m from the nucleus centre? The nuclei of atoms are of the order of the 10^{-14}m in diameter, so the nucleus can be considered a point charge.*

Soln.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$q_1 = 92.e = (92 \times 1.6 \times 10^{-19}C)$

$q_2 = 1.6 \times 10^{-19}C$

$r = 1 \times 10^{-11} m$

$$= \frac{(9\times10^9)(92\times1.6\times10^{-19})(1.6\times10^{-19})}{(1\times10^{-11})^2}$$

$= 2.1 \times 10^{-4}$N.

***Example 1.3** Two point charges q_1 and q_2 are 3m apart, and their combined charge is 20μC. What are the magnitude of the two charges if (a) one repels the other with a force of 0.075N, and (b) attracts the other with a force of 0.525N?*

Soln.

(a) Since the force is repulsive, we have from the relation

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2},$$

$$0.075 = (9 \times 10^9)\frac{q_1 q_2}{r^2}$$

or, $q_1 q_2 = \frac{0.075\times9}{9\times10^9} = 75 \times 10^{-12}\ C^2 = 75\mu C^2.$

Now $q_1 + q_2 = 20\mu C$, or, $q_2 = (20 - q_1)$

$\therefore\ q_1(20 - q_1) = 75;$

or, $q^2 - 20q_1 + 75 = 0;$ or, $q_1^2 - 5q_1 - 15q_1 + 75 = 0$

or, $q_1(q_1-5)-15(q_1-5)=0$, or, $(q_1-5)(q_1-15)=0$

or, $q_1=5$ or, $15\,\mu C$

Hence $q_2 = 15$ or, $5\mu C$.

(b) Force is attractive; hence

$$-0.525 = (9 \times 10^9)\frac{q_1 q_2}{9}$$

$$\text{or, } q_1q_2 = \frac{(-0.525)(9)}{9\times10^9} = -525\mu C^2.$$

Again substituting $q_2 = (20 - q_1)$

$$q_1(20-q_1) = 525$$

or, $q_1^2 - 20q_1 - 525 = 0$

or, $q_1^2 + 15q_1 - 35q_1 - 525 = 0$

or, $q_1(q_1+15) - 35(q_1+15) = 0$

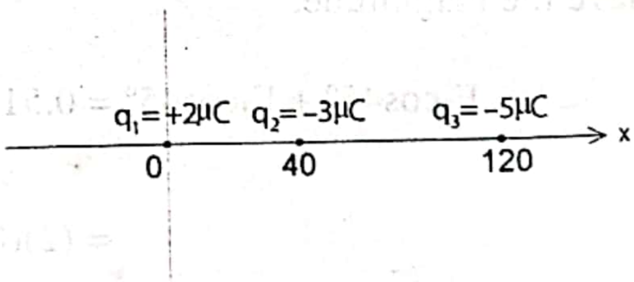
or, $(q_1+15)(q_1-35) = 0$

or, $q_1 = -15$ or, $+35\mu C$ and $q_2 = +35$ or, $-15\mu C$.

***Example 1.4** Three point charges are placed at the following points on the x-axis: $+2\mu C$ at $x = 0$, $-3\mu C$ at $x = 40cm$. $-5\mu C$ at $x = 120cm$. Find the force on the $-3\mu C$ charge.*

Soln.

The arrangement is shown in the adjoining figure. The force on q_2 is the vector sum of two contributions, the attractive force due to q_1 and the repulsive force due to q_3.



The direction of both the forces is towards the left *i.e.*, towards q_1. The sum of these two forces, taken algebraically, since they are along the same line, is

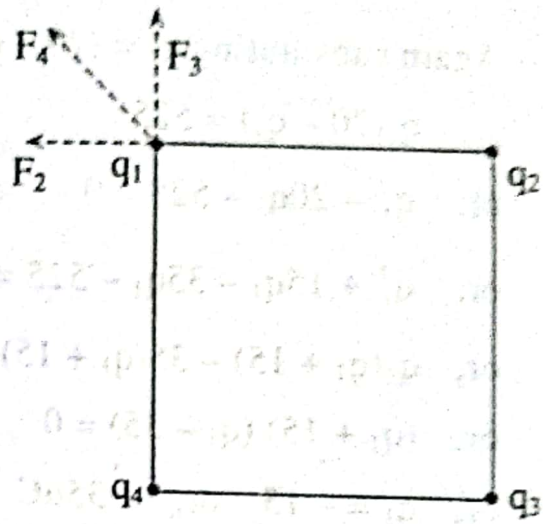
$$F = F_1 + F_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_1^2} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_2^2}$$

$$= 9 \times 10^9 \left[\frac{(2\times10^{-6})(3\times10^{-6})}{(0.40)^2} + \frac{(5\times10^{-6})(3\times10^{-6})}{(0.80)^2}\right]$$

$= 0.55$N towards left.

Example 1.5 Four equal point charges, +3mC, are placed at the four corners of a square that is 40 cm on a side. Find the force on any one of the charges.

Soln.



The situation is depicted in the adjoining figure. Let us consider the forces acting on q_1. Let F_2, F_3 and F_4 be the forces exerted on q_1 by q_2, q_3 and q_4 respectively.

By symmetry

$$F_2 = F_3 = (9 \times 10^9)\frac{(3\times10^{-6})(3\times10^{-6})}{(0.40)^2} = 0.51 \text{ N}.$$

Since the directions of F_2 and F_3 are along the edges as shown in the figure, their vector sum will lie along the diagonal from q_4 to q_1 and have the magnitude.

$$F_2\cos45^\circ + F_3\cos45^\circ = 0.51\frac{1}{\sqrt{2}} + 0.51\frac{1}{\sqrt{2}}$$

$$= (2)(0.51)\frac{1}{\sqrt{2}} = 0.72\text{N}.$$

The remaining force F_4 is also along this diagonal, and

$$F_4 = (9 \times 10^9)\frac{(3\times10^{-6})(3\times10^{-6})}{\left(\sqrt{0.40^2+0.40^2}\right)^2} = 0.25\text{ N}$$

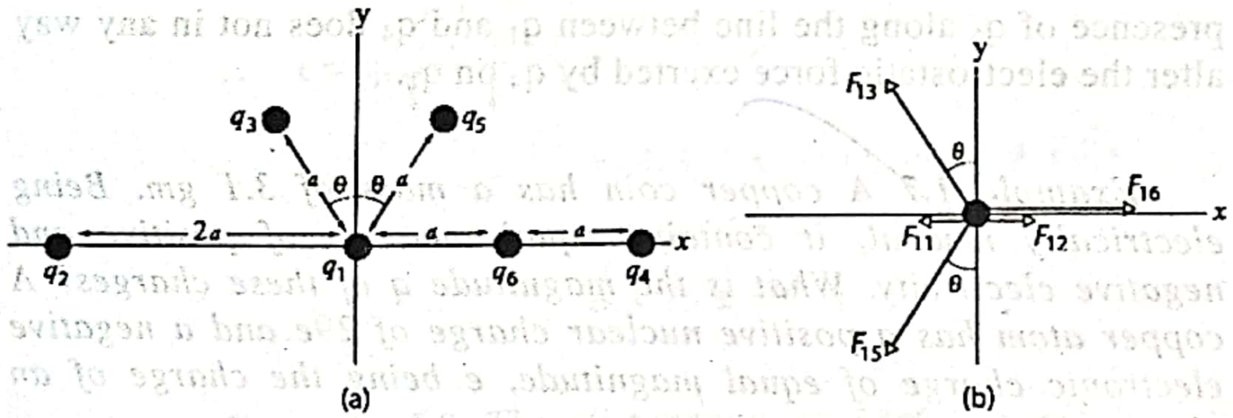
The resultant of all the three force acting along the diagonal away from the square.

$$= 0.72\text{ N} + 0.25\text{ N} = 0.97\text{ N.}$$

***Example 1.6** The figure below shows an arrangement of six fixed charged particles, where $a = 2.0$ cm and $\theta = 30^o$. All six particles have the same magnitude of charge $q = 3.0 \times 10^{-6}$C; their electrical signs are as indicated. What is the net electrostatic force F_1 acting on q_1 due to other charges?*

Soln.

From the figure it is obvious that F_1, the net electrostatic force acting on q_1 is the vector sum of F_{12}, F_{13} F_{14}, F_{15} and F_{16}, the exerted on q_1 due to other charges. Because q_2 and q_4 are equal in magnitude and are both a distance r = 2a from q_1, we have from eqn. (1.4)



$$F_{12} = F_{14} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{(2a)^2} \qquad \text{(i)}$$

Similarly, since q_3, q_5 and q_6 are equal in magnitude and are each a distance r = a from q_1, we have

$$F_{13} = F_{15} = F_{16} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_3}{a^2} \qquad \text{(ii)}$$

Fig. (b) is a free body diagram for q_1. It and eqn. (i) show that F_{12} and F_{14} are equal in magnitude but opposite in direction, thus they cancel each other. Inspection of the figure (b) and eqn. (ii) reveals that the y-components of F_{13} and F_{15} also cancel and that their x-components are identical in magnitude and both point in the direction of decreasing x. Fig. (b) also shows that F_{16} points in the direction of increasing x. Thus F_1 must be parallel to the x-axis; its magnitude is the difference between F_{16} and twice the x-component of F_{13}.

$$\therefore \quad F_1 = F_{16} - 2F_{13}\sin\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q_1q_6}{a^2} - 2.\frac{1}{4\pi\epsilon_0}\frac{q_1q_3}{a^2}\sin\theta$$

Now $q_3 = q_6$ and $\theta = 30°$,

Substituting these values we find

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_6}{a^2} - 2.\frac{1}{4\pi\epsilon_0}\frac{q_1q_6}{a^2}\sin 30° = 0$$

So the net force acting on q_1 is zero. It may be noted that the presence of q_6 along the line between q_1 and q_4 does not in any way alter the electrostatic force exerted by q_4 on q_1.

