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Evolutionary Identification of Hidden Chaotic Attractors

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Abstract

In this participation we discuss the possibility of mutual fusion of evolutionary algorithms and deterministic chaos. As demonstrated in previous research papers, evolutionary algorithms are capable of chaotic system control, identification or synthesis and vice versa, chaos can be observed in the evolutionary dynamics. More exactly, in this paper there is numerically demonstrated possible solution of the question whether identification of so called basin of attraction for hidden attractor can be done by evolutionary algorithms. Hidden attractors are a special kind of attractors, that are hidden in the system structure and if ignored (undiscovered), then can cause serious damages, as already observed in the real world. The research presented here is bivalent. At first it shows, that evolutionary algorithms are able to identify presence of hidden attractors in the system, but also it can be extended to study an existence of hidden attractors in the evolutionary algorithms dynamics. All numerical simulations are demonstrated on Chua's chaotic attractor that contains an example of hidden attractor and at the end there are discussed discrete systems (synthesized by evolution) that likely exhibit hidden attractors, too.

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1. Introduction

Evolutionary algorithms (EAs) and deterministic chaos which is complex behavior produced by complex as well as simple dynamical systems, are tightly joined and create interdisciplinary fusion of two interesting areas. This paper discusses use of EAs on numerical identification of the so called hidden attractors, that are part of chaotic dynamics. To understand this very specific topic, it is important to explain a little the importance of hidden attractors for industrial technology and also for dynamics of evolutionary algorithms, as well as previous use of EAs on chaotic dynamics control, identification and synthesis. In this section we will overview consequently a) hidden attractors, b) EAs use on chaotic dynamics and c) existence of chaos inside EAs and its impact on EAs performance.

The hidden attractors (see Fig. 4) are a special set of points that reflect dynamic of observed system as reported in [1]-[14]. In general and from a computational point of view attractors can be regarded as self-excited and hidden attractors. Self-excited attractors can be localized numerically by a standard computational procedure, in which after a transient process a trajectory, starting from a point of unstable manifold in a neighborhood of an equilibrium, reaches a state of oscillation, therefore one can easily identify it. In contrast, for a hidden attractor, a basin of attraction does not intersect with any small neighborhoods of equilibria. Hidden attractor can be chaotic as well as periodic solution - e.g. the case of coexistence of the only stationary point which is stable and a stable limit cycle (like in the counterexamples to the Kalman and Aizerman conjecture) [1]-[14]. Classical attractors are self-excited, attractors can therefore be obtained and identified numerically by the standard computational procedure as for example for the Lorenz system. It can easily predict the existence of self-excited attractor, while for hidden attractor the main problem is how to predict its existence in the phase space. Thus, for localization of hidden attractors it is important to develop special procedures, since there are no similar transient processes leading to such attractors. If the hidden attractor is present in the system dynamics and if coincidently reached, then device (airplane, el. circuit, etc...) starts to show quasi-cyclic behavior, that can, based on kind of device, cause real disasters. As an example can be used Gripen jet fighter crash[1] or F-22 raptor crash landing[2] caused by computer malfunction that lead into oscillations (called also wind-up in control theory). Hidden attractors as a part of deterministic chaos, can be studied in [1]-[14] and deterministic chaos itself for example in [27], [28], [65]. The latest research papers, that discuss hidden attractor topics and which are source of very good information are [15] - [26]. In those papers there are discussed topics like controlling of hidden attractors [19] and/or its theoretical background as for example in [18], [22] or [25].

In the last 15 years it has been demonstrated that evolutionary algorithms can be used successfully in deterministic chaos system control, its identification and/or synthesis. Deterministic chaos control, see [65] (this handbook can serve as a very good reference book to that topic and related areas) and control law synthesis is another area of EAs use. The interest in the control of chaotic systems has been an active area of research during the past decade. Numerous papers focused on chaos control with EAs are published frequently. As an example we can mention paper [35] where the basic ideas about chaos control, or about CML systems control, that is more complex and usually expect some preliminary information to derive control law (for classical controllers) were published. One of the first and important initial studies, of EAs for control by means of EAs (including CML systems control) use was reported in [36], [38] and [37], where the control law was based on the Pyragas method: Extended delay feedback control - ETDAS [39]. Those papers focused on the tuning of several parameters inside the control technique for a chaotic system. Some research in this field has been recently done using the evolutionary algorithms for optimization of local control of chaos based on a Lyapunov approach [40], [41]. But the approach by EA described in this paper is very different from

[1] https://www.youtube.com/watch?v=jP-QMmzGL5I
[2] https://www.youtube.com/watch?v=faB5bIdksi8

the classical one due to EA principles. EA searched for optimal setting of adjustable parameters of arbitrary control method to reach desired state or behavior of chaotic system. Another applications of unconventional control of chaotic systems by EA are described in [42] and [44, 43, 45]. EA synthesis of control law for discrete chaotic system is discussed there. Compared to that, published paper [44, 43, 45] shows possibility how to generate the whole control law (not only to optimize several parameters) in order to stabilize a chaotic system. The synthesis of control is inspired by the Pyragass delayed feedback control technique [46, 47]. Unlike the original OGY control method [35], it can be simply considered as a targeting and stabilizing algorithm together in one package. Another big advantage of the Pyragas method is the amount of accessible control parameters. Methods used in generating new chaotic systems from physical systems or from "manipulations" (e.g., control and parameter estimation) [48, 49] are based on classical mathematical analysis. Along with these classical methods, EAs are also applicable on chaos system synthesis, as reported in [50], [51]. Both papers introduce the chaos synthesis by means of novel EA method. This method is similar to genetic programming or/and grammatical evolution. As results in [50], [51] show that such approach is able to synthesize new and "simple" chaotic systems based on some elements contained in a pre-chosen existing chaotic system and a properly defined cost function. The research in [51] consists of 11 case studies: the aforementioned three EAs in 11 versions. For all the algorithms, 100 simulations of chaos synthesis were repeated and then averaged to guarantee the reliability and robustness of the proposed method. The most significant results are carefully selected, visualized and reported in [51].

