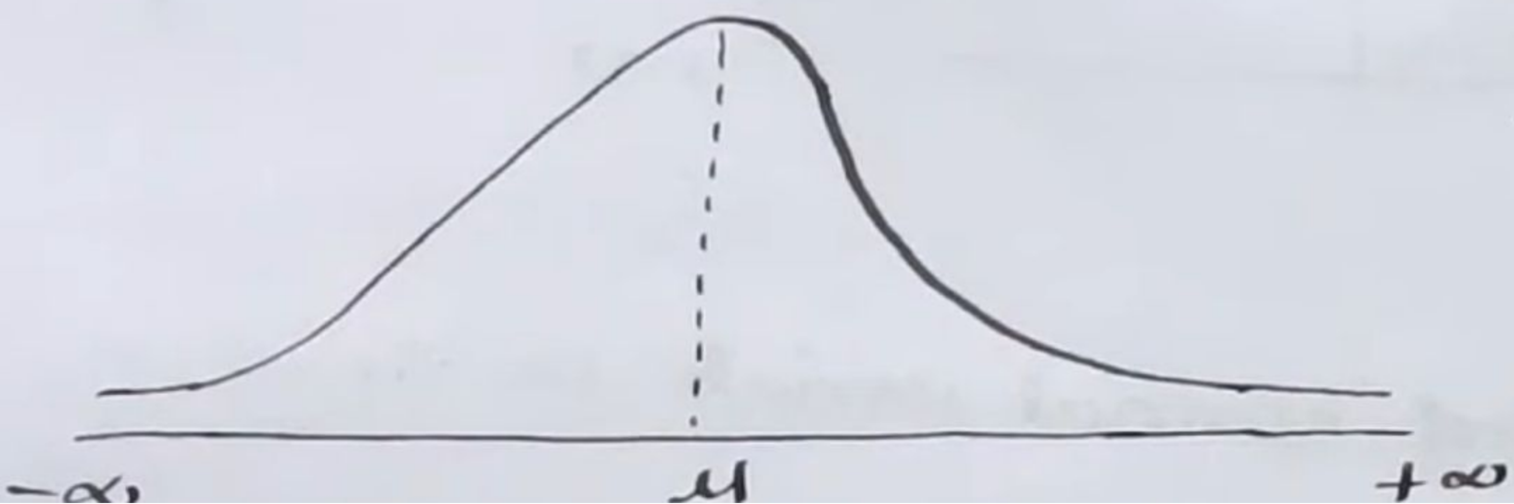
Normal Distribution

(Probability and Distributions)

Complete Concept

- A continuous random variable X is said to follow normal distribution with mean(μ) and standard deviation(σ), if its probability function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$-\infty < X < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

- The curve representing the normal distribution is called Normal Curve.
- The normal curve is "Bell-shaped" and is symmetrical about its mean.

- Probability of a Normal random variable in an Interval

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

Que① X is a normal variate with mean 30 and SD 5. Find the probability that

(i) $26 \leq X \leq 40$ (ii) $X \geq 45$ (iii) $|X - 30| > 5$

Sol.ⁿ Given: $\mu = 30$ and $\sigma = 5$

Standard Normal Variate, $Z = \dfrac{X-\mu}{\sigma} = \dfrac{X-30}{5}$

(i) $26 \leq X \leq 40$

when $X = 26$, $Z = \dfrac{26-30}{5} = -0.8$

Also, when $X = 40$, $Z = \dfrac{40-30}{5} = 2$

$\therefore P(26 \leq X \leq 40) = P(-0.8 \leq Z \leq 2)$

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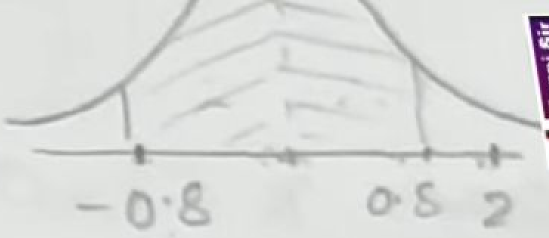
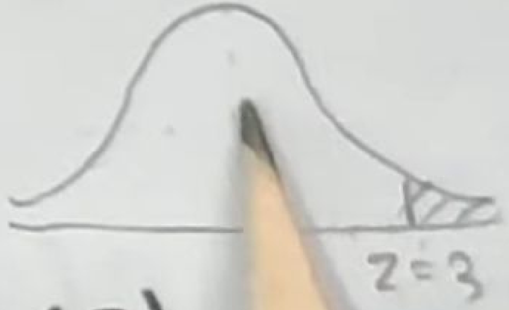