Example 1.7 *A copper coin has a mass of 3.1 gm. Being electrically neutral, it contains equal amounts of positive and negative electricity. What is the magnitude q of these charges? A copper atom has a positive nuclear charge of 29e and a negative electronic charge of equal magnitude, e being the charge of an electron.*

Soln.

The number N of copper atoms in the coin

$$N = \frac{m}{M}.N_o \qquad \text{where } m = \text{mass of the coin} = 3.1\text{ gm}$$

M = atomic weight of copper

$$= \frac{3.1}{64} \times 6.0 \times 10^{23} \qquad N_o = \text{Avogadro's number}$$

$= 2.9 \times 10^{22}$ atoms.

Charge (either positive or negative) of a copper atom

$= 29 \times 1.6 \times 10^{-19}C = 4.64 \times 10^{-18}C.$

Therefore, the charge

$q = (4.64 \times 10^{-18})(2.9 \times 10^{22}) = 1.345 \times 10^5 C.$

This is an enormous amount of charge. It can be verified that it will take 40 hrs. for a charge of this amount to pass through a 100 watt, 110 volt light bulb.

***Example 1.8** What should be the distance of separation between the total positive and negative charges of the copper coin of example 1.7 such that their force of attraction is 4.5N?*

Soln.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} \qquad q_1 = q_2 = 1.345 \times 10^5 C.$$

$$F = 4.5\ N$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

$$\therefore \quad r = q.\sqrt{\frac{1/4\pi\epsilon_0}{F}} = (1.345 \times 10^5)\sqrt{\frac{9\times10^9}{4.5}}$$

$$= (1.345 \times 10^5)(0.4472 \times 10^5)$$

$$= 6.0 \times 10^9 \text{ metres}$$

$$= 3.75 \times 10^6 \text{ miles.}$$

***Example 1.9** The distance r between the electron and the proton in the hydrogen atom is about 5.3×10^{-11} metre. What are the magnitudes of (a) the electrical force and (b) the gravitational force between these two particles?*

Soln.

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} \qquad q_1 = q_2 = 1.6 \times 10^{-19}C$$

$$r = 5.3 \times 10^{-11}m.$$

$$= (9 \times 10^9)\ \frac{(1.6\times10^{-19})^2}{(5.3\times10^{-11})^2}$$

$$= 8.1 \times 10^{-8}\ N.$$

The gravitational force is given by

$$F_g = G.\frac{m_1m_2}{r^2}$$

$$= \frac{(6.7\times10^{-11})(9.1\times10^{-31})(1.7\times10^{-27})}{(5.3\times10^{-11})}$$

$$= 3.7 \times 10^{-47}\ N.$$

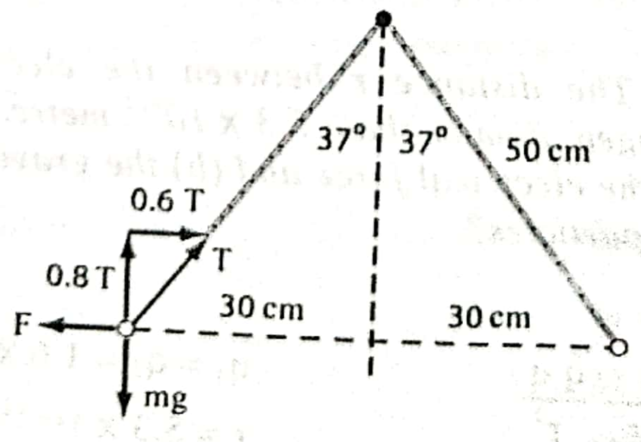
m_1 = electron mass
= 9.1×10^{-31}Kg
m_2 = proton mass
= 1.7×10^{-27}Kg
G = gravitational constant
= 6.7×10^{-11}N-m²/kg²
r = 5.3×10^{-11}m.

Thus the electrical force is approximately 2.2×10^{39} times stronger than the gravitational force.

Example 1.10 *The two balls shown in the figure below have identical masses of 0.20gm each. When suspended from 50cm long strings, they make an angle of 37° to the vertical. If the charges on each are the same, how large is each charge?*

Soln.

Since the system is at rest, let us apply the conditions for equilibrium to the ball on the left. Three forces act on this ball: its weight mg; the tension T in the string; and F, the repulsive force due to the charge on the other ball.



Applying the usual conditions for equilibrium:

$$\sum F_x = 0 \; ; \qquad i.e., \; F - 0.6T = 0 \qquad (i)$$

$$\sum F_y = 0 \; ; \qquad i.e., \; 0.8T - mg = 0$$

$$\text{or,} \quad 0.8T - (0.2 \times 10^{-3} kg)(9.8 m/s^2) = 0$$

$$\text{or,} \quad T = 2.45 \times 10^{-3} N \qquad (ii)$$

Substituting the value of T as given by (ii) in (i) we get

$$F - (0.6)(2.45 \times 10^{-3} N) = 0$$

$$\text{or,} \quad F = 1.47 \times 10^{-3} N \qquad (iii)$$

Substituting this value of F in Coulomb' law

$$1.47 \times 10^{-3} = (9 \times 10^9) \frac{q^2}{(0.60)^2}$$

$$\text{or,} \quad q^2 = \frac{(1.47 \times 10^{-3})(0.60)^2}{9 \times 10^9} = 0.0588 \times 10^{-12}$$

$$= 5.88 \times 10^{-14}$$

$$\text{or,} \quad q = 2.4248 \times 10^{-7} C$$

$$= 0.24248 \times 10^{-6} C$$

$$= 0.24248 \mu C.$$

1.3 Electric field and electric field strength

Suppose that a stationary positive charge q_o, placed at any point in a certain region of space, experiences a force F. We then say there is an *electric field* in the region, whose strength E at the point is given by

$$E = \frac{F}{q_0} \qquad (1.5)$$

The *electric field strength* E, also known as *electric intensity*, at a point is, therefore, defined as the *force per unit positive charge* (assumed positive for convenience). Here E is a vector because F is one, q_0 being scalar. The direction of **E** is the direction of **F**, that is, it is the direction in which a stationary positive charge placed at the point would tend to move. The condition that q_0 be stationary is necessary to distinguish electric field from magnetic field as moving charges give rise to magnetic fields. It is evident from eqn. (1.5) that the unit of electric field strength is *Newton per coulomb*.

If **E** itself is produced by a set of charges *fixed in space*, then its value as given by eqn. (1.5) is independent of the size of q_0 and depends on the positions and magnitudes of other charges. On the other hand, **E** may in practice be produced by charges on bodies such as conductors. In that case, when the charge q_0 is introduced, it may cause a redistribution of the source charges and the value of **E** will then depend on the size of q_0. While we might be interested in the field **E** for large values of q_0, the most useful concept is that of the electric field which would exist at a point due to the original undisturbed charges. This field is defined by making the magnitude of q_0, usually referred to as the *test charge*, as small as possible so that it does not disturb the field produced by the source charges. The electric field is, therefore, defined as

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\underline{\mathbf{F}}}{q_0} \tag{1.6}$$

q_0 cannot of course be smaller than the electronic charge. Unless otherwise specified, any value quoted for $\underline{E}$ will be that for an undisturbed source.

Example 1.11 *What is the magnitude of the electric field strength* **E** *such that an electron, placed in the field, would experience an electrical force equal to its weight?*

Soln.

$$E = \frac{F}{q_0} = \frac{mg}{q_0}$$

$m = 9.1 \times 10^{-31}$ kg

$g = 9.8$ m/s^2

$q_0 = 1.6 \times 10^{-19}$ C

$$= \frac{(9.1\times10^{-31})(9.8)}{1.6\times10^{-19}}$$

$$= 5.6 \times 10^{-11} \text{ N/C}$$

Lines of force

The concept of the electric field as a vector was not appreciated by Michael Faraday. In order to visualize the electric field in space, he introduced the concept of lines of force. The lines of force are supposed to originate from the positive charges and terminate in the negative charges. These lines may be straight or curved depending on the system of charge creating them. The relationship between the (imaginary) lines of force and the electric field strength vector is given below.

1. The tangent to the line of force at any point gives the *directio[n]* of **E** at that point.

2. The density of the lines of force (the number of lines of forc[e] crossing unit area perpendicular to the field) is proportional t[o] the magnitude of **E**. Where the lines are close together **E** i[s] large and where they are far apart **E** is small.

No two lines of force can cross each other; because in that cas[e] there will be two directions of the electric field strength at the point o[f] cross-over which is impossible. Lines of force are used for visualizin[g] electric field patterns – they are not employed quantitatively.