Also few research papers have demonstrated that chaos can also be observed in evolutionary dynamics [29], [51], or used instead of pseudorandom number generators like the logistic map like Persohn [52] who is not the only one who used the logistic map. Another paper [53] discusses use of logistic map like chaos-based true random number generator embedded in reconfigurable switched-capacitor hardware. Xing in [54] proposed an algorithm of generating pseudorandom number generator and combined the couple map lattice [65] and chaotic iteration. Authors also tested this algorithm in NIST 800-22 statistical test suits and it was used in image encryption. In [55] authors investigate interesting properties of chaotic systems in order to design a random bit generator (called CCCBG) in which two chaotic systems are cross-coupled with each other. For evaluation of the bit streams generated by the CCCBG, the four basic tests are performed: monobit test, serial test, auto-correlation, Poker test. Also the most stringent tests of randomness: the NIST suite tests have been used. Several studies have already dealt with the possibilities of integration of chaotic systems into the PSO algorithm and the performance of such algorithms, see [56], [57]. Papers [58]-[60] extends the previous experiments of [57] and investigates the impact of using different chaotic maps on the behavior of PSO algorithm especially in terms of convergence speed and premature convergence risk. Three different chaotic systems (maps) are used and their impact is compared in this study. The aim is to find a link between specific chaotic system and specific behavior of the PSO algorithm.

Above mentioned informations and references are only a fraction of existing research papers that discuss mutual fusion of EAs and chaos dynamics. In this paper there is discussed possibility on EA identification of hidden attractor existence, because, as written in many papers and books (e.g. [65], [27]), basin of attraction (i.e set of ”start” points that lead system dynamics into hidden attractor) is very hardly identifiable. This is also discussed in papers [5] and [7] that present identification of basin of attraction for hidden attractor (so in fact the hidden attractor existence) by means of classical numerical algorithms. Identification by EAs is based on suitable cost function definition that expresses quality of system state trajectory and that can be visualized as a surface. The global extreme on surface then represents optimal solution (or set of start/initial points ”leading” trajectories to the hidden attractor). As an example of complexity of such surfaces can be used Fig. 1, see also [51]. From the geometry of such surface is clear, that EAs are the most promising tools to solve such kind of problems due to their capability to avoid local extremes.

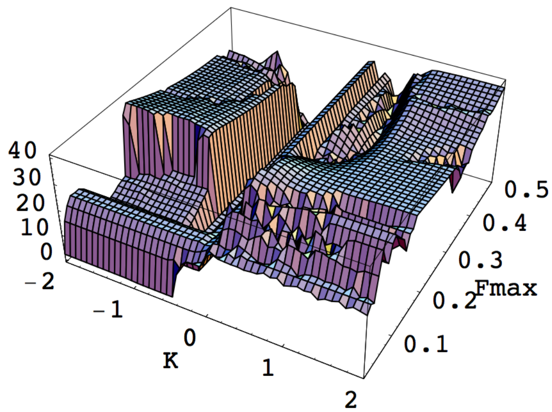

Figure 1: A typical example of cost function surface from experiments based on EAs and chaos synchronization in [51]. The surface typically exhibits lot of local extremes and is in general very complex.

2. Experiment Design

2.1. Used Algorithm and its Setting

All experiments reported here were done with selected evolutionary algorithms in repeated numerical simulations. The used algorithms were differential evolution (DERand1Bin) and [63], SOMA (AllToOne) [62]. Another algorithms like, genetic algorithms GA [61], simulated annealing (SA) [34], [33], PSO [32] or ES [30], [31] are matter of extension of this study with more systems that contain hidden attractors in our future research. Setting of the algorithm is in Table 1 - 2.

Both algorithms (i.e. SOMA + DE) have been applied 100 times in order to find the optimum of both case studies. The primary aim of this comparative study **is not to show** which algorithm is better or worst, but to show whether evolutionary algorithms are applicable on identification of basin of attraction related to hidden attractor. Comparing to the another case studies reported in the previous papers, population size is in this application set to quite low number (10) based on our experiences with EAs and chaotic systems.

Less/more straightforward dependance on population size is clearly visible in different research papers.

Differential Evolution [63] is a population-based optimization method that works on real-number coded individuals. For each individual $\vec{x}_{i,G}$ in the current generation G, differential evolution (DE) generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a randomly selected third individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as a perturbated vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, then $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$; otherwise $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$. Differential Evolution is robust, fast, and effective with a global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy, epistatic and time-dependent objective functions. Pseudocode for DE, especially for DERand1Bin, is

