Lecture: Introduction to Electricity

Ref book: Fundamentals of Physics - D. Halliday, R. Resnick & J. Walker (10th Ed.)

Ref https://www.youtube.com/watch?v=QpVxj3XrLgk to understand

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Web ref provided on slides

Concept of charge

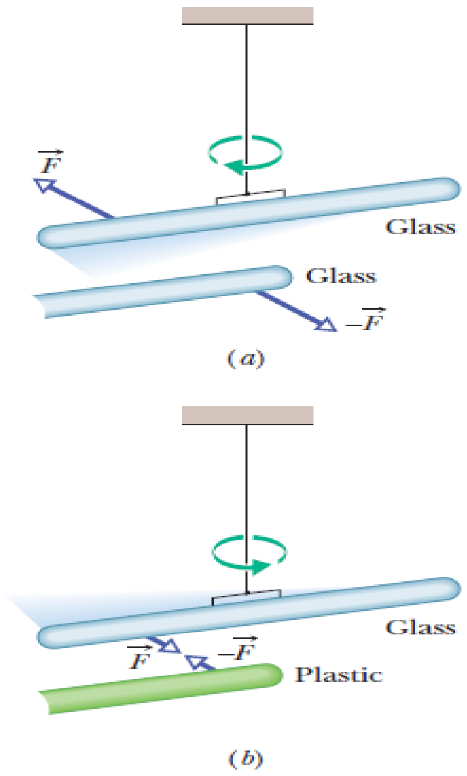


Figure 21-1 (*a*) The two glass rods were each rubbed with a silk cloth and one was suspended by thread. When they are close to each other, they repel each other. (*b*) The plastic rod was rubbed with fur. When brought close to the glass rod, the rods attract each other.

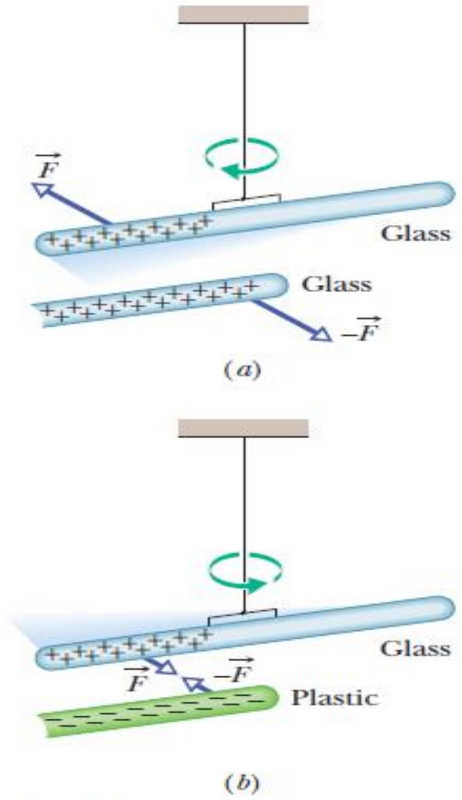
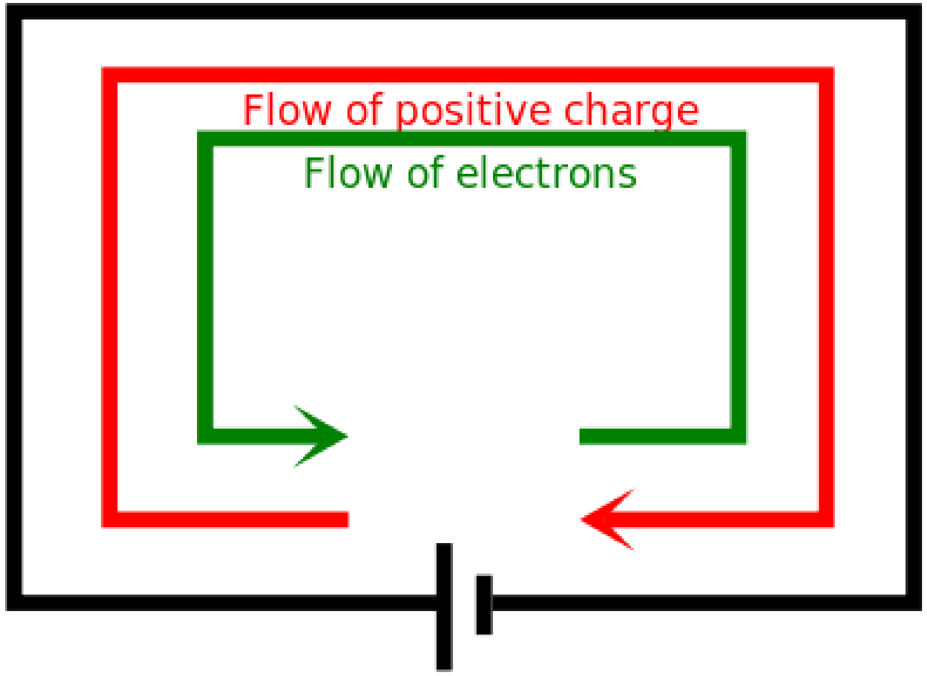


Figure 21-2 (*a*) Two charged rods of the same sign repel each other. (*b*) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

Electric Charge

Electric charge is a property of matter. There are two types of electric charge, which are conventionally labelled **positive** and **negative**. Two objects that have the same charge exert **repulsive** forces on each other. Two objects that have opposite charges exert **attractive** forces on each other.



Current is the **rate of flow of positive charge**. Current can be caused by the flow of electrons, ions or other charged particles. Electrons are negatively charged, so the direction electrons flow is the **opposite** direction to current.

The equation relating electric charge, current and time is:

electric charge=electric current×time

Q=Itelectric charge

Q=electric current×time=It

Ref: google image

https://byjus.com/physics/current-electricity/

https://isaacphysics.org/pages/gcse_ch3_22_text?stage=all

Electric Charge

In an electric circuit, electric current flows from the **positive** terminal of a power supply to the **negative** terminal or **ground**; or from the **ground** to a **negative** terminal.
Electric current is measured in amperes left bracket, A, right bracket,(A) and has the symbol I,*I*.

Static Electricity
Static electricity refers to the electric charges that build up on the surface of materials or substances. These charges remain static until they are grounded, or discharged. This type of electricity is formed due to fiction. Basically, the phenomenon of static electricityarises when the positive and negative charges are separated.

Current Electricity	**Static Electricity**
The electricity due to the flow of electrons is known as current electricity	The electricity built on the surface of a substance is known as static electricity
Current electricity is generated by powerplants and batteries	Static electricity is generated when objects are rubbed against each other resulting in charge transfer
Current Electricity is controlled	Static Electricity is uncontrolled
The electricity that is used to power up electronic devices is an example of current electricity	The shock experienced while touching a doorknob is an example of static electricity.

Ref: google image
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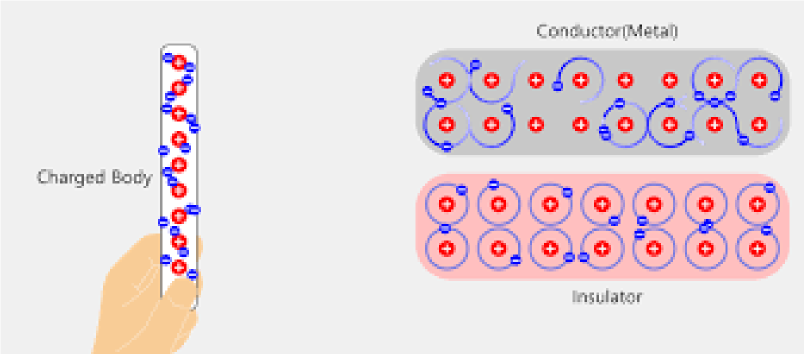
Electric Charge

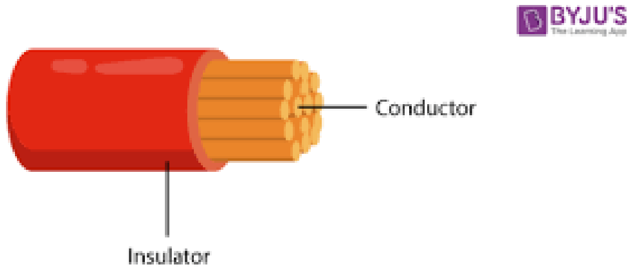
If a charge of 105, C,105C flows in 15, s,15s, calculate the current.

Number of electrons used to carry charge equals,= charge slash,/ charge of one electron
Number of electrons which flow past a point in one second equals,= charge flow in one second slash,/ charge of one electron equals,=current slash,/ charge of one electron

Calculate the number of electrons which are required to carry a charge of 105, C,105C.

7.0A; 6.6×10^{20}





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Conductors and Insulators

We differentiate the elements around us based on their physical properties such as malleability, phase, texture, colour, polarity, solubility, etc. But as we know, another very important classification of elements is done on the basis of their conductivity of electric charge, i.e. conductors and insulators.

In simple terms, an electrical conductor is defined as materials that allow electricity to flow through them easily. This property of conductors that allow them to conduct electricity is known as *conductivity*.

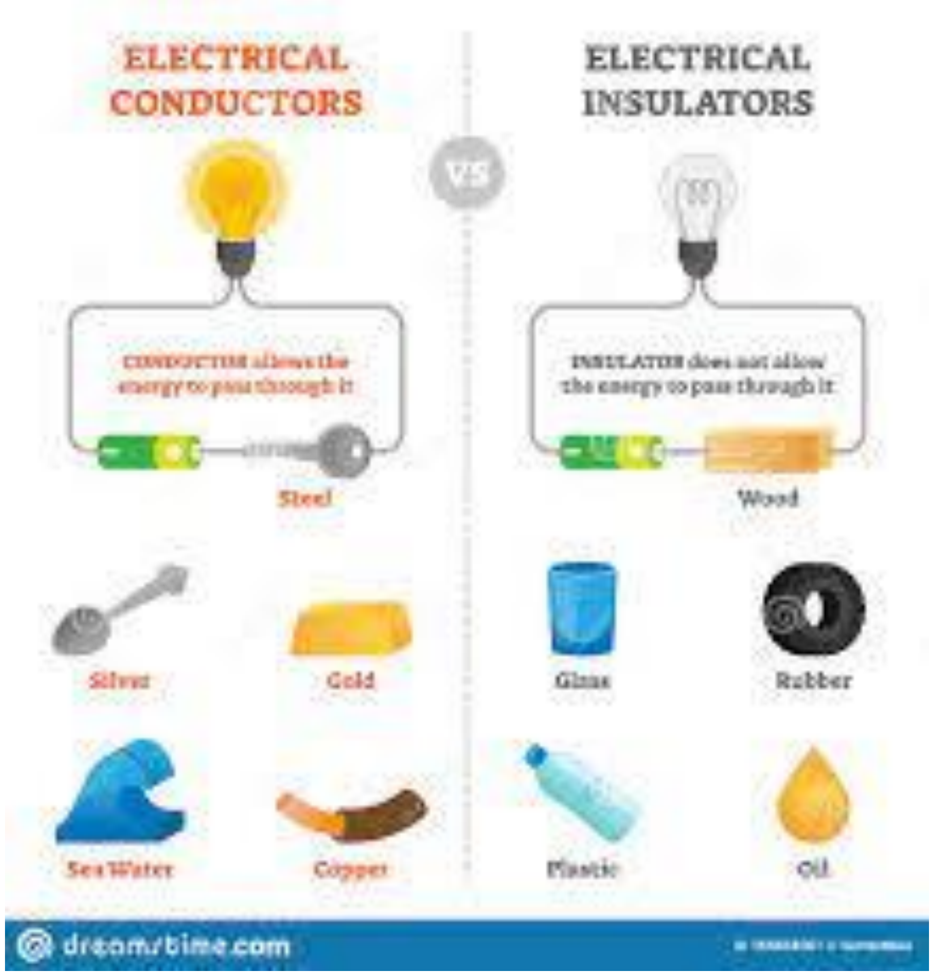
The flow of electrons in a conductor is known as the *electric current*. The force required to make that current flow through the conductor is known as *voltage*.

Graphite, the human body, and the earth are good conductors of electricity. Some of the common conductor examples include metals such as:

Copper

Gold

Iron



Ref: https://byjus.com/physics/conductors-insulators/

Conductors and Insulators

Insulators are materials that hinder the free flow of electrons from one particle of the element to another. If we transfer some amount of charge to such an element at any point, the charge remains at the initial location and does not get distributed across the surface. The most common process of charging of such elements is charging by rubbing (for some elements, with the help of suitable materials).

Some of the common insulator examples are given below:

Plastic

Wood

Glass

Ref: https://byjus.com/physics/conductors-insulators/

Conductors and Insulators

Conductor	**Insulator**
Materials that permit electricity or heat to pass through it	Materials that do not permit heat and electricity to pass through it
A few examples of a conductor are silver, aluminum, and iron	A few examples of an insulator are paper, wood, and rubber
Electrons move freely within the conductor	Electrons do not move freely within the insulator
The electric field exists on the surface but remains zero on the inside	The electric field doesn’t exist

Ref: https://byjus.com/physics/conductors-insulators/

Conductors and Insulators

Why are metals a preferred choice of material for making electrical wires?

Metals are a preferred choice of material for making electrical wire because they are good conductors of electricity.

The material that has a resistance of zero is known as a _____.?

Superconductor

What is a semiconductor?

A semiconductor is a material whose electrical conductivity falls between that of a conductor and an insulator. Example, Germanium and Silicon

What is the purpose of lightning rods?

The purpose of a lightning rod is to protect structures from lightning damages by blocking the surges and guiding their currents to the ground.

Which are the factors that affect the resistivity of a conductor?