Calculation of electric field strength

Let a test charge q_0 be placed at a distance r from a poin[t] charge. The magnitude of the force acting on q_0 as given b[y] Coulomb's law is

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

The strength of the electric field at the site of the test charge a[s] given by eqn. (1.5) is

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad (1.7)$$

The direction of **E** is on a radial line from q, pointing outwa[rd] if q_0 is positive and inward if q is negative.

If there are more than one charge in the field then (i) th[e] electric field strength at the given point should be calculated f[or] each charge *as if it were the only charge present* and (ii) these separately calculated fields should be added vectorially to find the resultant field strength **E** at the point. In equation form,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots\ldots = \sum \mathbf{E}_n \quad n = 1, 2, 3, \ldots \tag{1.8}$$

The sum is a vector sum, taken over all the charges. If the charge distribution is not discrete but continuous, then the charge must be divided into infinitesimal elements of charge dq. The field dE due to each element of charge at the point in question is then calculated. The magnitude of dE is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} \tag{1.9}$$

where r is the distance of the given point from the charge element dq. The resultant field at the point is then obtained by adding vectorially the field contributions due to all charge elements, but in most cases the addition needs to be carried out by integration.

or, $\mathbf{E} = \int dE$

In carrying out the integration, the only suitable method is to resolve the field contributions from the charge elements into components, add the components by integration giving say, $\mathbf{E}_x$, $\mathbf{E}_y$ and $\mathbf{E}_z$ from which **E** can be obtained.

i) **Field due to a uniformly charged wire (line of charge):**

Let us consider a section of an infinite line of charge having a charge density (*i.e.*, the charge per unit length) λ coulomb per metre-oriented as shown in Fig. 1.3. We would like to calculate the electric field **E** at a point P a distance y from the line.

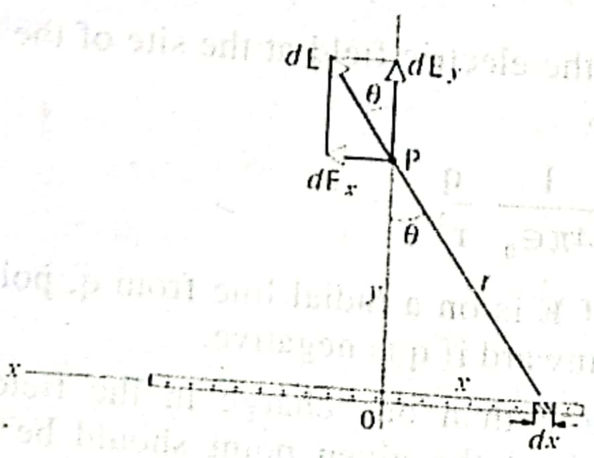


Fig. 1.3

The magnitude of the field contribution dE at the point P due to a charge element dq ($= \lambda.dx$) is given, using eqn. 1.8, by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

where r is the distance of the point P from the charge element dq.

Now $r^2 = x^2 + y^2$

$$\therefore \quad dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{x^2+y^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda.dx}{x^2+y^2} \qquad (1.10)$$

The vector dE can be resolved, as shown in the figure, into two components

$$dE_x = -\,dE\sin\theta \quad \text{and} \quad dE_y = dE\cos\theta$$

The minus sign in front of dE_x indicates that dE_x points in the negative x direction.

The x and y components of the resultant vector E at point P is given by

$$E_x = \int dE_x = -\int_{x=-\infty}^{x=+\infty} \sin\theta dE$$

and $$E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} \sin\theta dE$$

If the wire is infinitely long and be symmetrically placed on both right and left of the perpendicular PO, then for any element of charge dq = λ.dx to the right of O, there is an equal element of charge in a symmetrical position to the left of O. Consequently, the field contribution in the x direction made by these two symmetric elements of charge cancel each other. E_x must, therefore, be zero. Thus E points entirely in the y-direction. Because the contributions to E_y from the right and left halves are equal, E_y, and hence E, may be written as

$$E = E_y = 2\int_{x=0}^{x=+\infty} \sin\theta.dE \qquad (1.11)$$

It may be noted that the lower limit of the integration has been changed and a factor of 2 introduced.

From Fig. 1.3,

$x = y \tan\theta$ so that $dx = y \sec^2\theta . d\theta$

$r = y/\cos\theta = y\sec\theta$ so that $r^2 = x^2 + y^2 = y^2\sec^2\theta$

Now $dE_y = dE \cos\theta$

$$= \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda . dx}{x^2 + y^2}\right) \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda . y\sec^2\theta d\theta}{y^2\sec^2\theta} \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0 y} \cos\theta . d\theta$$

Thus,

$$E = 2\int_{x=0}^{x=+\infty} \cos\theta . dE = 2\int_{x=0}^{x=+\infty} \frac{\lambda}{4\pi\epsilon_0 y} \cos\theta . d\theta$$

$$= \frac{2\lambda}{4\pi\epsilon_0 y} \int_{\theta=0}^{\theta=\pi/2} \cos\theta d\theta$$

as $x = 0$ corresponds to $\theta = 0$ and $x = +\infty$ corresponds to $\theta = \pi/2$.

$$\therefore \quad E = \frac{\lambda}{2\pi\epsilon_0 y} \left|\sin\theta\right|_0^{\pi/2}$$

$$= \frac{\lambda}{2\pi\epsilon_0 y} \qquad (1.12)$$

Any actual line must have a finite length – not an infinite length. However, for points close enough to finite lines and far from their ends, eqn. (1.11) yields results that are so close to the correct values that the difference can be ignored in many practical situations.

This is the electric field produced by an infinite sheet of uniform charge, located on one side of a nonconductor such as plastic.

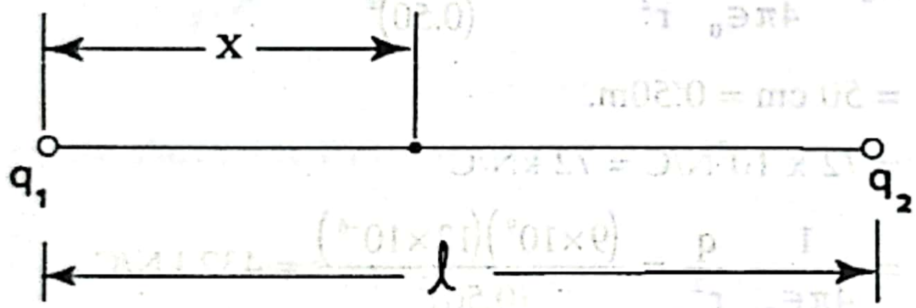
If we let $x \to 0$ in eqn. (1.15) while keeping R finite, we again obtain (1.16). This shows that at points very close to the disk, the electric field set up by the disk is the same as if the disk were infinite in extent. At points very far from the disk, the electric field produced by the disk is the same as if it were a point charge.

Example 1.12 Figure shows a charge $q_1 (= +1.0 \times 10^{-6}$ coul) 10 cm from a charge q_2 (= + 2.0 x 10^{-6} coul). At what point on the line joining the two charges is the electric field strength zero?

Soln.

The point must lie between the charges because only here do the forces exerted by q_1 and q_2 on the test charge oppose each other.

Let the point P be at a distance x from q_1. If $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric field strengths at P due to q_1 and q_2 respectively, we must have



$$E_1 = E_2$$

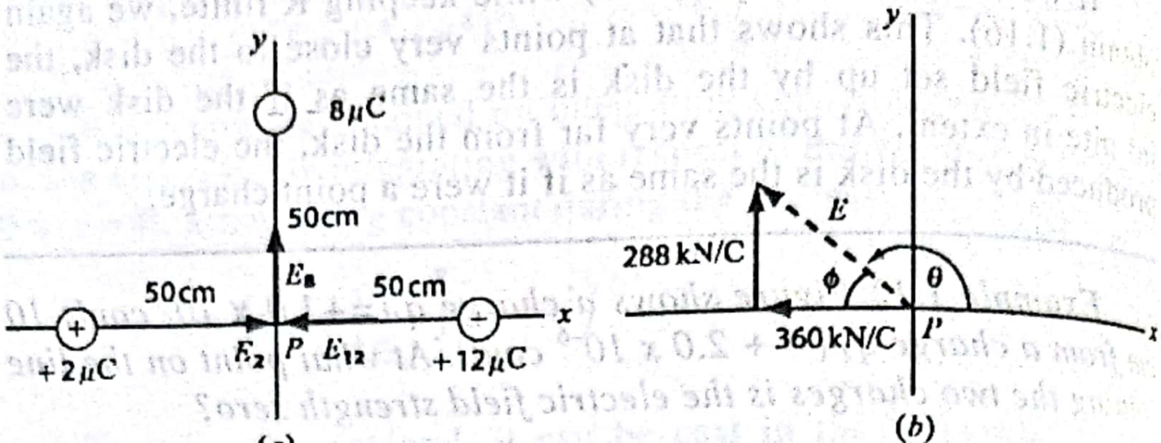
or, $$\frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(l-x)^2}$$ where $l = 10$ cm.

or, $$\frac{(l-x)^2}{x^2} = \frac{q_2}{q_1} = 2;$$ or, $$\frac{l-x}{x} = \sqrt{2}$$

or, $$\frac{10}{x} - 1 = \sqrt{2};$$ or, $$\frac{10}{x} = 1 + \sqrt{2}$$

or, $$x = \frac{10}{1+\sqrt{2}} = 4.1 \text{ cm}.$$

***Example 1.13** Find electric field at point P in figure below due to the charges shown.*



(a) (b)

Soln.