$$
\begin{array}{l}
\text{1.Input}: D, G_{\max}, NP \geq 4, F \in (0, 1+), CR \in [0,1], \text{ and initial bounds}: \vec{x}^{(lo)}, \vec{x}^{(hi)}. \\
\text{2.Initialize}: \left\{ \begin{array}{l} \forall i \leq NP \wedge \forall j \leq D : x_{i,j,G=0} = x_j^{(lo)} + rand_j\,[0,1] \bullet \left(x_j^{(hi)} - x_j^{(lo)} \right) \\ i = \{1,2,...,NP\}, j = \{1,2,...,D\}, G = 0, rand_j[0,1] \in [0,1] \end{array} \right. \\
\left\{ \begin{array}{l}
\text{3.While } G < G_{\max} \\
\forall i \leq NP \left\{ \begin{array}{l}
\text{4.Mutate and recombine}: \\
\text{4.1} r_1, r_2, r_3 \in \{1,2,....,NP\}, \text{randomly selected, except}: r_1 \neq r_2 \neq r_3 \neq i \\
\text{4.2} j_{rand} \in \{1,2,...,D\}, \text{randomly selected once each } i \\
\text{4.3} \forall j \leq D, u_{j,i,G+1} = \left\{ \begin{array}{l} x_{j,r_3,G} + F \cdot (x_{j,r_1,G} - x_{j,r_2,G}) \\ \text{if}(rand_j[0,1] < CR \vee j = j_{rand}) \\ x_{j,i,G} \text{otherwise} \end{array} \right. \\
\text{5.Select} \\
\vec{x}_{i,G+1} = \left\{ \begin{array}{l} \vec{u}_{i,G+1} \text{ if } f(\vec{u}_{i,G+1}) \leq f(\vec{x}_{i,G}) \\ \vec{x}_{i,G} \text{ otherwise} \end{array} \right.
\end{array} \right. \\
G = G + 1
\end{array} \right.
\end{array}
\tag{1}
$$

SOMA is a stochastic optimization algorithm that is modeled based on the social behaviour of competitive-cooperating individuals [62]. It was chosen because it has been proved that this algorithm has the ability to converge towards the global optimum [62]. SOMA works on a population of candidate solutions in loops, called migration loops. The population is initialized by uniform random distribution over the search space at the beginning of the search. In each loop, the population is evaluated and the solution with the best cost value becomes the Leader. Apart from the Leader, in one migration loop, all individuals will traverse the searched space in the direction of the leader. It ensures diversity amongst all the individuals and it also provides a means to restore lost information in a population. Mutation is different in SOMA as compared with other EAs. SOMA uses a parameter called PRT to achieve perturbations. The PRT vector defines the final movement of an active individual in the search space. The randomly generated binary perturbation vector controls the allowed dimensions for an individual. If an element of the perturbation vector is set to zero, then the individual is not allowed to change its position in the corresponding dimension. An individual will travel over a certain distance (called the $PathLength$) towards the leader in finite steps of the defined length. If the $PathLength$ is chosen to be greater than one, then the individual will overshoot the Leader. This path is perturbed randomly. Pseudocode for SOMA is

Table 1: DE setting.

Parameter	Value
NP	10
F	0.9
CR	0.3
Generations	500
Individual Length	1

$$\begin{array}{l}
\text{Input}: N, Migrations, PopSize \geq 2, PRT \in [0,1], Step \in (0,1], \text{MinDiv} \in (0,1], \\
\text{PathLength} \in (0,5], Specimen \text{ with uper and lower bound } x_j^{(hi)}, x_j^{(lo)} \\
\text{Inicialization}: \left\{ \begin{array}{l} \forall i \leq PopSize \wedge \forall j \leq N : x_{i,j,Migrations=0} = x_j^{(lo)} + rand_j\,[0,1] \bullet \left(x_j^{(hi)} - x_j^{(lo)}\right) \\ i = \{1,2,...,Migrations\}, j = \{1,2,...,N\}, M\text{igrations} = 0, rand_j[0,1] \in [0,1] \end{array} \right. \\
\left\{ \begin{array}{l} \text{While } Migrations < Migrations_{\max} \\
\forall i \leq PopSize \left\{ \begin{array}{l} \text{While} t \leq PathLength \\ if\ rnd_j < PRT\ pak\ PRTVector_j = 1\ \ else\ 0\ , \quad j = 1, \ldots, N \\ x_{i,j}^{ML+1} = x_{i,j,start}^{ML} + (x_{L,j}^{ML} - x_{i,j,start}^{ML})\, t\, PRTVector_j \\ f\left(x_{i,j}^{ML+1}\right) = \text{ if } f\left(x_{i,j}^{ML}\right) \leq f\left(x_{i,j,start}^{ML}\right) \text{ else } f\left(x_{i,j,start}^{ML}\right) \\ t = t + Step \end{array} \right. \\
Migrations = Migrations + 1 \end{array} \right.
\end{array} \tag{2}$$

All experiments were done in Mathematica 10, on MacBook Pro, 2.8 GHz Intel Core 2 Duo.

2.2. Used Hidden Attractor

Typical hidden attractor, used in this paper come from electronics and has been reported in [1]-[13] or in special issue [14] and another research papers[3], [15] - [26]. It is Chua's attractor that can be observed for example in the electronic

[3]http://www.math.spbu.ru/user/nk/

Table 2: SOMA setting.

Parameter	Value
PathLength	3
Step	.11
PRT	1
PopSize	10
Migrations	10
MinDiv	-0.1
Individual Length	1

circuit of Chua's oscillator. Electronic circuits are among the most popular systems used to demonstrate deterministic chaos. Their popularity stems from the fact that electronic circuits are easy to set up and provide fast response to control inputs and settings. Typical representatives of electronic circuits with deterministic chaos is Chua's oscillator, whose scheme, hardware design and behaviour are shown in Figs. 2-4. The core of Chua's circuit is a nonlinear resistor, sometimes called Chua's diode [68].

