

Objects

Sets

Subsets

Multisets

Binary strings

Paths

Maps

$I_n \rightarrow I_n$

- words 1223

permutations

- cycles

Distributions / arrangements of

objects into boxes

labelled unlabelled

$x \mid x \mid x \mid \dots$

- $x_1 + x_2 + \dots + x_m = n, x_i \geq 0$

of solutions

- $\dots \mid \dots \mid \dots \mid \dots, x_i \geq 0$

Compositions

Set partitions $S(n, k)$ / Bell #s

- surjections

Cyclic decomposition $= S(n, n)$

Partitions

Graphs

- Trees/bipartite/planar

- rooted binary trees.

Algebraic objects

$\mathbb{Z}_n / 2p / 2n^*$ mult. group

Formal power series / polynomials

permutation groups

cyclic group $\cong \mathbb{Z}_p$

Roots of unity

Algorithms

Euclidean's sieve

GCD algorithm

Chinese Remainder thm

Taylor expansions

Hall's thm / alt paths algo.

Theorems

Div algorithm.

\mathbb{Z}_p is a field iff

p is prime.

Fundamental theorem of arithmetic

Base n thm

Chinese Remainder thm

$\frac{1}{1-x} = 1 + x + x^2 + \dots$
 $\sum k, \sum k^2, \sum k^3, \dots, \sum k^n$

Tree / Bipartite thms

$\sqrt{2}$ is irr

Primes are infinite.

Tree/bipartite graphs thms

$K_{3,3}, K_5$ are not planar.

OVERVIEW DISCRETE MATH.

Techniques /

Problem solving ideas.

list small cases

OEIS

Counting in 2 different ways

Induction

WOP

Counting principles

- $|A \cup B| = |A| + |B| - |A \cap B|$

- PIE

- Equivalence relations

Generating functions $\left\{ \begin{array}{l} \text{OSF} \\ \text{ESP} \end{array} \right.$

Recurrence relations

- 2 km

- thm with 2 variables

- const coeff

GF \leftrightarrow pice (partition identities)

O calculating

Estimating sums / identity

Log derivatives

GF \rightarrow rational function \rightarrow partial fractions \rightarrow geometric series

Interchanging sums - Wilf's Stanley method.

Classics

Wilson's theorem

Fermat's little thm

- $a^{p-1} \equiv 1 \pmod{p}$

- $a^{p(n)} \equiv 1 \pmod{p}$

$\phi(a, n) = 1$

Newton's Binomial

$(1+x)^a = \sum_{k=0}^{\infty} \binom{a}{k} x^k$ for $|x| < 1$ nPB

Polya's change prob

Euler's odd = distinct

- other partition classes

q-binomial thm

Legendre thm: $\#H / \#G$

- $o(a) \mid \#G$

$La^k = 1$ (smallest k)

Königsberg Bridge Problem

Euler's $V - E + F = 2$

- Only 5 platonic solids

Perfect Matching theorem

Hall's / König - min/max thms

Catalan Counting

Cayley: n^{n-2} formula

Stirling's formula

Binet's formula (Fibonacci)

Binomial theorem

Derangement Problem

Chu-Vandermonde Convolution

P and not $P \Rightarrow$ any Q

Russell's paradox

Cantor's diagonal argument.