The field due to each charge are shown in the figure.

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{(9\times10^9)(2\times10^{-6})^2}{(0.50)^2}$$

where $r = 50$ cm $= 0.50$m.

$= 72 \times 10^3$N/C $= 72$ kN/C

$$E_{12} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{(9\times10^9)(12\times10^{-6})}{(0.50)^2} = 432 \text{ kN/C}$$

$$E_8 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{(9\times10^9)(8\times10^{-6})}{(0.50)^2} = 288 \text{ kN/C}$$

Of these, E_2 and E_{12} act along the x-axis in opposite directions. Their resultant is (432 − 72) = 360 kN/C which act along the negative x-axis, as shown in Fig. (b). E_8 acts along the positive y-axis, also shown in the figure. Their resultant is

$E = (E_2 + E_{12}) + E_8$

$= 461$ kN/C

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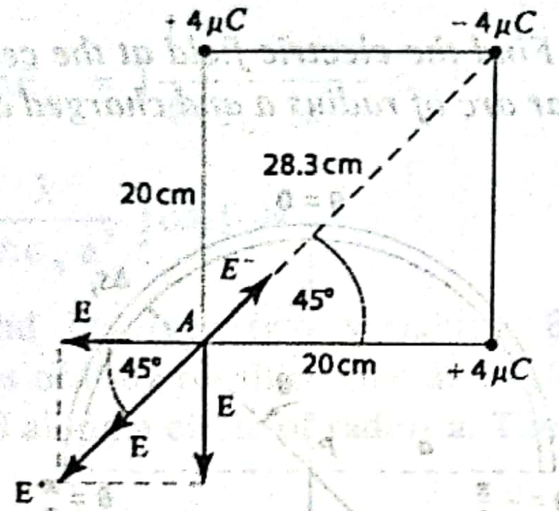
The resultant field acts in a direction making an angle of 39° with the –ve x-axis [$\cos\phi = \frac{288}{360}$; $\phi = 39°$].

***Example 1.14** Three charges are placed at three corners of a square of side 20 cm as shown in the figure below. Find the magnitude and direction of the electric field strength at point A.*



Soln.

The figure shows the fields produced at A by each of the three charges.

The fields due to +4μC charges are

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{(9\times10^9)(4\times10^{-6})}{(0.20)^2}$$

$$= 0.9 \text{ MN/C}.$$

Their directions are shown in the figure. Their resultant is

$$E^+ = \sqrt{(9\times10^5)^2 + (9\times10^5)^2} = 1.27 \text{ MN/C}.$$

and its direction is as shown in the figure. The field due to the negative charge is

$$E^- = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{(9\times10^9)(-4\times10^{-6})}{\left[\sqrt{(0.20)^2+(0.20)^2}\right]^2} = -0.45 \text{ MN/C}$$

and its direction is as shown.

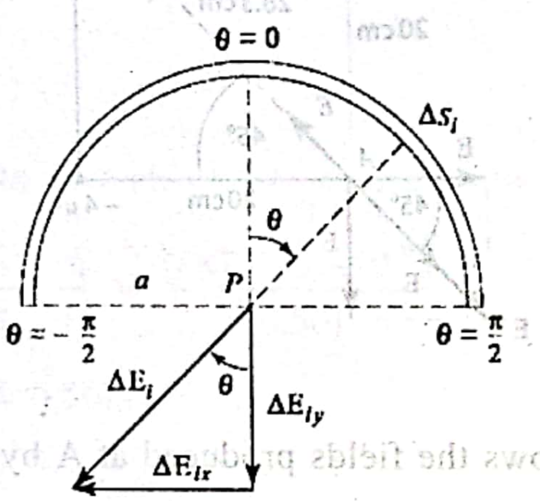
Both E^+ and E^- act along the same line but in opposite directions. Hence the net field at the point A is

$$E_t = E^+ + E^- = 0.82 MN/C$$

and its direction is away from the negative charge as shown in the figure.

***Example 1.15** Find the electric field at the centre of a uniformly charged semicircular arc of radius a and charged density of λ C/m.*

Soln.



Let the arc be split into small segments ΔSi. The segments are sufficiently small to be considered to be a point charge $\lambda \Delta Si$. The electric field due to a segment at the centre of the arc *i.e.*, at point P, is given by

$$\Delta E_i = \frac{1}{4\pi\epsilon_0}\frac{\lambda \Delta Si}{a^2}$$

Each little segment of the arc will give a ΔE_i in a different direction. The resultant field at the point P is the vector sum of the fields due to all the segments.

To add these fields vectorially, the fields shoud be resolved into components. The resultant component in the x-direction, E_x, will be zero at the point P. This is because the ΔE_{ix} shown in the figure will be cancelled by the contribution from a symmetrically placed ΔS on the left half of the arc. As a result, we need only compute E_y at point

CHAPTER III

ELECTRIC POTENTIAL

3.1 Electric Potential

The electric field around a charged particle can be described not only by the electric field strength E which is a vector quantity but also by a scalar quantity – the *electric potential V.* The two quantities are intimately related, and often it is a matter of convenience which is used in a given problem.

The idea of electric potential is related to work done in carrying a charge from one point to another in an electric field. Let us begin by placing a free positive charge q_0 in an electrostatic field. It will experience a force in the direction of the electric intensity. A free negative charge will move in the opposite direction. Work has to be done in moving the free positive charge – referred to as the *test charge*, against the direction of the field. The work done in moving a test charge q_0 from a point A to a point B with constant speed, *i.e.*, always keeping the charge in *equilibrium*, is called the *electric potential difference* between the points. If the electric potential difference between the points is $V_B - V_A$, then it is defined by the equation

$$V_B - V_A = \frac{W_{AB}}{q_0} \tag{3.1}$$

where W_{AB} is the work done by an agent to move the charge from A to B. The work W_{AB} may be (i) positive, (ii) negative, or (iii) zero. In these cases the electric potential at B will be (i) higher, (ii) lower, or (iii) the same as the electric potential at A.

The mks unit of potential difference that follows from eqn. (3.1) is joule/coulomb. However, this combination occurs so often that a special unit, the *volt*, is used to represent it. Or,

$$1 \text{ volt} = 1 \text{ joule/coul}$$

The potential difference between two points is 1 volt if one joule of work is done in moving one coulomb of charge from one point to the other.

W_{AB} is also the increase in potential energy because there is no increase in kinetic energy as the test charge is made to move at constant speed. Thus we expect the potential to rise for displacements in opposition to **E** and to fall for displacements along **E**.

Strictly speaking, only differences of potential can be defined. We can, however, always choose the point A to be at large (strictly an infinite) distance from all charges, and the electric potential V_A at this infinite distance is arbitrarily taken as zero. Putting $V_A = 0$ in eqn. (3.1) and dropping the subscript leads to

$$V = \frac{W}{q_0} \qquad (3.2)$$

where W is the work that an external agent must do to move that test charge q_0 from infinity to the point in question. Thus the *electric potential at any point is defined as the work that must be done in bringing a unit positive charge from infinity up to the point.* Since both W and q_0 in eqn. (3.2) are scalar quantities, electric potential is a scalar quantity.

Bearing in mind the assumptions made about the reference position, electric potential near an isolated positive charge is positive since positive work must be done by the outside agent to push a (positive) test charge in from infinity. Similarly, the potential near an isolated negative charge is negative because an outside agent must exert a restraining force *i.e.*, must do negative work on positive test charge as it comes in from unity.

It can be easily proved that both W_{AB} and $V_B - V_A$ are independent of the path followed in moving the test charge from point A to point B. If this were not so, point B would not have a unique electric potential (with respect to point A as a defined reference position) and the concept of potential would have limited usefulness.

3.2 Potential and Electric Field Strength

Fig. 3.1 shows two points A and B in a uniform electric field E, set up by an arrangement of charge not shown in the figure. Let the distance of A from B in the field direction be d. Assume that a positive test charge q_0 is being moved by an external agent from A to B along the straight line connecting them under the condition of equilibrium *i.e.*, q_0 moves in such a way that it is not accelerated; it moves with a constant velocity.