In Fig. 4 Chua's attractor visualized by the program Mathematica together with his hidden sttractor. Chua's circuit can be described mathematically by Eq. (3), which can be used to simulate the behavior of the circuit:

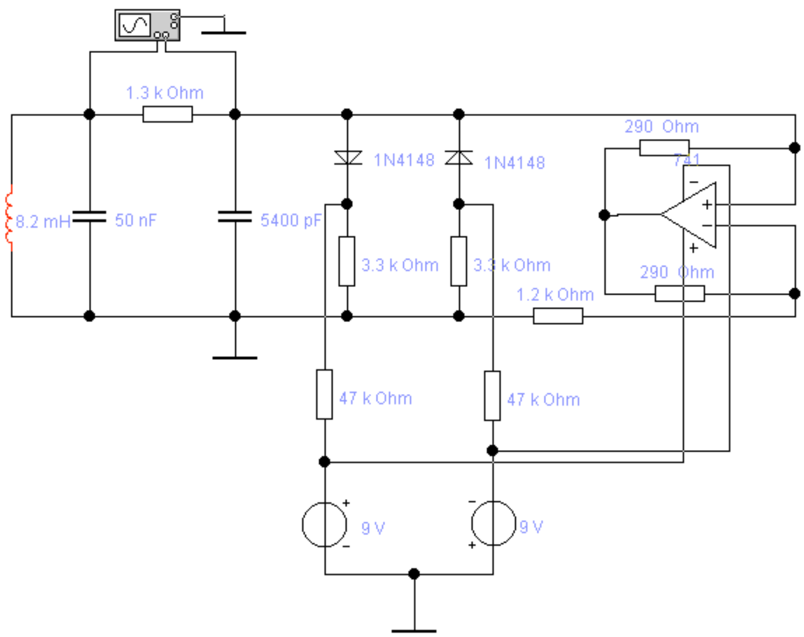

Figure 2: Scheme of the Chua's circuit ...

Chua$'$s system

$$x'(t) = \alpha(y(t) - x(t) - m_1 x(t) - 0.5(m_0 - m_1)sat(x))$$

$$y'(t) = x(t) - y(t) + z(t)$$

$$z'(t) = -\beta y(t) - \gamma z(t)$$

with saturation function

$$sat(x) = |x(t) + 1| - |x(t) - 1| \tag{3}$$

initial conditions

$$x(0) \in [-10, 10], y(0) \in [-10, 10], z(0) \in [-15, 15]$$

and parameter setting

$$\alpha = 8.4562, \beta = 12.0732, \gamma = 0.0052$$

$$m_0 = -0.1768, m_1 = -1.1468$$

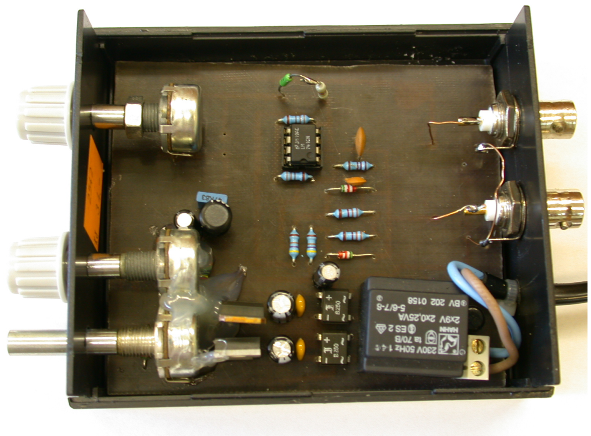

Figure 3: ... and hardware design of Chua's circuit.

If suitable initial conditions are set as described in (3), a chaotic attractor can be found in the system (Fig. 4). In our experiments Chua's and hidden attractor have been set according to [1] and [14]. It is reported in Eq. (3).

2.3. Cost Function and its Visualization

The most important part of the experiments was the cost function definition. The cost function was based on Chua's circuit with setting for its hidden attractor. For experiments reported here few simplifications have been done, because in fact, in Chua's system there are 8 adjustable parameters: 3 for initial conditions ($x(0)$, $y(0)$ and $z(0)$) and 5 for parameter settings (m_0, m_1, α, β and γ). So in total it is possible to search in $8th$ dimensional space (compare complexity of simpler chaotic case in Fig. 1). For simplicity we fixed 5 parameters (so that hidden attractor would exist) and only 3 initial conditions were under investigation by EAs in order to find points belonging to the domain of its attraction. The cost function was set up as in Eq. (4). System (3) of three differential equations has been calculated for each evaluation of Eq. (4) in time $t \in [0, 200]$ and trajectories data from the last 50s has been used (to avoid possible initial trajectory transition with bigger amplitudes, as can be observed in Fig. 10-11). The parameter Diam(eter) has been set empirically to be $\in [3, 11]$

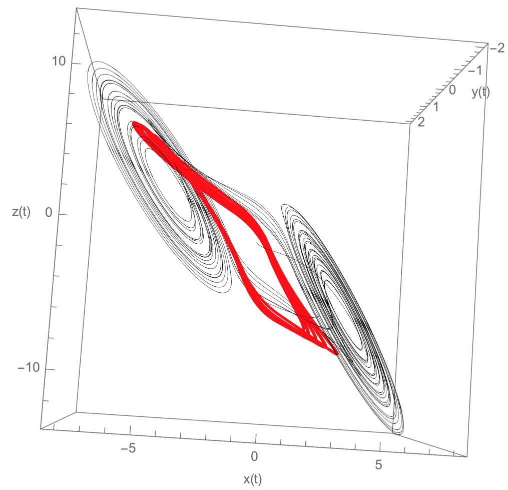

Figure 4: Chua's attractor with hidden one. Hidden attractor is red "circle" set of points in bigger Chua's attractors. Both attractors has been obtained for different parameter set.