The resistivity of a conductor depends on

Temperature

The material with which the conductor is made of

Ref: https://byjus.com/physics/conductors-insulators/

Coulombs Law

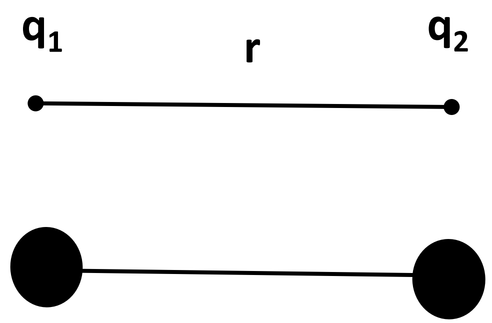
Lab Experiment

In 1785 Charles Augustin Coulomb reported in the Royal Academy Memoires using a torsion balance two charged mulberry pithballs repelled each other with a force that is inversely proportional to the distance.

$$F=\frac{kq_1q_2}{r^2}$$

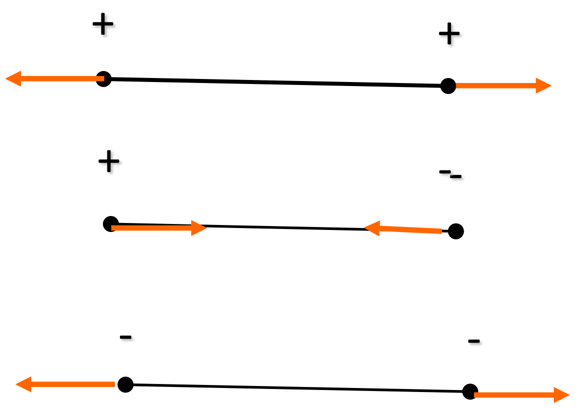
where $k=8.99*10^9$ Nm^2/C^2 in SI unit

$k \sim 10^{10}$ Nm^2/C^2



Point charges

Spheres same as points

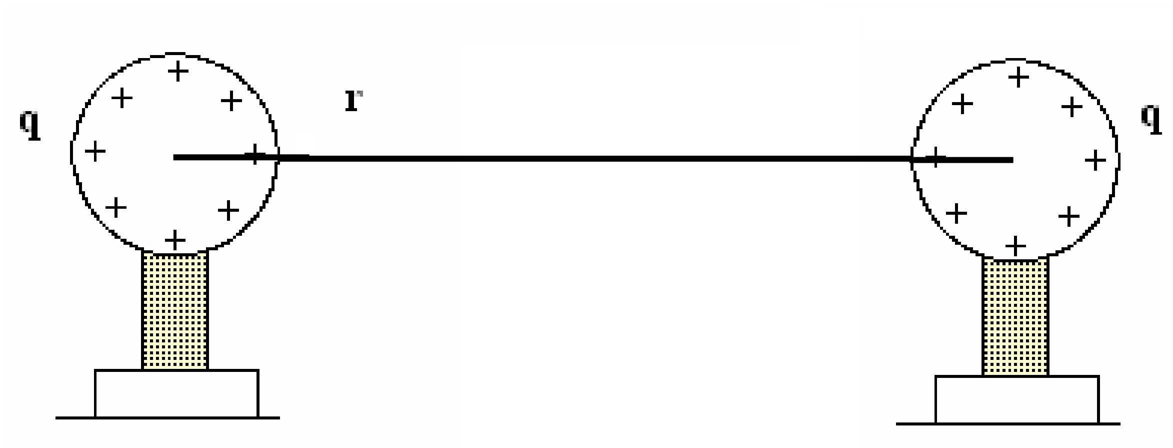


Repulsion

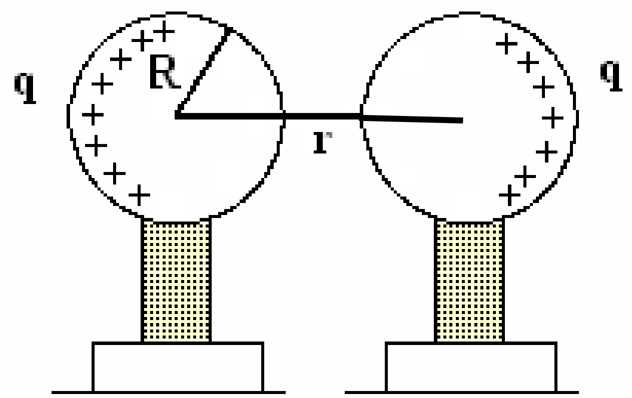
Attraction

Repulsion

Uniformly charged metal spheres of Radius R



$$F=\frac{kq^2}{(r)^2}$$



$$F- \frac{kq^2}{(r+2R)^2}$$

Demo: Show uniformity of charge around sphere using electrometer.
Demo: Show charging spheres by induction using electrometer

Coulomb's Law

If two charged particles are brought near each other, they each exert an **electrostatic force** on the other. The direction of the force vectors depends on the signs of the charges. If the particles have the same sign of charge, they repel each other. That means that the force vector on each is directly away from the other particle.

$$\vec{F} = k\,\frac{q_1 q_2}{r^2}\,\hat{\mathrm{r}} \qquad \text{(Coulomb's law)}$$

$$\vec{F} = G\,\frac{m_1 m_2}{r^2}\,\hat{\mathrm{r}} \qquad \text{(Newton's law)}$$

G is the gravitational constant and *k* is a positive constant called the *electrostatic constant* or the *Coulomb constant*

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}$$

Coulomb’s Law

- Coulomb’s law holds for stationary charges only which are point sized. This law obeys Newton’s third law
- **Application of Coulomb’s Law**

To determine the distance and force between the two charges.

To determine the force on one point due to the presence of several points.

- **Limitations of the Law**

The law applies just for the point charges at rest.

Coulomb’s Law can be just applied in those cases where the inverse square law is complied with.

It is challenging to execute Coulomb’s law where charges remain in approximate shape since in such cases, we can not identify the distance in between the charges.

The law can’t be utilized directly to determine the charge on the big planets or objects.

Ref: https://byjus.com/jee/coulombs-law/
https://www.guyhowto.com/coulombs-law/

Coulomb's Law

- Between two-electrons separated by a certain distance: Electrical force/Gravitational force = 10^{42}
- Between two protons separated by a certain distance: Electrical force/Gravitational force = 10^{36}
- Between a proton and an electron separated by a certain distance: Electrical force/Gravitational force = 10^{39}
- Between two-electrons separated by a certain distance: Electrical force/Gravitational force = 10^{42}
- Between two protons separated by a certain distance: Electrical force/Gravitational force = 10^{36}
- Between a proton and an electron separated by a certain distance: Electrical force/Gravitational force = 10^{39}
- The relationship between the velocity of light, the permeability of free space and permittivity of free space is given by the expression $c = 1 / \sqrt{(\mu_o \varepsilon_o)}$

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Electric charge is quantized

Any positive or negative charge *q* that can be detected can be written as

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots,$$

in which e, the **elementary charge,** has the approximate value

$$e = 1.602 \times 10^{-19}\ \mathrm{C}.$$

When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is **quantized.**

Charge Is Conserved

$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He},$$

the *parent* nucleus ^{238}U contains 92 protons (a charge of $+92e$), the *daughter* nucleus ^{234}Th contains 90 protons (a charge of $+90e$), and the emitted alpha particle ^{4}He contains 2 protons (a charge of $+2e$). We see that the total charge is $+92e$ before and after the decay; thus, charge is conserved. (The total number of protons and neutrons is also conserved: 238 before the decay and $234 + 4 = 238$ after the decay.)

$$\gamma \rightarrow e^- + e^+ \quad \text{(pair production)}$$

$$e^- + e^+ \rightarrow \gamma + \gamma \quad \text{(annihilation)}$$

Superposition of charges

As with all forces in this book, the electrostatic force obeys the principle of superposition. Suppose we have *n* charged particles near a chosen particle called particle 1; then the net force on particle 1 is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}$$

Problems on Coulomb's Law

Problem 1: Charges of magnitude 100 microcoulomb each are located in vacuum at the corners A, B and C of an equilateral triangle measuring 4 meters on each side. If the charge at A and C are positive and the charge B negative, what is the magnitude and direction of the total force on the charge at C?

- The situation is shown in fig. Let us consider the forces acting on C due to A and B.

Now, from Coulomb's law, the force of repulsion on C due to A i.e., FCA in direction AC is given by

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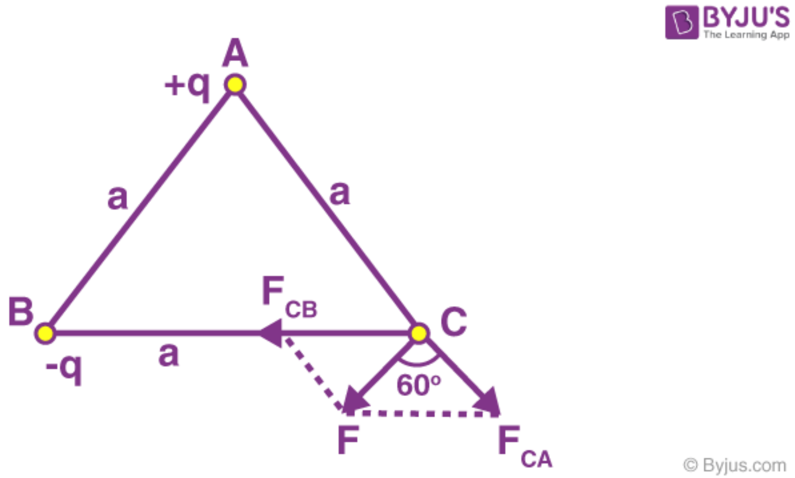
Problems on Coulomb's Law

F_{CA} =
along AC
The force of attraction on C due to B i.e., F_{CB} in direction CB is given by
F_{CB} =
along CB
Thus the two forces are equal in magnitude. The angle between them is 120º. The resultant force F is given by
F =
= (use formula)
= 5.625 Newton
This force is parallel to AB.



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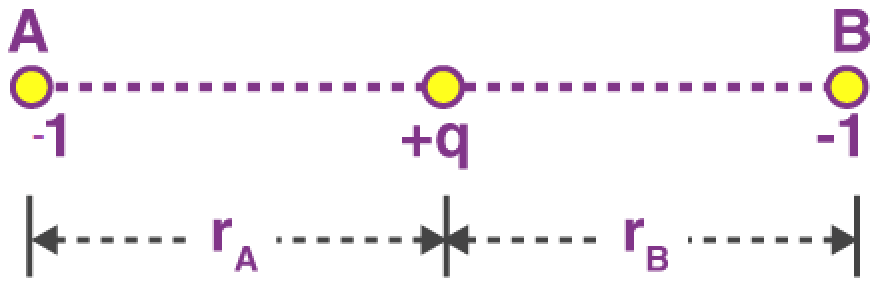
Problems on Coulomb's Law

Problem 2: The negative point charges of unit magnitude and a positive point charge q are placed along the straight line. At what position and for what value of q will the system be in equilibrium? Check whether it is stable, unstable or neutral equilibrium.
Sol.
The two negative charges A and B of unit magnitude are shown in fig.
Let the positive charge q be at a distance $r_A$from A and at a distance r_B from B.
Now, from coulombs law, Force on q due to A
FqA towards A (use formula)
Force on q due to B
FqB towards B (use formula).
These two forces acting on q are opposite and collinear. For the equilibrium of q, the two forces must also be equal i.e.
$|F_{qA}| = |F_{qB}|$ (use formula)
Hence rA = rB
So for the equilibrium of q, it must be equidistant from A & B i.e. at the middle of AB
Now for the equilibrium of the system, A and B must be in equilibrium.



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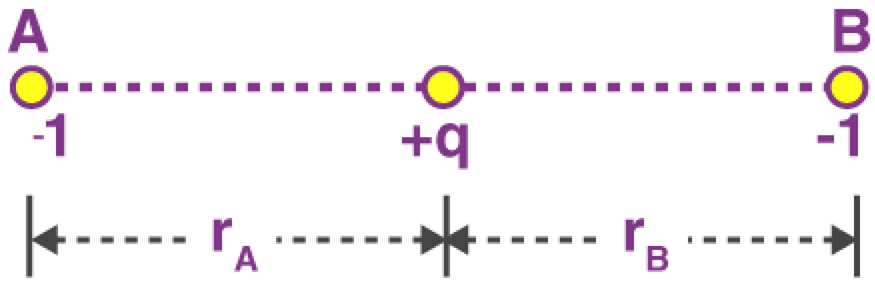
Problems on Coulomb's Law

For the equilibrium of A
Force on A by q = (use formula)
towards q

Force on A by B =
= (use formula)
away from q

The two forces are opposite and collinear. For equilibrium the forces must be equal, opposite and collinear. Hence
or q = 1/4 in magnitude of either charge.
It can also be shown that for the equilibrium of B, the magnitude of q must be 1/4 of the magnitude of either charge.



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Problems on Coulomb's Law

A positive charge of 6×10^{-6} C is 0.040m from the second positive charge of 4×10^{-6} C. Calculate the force between the charges.

Given

$q_{1} =$ 6×10^{-6} C

$q_2 = 4\times10^{-6}$ C

r = 0.040 m

Sol.

$F_e = 134.85$ N

Ref: https://byjus.com/jee/coulombs-law/
https://www.guyhowto.com/coulombs-law/

Problems on Coulomb's Law

Two-point charges, q_1 = +9 μC and q_2 = 4 μC, are separated by a distance r = 12 cm. What is the magnitude of the electric force?
given
$k = 8.988 \times 10^9$ Nm^{2}C^{-2}

$q_1 = 9 \times 10^{-6}$ C
$q_2 = 4 \times 10^{-6}$ C
r = 12cm = 0.12 m
Sol:
F_e = 22.475 N

Ref: https://byjus.com/jee/coulombs-law/
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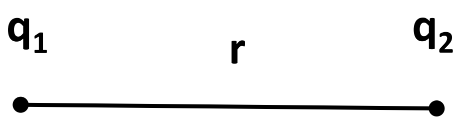
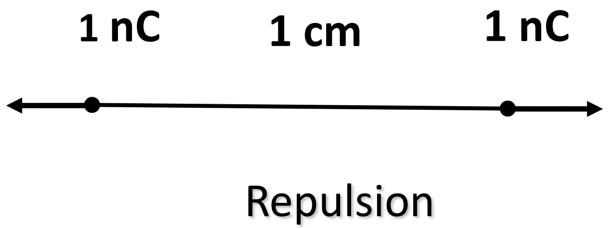
Coulombs Law
Two Positive Charges

Example: What is the force between two positive charges each 1 nano Coulomb, 1cm apart in a typical demo? Why is the force so weak here?

Solution:





$$F=\frac{kq_1q_2}{r^2}$$

$$F=\frac{\left(10^{10}\,\frac{Nm^2}{C^2}\right)\left(10^{-9}\,c\right)^2}{\left(10^{-2}\,m\right)^2}=10^{-4}\,N$$

(equivalent to a weight of something with a mass of 10^{-5} kg = 10^{-2} gm or 10 mg - long strand of hair)

Coulombs Law
Two Pennies without electrons

Example: (i) What is the force between two 3 gm pennies one meter apart if we remove all the electrons from the copper atoms? [Modeling] (ii) What is their acceleration as they separate?