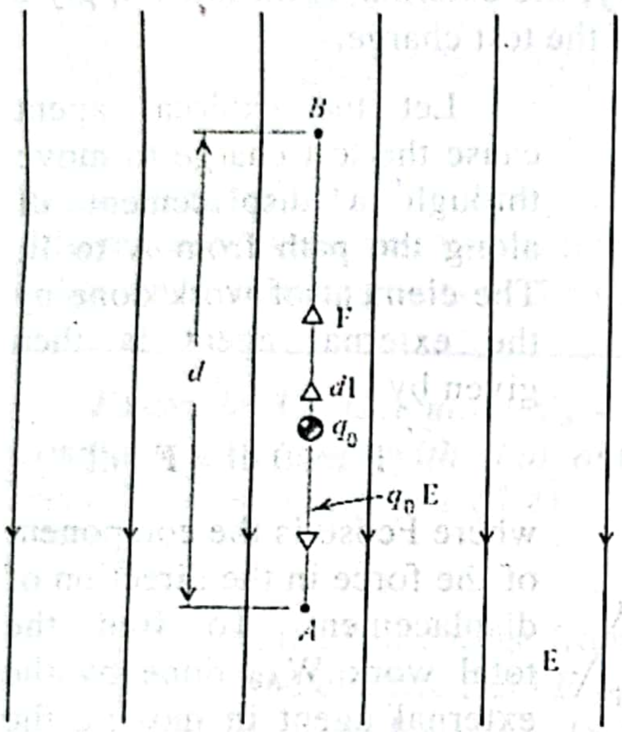


Fig. 3.1

The electric force on the charge is q_0E and points downward. To move the charge in the manner described above, the force on q_0 must be countered by an external force **F** of the same magnitude but directed upward. The work done by the agent that supplies this force is

$$W_{AB} = F.d = q_0Ed \qquad (3.3)$$

Substituting this in eqn. 3.1, we have

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q_0Ed}{q_0} = Ed \qquad (3.4)$$

Eqn. (3.4) gives the relation connecting potential difference and field strength for a simple special case. From eqn. (3.4) it appears that another unit for E $\left(= \frac{V_B - V_A}{d}\right)$ is *volt per metre.* It can be proved that volt/metre is identical with newton/coulomb. Or,

1 volt/metre = 1 nt/coulomb.

In Fig. 3.1 B is at a higher potential than A. This is expected because the external agent would have to do positive work to push a positive charge from A to B against the direction of the field.

Let us now investigate the relation between V and **E** in the more general case in which the field is *not* uniform and in which the test body is made to move along a path which is *not* straight. The electric field exerts a force $q_0\mathbf{E}$ on the test charge as shown in Fig. 3.2. To keep the test charge from accelerating *i.e.*, if the test charge is to move with a constant velocity, the external agent must apply a force $\mathbf{F} = -q_0\mathbf{E}$ for all positions of the test charge.

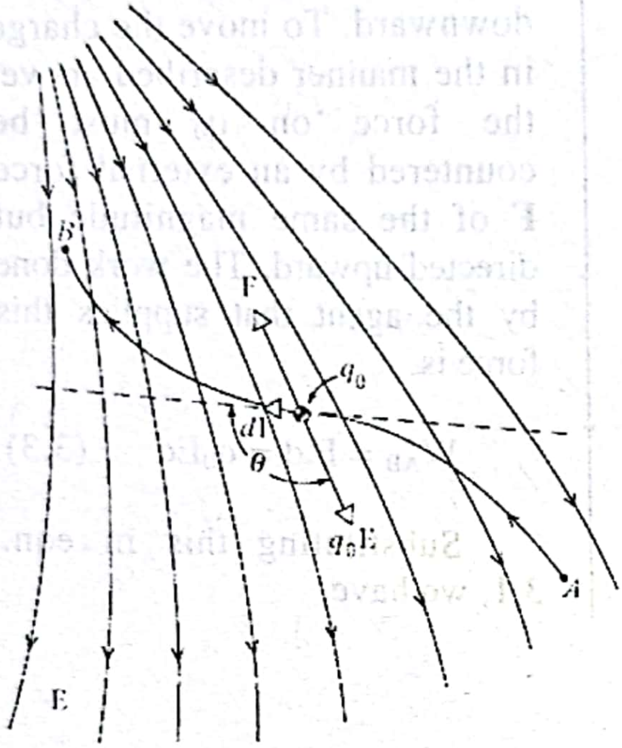


Fig. 3.2

Let the external agent cause the test charge to move through a displacement dl along the path from A to B. The element of work done by the external agent is then given by

$$F\cos\theta\, dl = \mathbf{F} \cdot \mathbf{dl}$$

where $F\cos\theta$ is the component of the force in the direction of displacement. To find the total work W_{AB} done by the external agent in moving the test charge from A to B, we integrate (add up) the work contributions for all infinitesimal segments into which the path is divided. This leads to

$$W_{AB} = \int_A^B \mathbf{F}.\mathbf{dl} = -q_0 \int_A^B \mathbf{F}.\,\mathbf{dl} \qquad (3.5)$$

since $\mathbf{F} = -q_0\,\mathbf{E}$

The type of integral as given by eqn. (3.5) is called a *line integral*.

Substituting this expression for W_{AB} in eqn. (3.1), we obtain

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{-q_0 \int_A^B \mathbf{E}.\mathbf{dl}}{q_0}$$

$$= -\int_A^B \mathbf{E}.d\mathbf{l} \quad (3.6)$$

If the point A is at infinite distance then the potential V_A at infinity is taken as zero. Eqn. (3.6) then gives the potential V at the point B, or, dropping the subscript B,

$$V = -\int_\infty^B \mathbf{E}.d\mathbf{l} \quad (3.7)$$

Eqns. (3.6) and (3.7) allow us to calculate either the potential difference between any two points or the potential at any point if **E** is known at various points in the field.

Example 3.1 Calculate $V_B - V_A$ in Fig. 3.1 using eqn. (3.6). Compare the result with that obtained by direct analysis of this special case by using eqn. (3.4)

Soln.

In moving the test charge from A to B the path **dl** always points upward while the electric field **E** points downward (Fig. 3.1), so that the angle θ between **E** and **dl** is 180°.

Eqn. (3.6) then becomes

$$V_B - V_A = -\int_A^B \mathbf{E}.\,d\mathbf{l} = -\int_A^B E\cos 180^\circ\, dl = \int_A^B E dl$$

E is constant for all parts of the path in this problem and hence can be taken outside the integral sign, giving

$$V_B - V_A = E\int_A^B dl = Ed$$

the same as that given by eqn. (3.4).

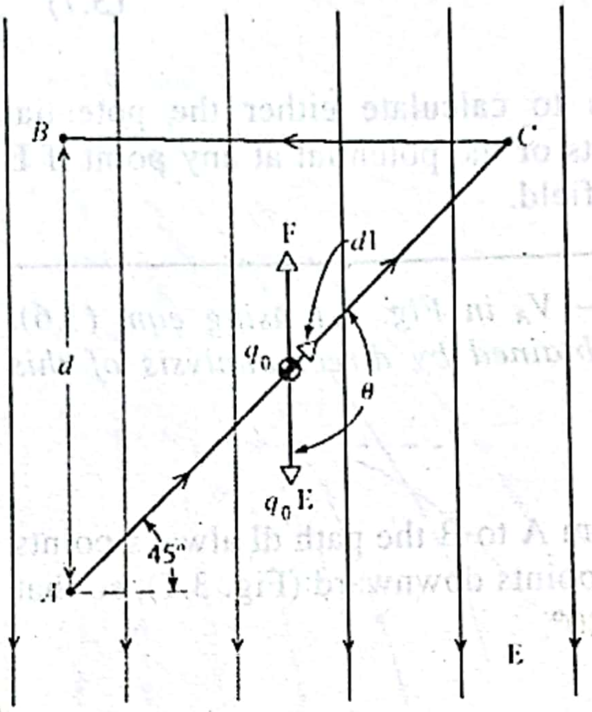
***Example* 3.2** *Let a test charge q_0 be moved without acceleration from A to B over the path as shown in Fig. 3.3 Compute the potential difference between A and B.*

Soln.

For the path AC, $\theta = 135°$ and, from eqn. (3.6), we obtain

$$V_C - V_A = -\int_A^C \mathbf{E}\cdot d\mathbf{l} = -\int_A^C E\cos 135° dl$$

$$= \frac{E}{\sqrt{2}}\int_A^C dl$$



The integral is the length of the line AC which is $\sqrt{2}\,d$. Thus

$$V_C - V_A = \frac{E}{\sqrt{2}}(\sqrt{2}\,d) = Ed$$

Points B and C are at the same potential because no work is done in moving a charge between them, **E** and **dl** being at right angles for all points on the line CB. In other words B and C lie on the same equipotential surface at right angles to the lines of force. Thus

$$V_B - V_A = V_C - V_A = Ed$$

This is the same value derived for a direct path connecting A and B. This is to be expected because the potential difference between two points is path independent.

Example 3.3 *The electric field outside a long charged wire is given by $E = -5000/r$ V/m and is radially inward. What is the sign of the charge on the wire? Find the value of $V_B - V_A$ if $r_B = 60$ cm and $r_A = 30$cm. Which point is at the higher potential?*

Soln.

Since the field is directed toward the wire, the wire is charged negatively. Going from A to B is opposite to the direction of the field, so B is at a higher potential than A. Thus

$$V_B - V_A = -\int_{0.3}^{0.6} \frac{-5000}{r}\, dr = 5000\, ln2 = 3470 \text{ V}.$$

3.3 Potential due to a point charge

As shown in Fig. 3.3, A and B are two points near an isolated point charge q. For simplicity we assume that A, B and q lie on a straight line. We shall calculate the potential difference between points A and B, assuming that a test charge q_0 is moved without acceleration along a radial line from A to B.