and guarantee that if trajectory fulfill Eq. (4), then trajectory is trapped in hidden attractor. Otherwise return value of Eq. (4) was usually extremely big as 10^8, 10^{153} etc. due to the trajectory escaping to the infinity. Thus, individuals (i.e. coordinates of the start trajectory) with CF = 0 represented solutions.

$$\begin{aligned} &CF_{temp} = Max(\sum_{t=150s}^{200} \sqrt{x(t)^2 + y(t)^2 + z(t)^2}) \\ &IF\ \mathrm{CF_{temp}} \in \mathrm{Diam\ then\ CF} = 0 \mathrm{\ else\ CF} = \mathrm{CF_{temp}} \\ &Diam \in [3, 11] \end{aligned} \tag{4}$$

Dynamics of the Eq. (3) according to Eq. (4), can be visualized in few different ways and as an example, few typical visualizations are depicted here. Because there were 3 variables (i.e. cost function surface is in 4 dimensional space), can be cost function surface visualized so that one variable is set to constant value and remaining two are changed in allowed interval. Typical examples of figures are then Fig. 5 - 7. In Fig. 5 is visualized Eq. (4) for parameters in $x, y \in [-10, 10]$. In Fig. 5 the black area represent the basin of attraction, i.e. set of initial points whose trajectory lead to the standard Chua's attractor (see bigger attractor in Fig. 4). On the contrary, when parameters (m_0, m_1, α, β and γ) are set for hidden attractor existence, then basin of attraction is very small, as is shown in Fig. 6, or even complex as Fig. 7 suggest. Both figures shows, how basin of attraction change its shape and structure, when for example $z(0)$ is changed (in fact they are xy slice of the attraction domain for a fixed $z(0)$.). It is also important to mention that graphics visualization of basins of attraction is only an approximation of the real one and depend on setting of graphical resolution, that is of course limited on PCs.Confront with Fig. 7, where high resolution has to be set in numerical computation in order to made those few points visible.

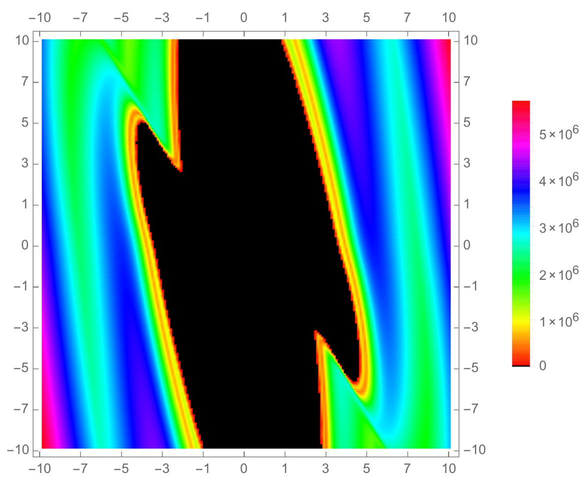

Figure 5: Basin of attraction (black area) for Chua's standard (i.e. non-hidden) attractor. Black points are initial conditions - trajectories starting from black area ends in standard Chua attractor.

It is logical expect, that structure of the complete basin of attraction based on Eq. (4), will be more complex and changed as all 8 parameters of the Eq. (3) will be changed. In this paper, as in initial study in this matter, we are concerned on EAs identification of initial conditions that lead to hidden attractor, when parameters $x(0)$ and $y(0)$ searched in interval [-10, 10], for $z(0) = 8.7739$ and -13.4705, i.e. individual has "only" dimension 2.

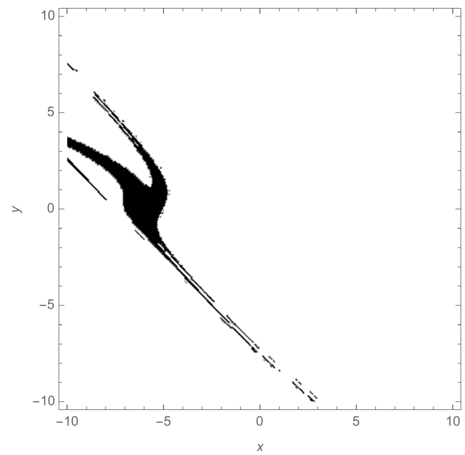

Figure 6: Basin of attraction (black area) for Chua's hidden attractor with specific initial conditions in 2D ($z(0) = 8.7739$).

3. Results

Results of EAs identification of basin of attraction come from experiments with SOMA and DE. Both algorithms were repeated 100 times and for each individual and its cost function evaluation was calculated by Eq. (3). All visualization has been done by means of Mathematica 10. In both algorithms and all experiments were successfully located initial points that belong to the basin of attraction. As histogram in Fig. 8 shows then for algorithm SOMA (for example) has basin of attraction been located after a few migrations. The same can be stated for DE.

To do simulations more trickier, z(0) has been set to initial value -13.4705 and basin of attraction looks as is approximated at Fig. 7. Again 100 simulations for SOMA and DE has been done and results are captured in histogram in Fig. 9. Despite fact that basin of attraction was sparse, both EAs again successfully located points belonging to basin of attraction.