Solution: (i) We know,

$$F=\frac{kq_1q_2}{r^2}=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)q^2}{(1m)^2}$$

The force is

$$F=\frac{\left(10^{10}\ \frac{Nm^2}{C^2}\right)\left(1.4\ \times 10^5 C\right)^2}{1m^2}=2\times 10^{20}N$$

The atom Cu has 29 protons and a 3 gm penny has

$$=\ \left(\frac{3gm}{63.5gm}\right)\times 6\times 10^{23}\,atoms = 3\times 10^{22}\,atoms$$

The total charge is $q = 29\times 3\times 10^{22}\,atoms\times 1.6\times 10^{-19}C = 1.4\times 10^5 C$

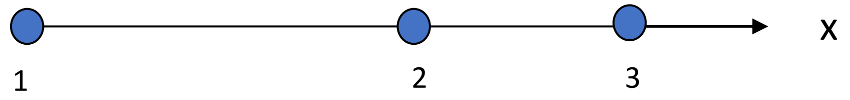
(ii) What is their acceleration as they separate?

$$a=\frac{F}{m}=\frac{2x10^{20}N}{3x10^{-3}kg}=7x10^{22}\ \frac{m}{s^2}$$

Principle of Superposition
Three charges In a line

- In the previous example we tacitly assumed that the forces between nuclei simply added and did not interfere with each other. That is the force between two nuclei in each penny is the same as if all the others were not there. This idea is correct and is referred to as the Principle of Superposition.

Example: Example of charges in a line.



Three charges lie on the x axis: q_1=+25 nC at the origin, q_2= -12 nC at x =2m, q_3=+18 nC at x=3 m. What is the net force on q_1?

Solution: We simply add the two forces keeping track of their directions. Let a positive force be one in the + x direction.

$$F=-kq_1\left(\frac{q_2}{(2m)^2}+\frac{q_3}{(3m)^2}\right)$$

$$=-\left(10^{10}\,\frac{Nm^2}{C^2}\right)(25\times10^{-9}C)\left(\frac{-12\times10^{-9}C}{(2m)^2}+\frac{18\times10^{-9}C}{(3m)^2}\right)$$

$$=2.5\times10^{-7}\,N$$

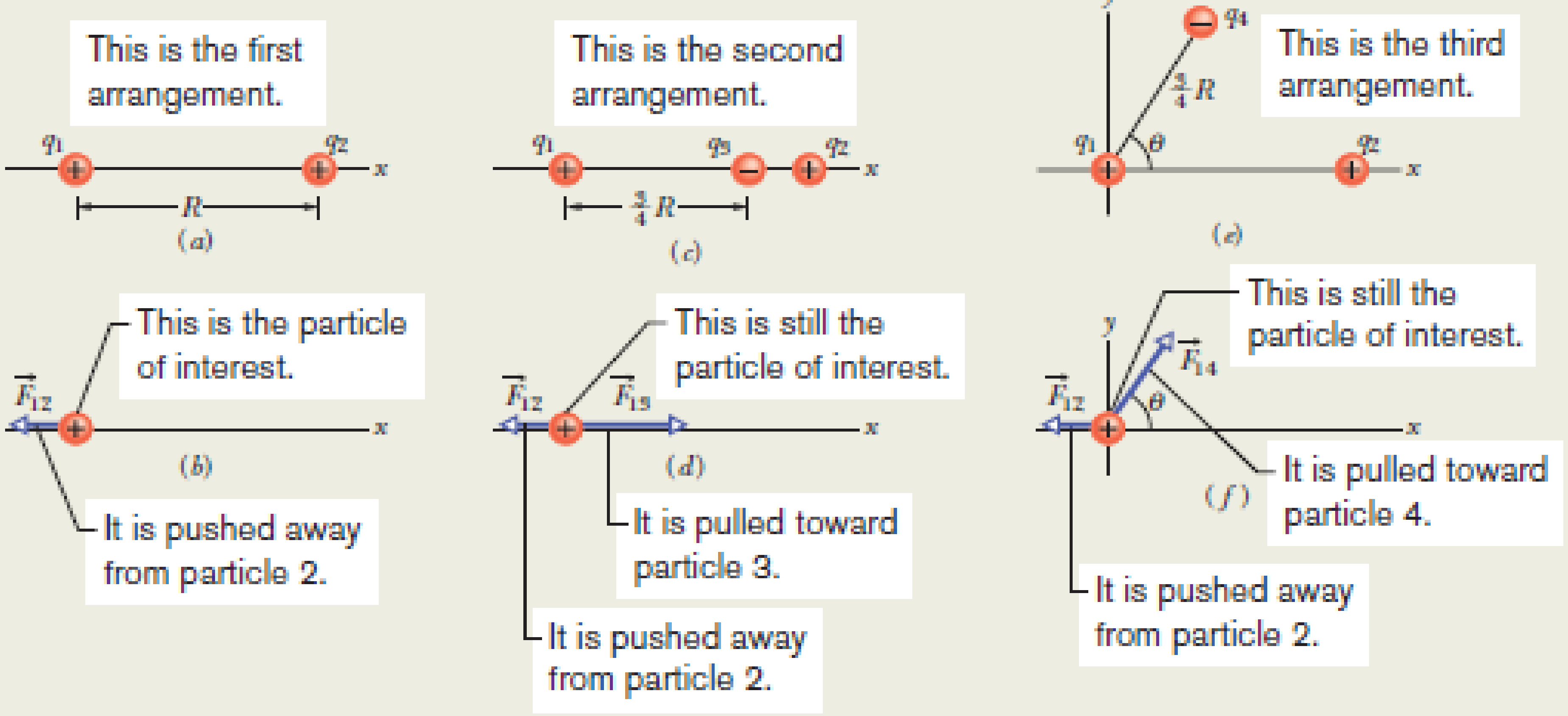


Figure 21-7 (*a*) Two charged particles of charges q_1 and q_2 are fixed in place on an x axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

This sample problem actually contains three examples, to build from basic stuff to harder stuff. In each we have the same charged particle 1. First there is a single force acting on it (easy stuff). Then there are two forces, but they are just in opposite directions (not too bad). Then there are again two forces but they are in very different directions (ah, now we have to get serious about the fact that they are vectors). The key to all three examples is to draw the forces correctly *before* you reach for a calculator, otherwise you may be calculating nonsense on the calculator. (Figure 21-7 is available in *WileyPLUS* as an animation with voiceover.)

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(0.0200\ \text{m})^2}$$

$$= 1.15 \times 10^{-24}\ \text{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24}\ \text{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

We can also write $\vec{F}_{12}$ in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$

$$= 2.05 \times 10^{-24}\ \text{N}.$$

We can also write $\vec{F}_{13}$ in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}.$$

The net force $\vec{F}_{1,\text{net}}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$

$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}} + (2.05 \times 10^{-24}\ \text{N})\hat{\text{i}}$$

$$= (9.00 \times 10^{-25}\ \text{N})\hat{\text{i}}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,\text{net}}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25}\ \text{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

Four particles: We can rewrite Eq. 21-4 as

$$F_{14} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)$$

$$\times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$

$$= 2.05 \times 10^{-24}\ \text{N}.$$

Then from Eq. 21-7, we can write the net force $\vec{F}_{1,\text{net}}$ on particle 1 as

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}.$$

Because the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are not directed along the same axis, we *cannot* sum simply by combining their magnitudes. Instead, we must add them as vectors, using one of the following methods.

Method 1. Summing directly on a vector-capable calculator. For $\vec{F}_{12}$, we enter the magnitude 1.15×10^{-24} and the angle 180°. For $\vec{F}_{14}$, we enter the magnitude 2.05×10^{-24} and the angle 60°. Then we add the vectors.

Method 2. Summing in unit-vector notation. First we rewrite $\vec{F}_{14}$ as

$$\vec{F}_{14} = (F_{14}\cos\theta)\hat{\text{i}} + (F_{14}\sin\theta)\hat{\text{j}}.$$

Substituting 2.05×10^{-24} N for F_{14} and 60° for θ, this becomes

$$\vec{F}_{14} = (1.025 \times 10^{-24}\ \text{N})\hat{\text{i}} + (1.775 \times 10^{-24}\ \text{N})\hat{\text{j}}.$$

Then we sum:

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{14}$$

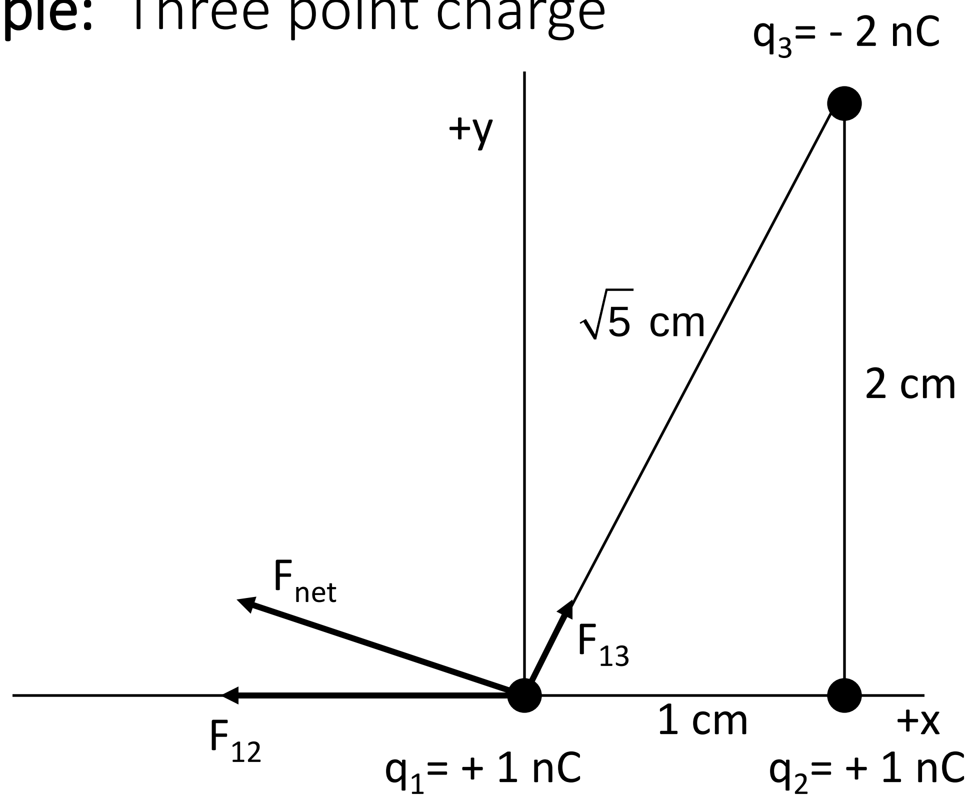
$$= -(1.15 \times 10^{-24}\ \text{N})\hat{\text{i}}$$

$$+ (1.025 \times 10^{-24}\ \text{N})\hat{\text{i}} + (1.775 \times 10^{-24}\ \text{N})\hat{\text{j}}$$

$$\approx (-1.25 \times 10^{-25}\ \text{N})\hat{\text{i}} + (1.78 \times 10^{-24}\ \text{N})\hat{\text{j}}.$$

(Answer)

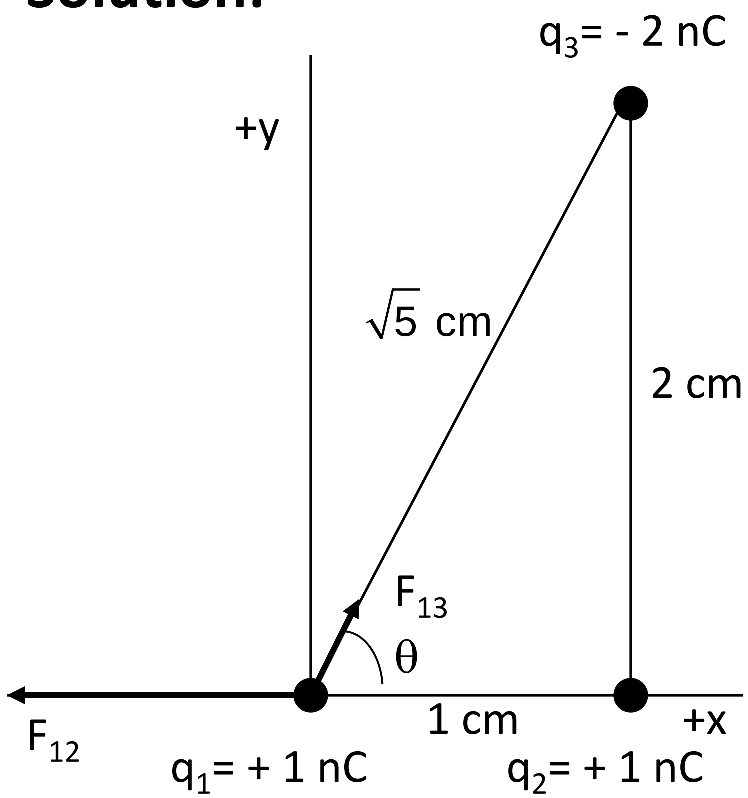
Example: Three point charge



Question: What is the net force on q_1 and in what direction?

Hint : Find x and y components of force on q_1 due to q_2 and q_3 and add them up.