$\therefore P(26 \leq X \leq 40) = P(-0.8 \leq Z \leq 2)$

$= P(-0.8 \leq Z \leq 0) + P(0 \leq Z \leq 2) = P(0 \leq Z \leq 0.8) + P(0 \leq Z$

$= 0.2881 + 0.4772 = 0.7653$. Ans

(ii) $X \geqslant 45$



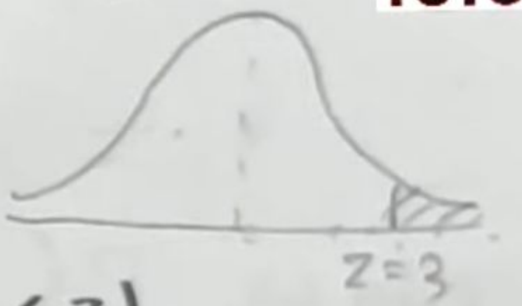
when $X = 45$, $Z = \frac{45-30}{5} = 3$

$\therefore P(X \geqslant 45) = P(Z \geqslant 3) = 0.5 - P(0 \leq Z \leq 3)$

$= 0.5 - 0.4987 = 0.0013$. Ans

$= 0\cdot2881 + 0\cdot4772 = 0\cdot7653$. Ans

(ii) $X \geq 45$



when $X = 45$, $Z = \dfrac{45-30}{5} = 3$

$\therefore P(X \geq 45) = P(Z \geq 3) = 0\cdot5 - P(0 \leq Z \leq 3)$

$= 0\cdot5 - 0\cdot4987 = 0\cdot0013$. Ans

(iii) $|X-30| \leq 5$

$\Rightarrow 25 \leq X \leq 35$

when $X = 25$, $Z = -1$

when $X = 35$, $Z = 1$

$\therefore P(25 \leq X \leq 35) = P(-1 \leq Z \leq 1) = 2\left[P(0 \leq Z \leq 1)\right]$

$= 2 \times 0\cdot3413 = 0\cdot6826$.

Now

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(iii) $|X - 30| \leq 5$

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$= 2 \times 0.3413 = 0.6826.$

Now, $P\{|X - 30| > 5\} = 1 - P\{|X - 30| \leq 5\}$

$= 1 - 0.6826$

$= 0.3174$

Que(4) Find the eq.^{n} of normal probability curve that may be fitted to the following data:

x :	0	1	2	3	4	5
f :	13	23	34	15	11	4.

Sol^n

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} \quad \text{and} \quad SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Sol^n Mean $= \frac{\sum fx}{\sum f}$ and $SD = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

x	x^2	f	fx	fx^2
0	0	13	0	0
1	1	23	23	23
2	4	34	68	136
3	9	15	45	135
4	16	11	44	176
5	25	4	20	100
		$\sum f = 100$	$\sum fx = 200$	$\sum fx^2 = 570$

5	25	4	20	100
		$\Sigma f = 100$	$\Sigma fx = 200$	$\Sigma fx^2 = 570$

$$\therefore \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{200}{100} = 2 = \mu$$

$$\text{and SD} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{570}{100} - \left(\frac{200}{100}\right)^2} = \sqrt{5.7 - 4}$$

$$= \sqrt{1.7} = 1.3 = \sigma$$

The eq.^{n} of normal curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{(1.3)\sqrt{2\pi}} e^{-\frac{(x-2)^2}{2(1.3)^2}}$$

$$y = \frac{1}{(1.3)\sqrt{2\pi}} \cdot e^{-\frac{(x-2)^2}{3.38}}$$

$\therefore$ Mean $= \dfrac{\Sigma fx}{\Sigma f} = \dfrac{200}{100} = 2 = \mu$

and SD $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \left(\dfrac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\dfrac{570}{100} - \left(\dfrac{200}{100}\right)^2} = \sqrt{5 \cdot 7 - 4}$

$= \sqrt{1 \cdot 7} = 1 \cdot 3 = \sigma$

The eq.^{n} of normal curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{(1 \cdot 3)\sqrt{2\pi}} e^{-\frac{(x-2)^2}{2(1 \cdot 3)^2}}$$

$$\boxed{y = \frac{1}{(1 \cdot 3)\sqrt{2\pi}} \cdot e^{-\frac{(x-2)^2}{3 \cdot 38}}}$$

which is the reqd. equation.