Another interesting result was that both EAs, during its evolution, has identified not only one hidden attractor, as reported in [1], but also stable limit cycle in it, as is depicted in Fig. 10 and Fig. 11. In both figures it is visible that

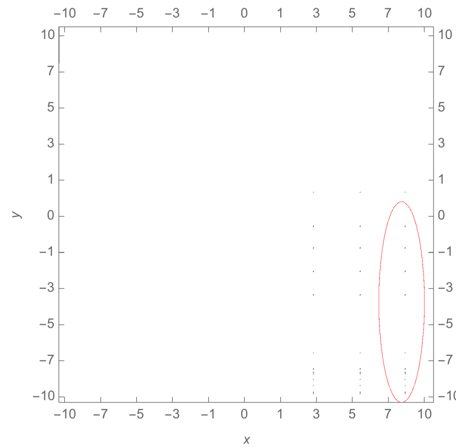

Figure 7: Basin of attraction (black dots) for Chua's hidden attractor with specific initial conditions in 2D ($z(0) = -13.4705$). Black dots are scattered through space of possible solutions (see red oval capturing a few of them).

despite fact that both kind of trajectories are very close at the start, they ends up in different behavior as hidden attractor or limit cycle is. This just confirm that basin of attraction shall be really complex and tiny.

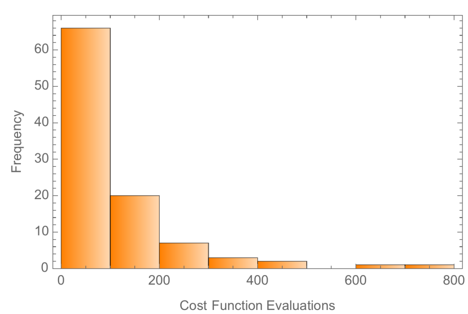

Figure 8: Histogram of SOMA algorithms exhibiting No. of cost function evaluations needed to find initial points belonging to hidden attractor, see Fig. 6.

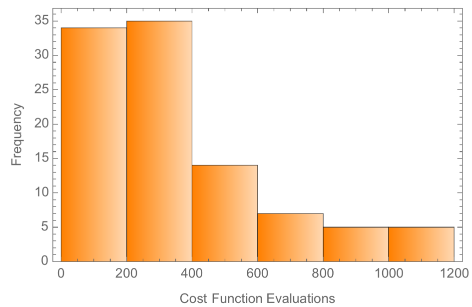

Figure 9: Histogram of SOMA algorithms exhibiting No. of cost function evaluations needed to find initial points belonging to hidden attractor, see Fig. 7.

4. Conclusion

The main motivation of the research on existence of hidden attractors is not only its theoretical importance, but also its impact on technological and industrial devices, as mentioned above in the Introduction. The main question was whether EAs can be used on evolutionary identification of basin of attraction that belongs to hidden attractor. Successful identification then can help avoid critical moments and device malfunctions. For the numerical identifications there are already published representative papers like [5] and [7], based on classical (i.e. non-evolutionary) algorithms. On the other side, EAs are well known for their very good performance (including on black-box systems) and ability to avoid local extremes as on Fig. 1, shall be very promising candidates to solve such task also. Based on results obtained from our initial experiments it can be stated that EAs are applicable on such problems and their performance is very good. Experiments here were simplified so that for identification of the basin of attraction were 5 of 8 possible parameters set to known values and EAs estimated remaining 3 (the initial conditions). That seems to be low dimensional number, however as [51] discuss and Fig. 1 shows, even low dimensional problems can be very complex when chaos is involved in it.

It has to be noted that the next step, expanding experiments here, will be focused on

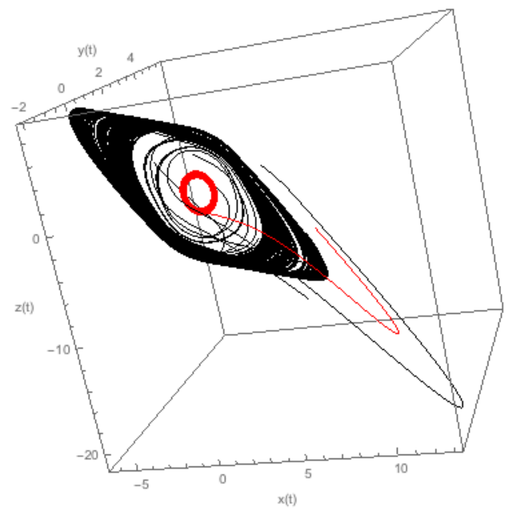

Figure 10: Hidden attractor (black) and limit cycle (red).

1. All 8 parameters of Chua's system, Eq. (3), estimation: i.e. initial as well as structural parameters will be estimated.
2. More nonlinear dynamical systems will be investigated for the hidden attractor identification by means of EAs. Continuous as well as discrete.
3. More EAs will be used with 1. and 2.

Concerning to item 2., it has to be noted that there is big class of discrete systems that definitely shall contain hidden attractor. The behavior of such systems is demonstrated (a few of many) in Fig. 12-16, for more see [51]. Candidates for hidden attractors can be for example very small regions as demonstrated in red circle in Fig. 12 that exhibit chaos for very small range of control parameter $A \in [0.32\text{-}0.35]$, compare with logistic equation with $A \in [1\text{-}4]$, the same can be observed in Fig. 13-16. The synthesis as well as identification of continuous systems has been discussed in [69] for example. The discrete systems, that are more easily to simulate are thus interesting candidates for hidden attractor identification and search.