Solution:



x and y Components of force due to q_2

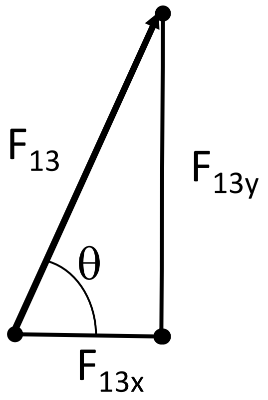
$$F_{12x} = -10^{10}\,\tfrac{\mathrm{Nm}^2}{\mathrm{C}^2}\frac{(10^{-9}\mathrm{C})^2}{(10^{-2}\mathrm{m})^2} = -1\times10^{-4}\mathrm{N}$$

$$F_{12y} = 0$$

x and y Components of force due to q_3

Magnitude of Force due to q_3

$$\left|F_{13}\right| = 10^{10}\,\tfrac{\mathrm{Nm}^2}{\mathrm{C}^2}\frac{(2\times10^{-9}\mathrm{C})(1\times10^{-9}\mathrm{C})}{(\sqrt{5}\times10^{-2}\mathrm{m})^2} = 0.40\times10^{-4}\mathrm{N}$$

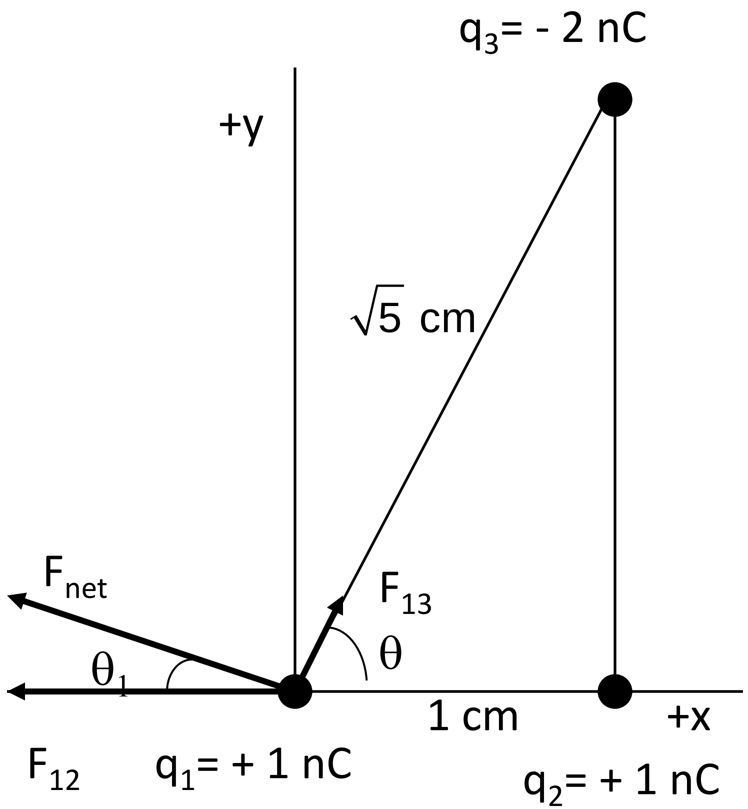


$$\tan\theta = \frac{\text{y - axis value}}{\text{x - axis value}} \quad \Rightarrow \theta = \tan^{-1}(\tfrac{2}{1}) = 63.43\ \mathrm{deg}$$

$$F_{13x} = F\cos\theta = (0.40\mathrm{N})(\cos 63.43) = 0.179\times10^{-4}\mathrm{N}$$

$$F_{13y} = F\sin\theta = (0.40\mathrm{N})(\sin 63.43) = 0.358\times10^{-4}\mathrm{N}$$

Example Cont.



Total force along the x-axis

$$F_{x_{net}} = F_{12x} + F_{13x} = (-1\times10^{-4} + 0.179\times10^{-4})N = -0.821\times10^{-4}N$$

Total force along the y-axis

$$F_{y_{net}} = F_{12y} + F_{13y} = (0 + 0.358\times10^{-4})N = 0.358\times10^{-4}N$$

$$F_{net} = \sqrt{F_{x_{net}}^2 + F_{y_{net}}^2} = \sqrt{((0.821)^2 + (0.358)^2)\times(10^{-4})^2}N$$

$$F_{net} = +0.802\,10^{-4}N$$

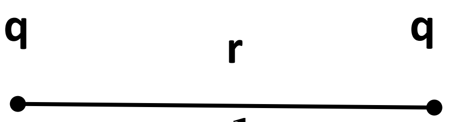
$$\tan\theta_1 = \frac{F_y}{F_x}$$

$$\theta_1 = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad \theta_1 = \tan^{-1}\left(\frac{0.358\times10^{-4}N}{-0.821\times10^{-4}N}\right)$$

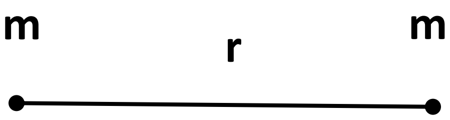
$$\theta_1 = 23.6^\circ \text{ from the } -x \text{ axis}$$

Example-6: In an atom can we neglect the gravitational force between the electrons and protons? What is the ratio of Coulomb's electric force to Newton's gravity force for 2 electrons separated by a distance r ?

Solution:



$$F_c = \frac{kee}{r^2}$$



$$F_g = \frac{Gmm}{r^2}$$

$$\frac{F_c}{F_g} = \frac{ke^2}{Gm^2}$$

$$\frac{F_c}{F_g} = \frac{\left(10^{10}\, Nm^2/C^2\right)\left(1.6\times 10^{-19}\, C\right)^2}{\left(6.67\times 10^{-11}\, Nm^2/kg^2\right)\left(9.1\times 10^{-31}\, kg\right)^2}$$

$$= 4.6\times 10^{42}$$

Huge number, pure ratio

Electric Fields

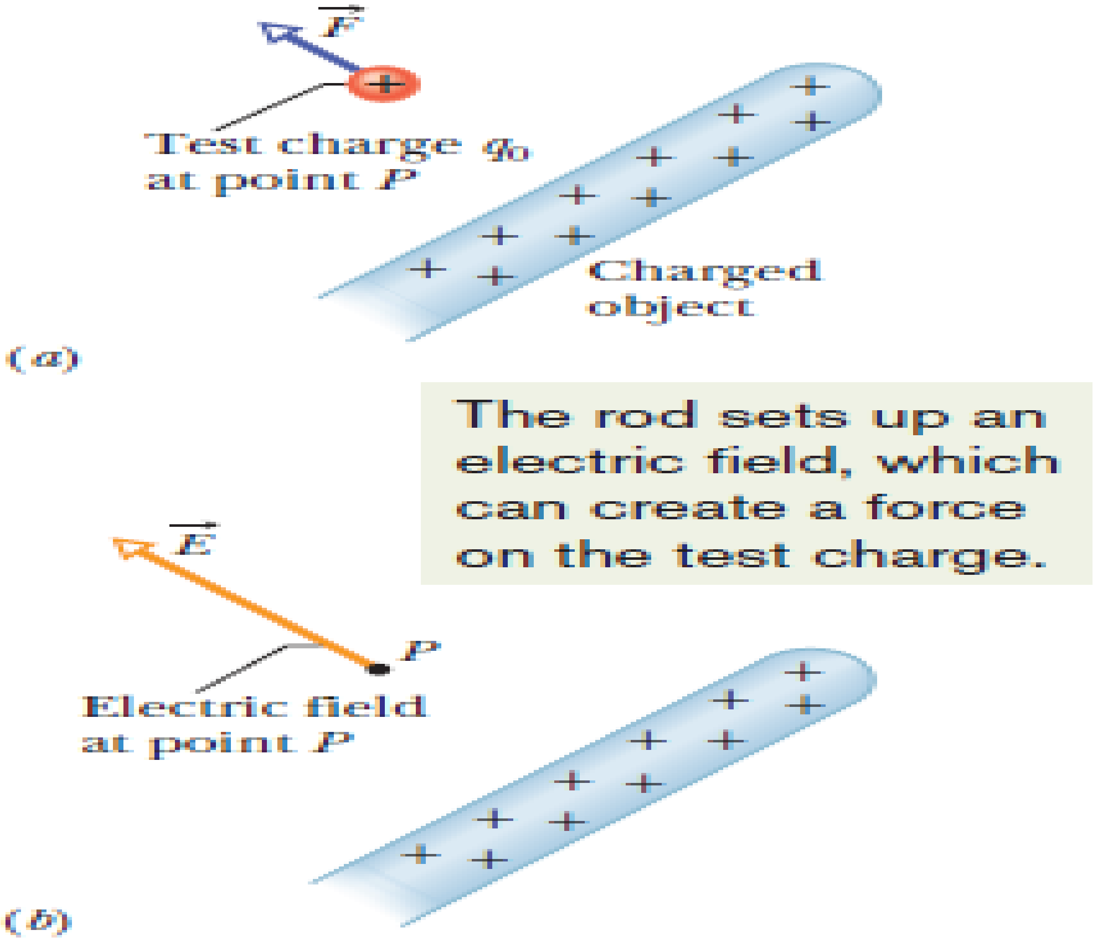


Fig. 22-1 (a) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (b) The electric field $\vec{E}$ at point P produced by the charged object.

22-2 The Electric Field

The temperature at every point in a room has a definite value. You can measure the temperature at any given point or combination of points by putting a thermometer there. We call the resulting distribution of temperatures a *temperature field*. In much the same way, you can imagine a *pressure field* in the atmosphere; it consists of the distribution of air pressure values, one for each point in the atmosphere. These two examples are of *scalar fields* because temperature and air pressure are scalar quantities.

The electric field is a *vector field;* it consists of a distribution of *vectors,* one for each point in the region around a charged object, such as a charged rod. In principle, we can define the electric field at some point near the charged object, such as point P in Fig. 22-1a, as follows: We first place a *positive* charge q_0, called a *test charge*, at the point. We then measure the electrostatic force $\vec{F}$ that acts on the test charge. Finally, we define the electric field $\vec{E}$ at point P due to the charged object as

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).} \tag{22-1}$$

Thus, the magnitude of the electric field $\vec{E}$ at point P is $E = F/q_0$, and the direction of $\vec{E}$ is that of the force $\vec{F}$ that acts on the *positive* test charge. As shown in Fig. 22-1*b*, we represent the electric field at P with a vector whose tail is at P. To define the electric field within some region, we must similarly define it at all points in the region.

The SI unit for the electric field is the newton per coulomb (N/C). Table 22-1 shows the electric fields that occur in a few physical situations.

Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

22-3 Electric Field Lines

Michael Faraday, who introduced the idea of electric fields in the 19th century, thought of the space around a charged body as filled with *lines of force*. Although we no longer attach much reality to these lines, now usually called **electric field lines,** they still provide a nice way to visualize patterns in electric fields.

The relation between the field lines and electric field vectors is this: (1) At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of $\vec{E}$ at that point, and (2) the field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the *magnitude* of $\vec{E}$. Thus, E is large where field lines are close together and small where they are far apart.

Figure 22-2*a* shows a sphere of uniform negative charge. If we place a *positive* test charge anywhere near the sphere, an electrostatic force pointing *toward* the center of the sphere will act on the test charge as shown. In other words, the electric field vectors at all points near the sphere are directed radially toward the sphere. This pattern of vectors is neatly displayed by the field lines in Fig. 22-2*b*, which point in the same directions as the force and field vectors. Moreover, the spreading of the field lines with distance from the sphere tells us that the magnitude of the electric field decreases with distance from the sphere.

If the sphere of Fig. 22-2 were of uniform *positive* charge, the electric field vectors at all points near the sphere would be directed radially *away from* the sphere. Thus, the electric field lines would also extend radially away from the sphere. We then have the following rule:

