

Objects

- Sets
- Subsets
- Multisets
- Binary strings
- Palindromes
- maps
- $I^n \rightarrow I^m$
- words
- permutations
- cycles
- Distributions / arrangements
- Objects into boxes
- labelled
- unlabelled
- admissible
- yes/no
- $x_1 + x_2 + \dots + x_m = n, x_i \geq 0$
- # of solutions
- ... | ... | ... $x_i > 0$
- Compositions
- Set partitions
- injections
- Cyclic decompositon
- $\approx S(n, m)$
- Partitions
- Graphs
- Trees / bipartite / planar
- rooted binary trees.
- Algebraic objects
- field
- $\mathbb{Z}_p / \langle p \rangle / \mathbb{Z}_p^\times \in \text{mult. group}$
- Fundamental series / polynomials
- Permutation groups
- Cyclic group $\cong \mathbb{Z}_p$
- Roots of unity

Algorithms

- Eratosthenes sieve
- GCD algorithm
- Chinese Remainder thm
- Taylor expansion
- Hall's thm / alt. pallo algor.

OVERVIEW DISCRETE MATH.

Techniques / Problem solving ideas.

- list small cases
- OEIS

- Counting in different ways
- Induction
- W.P.

Counting principles

- $|A \cup B| = |A| + |B| - |A \cap B|$
- PIE
- Equivalence relations

- generating functions
- GF
- ESP

Recurrence relations

- 2 term
- function with 2 variables
- const coeff

- GF \leftrightarrow pics (partition identities)
- O calculating
 - Estimating sums / identities
 - Log derivatives

- GF \rightarrow rational function \rightarrow partial fractions \rightarrow geometric series

- Interchanging sums - Wilf's Shuffling method.

Theorems

- Dirichlet's
- \mathbb{Z}_p is a field iff p is prime.

Fundamental theorem of arithmetic

Basic up thm

Chinese Remainder thm

$$\frac{1}{1-q} = 1 + q + q^2 + \dots$$

$$\sum k^0, \sum k^1, \sum k^2, \dots, \sum_{k=0}^n q^k$$

Trees / Bipartite trees

V_2 is tree

Primes are infinite.

Trees / bipartite graphs thm
 $K_{3,3}, K_5$ are not planar.

Classes

- Wilson's theorem
- Fermat's Little thm
- $a^{p-1} \equiv 1 \pmod{p}$
- $a^{p(p-1)} \equiv 1 \pmod{p}$
- $\varphi(a, n) = 1$

Newton's Binomial

$$(1+x)^n = \sum_{k=0}^{\infty} \binom{n}{k} x^k \text{ for } |x| < 1 \text{ n.P.B.}$$

Polya's Counting prob

- Euler's odd d = distinct
- other partition classes

q-binomial thm

- Lagrange thm: # H / # G
- $\binom{n}{a} \mid \# G$
 - $\binom{n}{a} = 1$ (smallest k)

Königsberg Bridge Problem

- Euler's $V - E + F = 2$
- Only 5 platonic solids

Perfect Matching theorem

Hall's / König - min/max thms

Catalan Counting

Cayley: n^{n-2} formula

Stirling's formula

Binet's formula (Fibonacci)

Birthday paradox

Derangement Problem

Char-Vandermonde Convolution

P and not P \Rightarrow any Q

Russells paradox

Cantor's diagonal argument