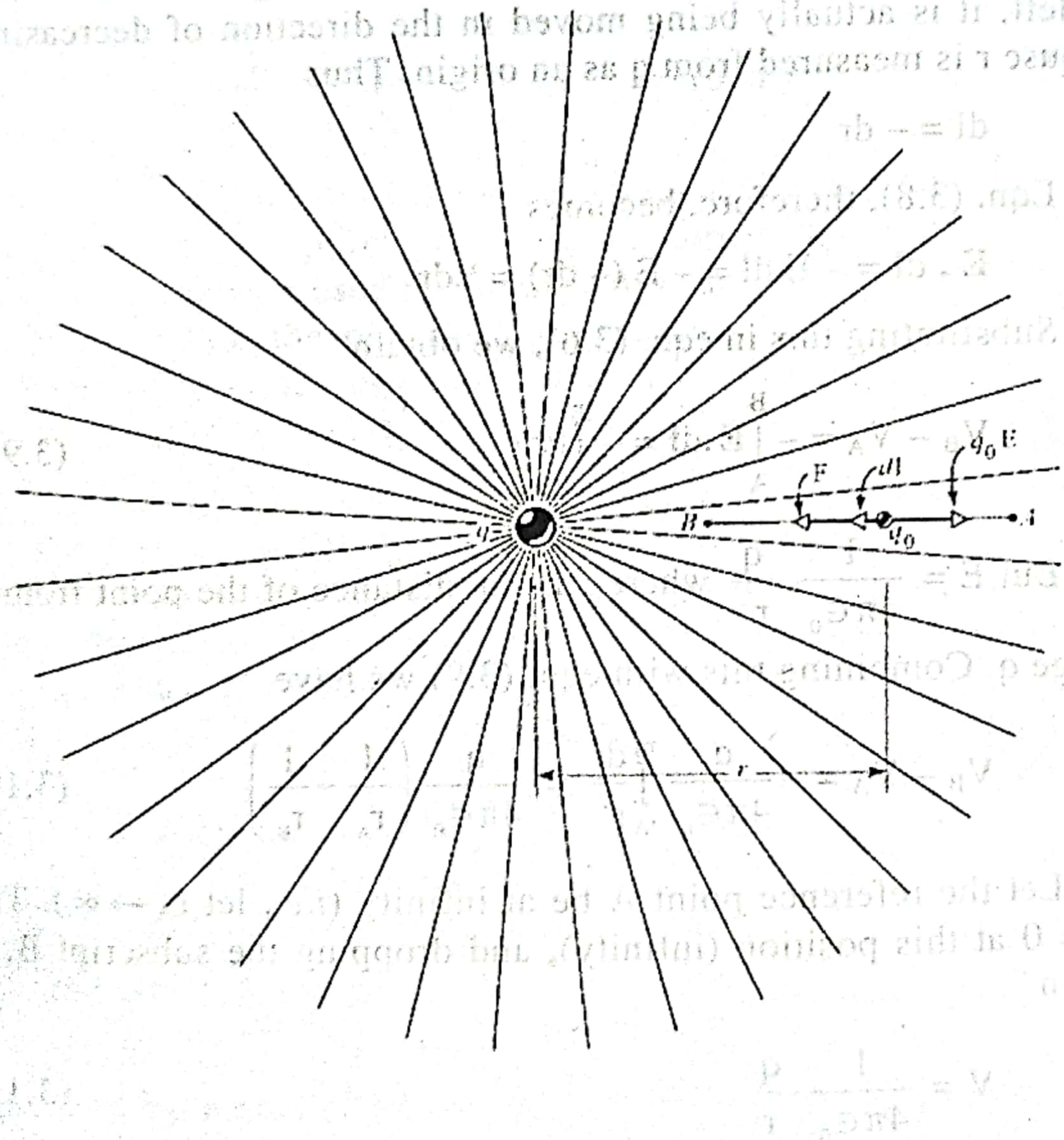


Fig. 3.3

The potential difference between the points A and B is given by (eqn. 3.6),

$$V_B - V_A = -\int_A^B \mathbf{E}.d\mathbf{l}$$

As can be seen in the figure, **E** points to the right whereas dl, which is always in the direction of motion, points to the left; so that the angle between them is $\theta = 180°$. Therefore,

$$\mathbf{E}\cdot d\mathbf{l} = E\cos\theta\, dl = -Edl \tag{3.8}$$

However, as the test charge is moved through a distance dl to the left, it is actually being moved in the direction of decreasing r because r is measured from q as an origin. Thus

$$dl = -dr$$

Eqn. (3.8), therefore, becomes

$$\mathbf{E}\cdot d\mathbf{l} = -E\,dl = -E\,(-dr) = Edr$$

Substituting this in eqn. (3.6), we obtain

$$V_B - V_A = -\int_A^B \mathbf{E}.d\mathbf{l} = -\int_{r_A}^{r_B} Edr \tag{3.9}$$

But $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$ where r is the distance of the point from the charge q. Combining this with eqn. (3.9) we have

$$V_B - V_A = -\frac{q}{4\pi\epsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \tag{3.10}$$

Let the reference point A be at infinity (*i.e.*, let $r_A \rightarrow \infty$). Then $V_A = 0$ at this position (infinity), and dropping the subscript B, we obtain

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{3.11}$$

Eqn. (3.11) clearly shows that *equipotential surfaces for an isolated point charge are spheres concentric with the point charge.*

3.4 Potential due to collection of charges

For a collection of discrete charges, the potential at a point is calculated due to each individual charge, as if the other charges were not present. These potentials are then added and we obtain

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} + \text{........} \text{ (zero at infinity)}$$

where r_1 is the distance of the point from the charge q_1, r_2 from q_2, etc.

Or,

$$V = \frac{1}{4\pi\epsilon_0}\sum_n \frac{q_n}{r_n} \qquad \text{(zero at infinity)} \qquad (3.12)$$

where q_n is the value of the n^{th} charge and r_n is the distance of this charge from the point in question. Since potentials are scalar quantities, the sum used to calculate V is an *algebraic sum* and not a vector sum like the one used to calculate **E** for a group of point charges. This is an important advantage of potential over electric field strength for making calculations.

If the charge distribution is continuous, instead of being discrete, then eqn. (3.11) can be used to find dV at a point due to a typical element of charge dq. The total potential V at the point is then obtained by integrating over the whole region occupied by the charge. Thus, if r is the distance of the point from the charge dq, then

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} \qquad (3.13)$$

3.5 Potential due to a dipole

As discussed in Art. 1.5, two equal charges, q, of opposite sign, separated by a small distance 2a, constitute an electric dipole. The electric dipole moment **p** has the magnitude 2aq and points from the negative charge to the positive charge. We would like to derive an expression for the electric potential V at any point of space due to a dipole, provided only that the point is not too close to the dipole.

where q ($= \sigma\pi a^2$) is the total charge on the disk. This limiting result is expected because for r >> a the disk behaves like a point charge.

3.9 Electric potential energy

Fig. 3.8 shows two charges q_1 and q_2 a distance r apart. If the separation between them is increased, work must be done by an external agent. The work will be positive if the charge are opposite in sign and negative otherwise. The energy represented by this work can be thought of as stored in the system q_1+q_2 as *electric potential energy*. Like all forms of potential energy, this energy can also be transformed into other forms. For example, if q_1 and q_2 are charges of opposite sign and we release them, they will accelerate towards each other, transforming the stored potential energy into kinetic energy of the accelerating masses.

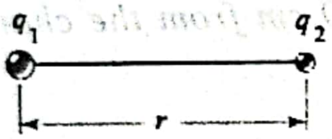


Fig. 3.8

The electric potential energy of a system of point charges may be defined as the work required to assemble this system of charges by bringing them in from an infinite distance. We assume that the charges are all at rest when they are infinitely separated, that is, they have no initial kinetic energy.

Let us imagine that q_2 in Fig. 3.8 to be removed to infinity and at rest. The *electric potential* at the original site of q_2 due to q_1 is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r} \tag{3.30}$$

If q_2 is now moved from infinity to its original position at a distance r from q_1, the work required is, from the definition of electric potential ($V = W/q$), given by

$$W = V.\, q_2 \tag{3.31}$$

Combining eqns. 3.30 and 3.31

we obtain

$$W = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \tag{3.32}$$

But the work is precisely the *electric potential energy* U of the system $q_1 + q_2$. Thus

$$U\ (= W) = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

The subscript of r emphasizes that the distance involved is that between the point charges q_1 and q_2.

For a system containing more than two charges, the procedure is to compute the potential energy for every pair of charge separately. The results are then added separately. This procedure rests on a physical picture in which (i) charge q_1 is brought into position, (ii) q_2 is brought from infinity to its position near q_1, (iii) q_3 is brought from infinity to its position near q_1 and q_2, etc.

***Example 3.4** What must the magnitude of an isolated positive point charge be for the electric potential at 10 cm from the charge to be +100 volts?*

Soln.