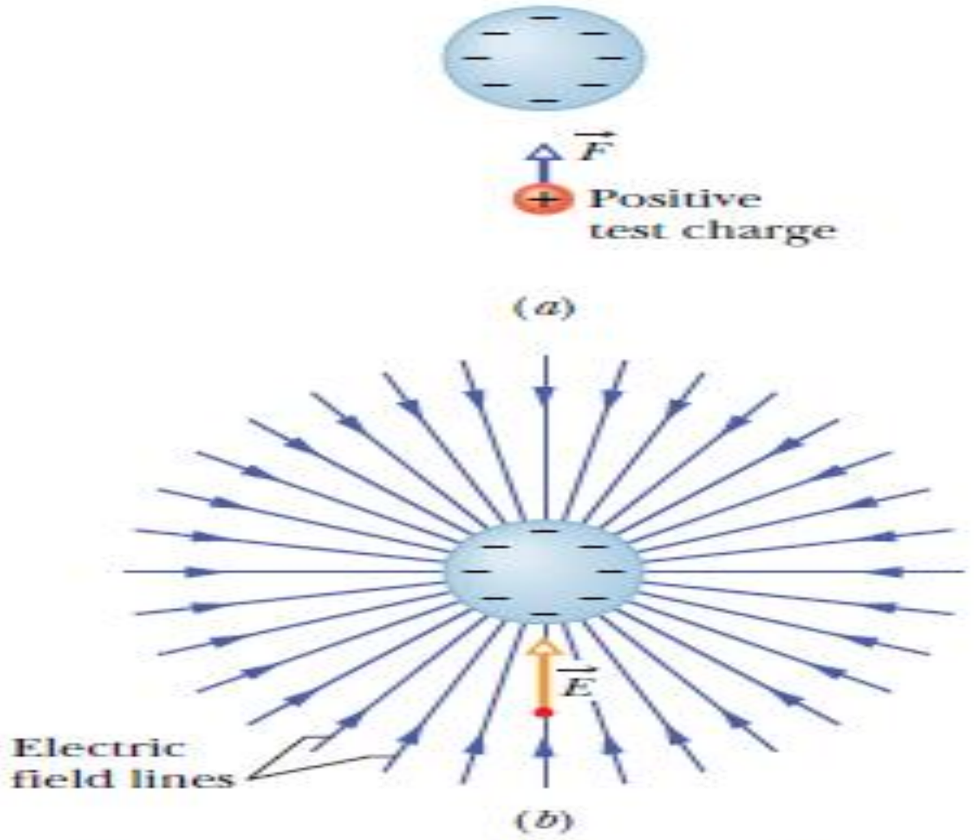


Fig. 22-2 (*a*) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

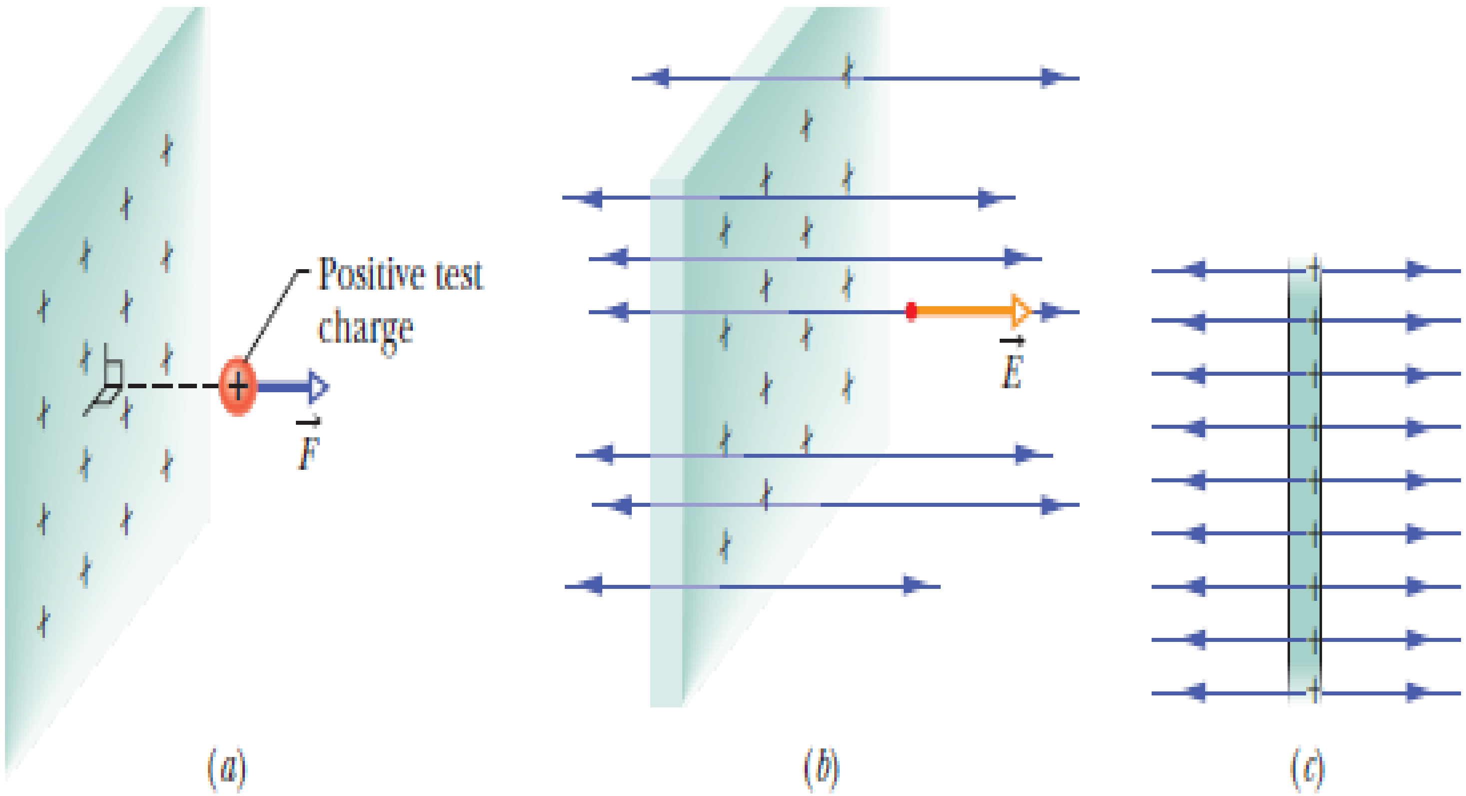
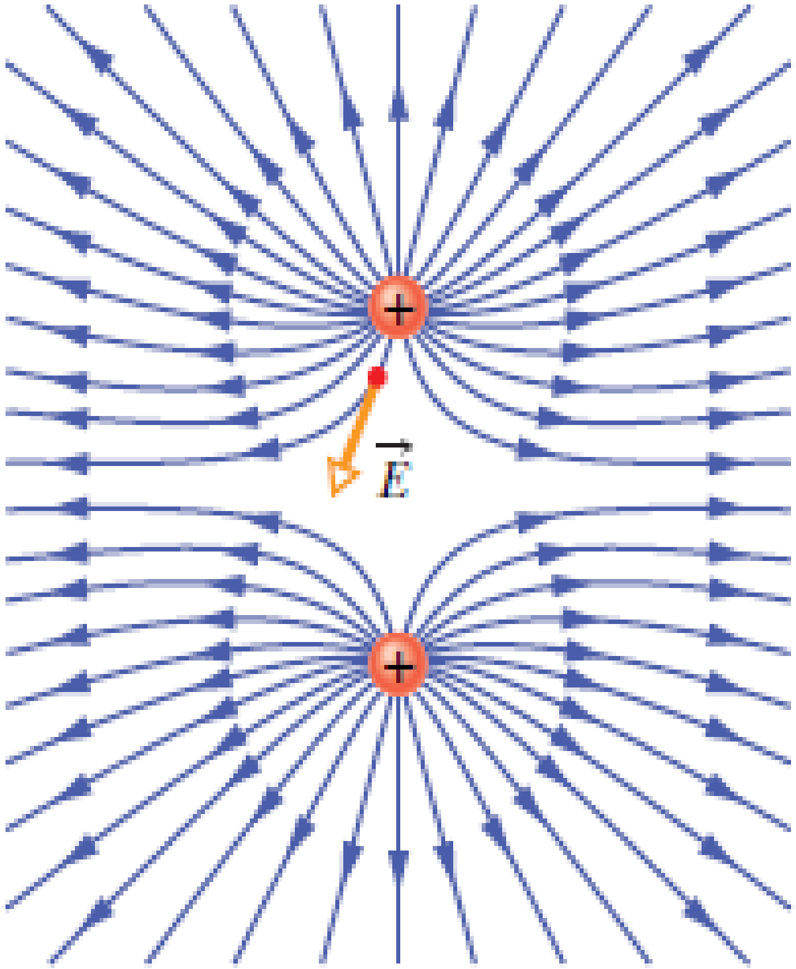


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.



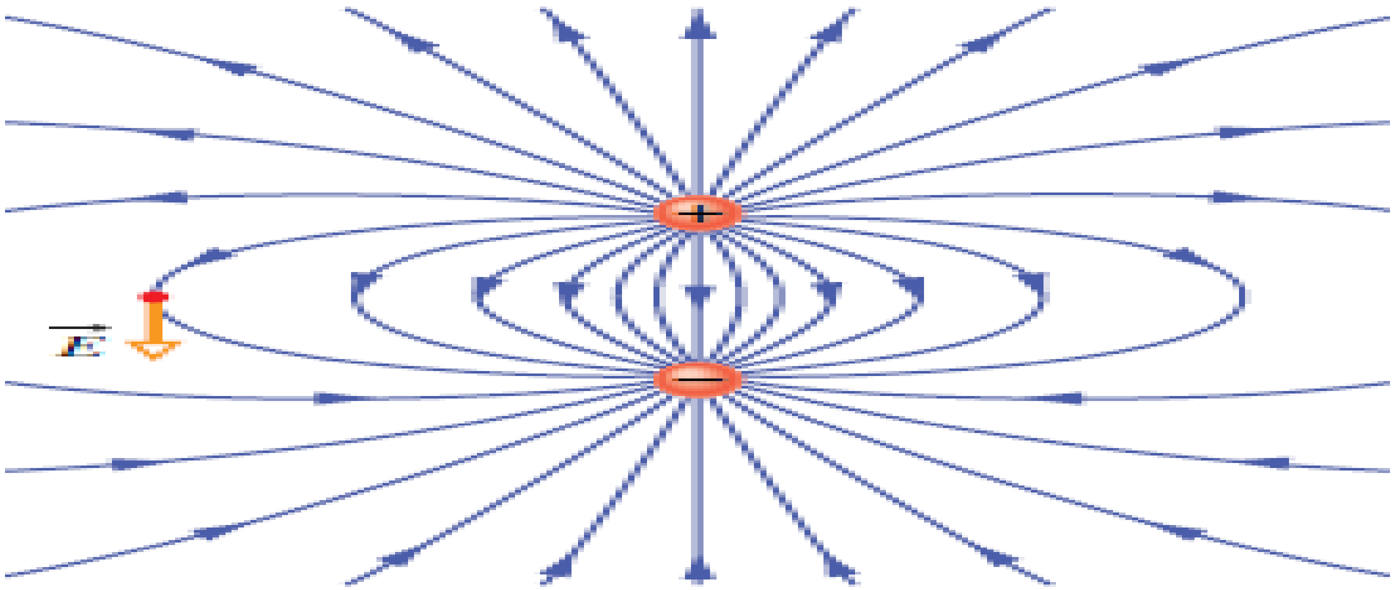


Fig. 22-5 Field lines for a positive point charge and a nearby negative point charge that are equal in magnitude. The charges attract each other. The pattern of field lines and the electric field it represents have rotational symmetry about an axis passing through both charges in the plane of the page. The electric field vector at one point is shown; the vector is tangent to the field line through the point.

The relation between the field lines and electric field vectors is this: (1) At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of $\vec{E}$ at that point, and (2) the field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the *magnitude* of $\vec{E}$. Thus, E is large where field lines are close together and small where they are far apart.

22-4 The Electric Field Due to a Point Charge

To find the electric field due to a point charge q (or charged particle) at any point a distance r from the point charge, we put a positive test charge q_0 at that point. From Coulomb's law (Eq. 21-1), the electrostatic force acting on q_0 is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qq_0}{r^2} \hat{\mathrm{r}}. \tag{22-2}$$

The direction of $\vec{F}$ is directly away from the point charge if q is positive, and directly toward the point charge if q is negative. The electric field vector is, from Eq. 22-1,

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\mathrm{r}} \quad \text{(point charge).} \tag{22-3}$$

The direction of $\vec{E}$ is the same as that of the force on the positive test charge: directly away from the point charge if q is positive, and toward it if q is negative.

Because there is nothing special about the point we chose for q_0, Eq. 22-3 gives the field at every point around the point charge q. The field for a positive point charge is shown in Fig. 22-6 in vector form (not as field lines).

We can quickly find the net, or resultant, electric field due to more than one point charge. If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then, from Eq. 21-7, the net force $\vec{F}_0$ from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

Therefore, from Eq. 22-1, the net electric field at the position of the test charge is

$$\begin{aligned} \vec{E} &= \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n. \end{aligned} \tag{22-4}$$

Here $\vec{E}_i$ is the electric field that would be set up by point charge i acting alone. Equation 22-4 shows us that the principle of superposition applies to electric fields as well as to electrostatic forces.

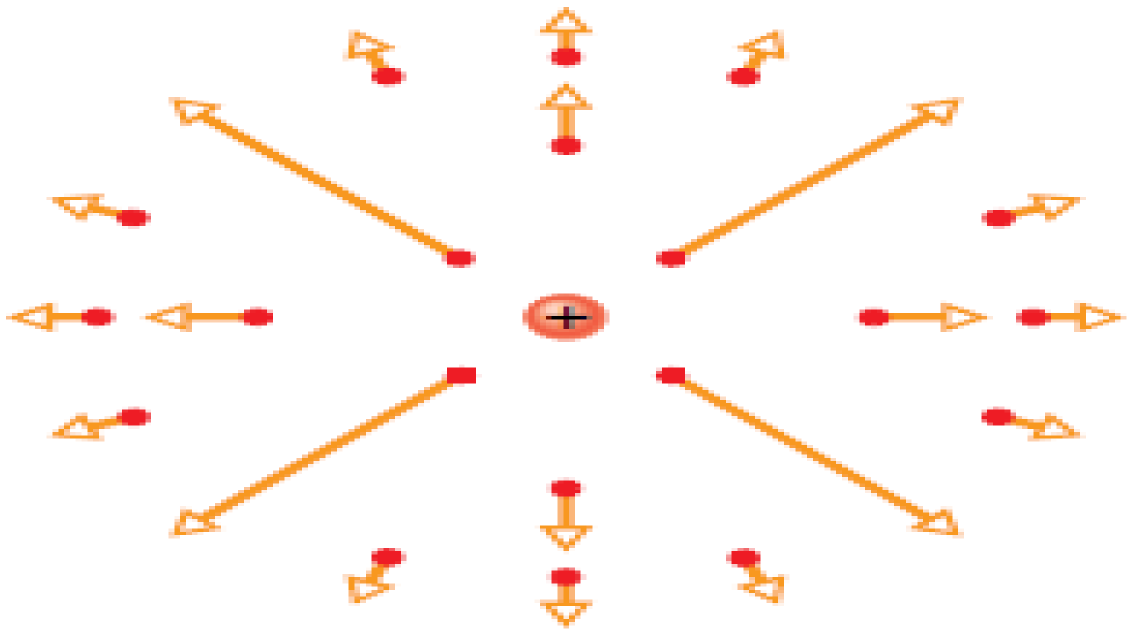


Fig. 22-6 The electric field vectors at various points around a positive point charge.

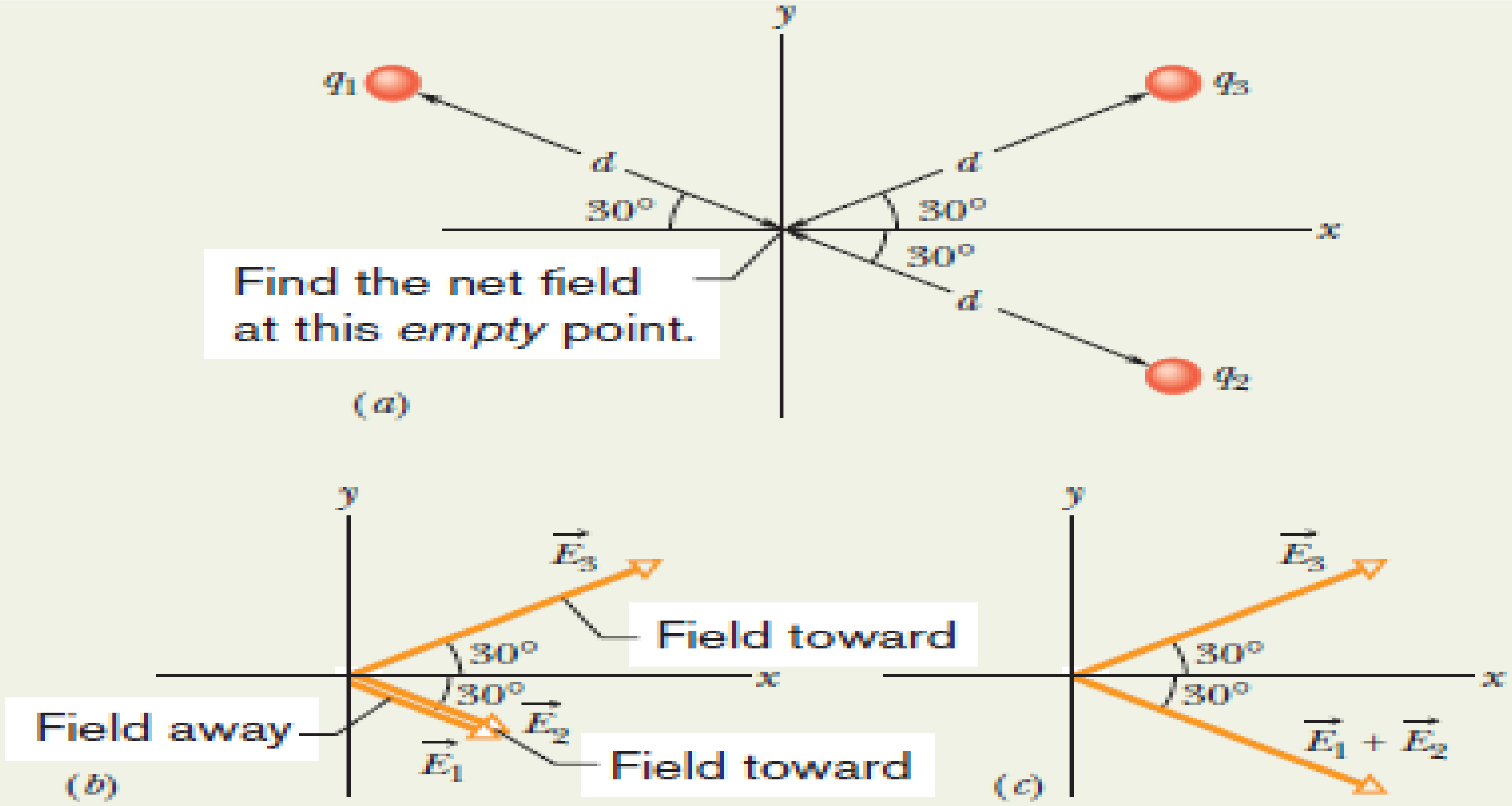


Fig. 22-7 (a) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (b) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (c) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

