ADVANCED ENGINEERING MATHEMATICS

IT (III SEM)

Course Objectives:

CO1 To prepare many optimization tools for applying them into different research areas as per the requirement.

CO2 To prepare important strategies of linear programming for applying them into solving the problems of transportation and assignment.

CO3 To establish different theorems and properties of random variables for understanding expectation, moments, moment generating function etc.

CO4 To analyze the various discrete and continuous distributions with their appropriate applications.

ASSIGNMENT PROBLEMS (CO2)

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INTRODUCTION

It is a special type of linear programming problem, in which **we assign n jobs *(tasks)* to *n* persons (facilities)**, each person being assigned exactly one job to get *the best results*. Here we make the assumption that each person can perform each *job* but with varying degree of efficiencies.

The assigned problem can be stated in the form of $n \times n$ matrix $(\mathbf{c_{ij}})$ called the **cost or effectiveness matrix**, where c_{ij} is the cost of assigning i^{th} person to j^{th} job.

Jobs

$$
\textbf{Persons}\quad
\begin{array}{c|cccccc|}
 & 1 & 2 & 3 & \cdots & \cdots & n \\
\hline
1 & C_{11} & C_{12} & C_{13} & \cdots & \cdots & C_{1n} \\
2 & C_{21} & C_{22} & C_{23} & \cdots & \cdots & C_{2n} \\
3 & C_{31} & C_{32} & C_{33} & \cdots & \cdots & C_{3n} \\
\vdots & \vdots & & & & & \\
n & C_{n1} & C_{n2} & C_{n3} & \cdots & \cdots & C_{nn} \\
\hline
\end{array}
$$

Effectiveness Matrix

HUNGARIAN ASSIGNMENT ALGORITHM FOR SOLVING MINIMAL ASSIGNMENT PROBLEM

Step 1: (i) Subtract minimum element of each row in the matrix (c_{ij}) from all the elements of the corresponding row.

(ii) Now substract the minimum element of each column from all the elements of the corresponding column.

Step 2: (a) Starting with the first row of the matrix so obtained, examine rows successively, until a row with exactly one zero (unmarked) is found. Mark at this zero and cross-out all other zeros (if any) in the corresponding (in which □ is marked) column. Proceed, until the last row is examined.

(b) Now examine columns, starting with column one, until a column containing exactly one unmarked zero is found. Mark □ at this zero and cross-out all other zeros, in the corresponding row. Proceed, until the last column is examined.

(c) Continue the operations (a) and (b) until either (i) all zeros are marked or crossed-out or (ii) The remaining unmarked zeros lies at least two in a row/column .In (ii) choose any one of the zeros and continue (a) & (b) until all zeros are either marked or crossed-out.

Step 3: If there is exactly one marked zero in every row and every column, the complete optimal assignment is obtained. (The total marked □ zeros are exactly n). We stop at this stage. If not, we pass onto step 4.

Step 4: (i) Mark (✓) all rows for which assignment have not been made

(ii) Mark (✓) columns which have zeros in marked rows.

(iii) Mark (✓) rows (not already marked) which have assignment in marked columns.

(iv) Repeat (ii) and (iii) until the chain of marking ends.

(v) Draw lines through unmarked rows and marked columns.

Step 5 : Select the smallest of the elements those do not have a line through them. Subtract it from all the elements that do not have a line through them, add it to every element that lies at the intersection of two lines and leave the remaining elements of the matrix unchanged.

Step 6 : Go to step 2.

Remarks:
1. Mark □ represents the assignment of the job to the respective person.
2. In step 5 , number of zeros are not decreased.
3. The lines drawn, according to step 4,gives us the minimum number of lines through all the zeros.

Q.1. *Four different jobs can be done on four different machines. The matrix below gives the cost in rupees of producing job i on machine j. How should the jobs be assigned to the various machines so that the total cost is minimized.*

Machines

Jobs	M_1	M_2	M_3	M_4
J_1	12	30	21	15
J_2	18	33	9	31
J_3	44	25	24	21
J_4	23	30	28	14

Sol. Subtracting smallest element of each row from the other elements of the row, we get

	M_1	M_2	M_3	M_4
J_1	0	18	9	3
J_2	9	24	0	22
J_3	23	4	3	0
J_4	9	16	14	0

Subtract, smallest element of each column, from the other elements of the column, we get

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

Now, assigning zeros in rows and columns, we get

	M_1	M_2	M_3	M_4
J_1	$\boxed{0}$	14	9	3
J_2	9	20	$\boxed{0}$	22
J_3	23	$\boxed{0}$	3	$\cancel{0}$
J_4	9	12	14	$\boxed{0}$

Since every row and every column have one assignment. This is the optimal assignment.

Final assignment : Job 1 on Machine 1, Job 2 on Machine 3, Job 3 on Machine 2 and Job 4 on Machine 4. Total Cost = 12 + 9 + 25 + 14 = Rs. 60.

Q.2. *Solve the following assignment problem*:

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

Sol. Subtracting minimum elements of each row and then each column from respective rows and columns, we get

	I	II	III	IV	V
A	**0**	**2**	**1**	**2**	**4**
B	**1**	**3**	**2**	**0**	**3**
C	**2**	**3**	**0**	**1**	**2**
D	**2**	**0**	**3**	**1**	**0**
E	**0**	**4**	**5**	**4**	**2**

Now, assigning zeros in rows and columns, we get

	I	II	III	IV	V
A	$\boxed{0}$	2	1	2	4
B	1	3	2	$\boxed{0}$	3
C	2	3	$\boxed{0}$	1	2
D	2	$\boxed{0}$	3	1	$\cancel{0}$
E	$\cancel{0}$	4	5	4	2

Since row 5 and column 5 have no assignments, we draw minimum number of lines by adopting step - 4 of the algorithm.

