ADVANCED ENGINEERING MATHEMATICS

IT (III SEM)

Course Objectives:

CO1 To prepare many optimization tools for applying them into different research areas as per the requirement.

CO2 To prepare important strategies of linear programming for applying them into solving the problems of transportation and assignment.

CO3 To establish different theorems and properties of random variables for understanding expectation, moments, moment generating function etc.

CO4 To analyze the various discrete and continuous distributions with their appropriate applications.

INTRODUCTION & HISTORICAL DEVELOPMENT OF OPTIMIZATION TECHNIQUES (CO1)

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INTRODUCTION

Optimization is the technique of obtaining best results under given circumstances. ***The 'Optimization Techniques' also known as 'Mathematical programming techniques' are the methods which gives the best results, under the given conditions, to the given programming problems.*** The optimum seeking methods (optimization techniques) are generally studied as a part of 'Operations Research'. Operations Research is a branch of mathematics, in which we deal with the applications of scientific methods and techniques to the complicated decision making problems arising in Engineering, Science, Industry etc., for establishing the best or optimal solution.

HISTORY OF OPTIMIZATION

The main origin is during the second world war. At that time the scientist of England were asked to study the strategic and tactical problems related to air and land defence of the country. Due to the limited resources (military etc.), it was necessary to make the most utilization of them.

During world-war II, the military commands of U.S.A. and U.K. form a inter-diciplinary teams of scientists for scientific research into strategic and tactical military operations. Their mission was to formulate a plan for military commands so that they can make the best use of their scarce military resources, food and effords and also to inplement the decision effectively. These scientists were not actually engaged in fighting the war of militairy operations but their strategical initiatives of military commands and other intellectual support helped them to win the war.

After the end of the war, because of the success of the military team in war, the industrial managers were attracted towards this team of scientists. They wanted that this team of scientists should help them, to find a way to minimize the cost and maximize the profit. The first mathematical technique in this field was developed as 'Simplex Method in Linear Programming" in 1947. Since then a lot of new techniques and applications have been developed in this field.

ENGINEERING APPLICATIONS OF OPTIMIZATION

Optimization can solve almost all the problems in engineering. Some of the applications from different branches of engineering are given below :

1. In bringing out new design (more efficient) of a machine.
2. Finding the optimal trajectories of space vehicles.
3. Design of water resources system for maximum benefit.
4. Minimum weight design of structures for earth quake, wind etc.
5. Optimum designs of gears, machine tools and other mechanical equipments.
6. Optimum design of electrical equipments.
7. Electrical Networking
8. Design of Aircraft and aerospace structure for minimum weight.
9. Design of structures : frames, bridges, towers etc. for minimum cost.
10. Production planning , scheduling and controlling.
11. Travelling Salesman Problem.
12. Design of chemical processing equipments and plants.
13. Planning of maintenance and replacement of equipment to reduce operating costs.
14. Inventory Control
15. Design of control system.
16. Controlling the idle and waiting times and queueing in production lines to reduce the costs etc.

There are many more applications of optimization in engineering and also in other fields of studies.

OPTIMIZATION PROBLEM

An optimization or a mathematical programming problem can be stated as follows:

$$\left.\begin{array}{lll} \textit{Minimize} & f(X) & \\ \textit{Subject to} & g_j(X) \le O & j = 1,2,\ldots,m \\ & h_k(X) = O & k = 1,2,\ldots,p \end{array}\right\} \tag{1.1}$$

where X is an n-dimensional vector $(x_1, x_2, \ldots.. x_n)^T$, called the decision vector, $f(x)$ is the objective function and $g_j(x) \le 0$ and $h_k(x) = 0$ are the inequality and equality constraints respectively. **The problem (1.1) is a constrained optimization problem,**

Note that n, m and p are not related by any relation but they are according to the requirements of the problem.

If there are no constraints present, the optimization problem becomes

$$\left.\begin{array}{ll} \underset{X}{Minimize} & f(X) \\ where & X = (x_1,\ x_2,\ \ldots\ldots, x_n)^T \end{array}\right\}$$

This is called an unconstratined optimization problem.

.1 Decision Variables

In any engineering system we deal with a numer of parameters (quantities). Some of these parameters have preassigned values where as others are not fixed.

These parameters which are not fixed are called the **decision variables**. The decision variables are collectively represented as a decision vector $X = (x_1\ x_2 \ldots x_n\ )T$

.2 Objective Function

Every problem has a criterion (aim) to be satisfied e.g. to produce the acceptable design or to have a profit or cost should be less etc. These all criterions can be expressed with the help of the decision variables called the **objective funcion**. In general a function to be optimized is called the objective function.

In some problems, there may be more than one criterion to be satisfied, e.g.consider a salesman travelling different cities to collect money from his customers. His criterion will be to maximize the collection travelling minimum distance . An optimization problem involving multiple objective function is known as **multiobjective programming problem.**

.3 Constraints

In most of the problems, decision variables depend on certain conditions/ requirements and can not be chosen arbitrarily. The restrictions that must be satisfied by the decision variables are called the constraints.