Problem-1:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

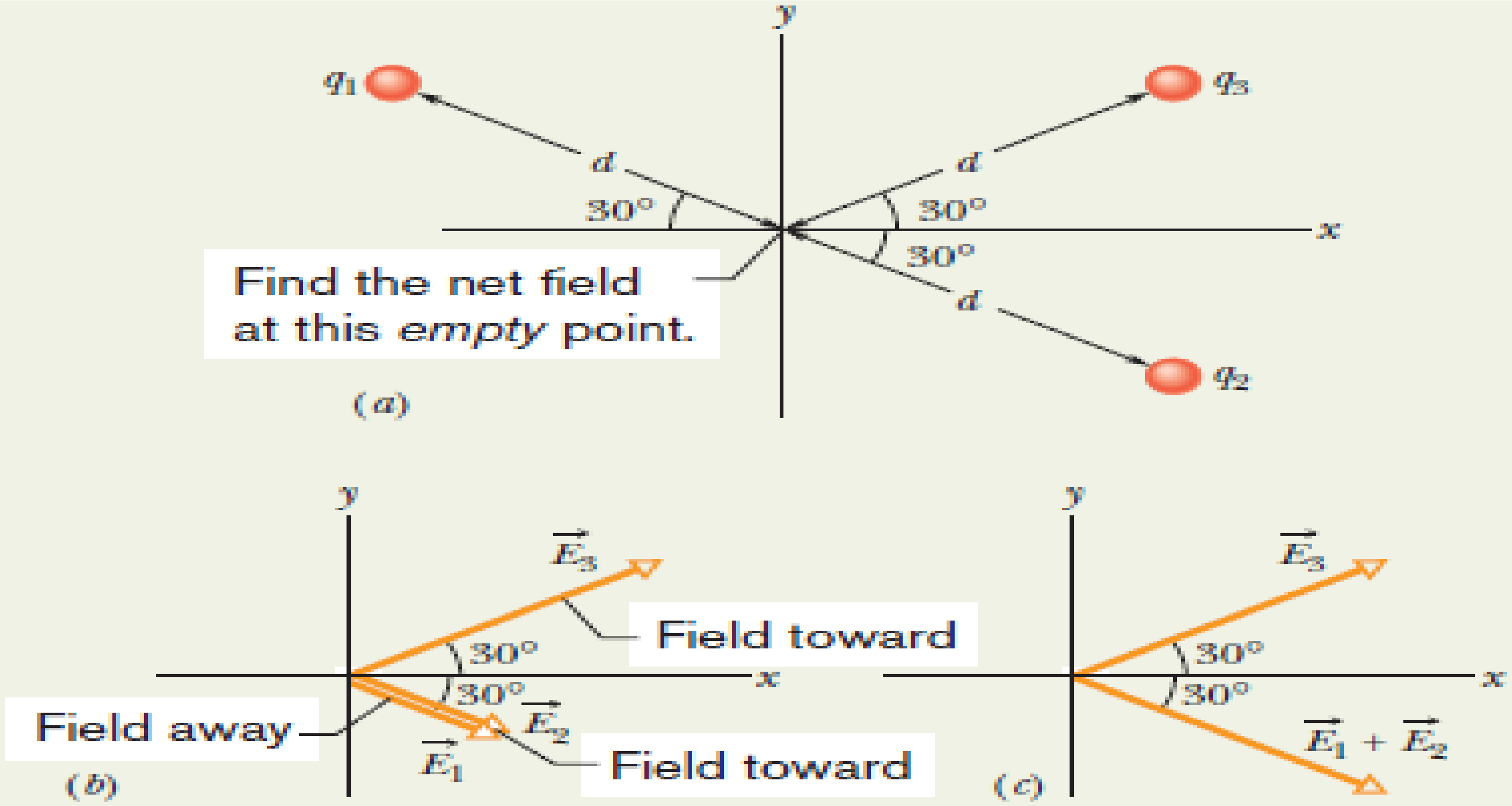


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7c. From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^\circ$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

22-5 The Electric Field Due to an Electric Dipole

Figure 22-8*a* shows two charged particles of magnitude q but of opposite sign, separated by a distance d. As was noted in connection with Fig. 22-5, we call this configuration an *electric dipole*. Let us find the electric field due to the dipole of Fig. 22-8*a* at a point P, a distance z from the midpoint of the dipole and on the axis through the particles, which is called the *dipole axis*.

From symmetry, the electric field $\vec{E}$ at point P—and also the fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ due to the separate charges that make up the dipole—must lie along the dipole axis, which we have taken to be a z axis. Applying the superposition principle for electric fields, we find that the magnitude E of the electric field at P is

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\varepsilon_0(z-\frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z+\frac{1}{2}d)^2}. \end{aligned} \qquad (22\text{-}5)$$

After a little algebra, we can rewrite this equation as

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\dfrac{d}{2z}\right)^2} - \frac{1}{\left(1+\dfrac{d}{2z}\right)^2}\right). \tag{22-6}$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1-\left(\dfrac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1-\left(\dfrac{d}{2z}\right)^2\right)^2}. \tag{22-7}$$

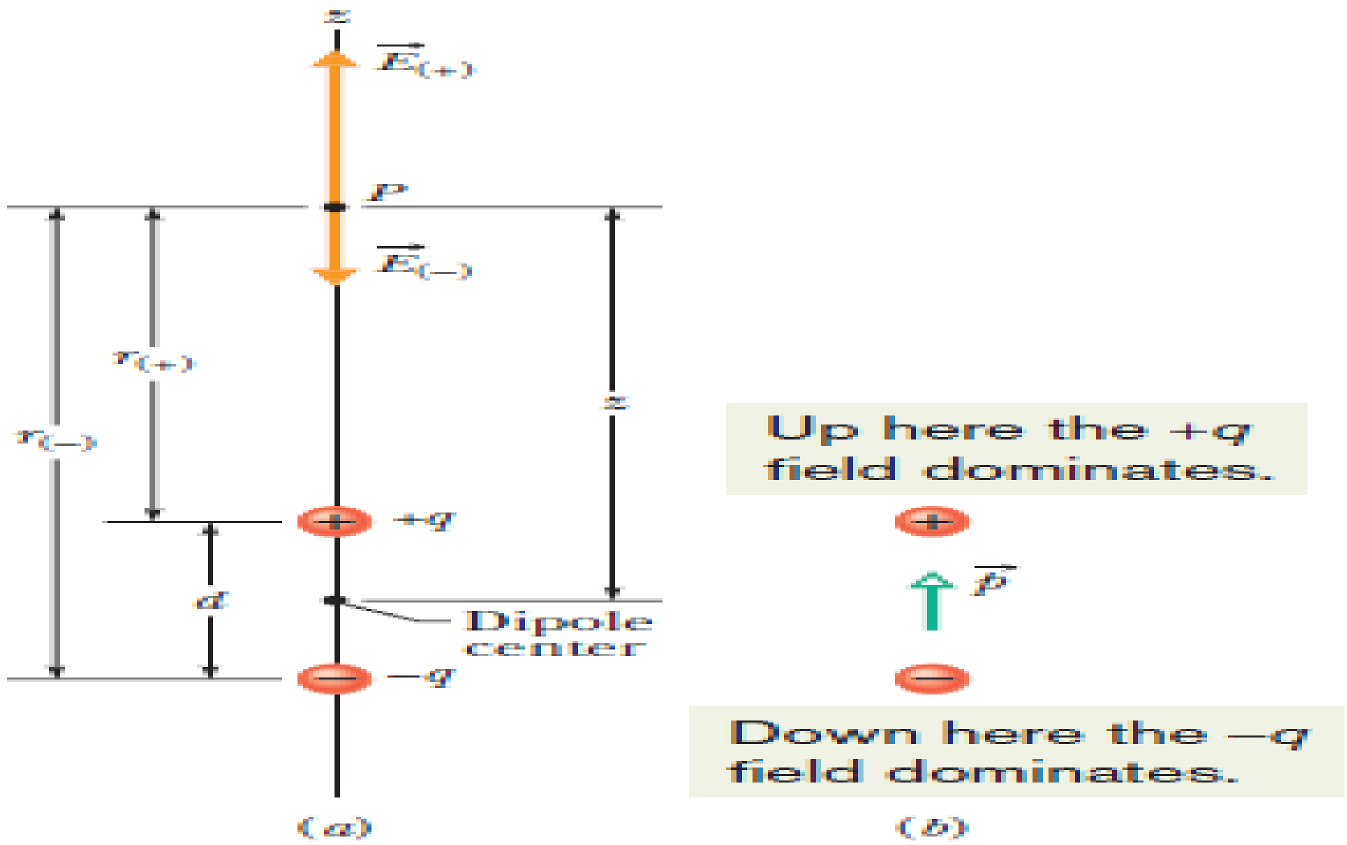


Fig. 22-8 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq. 22-7. Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \qquad (22\text{-}8)$$

The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the **electric dipole moment** $\vec{p}$ of the dipole. (The unit of $\vec{p}$ is the coulomb-meter.) Thus, we can write Eq. 22-8 as

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).} \qquad (22\text{-}9)$$

The direction of $\vec{p}$ is taken to be from the negative to the positive end of the dipole, as indicated in Fig. 22-8*b*. We can use the direction of $\vec{p}$ to specify the orientation of a dipole.

22-6 The Electric Field Due to a Line of Charge

We now consider charge distributions that consist of a great many closely spaced point charges (perhaps billions) that are spread along a line, over a surface, or within a volume. Such distributions are said to be **continuous** rather than discrete. Since these distributions can include an enormous number of point charges, we find the electric fields that they produce by means of calculus rather than by considering the point charges one by one. In this section we discuss the electric field caused by a line of charge. We consider a charged surface in the next section. In the next chapter, we shall find the field inside a uniformly charged sphere.

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a *charge density* rather than as a total charge. For a line of charge, for example, we would report the *linear charge density* (or charge per unit length) λ, whose SI unit is the coulomb per meter. Table 22-2 shows the other charge densities we shall be using.

Figure 22-10 shows a thin ring of radius R with a uniform positive linear charge density λ around its circumference. We may imagine the ring to be made of plastic or some other insulator, so that the charges can be regarded as fixed in place. What is the electric field $\vec{E}$ at point P, a distance z from the plane of the ring along its central axis?

To answer, we cannot just apply Eq. 22-3, which gives the electric field set up by a point charge, because the ring is obviously not a point charge. However, we can mentally divide the ring into differential elements of charge that are so small that they are like point charges, and then we can apply Eq. 22-3 to each of them. Next, we can add the electric fields set up at P by all the differential elements. The vector sum of the fields gives us the field set up at P by the ring.

Let ds be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds. \tag{22-10}$$

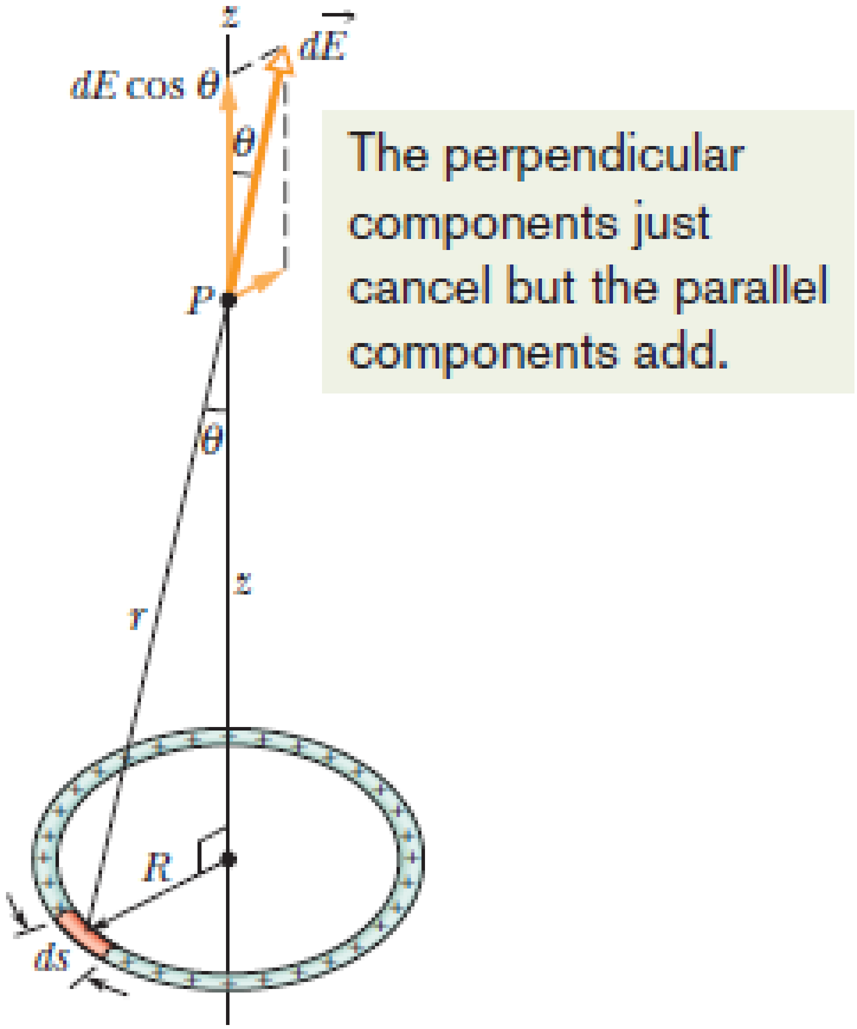


Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point *P*. The component of $d\vec{E}$ along the central axis of the ring is $dE \cos \theta$.

This differential charge sets up a differential electric field $d\vec{E}$ at point P, which is a distance r from the element. Treating the element as a point charge and using Eq. 22-10, we can rewrite Eq. 22-3 to express the magnitude of $d\vec{E}$ as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{22-11}$$

From Fig. 22-10, we can rewrite Eq. 22-11 as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}. \tag{22-12}$$

Figure 22-10 shows that $d\vec{E}$ is at angle θ to the central axis (which we have taken to be a z axis) and has components perpendicular to and parallel to that axis.