Many chaos based discrete systems are derived from natural as well as social systems and thus are related to real world. On the other side, another set of

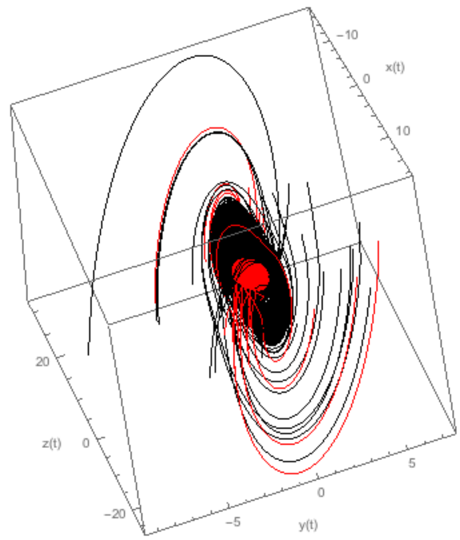

Figure 11: Hidden attractor (black) and limit cycle (red) - more detailed view.

artificially made discrete chaotic systems can be generated (synthesized) for technical / technological purposes as demonstrated in [51]. In this paper there have been used evolutionary algorithms in order to synthesize discrete chaotic systems with interesting properties. This suggests that hidden attractors can not only be observed in existing systems, but also artificially synthesized on demand, if necessary. Thus, evolution can be used in two complementary ways: identification (outlined here and in the [5] and [7] for example) and design (see an initial study in [51].). However, it deserve more deeper research that is out of scope of this paper.

Another, unanswered question coming from this research (and mutual intersections discussed in [51]) is, whether one can observe hidden attractors also in evolutionary algorithms dynamics and what impact it has on EAs performance. The existence of chaos inside EAs has been "proven" and numerically demonstrated in [29] and thus possible existence of hidden attractors is theoretically almost sure. This question can be expanded also for control and EAs dynamics, as discussed in [66], [67] and [65], that allows in principle to analyze and control

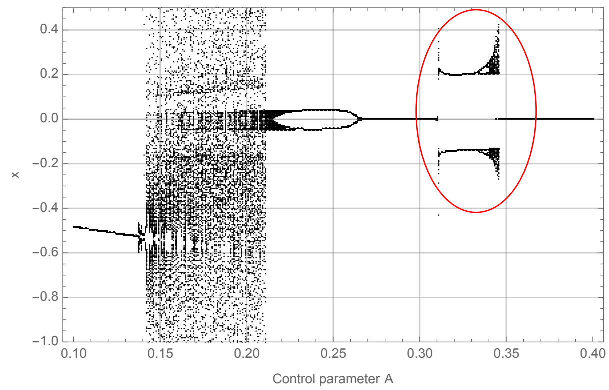

Figure 12: Bifurcation diagram of discrete chaotic system [51] that exhibit possible candidate of hidden attractor.

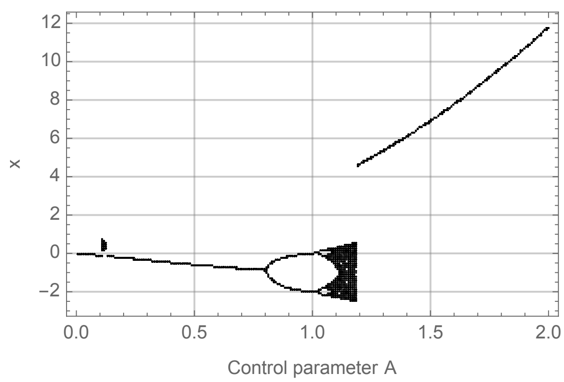

Figure 13: Another bifurcation diagram of discrete chaotic system [51] exhibiting small structure in $A \in [0.1, 0.13]$.

EAs dynamics.

Acknowledgment

The following grants are acknowledged for the financial support provided for this research: Grant Agency of the Czech Republic - GACR P103/15/06700S.

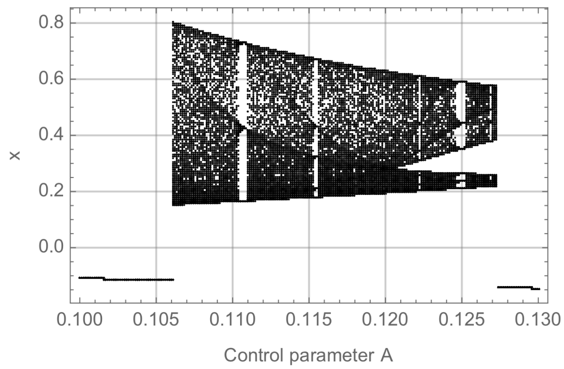

Figure 14: Zoom of the Fig. 13.

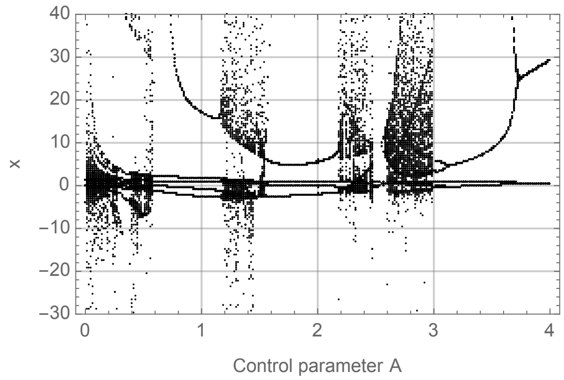

Figure 15: Bifurcation diagram of discrete chaotic system [51] that exhibit complex set of (hidden?) attractors.

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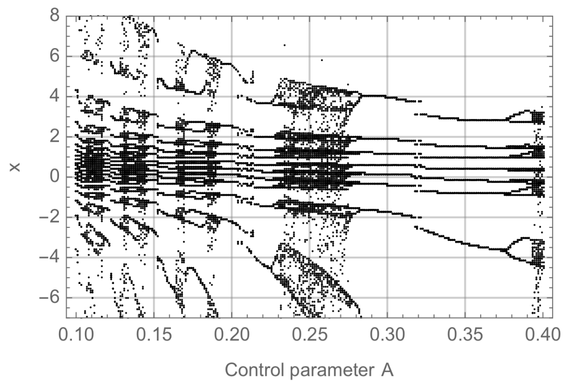

Figure 16: Zoom of the Fig. 15, again a complex set of bifurcation diagrams is visible in it.