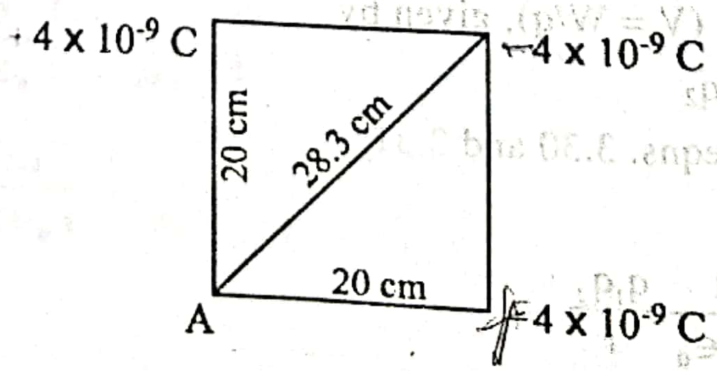
From $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ we obtain

$$q = (V)\,(4\pi\epsilon_0)\,(r)$$

$$= (100 \text{ volts})\,(4\pi)\,(8.9 \times 10^{-12} \text{ coul}^2/\text{nt-m}^2)\,(0.10 \text{ m})$$

$$= 1.1 \times 10^{-9} \text{ coul.}$$

***Example 3.5** Three charges are placed at three corners of a square as shown in the figure. Find the potential at point A.*



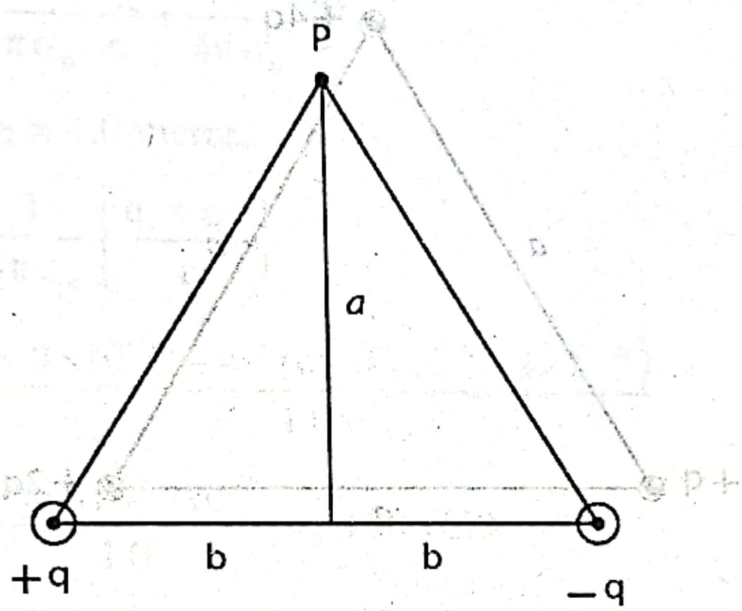
Soln.

$$V_A = \frac{1}{4\pi\epsilon_0}\left[\frac{+4\times10^{-9}C}{0.20m}+\frac{-4\times10^{-9}C}{0.283m}+\frac{+4\times10^{-9}C}{0.20m}\right]$$

$$= (9 \times 10^9)\left[\frac{4\times10^{-9}C}{0.20m}+\frac{-4\times10^{-9}C}{0.283m}+\frac{4\times10^{-9}C}{0.20m}\right]$$

$$= 233 \text{ V.}$$

Example 3.6 *Show that the absolute potential at point P in the figure below is zero.*



Soln.

$$V_P = \frac{1}{4\pi\epsilon_0}\frac{+q}{\sqrt{a^2+b^2}} + \frac{1}{4\pi\epsilon_0}\frac{-q}{\sqrt{a^2+b^2}}$$

$$= 0.$$

Example 3.7 *Two protons in a nucleus of U^{238} are 6.0×10^{-15} metre apart. What is their mutual electric potential energy?*

Soln.

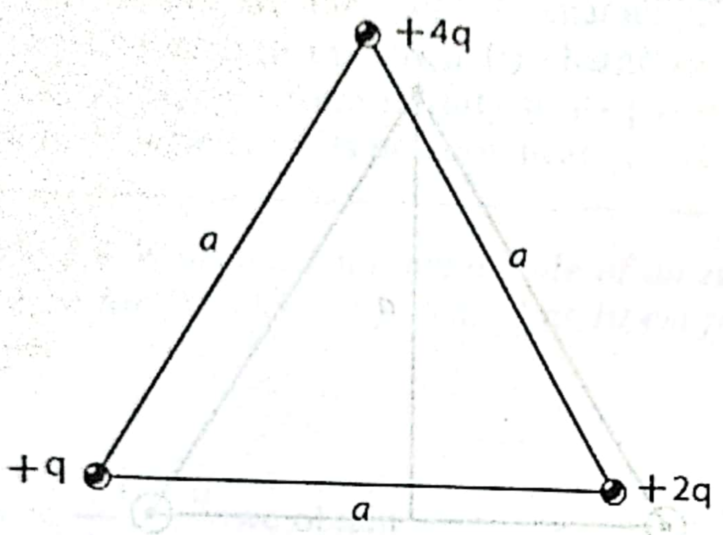
$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} = \frac{(9.0\times10^9 \text{ nt} - \text{m}^2/\text{coul}^2)(1.6\times10^{-19}\text{ coul})^2}{6.0\times10^{-15}\text{ m}}$$

$= 3.8 \times 10^{-14}$ Joule

$$= \frac{3.8\times10^{-14}}{1.6\times10^{-19}}\ eV$$

$= 2.4 \times 10^{5}$ eV.

Example 3.8 Three charges are arranged as in the figure. What is their mutual potential energy? Assume that $q = 1.0 \times 10^{-7}$ coul and $a = 10$ cm.



Soln.

The total energy of the configuration is the sum of the energies of each pair of particles. Therefore

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-4q)(+q)}{a} + \frac{(+q)(+2q)}{a} + \frac{(+2q)(-4q)}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{-10q^2}{a}\right]$$

$$= -\frac{(9.0\times10^{9}\,\text{nt}-\text{m}^2/\text{coul}^2)(10)(1.0\times10^{-7}\,\text{coul})^2}{0.10\,\text{m}}$$

$= -9.0 \times 10^{-3}$ Joule.

Example 3.9 Two metal spheres are 3.0 cm in radius and carry charges of +1.0 x 10^{-8} coul and –3.0 x 10^{-8} coul respectively, assumed to be uniformly distributed. If their centre are 2.0m apart, calculate (a) the potential of the point halfway between their centres and (b) the potential of each sphere.

Soln.

The charges may be assumed to be located at the centres of each sphere. The midway point will be 1.0 metre from either centres. The potential at the mid-point is then given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

But $r_1 = r_2 = 1.0$ metre.

$$\therefore \; V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 + q_2}{r}\right]$$

$$= \frac{(9.0\times10^9\,N-m^2/c^2)(1\times10^{-8} - 3\times10^{-8})}{1.0m}$$

$$= \frac{-18\times10^9\times10^{-8}}{1.0} = -180 \text{ volts.}$$

(b) The potential at the surface of sphere 1 is due to its own charge q_1 plus that due to charge q_2 of sphere 2 at a distance r.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r} \qquad R = 3\text{ cm} = 0.03\text{ m}, \; r = 2\text{m}$$

$$= (9 \times 10^9\text{ N-m}^2/\text{coul}^2)\left[\frac{1\times10^{-8}c}{0.03} + \frac{-3\times10^{-8}c}{2.0m}\right]$$

$= 2864$ volts.

The potential at the surface of sphere 2 is due to its own charge q_2 plus that due to charge q_1 of sphere 1 at a distance r.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r} + \frac{1}{4\pi\epsilon_0}\frac{q_1}{R}$$

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$$= (9 \times 10^9 \text{ N-m}^2/\text{coul}^2)\left[\frac{-3\times10^{-8}\text{c}}{0.03\text{m}} + \frac{1\times10^{-8}\text{c}}{2\text{m}}\right]$$

$$= -8955 \text{ volts.}$$

***Example 3.10** Calculate (i) the electric potential established by the nucleus of a hydrogen atom at the mean distance of the circulating electron ($r = 5.3 \times 10^{-11}$m), (ii) the electric potential energy of the atom when the electron is at its radius and (iii) the kinetic energy of the electron assuming it to be moving in a circular orbit of this radius centred on the nucleus. (iv) how much energy is required to ionize the hydrogen atom? Express all energies in electron-volts.*

Soln.

(i) The electric potential is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{(9\times10^9 \text{ N}-\text{m}^2/\text{c}^2)(1.6\times10^{-19}\text{c})}{5.3\times10^{-11}\text{m}}$$

$$= 27.1 \text{ volts.}$$

(ii) Electric potential energy of the atom is given by

$$U = qV = -e.V = -(1.6 \times 10^{-19}\text{c})\ (27.1 \text{ volts})$$

$$= -\frac{(1.6\times10^{-19}\text{c})(27.1\text{volts})}{1.6\times10^{-19}} \text{ eV}$$

$$= -27.1 \text{ eV.}$$

(iii)From the relation of electrostatic force balancing centripetal force, we obtain

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$\text{or, } mv^2 = \frac{e^2}{4\pi\epsilon_0 r^2} = 27.1 \text{ eV}$$

$$\therefore \text{ K. E.} = \frac{1}{2}mv^2 = 13.55 \text{ eV.}$$

3.10 Equipotential surfaces

If a surface can be imagined in such a way that it is everywhere at right angles to an electric field, then any path connecting two points on that surface is always at right angles to the field. The potential difference between the two points is given by

$$V_B - V_A = \int_A^B \mathbf{E}.d\mathbf{l} = \int_A^B E \cos 90^\circ \, dl = 0$$

In other words, all points on that surface are at the same potential. Such a surface is called an *equipotential surface*. Alternately, the locus of points, all of which have the same electric potential, is called an equipotential surface.