Every charge element in the ring sets up a differential field $d\vec{E}$ at P, with magnitude given by Eq. 22-12. All the $d\vec{E}$ vectors have identical components parallel to the central axis, in both magnitude and direction. All these $d\vec{E}$ vectors have components perpendicular to the central axis as well; these perpendicular components are identical in magnitude but point in different directions. In fact, for any perpendicular component that points in a given direction, there is another one that points in the opposite direction. The sum of this pair of components, like the sum of all other pairs of oppositely directed components, is zero.

Thus, the perpendicular components cancel and we need not consider them further. This leaves the parallel components; they all have the same direction, so the net electric field at P is their sum.

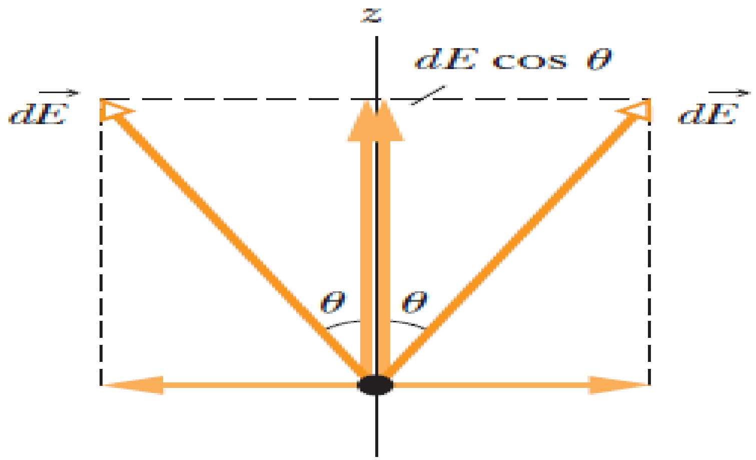


Figure 22-12 The electric fields set up at P by a charge element and its symmetric partner (on the opposite side of the ring). The components perpendicular to the z axis cancel; the parallel components add.

The parallel component of $d\vec{E}$ shown in Fig. 22-10 has magnitude $dE \cos \theta$. The figure also shows us that

$$\cos \theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \tag{22-13}$$

Then multiplying Eq. 22-12 by Eq. 22-13 gives us, for the parallel component of $d\vec{E}$,

$$dE \cos \theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds. \tag{22-14}$$

To add the parallel components $dE \cos \theta$ produced by all the elements, we integrate Eq. 22-14 around the circumference of the ring, from $s = 0$ to $s = 2\pi R$. Since the only quantity in Eq. 22-14 that varies during the integration is s, the other quantities can be moved outside the integral sign. The integration then gives us

$$E = \int dE \cos \theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}. \tag{22-15}$$

Since λ is the charge per length of the ring, the term $\lambda(2\pi R)$ in Eq. 22-15 is q, the total charge on the ring. We then can rewrite Eq. 22-15 as

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).} \tag{22-16}$$

If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still given by Eq. 22-16. However, the electric field vector then points toward the ring instead of away from it.

Let us check Eq. 22-16 for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ in Eq. 22-16 can be approximated as z^2, and Eq. 22-16 becomes

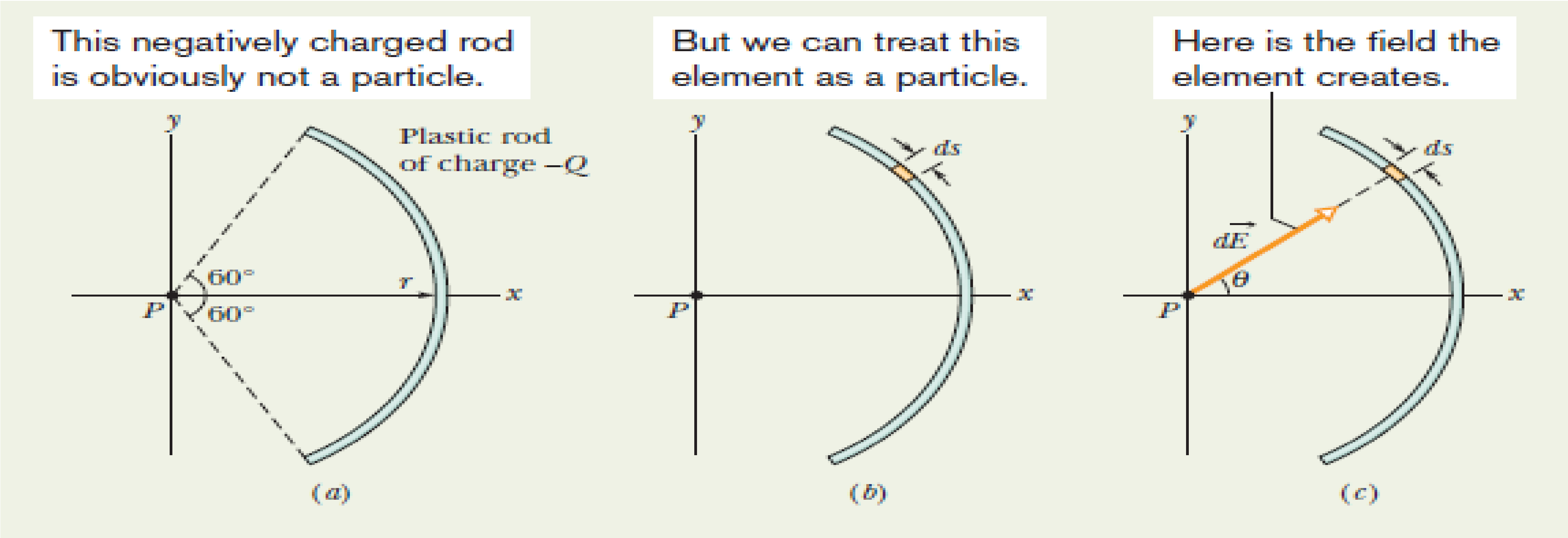
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \quad \text{(charged ring at large distance).} \tag{22-17}$$

This is a reasonable result because from a large distance, the ring "looks like" a point charge. If we replace z with r in Eq. 22-17, we indeed do have Eq. 22-3, the magnitude of the electric field due to a point charge.

Let us next check Eq. 22-16 for a point at the center of the ring—that is, for $z = 0$. At that point, Eq. 22-16 tells us that $E = 0$. This is a reasonable result because if we were to place a test charge at the center of the ring, there would be no net electrostatic force acting on it; the force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring. By Eq. 22-1, if the force at the center of the ring were zero, the electric field there would also have to be zero.

Problem-2:

Figure 22-11*a* shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?



Calculation:

$$dq = \lambda \, ds.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda \, ds}{r^2}.$$

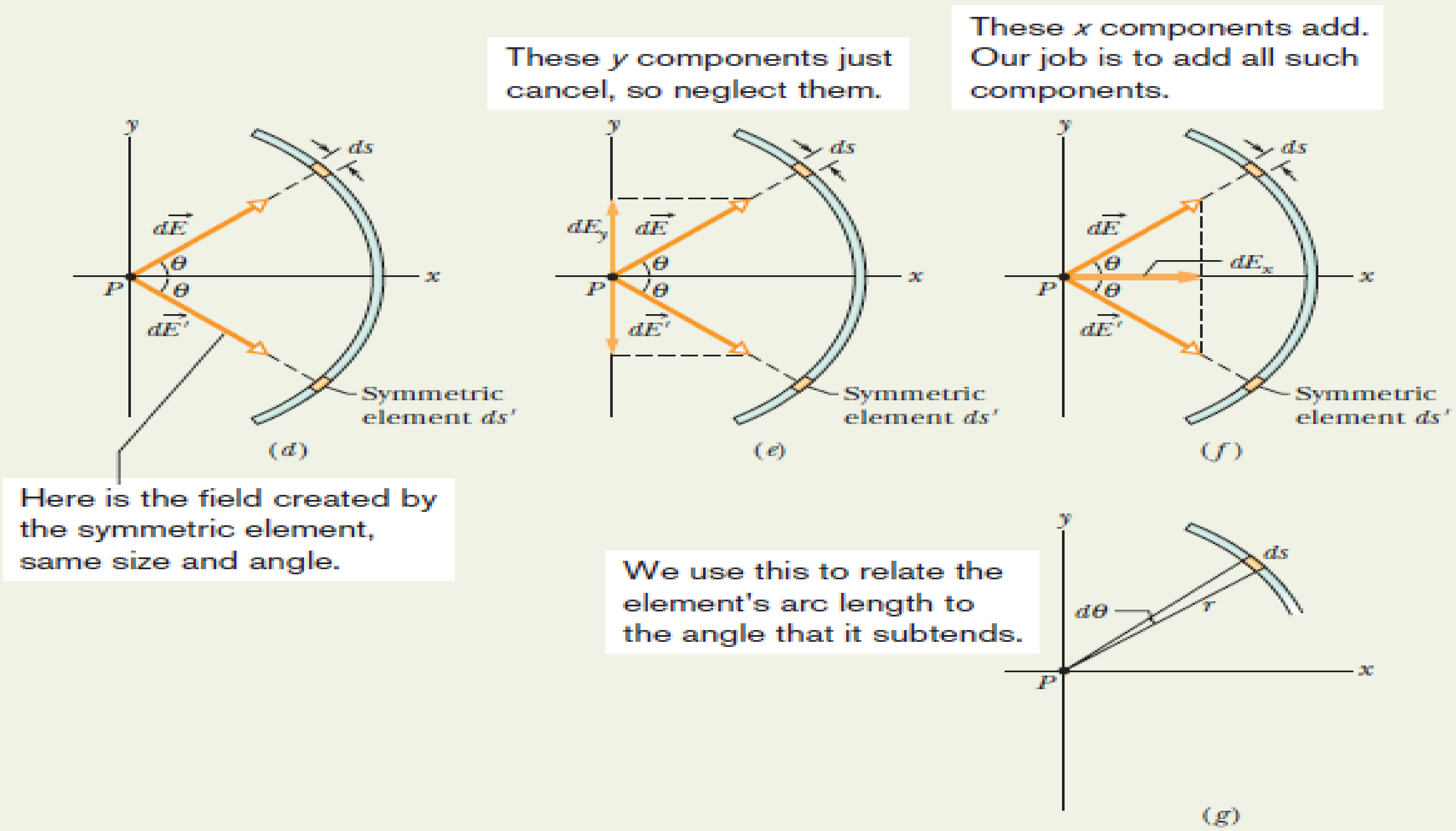


Fig. 22-11 (a) A plastic rod of charge $-Q$ is a circular section of radius r and central angle 120°; point P is the center of curvature of the rod. (b)–(c) A differential element in the top half of the rod, at an angle θ to the x axis and of arc length ds, sets up a differential electric field $d\vec{E}$ at P. (d) An element ds', symmetric to ds about the x axis, sets up a field $d\vec{E}'$ at P with the same magnitude. (e)–(f) The field components. (g) Arc length ds makes an angle $d\theta$ about point P.

22-9 A Dipole in an Electric Field

We have defined the electric dipole moment $\vec{p}$ of an electric dipole to be a vector that points from the negative to the positive end of the dipole. As you will see, the behavior of a dipole in a uniform external electric field $\vec{E}$ can be described completely in terms of the two vectors $\vec{E}$ and $\vec{p}$, with no need of any details about the dipole's structure.

Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions (as shown in Fig. 22-19*a*) and with the same magnitude $F = qE$. Thus, *because the field is uniform*, the net force on the dipole from the field is zero and the center of mass of the dipole does not move. However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance x from one end and thus a distance $d - x$ from the other end. From Eq. 10-39 ($\tau = rF \sin \phi$), we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta. \tag{22-32}$$

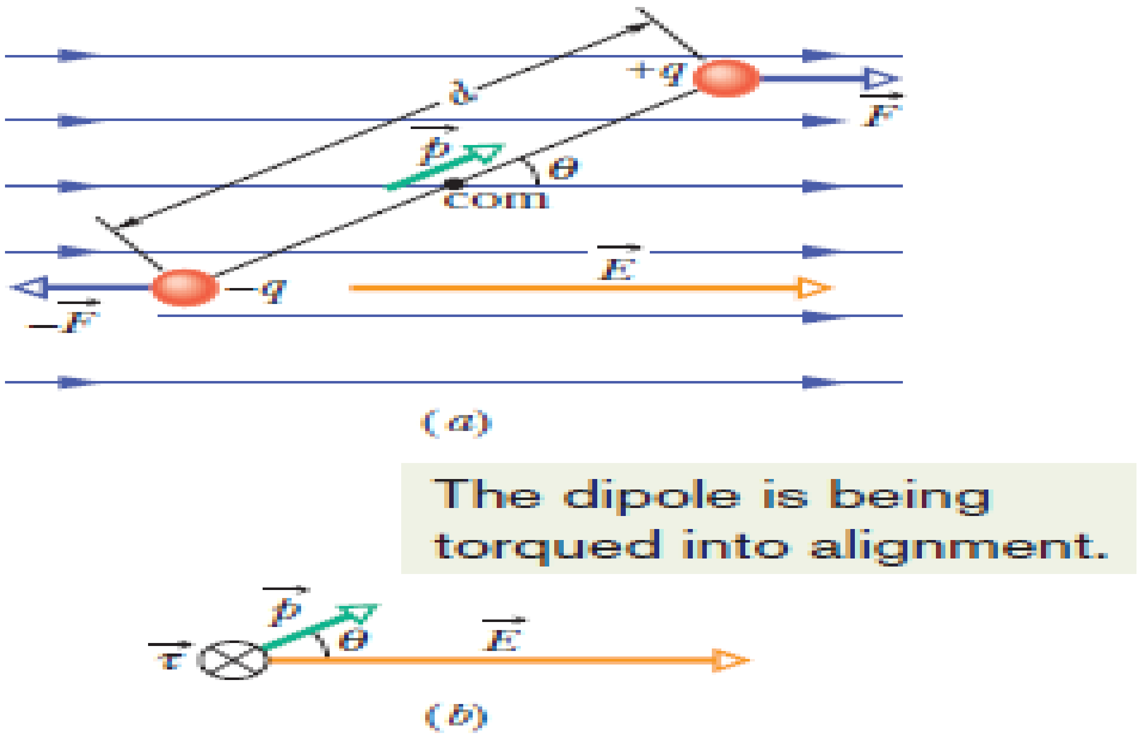


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol $\otimes$.