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***Response to Reviewers**

Dear Reviewers,

many thanks for your constructive reviews. I did my best and here are my responses.

General: I followed all your comments and also whole text was checked for the English grammar by professional corrector and also all references were checked and reformatted, when needed.

Review er #1: The author of the submission is w ell know n expert in the field of EA, thus his opinion on EA usage f or the study of hidden Chua attractors is very interesting. Hidden Chua attractors are an actual topic now - they w ere discussed during two recent plenary lectures by L. Chua and R.Chen at IFAC Chaos 2015.

Minor comments:
I have never w ork with Mathematica but I w onder if there exists a corresponding file exchange service (like http://se.mathw orks.com/matlabcentral/ for Matlab) w here all the code used in the paper can be presented

In the case of Mathematica I think not. There is Wolfram Demonstration project where can be uploaded files, but as ready to use school examples. On the other side, at the moment, we are now converting all into C language and on parallel platform for faster and more massive calculation, so after then, this code can be released for purposes, if needed. Mathematica code was mostly for prototyping…

I have minor remarks about grow ing number of papers devoted to hidden Chua attractors and hidden attractors:

1) I may suggest to mention The European Physical Journal Special Topics: Multistability: Uncovering Hidden Attractors, 2015 http://link.springer.com/journal/11734/topicalCollection/AC_9fcd6cfa51cbf272d9e9981e7d184d22/page/1

2) I may suggest to consider the follow ing recent papers on hidden Chua attractors
* Chen, M. and Yu, J. and Bao, B.-C., Finding hidden attractors in improved memristor-based Chua's circuit, Electronics Letters, 51(6), 2015, 462-464
* Bocheng Bao, Fengw ei Hu, Mo Chen, Quan Xu, Yajuan Yu, Self-Excited and Hidden Attractors Found simultaneously in A Modified Chua's Circuit, Int. J. Bifurcation Chaos 25, 1550075 (2015) [10 pages] DOI: 10.1142/S0218127415500753
* M. Chen, M. Li, Q. Yu, B. Bao, Q. Xu, J. Wang, Dynamics of self-excited attractors and hidden attractors in generalized memristor-based Chua's circuit, Nonlinear Dyn, 2015 DOI 10.1007/s11071-015-1983-7
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* N.V. Kuznetsov, G.A. Leonov, Hidden attractors in dynamical systems: systems with no equilibria, multistability and coexisting attractors, IFAC Proceedings Volumes (IFAC-PapersOnline), 19, 2014, pp. 5445-5454 (doi: 10.3182/20140824-6-ZA-1003.02501)

Many thanks, it was added into references and referred in the text where appropriate.

Reviewer #2: The paper is well written. The EA application for the search of hidden attractor turns out to be effective and deserves to be published. The topic is within the journal scope. I suggest to accept the paper. Please take into account in the final version the following remarks:
Remarks
1. Page 3 "it is necessary it is important"- I suggest to choose one

Corrected…

2. Text in Fig. 2 is not readable

Well, figure, when bigger, it is readable, it depend on Journal setting of its size. In worst case I case redraw it or we can reject it because such schemata can be easily found anywhere in Internet.

3. Please check the references style: e.g." (2008), p. 911 942" "2012, 238-245." "70: pp 1589-1592"

I did my best to correct it… Hope that rest help me typesetting process (if accepted)

Reviewer #3: Dear Editor,

The following is my referee's report for "Evolutionary Identification of Hidden Chaotic Attractors" by Ivan Zelinka

The topic is interesting: Hidden attractor is a new topic in chaos researches which has attracted lots of interest in recent years.
The paper is well written and the results are solid.
The artwork is not proper: I liked to see better color figures with higher quality.
The English needs improvement: Although I am not expert, it seems to me that the English can be improved.
There are some references about hidden attractors that could eventually be cited.

Corrected… Thanks

References

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Added, commented and referred.…

Reviewer #4: The manuscript is devoted to a very interesting subject of the evolutionary identification of hidden chaotic attractors. The title of the paper is explicit and very attractive, while the abstract is clear and to the point, stressing both the specific application and the generic aspects of the work.

Also, the Introduction clearly state the application area but it should be more brief.

Forgive me please I am afraid that it will be more brief, then it will not be so informative. Hidden att. Are quite fresh topic, so that is why…

Furthermore, the authors give the real evidence of the practical industrial benefits of the presented methodologies.

The paper has a lot of grammatical and syntactical errors.

Corrected…

A more detailed description of the used algorithms is needed.

For these reasons there is for more detailed reading references to books where are explained in all details… I added paragraphs about its brief description. Is that acceptable?

Also, the paper has a mistake in Chua's circuit description. The system equations does not describe the Chua circuit but the circuit of Chua's oscillator which has a resistor to the inductor branch.

Corrected, thanks…

Finally, in Conclusion Section a more detailed description of discrete systems that contain hidden attractors should be done.

I added a few info but this topic is open research and I am afraid to talk about fact that are not proven yet. I can only suggest ideas, presumptions and “week” evidences at the moment to inspire and modify other readers…

So, this paper represents an interesting piece of work and is acceptable for publication after a major revision.

Many thanks to all of you for time and work on my paper. Hope that you will be satisfied with its actual state.

Ivan Zelinka