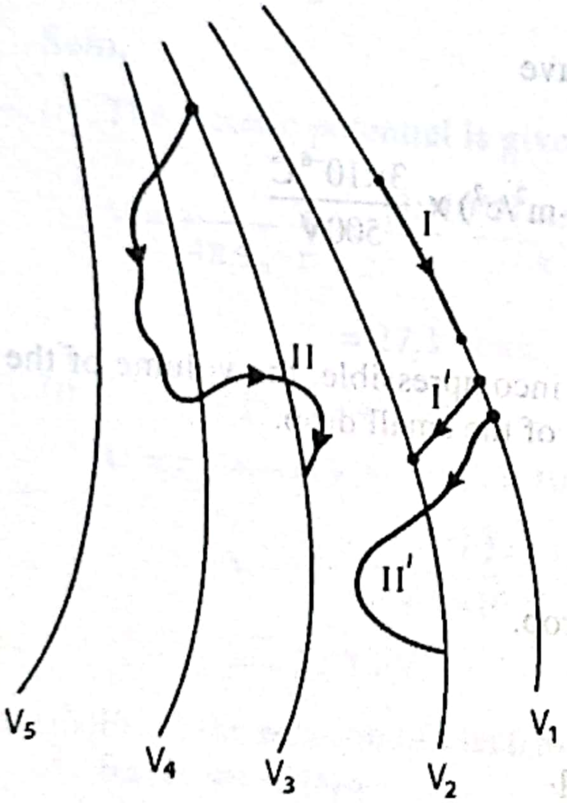

Fig. 3.9

A family of equipotential surfaces, each surface corresponding to a different value of the potential is shown in Fig. 3.9. The work to move a charge along path I and II is zero because all these paths begin and end on the same potential. But, on the other hand, the work to move a charge along paths I' and II' is not zero. Moreover, the amount of work done is also same because the initial and final potentials are identical; paths I' and II' connect the same pair of equipotential surfaces.

For an isolated point charge the potential at any point is given by

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Thus for an isolated point charge the equipatential surfaces are spheres concentric with the point charge (Fig. 3.10). It can be seen that the electric field strength **E** is everywhere normal to the equipotential surface.

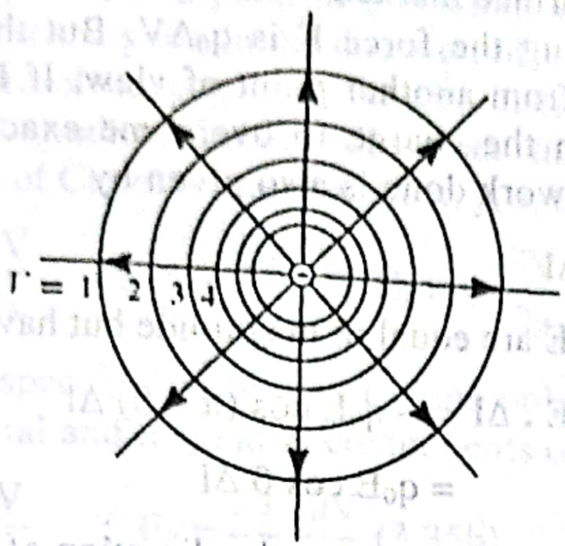

Fig. 3.10

Calculation of the electric field (E) from the potential (V)

It was mentioned at the very begining of this chapter that the electric field around a charged particle can be described by both electric field strength (E) and the electric potential (V). In Art. 3.3 we have shown how to calculate V from E. In this section, we propose to go the other way, that is, to find the electric field when we know the potential.

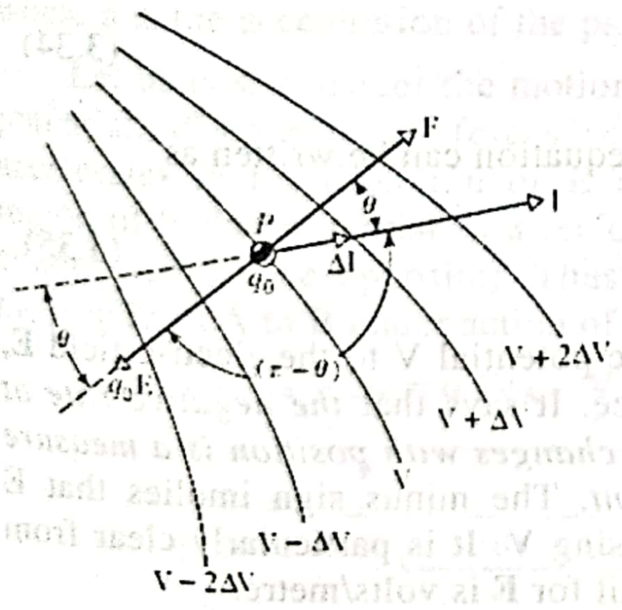

Fig. 3.11

Fig. 3.11 shows the intersection with the plane of the figure of a family of equipotential surfaces, differing in potential by the amount dV. The figure also shows the electric field **E** at a point P on the equipotential surface V. From the definition of equipotential surface, **E** should be at right angles to the equipotential surface through P.

Let us move a test charge q_0 from P along the path marked Δl to the equipotential surface marked V+ΔV. The work that must be done by the agent exerting the force **F** is $q_0\Delta V$. But the work done can also be calculated from another point of view. If **F** is the force that must be exerted on the charge to overcome exactly the electricla force q_0**E**, then the work done is also given by

$$\Delta W = \mathbf{F} \cdot \Delta \mathbf{l}$$

Since **F** and q_0**E** are equal in magnitude but have opposite signs,

$$\Delta W = -q_0\mathbf{E} \cdot \Delta \mathbf{l} = -q_0E\cos(\pi-\theta)\,\Delta l$$

$$= q_0E\cos\theta\,\Delta l$$

where $(\pi-\theta)$ is the angle between the direction of the force (**E**) and the direction of the displacement (Δ**l**).

Since the two expressions for work done must be equal,

$$q_0\,\Delta V = q_0\,E\cos\theta\,\Delta l$$

$$\text{or, } E\cos\theta = \frac{\Delta V}{\Delta l} \tag{3.33}$$

Now $E\cos\theta$ is the component of **E** in the direction of $-l$ in Fig. 3.11. Therefore, the quantity $-E\cos\theta$, which we call E_l, would be the component of **E** in the $+l$ direction. We then obtain

$$E_l = -\frac{\Delta V}{\Delta l} \tag{3.34}$$

In the differential limit this equation can be written as

$$E_l = -\frac{dV}{dl} \tag{3.35}$$

Eqn. 3.35 relates the electric potential V to the electric field E and is of fundamental importance. It says that *the negative rate at which the potential at any point changes with position is a measure of the electric field at that point*. The minus sign implies that E points in the direction of decreasing V. It is particularly clear from eqn. 3.35, that the appropriate unit for **E** is volts/metre.

The value of dV/dl at a point in a non-uniform field is called the *potential gradient* at the point in the direction of increasing distance. In other words, the resolved part of **E** in any direction is equal to the negative potential gradient in that direction. In particular, if V is expressed in terms of Cartesian coordinates x, y, z, then

$$E_x = -\frac{\partial V}{\partial x}\ ; \quad E_y = -\frac{\partial V}{\partial y}\ ; \quad E_z = -\frac{\partial V}{\partial z} \qquad (3.35a)$$

If V can be specified in terms of plane polar coordinates (r, θ) only, then the radial and tangential components of **E** are

$$E_r = -\frac{\partial V}{\partial r}\ ; \quad E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \qquad (3.35b)$$

3.12 The motion of charged particles in electric field

Let us consider a particle of mass m and charge q moving with velocity **v** in an electric field **E**. We shall assume that the charge is small enough not to affect the field in which it is placed or that the field can be contained constant by its sources. If no other forces act on the particle, then its equation of motion is

$$q\mathbf{E} = m\mathbf{a}\ ; \quad \text{or,} \quad \mathbf{a} = \frac{q\mathbf{E}}{m}$$

where **a** is the acceleration of the particle.

Let us now consider the motion between two points in terms of potential. If no external forces act, a positively charged particle accelerates in the direction of **E** and in doing so moves from a region of higher potential to a region of lower potential (a negative charge will do the opposite). Thus the loss in potential energy in moving from A to B under action of **E** alone (Fig. 3.12) is

$$U_{AB} = U_A - U_B = q\,(V_A - V_B)$$

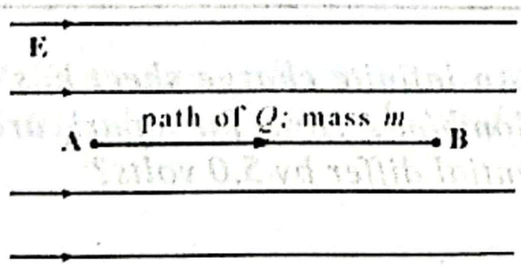

Fig. 3.12