We can also write the magnitude of $\vec{\tau}$ in terms of the magnitudes of the electric field E and the dipole moment $p = qd$. To do so, we substitute qE for F and p/q for d in Eq. 22-32, finding that the magnitude of $\vec{\tau}$ is

$$\tau = pE \sin \theta. \tag{22-33}$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).} \tag{22-34}$$

Vectors $\vec{p}$ and $\vec{E}$ are shown in Fig. 22-19b. The torque acting on a dipole tends to rotate $\vec{p}$ (hence the dipole) into the direction of field $\vec{E}$, thereby reducing θ. In Fig. 22-19, such rotation is clockwise. As we discussed in Chapter 10, we can represent a torque that gives rise to a clockwise rotation by including a minus sign with the magnitude of the torque. With that notation, the torque of Fig. 22-19 is

$$\tau = -pE \sin \theta. \tag{22-35}$$

Potential Energy of an Electric Dipole

Potential energy can be associated with the orientation of an electric dipole in an electric field. The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up with the field $\vec{E}$ (then

$\vec{\tau} = \vec{p} \times \vec{E} = 0$). It has greater potential energy in all other orientations. Thus the dipole is like a pendulum, which has *its* least gravitational potential energy in *its* equilibrium orientation—at its lowest point. To rotate the dipole or the pendulum to any other orientation requires work by some external agent.

The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ in Fig. 22-19 is 90°.We then can find the potential energy U of the dipole at any other value of θ with Eq. 8-1 ($\Delta U = -W$) by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.With the aid of Eq. 10-53 ($W = \tau d\theta$) and Eq. 22-35, we find that the potential energy U at any angle θ is

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta. \tag{22-36}$$

Evaluating the integral leads to

$$U = -pE\cos\theta. \tag{22-37}$$

We can generalize this equation to vector form as

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).} \tag{22-38}$$

Equations 22-37 and 22-38 show us that the potential energy of the dipole is least ($U = -pE$) when $\theta = 0$ ($\vec{p}$ and $\vec{E}$ are in the same direction); the potential energy is greatest ($U = pE$) when $\theta = 180°$ ($\vec{p}$ and $\vec{E}$ are in opposite directions).

When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i), \tag{22-39}$$

where U_f and U_i are calculated with Eq. 22-38. If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$W_a = -W = (U_f - U_i). \tag{22-40}$$

Water Molecule (Dipole (H_2O)

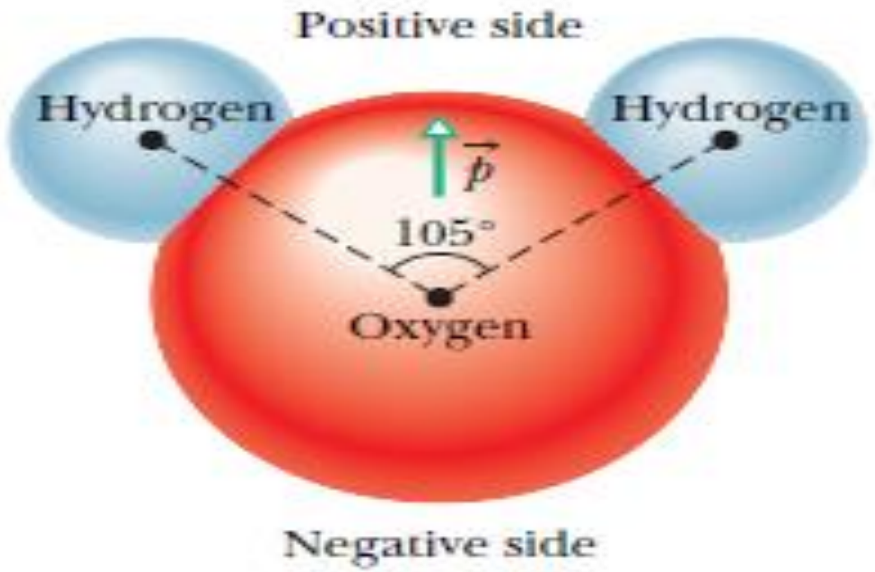


Fig. 22-18 A molecule of H_2O, showing the three nuclei (represented by dots) and the regions in which the electrons can be located. The electric dipole moment $\vec{p}$ points from the (negative) oxygen side to the (positive) hydrogen side of the molecule.

Microwave Cooking

Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles. When you turn on the oven, the microwave source sets up a rapidly oscillating electric field $\vec{E}$ within the oven and thus also within the food. From Eq. 22-34, we see that any electric field $\vec{E}$ produces a torque on an electric dipole moment $\vec{p}$ to align $\vec{p}$ with $\vec{E}$. Because the oven's $\vec{E}$ oscillates, the water molecules continuously flip-flop in a frustrated attempt to align with $\vec{E}$.

Energy is transferred from the electric field to the thermal energy of the water (and thus of the food) where three water molecules happened to have bonded together to form a group. The flip-flop breaks some of the bonds. When the molecules reform the bonds, energy is transferred to the random motion of the group and then to the surrounding molecules. Soon, the thermal energy of the water is enough to cook the food. Sometimes the heating is surprising. If you heat a jelly donut, for example, the jelly (which holds a lot of water) heats far more than the donut material (which holds much less water). Although the exterior of the donut may not be hot, biting into the jelly can burn you. If water molecules were not electric dipoles, we would not have microwave ovens.

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude $6.2 \times 10^{-30}\ C \cdot m$.

(a) How far apart are the molecule's centers of positive and negative charge?

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \mathrm{C\cdot m}}{(10)(1.60 \times 10^{-19}\ \mathrm{C})}$$

$$= 3.9 \times 10^{-12}\ \mathrm{m} = 3.9\ \mathrm{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\begin{aligned}\tau &= pE \sin\theta \\ &= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^{4}\ \text{N/C})(\sin 90°) \\ &= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}\end{aligned}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

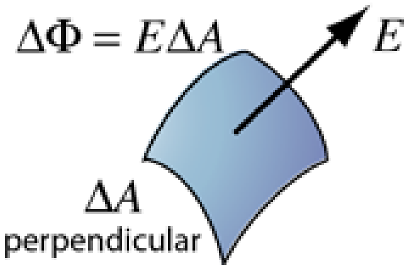
Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$
$$= (-pE\cos 180°) - (-pE\cos 0)$$
$$= 2pE = (2)(6.2 \times 10^{-30}\ \mathrm{C\cdot m})(1.5 \times 10^{4}\ \mathrm{N/C})$$
$$= 1.9 \times 10^{-25}\ \mathrm{J}. \qquad \text{(Answer)}$$

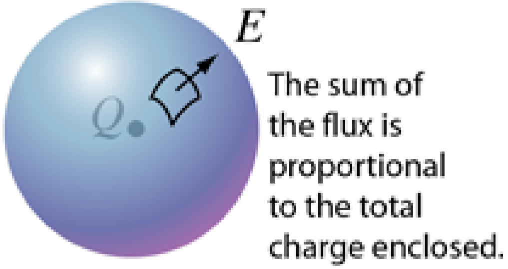
Gauss's Law

Gauss's Law

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.



$$\Phi_{electric} = \frac{Q}{\varepsilon_0}$$



The electric flux through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field. Gauss's Law is a general law applying to any closed surface. It is an important tool since it permits the assessment of the amount of enclosed charge by mapping the field on a surface outside the charge distribution. For geometries of sufficient symmetry, it simplifies the calculation of the electric field.

Another way of visualizing this is to consider a probe of area A which can measure the electric field perpendicular to that area. If it picks any closed surface and steps over that surface, measuring the perpendicular field times its area, it will obtain a measure of the net electric charge within the surface, no matter how that internal charge is configured.

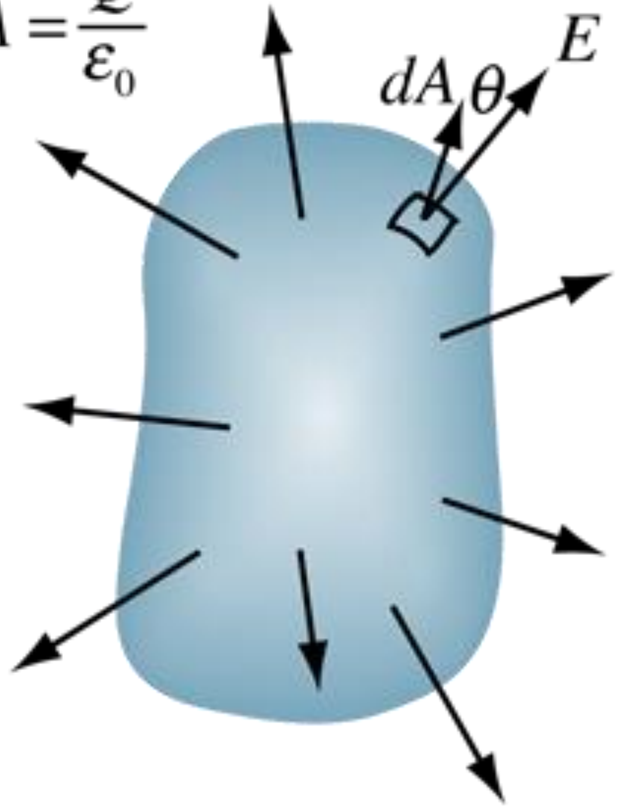
Ref: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/gaulaw.html

Gauss's Law

The area integral of the electric field over any closed surface is equal to the net charge enclosed in the surface divided by the permittivity of space. Gauss' law is a form of one of Maxwell's equations, the four fundamental equations for electricity and magnetism.

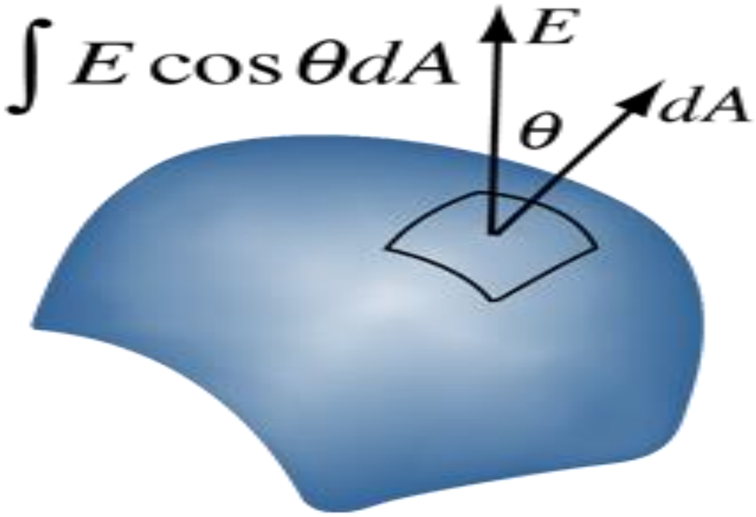
Gauss' law permits the evaluation of the electric field in many practical situations by forming a symmetric Gaussian surface surrounding a charge distribution and evaluating the electric flux through that surface.

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$$



Electric flux:

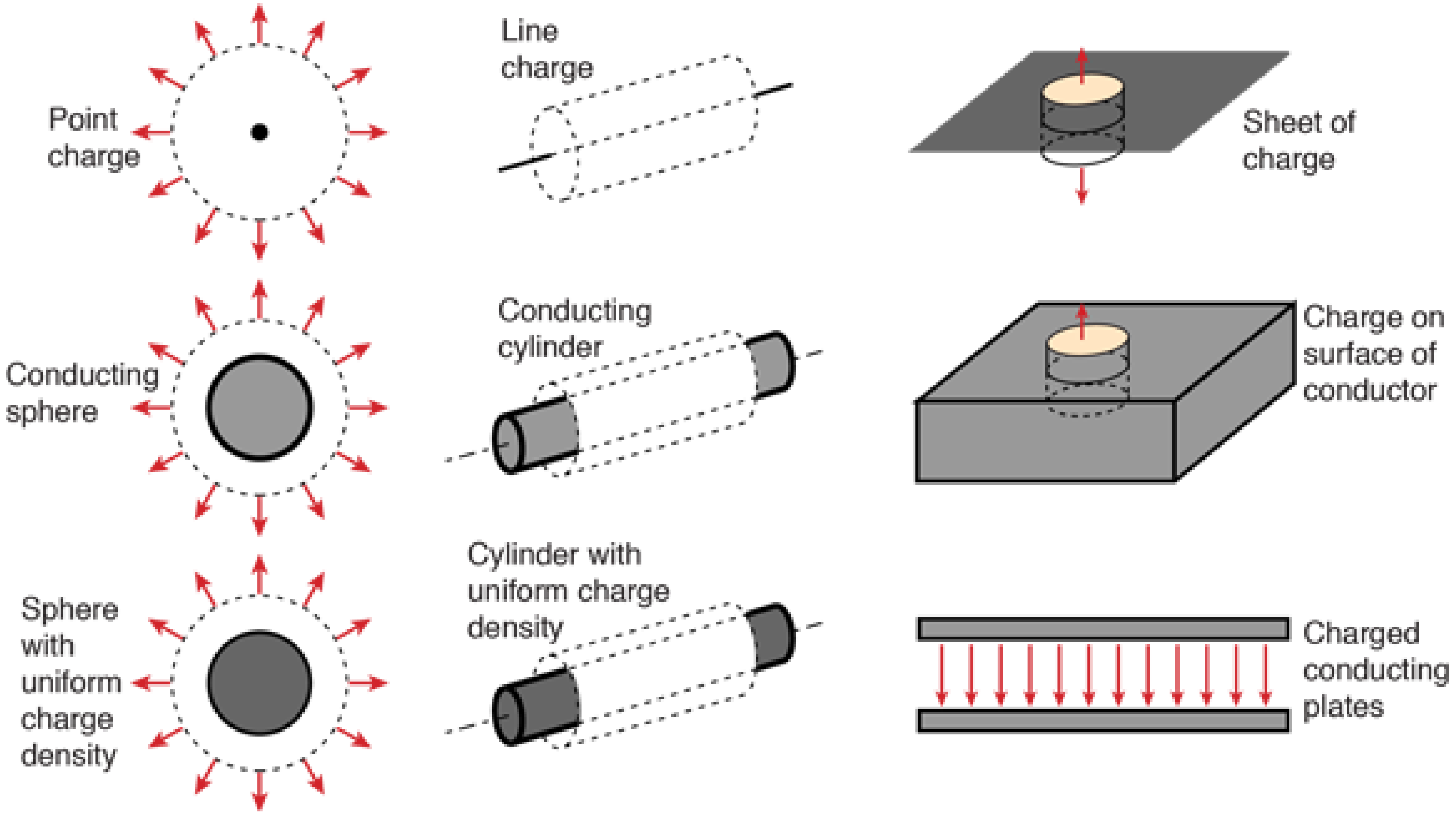
$$\Phi = \int E \cos\theta \, dA$$



Ref: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/gaulaw.html

Applications of Gauss' Law

Gauss' law is a powerful tool for the calculation of electric fields when they originate from charge distributions of sufficient symmetry to apply it.



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