4 Objective function Surfaces

Let the objective function is $f(X)$ where X is a vector. The locus of all points satisfying $f(X) = C$ (constant) is a hypersurface in n-dimentional space. Each value of C gives a different member of the family of surfaces. These surfaces are called objective function surfaces.

CLASSIFICATION OF AN OPTIMIZATION PROBLEM

Optimization problem can be classified in following ways :

1. Classification Based on the Existence/Non-existence of Constraints :

(*a*) **Constrained optimization problem :** If an optimization problem has constraints then it is known as constrained optimization problem. The problem

Minimize $f(X)$

Subject to $g_i(X) \leq 0 \; ; \quad i = 1, 2, \ldots\ldots\ldots, m$

$h_j(X) = 0 \; ; \quad j = 1, 2, \ldots\ldots\ldots, p$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

is a general constrained optimization problem.

(*b*) **Unconstrained optimization problem :** If an optimization problem has no constraints, then it is known as unconstrained optimization problem. The problem

Minimize $f(X)$

$$\text{where} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

is a general unconstrained optimization problem.

2. Classification Based on the Nature of the Decision Variables : The optimization problem based on this can be classified into two broad categories :

(*a*) Static optimization problem

(*b*) Dynamic optimization problem

(*a*) **Static (Parameter) Optimization Problem :** In this case, the objective is to compute a set of design parameters that make a prescribed function of these parameters minimum subject to certain constraints.

(*b*) **Dynamic (Trajectory) Optimization Problem :** Under this category, the objective is to find a set of design parameters, which are all continuous functions of some other parameter, that minimizes an objective function subject to a set of constraints.

3. Classification Based on the Nature of the Functions Involved :
Based on the nature of the function for the objective function and the constraints, optimization problems can be classified as linear, non-linear, geometric and quadratic programming problems.

(a) Linear Programming Problem : If the objective function and all the constraints are linear functions of the decision variables, then the mathematical programming problem is called a linear programming problem.

The general linear programming problem is :

$$\text{Min } f(x) = \sum_{i=1}^{n} C_i x_i$$

$$\text{subject to} \quad \sum_{i=1}^{n} a_{ij}\, x_i \le b_j \; ; \quad j = 1, 2, \ldots\ldots., m$$

$$\text{and} \quad x_i \ge 0 \; ; \quad i = 1, 2, \ldots\ldots.., n$$

where c_i, a_{ij}, b_j are constants.

(b) Non-Linear Programming Problem : If any of the functions among the objective and constraint functions is non-linear, then the problem is known as non-linear programming problem.

(c) Geometric Programming Problem : A geometric programming problem is one in which the objective function and the constraints are expressed as posynomials in X.

A function $h(X)$ is said to be a posynomial if it can be expressed as the sum of power terms each of the form $c_i x_i^{a_{i1}} x_2^{a_{i2}} \ldots\ldots x_n^{a_{in}}$ where c_i, a_{ij}'s are constants with positive c_i and x_j. A posynomial with N terms can be expressed as

$$h(X) = c_1 x_1^{a_{11}} x_2^{a_{12}} \ldots\ldots\ldots x_n^{a_{1n}} + \ldots\ldots\ldots + \ldots\ldots\ldots + C_N x_1^{a_{N1}} x_2^{a_{N2}} \ldots\ldots\ldots x_n^{a_{Nn}}$$

Thus, a general geometric programming problem can be expressible as follows :

Min $\quad f(X) = \sum_{i=1}^{N_0} c_i \left(\prod_{j=1}^{n} x_j^{p_{ij}} \right), \; c_i > 0, \; x_j > 0$

subject to $\quad g_K(X) = \sum_{i=1}^{N_k} a_{ik} \left(\prod_{j=1}^{n} x_j^{q_{ik}} \right) > 0; \; a_{ik} > 0, \; x_j > 0$

$K = 1, 2, \ldots, m$

where N_0 and N_k denote the number of posynomial terms in the objective and k^{th} constraint function respectively.

(d) Quadratic Programming Problem : A non-linear programming problem with a quadratic objective function and linear constraints is known as quadratic programming problem.

A general quadratic programming problem can be expressible as follows :

Min $\quad f(X) = c + \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} Q_{ij} \, x_i x_j$

subject to $\quad \sum_{i=1}^{n} a_{ij} x_i = b_j \,; \quad j = 1, 2, \ldots\ldots, m$

and $\quad x_i \geq 0 \,; \quad i = 1, 2, \ldots\ldots\ldots, n$

where Q_{ij}, q_i, c, a_{ij} and b_j are constants.

4. Classification based on the Permissible Values of the Decision Variables : Under this classification, optimization problems can be classified as integer and real valued programming problems; depending on the values permitted for the design variables.

(*a*) **Integer Programming Problem :** If some or all of the design variables of an optimization problem are restricted to take only discrete (or integer) values, then the problem is called an integer programming problem.

(*b*) **Real Valued Programming Problem :** If all the design variables are permitted to take values from within an allowed set; where the allowed set contains only real values, then it is called a real valued programming problem.

THANKS