	I	II	III	IV	V
A	$\boxed{0}$	2	1	2	4
B	1	3	2	$\boxed{0}$	3
C	2	3	$\boxed{0}$	1	2
D	2	$\boxed{0}$	3	1	$\cancel{0}$
E	$\cancel{0}$	4	5	4	2

Now, out of all numbers uncovered by any line, 1 is the smallest number. We add 1 to the enteries at the intersection of horizontal and vertical lines and subtract 1 from the enteries, which exist on any horizontal or vertical line.

	I	II	III	IV	V
A	0	1	0	1	3
B	2	3	2	0	3
C	3	3	0	1	2
D	3	0	3	1	0
E	0	3	4	3	1

Now, assigning zeros in rows and columns, we get

	I	II	III	IV	V
A	$\boxed{0}$	1	$\cancel{0}$	1	3
B	2	3	2	$\boxed{0}$	3
C	3	3	$\boxed{0}$	1	2
D	3	$\boxed{0}$	3	1	$\cancel{0}$
E	$\cancel{0}$	3	4	3	1

Since row 5 and column 5 have no assignments, we draw minimum number of lines by adopting step - 4 of the algorithm.

	I	II	III	IV	V	
A	$\boxed{0}$	1	$\cancel{0}$	1	3	✓
B	2	3	2	$\boxed{0}$	3	
C	3	3	$\boxed{0}$	1	2	✓
D	3	$\boxed{0}$	3	1	$\cancel{0}$	
E	$\cancel{0}$	3	4	3	1	✓
	✓		✓			

Now, out of all numbers uncovered by any line, 1 is the smallest number. We add 1 to the enteries at the intersection of horizontal and vertical lines and subtract 1 from the enteries, which exist on any horizontal or vertical line.

	I	II	III	IV	V
A	0	0	0	0	2
B	3	3	3	0	3
C	3	2	0	0	1
D	4	0	4	1	0
E	0	2	4	2	0

Now, assigning zeros in rows and columns, we get

	I	II	III	IV	V
A	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	2
B	3	3	3	$\boxed{0}$	3
C	3	2	$\boxed{0}$	$\cancel{0}$	1
D	4	$\boxed{0}$	4	1	$\cancel{0}$
E	$\cancel{0}$	2	4	2	$\boxed{0}$

Since every row and every column have one assignment. Above assignment is optimal.

Final assignment : A → I, B → IV, C → III, D → II, E → V

Toal cost = 1 + 1 + 3 + 1 + 4 = Rs. 10

We may have one more optimal solution as :

Final assignment : A → II, B → IV, C → III, D → V, E → I

Toal cost = 3 + 1 + 3 + 2 + 1 = Rs. 10

Q.3. Solve the following maximization problem of assignment :

Salesman ↓ / Territory →	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

Sol. To convert given maximization problem into a minimization problem, each element of the matrix is multiplied by -1 as shown below :

Salesman ↓ / Territory →	I	II	III	IV
A	−42	−35	−28	−21
B	−30	−25	−20	−15
C	−30	−25	−20	−15
D	−24	−20	−16	−12

Now subtracting the minimum element of each row from that row and then subtracting the minimum element of each column from that column, we get the following matrix respectively.

	I	II	III	IV
A	0	3	6	9
B	0	1	2	3
C	0	1	2	3
D	0	0	0	0

Now, assigning zeros in rows and columns and then by drawing minimum number of lines because of incomplete assignment continuously, we get the following matrices :

$$
\begin{array}{c|cccc|}
 & \text{I} & \text{II} & \text{III} & \text{IV} \\
\hline
A & \boxed{0} & 3 & 6 & 9 \\
B & \cancel{0} & 1 & 2 & 3 \\
C & \cancel{0} & 1 & 2 & 3 \\
D & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \\
\hline
\end{array}
\Rightarrow
\begin{array}{c|cccc|}
 & \text{I} & \text{II} & \text{III} & \text{IV} \\
\hline
A & \cancel{0} & 2 & 5 & 8 \\
B & \cancel{0} & \boxed{0} & 1 & 2 \\
C & \cancel{0} & \cancel{0} & 1 & 2 \\
D & 1 & \cancel{0} & \boxed{0} & \cancel{0} \\
\hline
\end{array}
$$

$$\Rightarrow \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline A & \boxed{0} & 2 & 4 & 7 \\ B & \cancel{0} & \cancel{0} & \boxed{0} & 1 \\ C & \cancel{0} & \boxed{0} & \cancel{0} & 1 \\ D & 2 & 1 & \cancel{0} & \boxed{0} \end{array}$$

Since every row and every column have one assignment. Above assignment is optimal.

Final assignment : A→I, B→III, C→II, D→IV

Total sale = 42 + 20 + 25 + 12 = 99 units

We may have one more optimal solution as :

Final assignment : A→I, B→II, C→III, D→IV

Total sale = 42 + 25 + 20 + 12 = 99 units

Exercise

1. Solve the following assignment problem:

Men → / Jobs ↓	I	II	III	IV	V
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

2. Solve the following assignment problem:

Men → / Jobs ↓	I	II	III	IV	V
A	6	10	15	15	15
B	6	10	15	15	15
C	4	6	16	16	16
D	4	6	16	16	16
E	12	5	8	8	8

3. Solve the following maximization problem of assignment:

Men → / Jobs ↓	I	II	III	IV	V
A	32	38	40	28	40
B	40	24	28	21	36
C	41	27	33	30	37
D	22	38	41	36	36
E	29	33	40	35	39

Answers

1. 9

2. 48

3. 191

THANKS