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Introduction to Music Processing

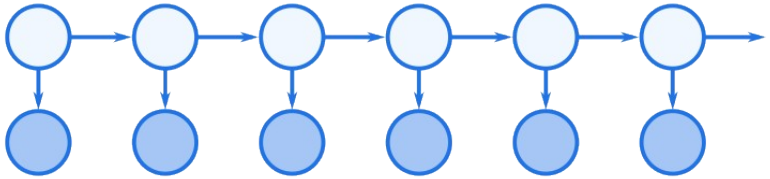
Hierarchical Models

Metrical Grid and Rhythm Grammars

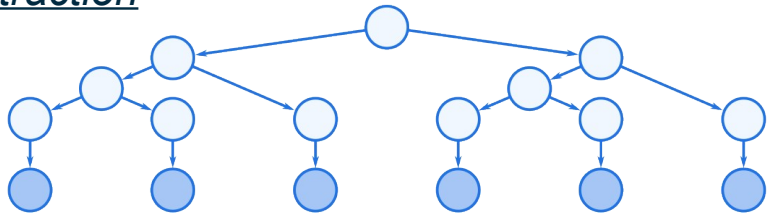
Dr Robert Lieck
robert.lieck@durham.ac.uk

Computational Models of Music

- **Sequential Models** (*n*-gram and (hidden) Markov)
  - Look at music *horizontally* (left-to-right)
  - Focus on *transitions*



- **Hierarchical Models** (context-free grammars)
  - Look at music *vertically* (bottom-up/top-down)
  - Focus on *abstraction*



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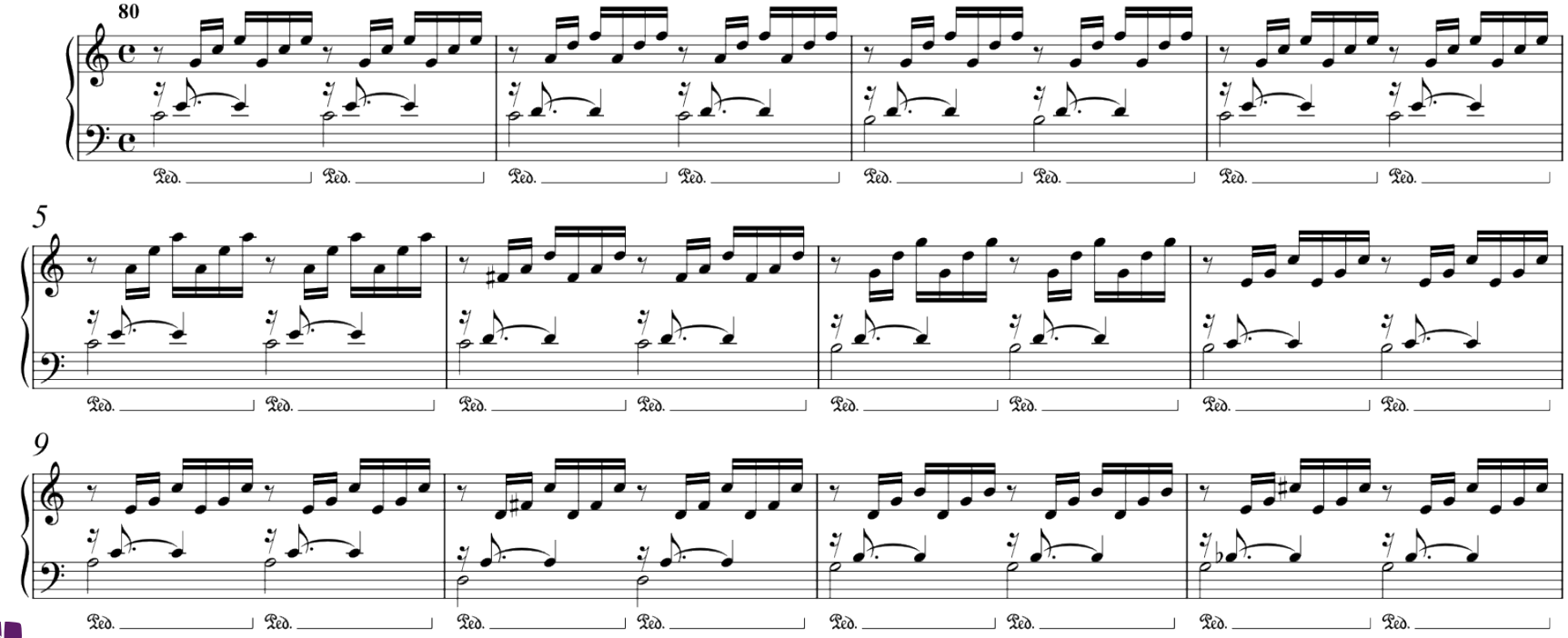
Music as a Hierarchy

Prélude No. 1 in C Major

from “Das Wohltemperierte Klavier” Book I
BWV 846

Johann Sebastian Bach
(1685 - 1750)



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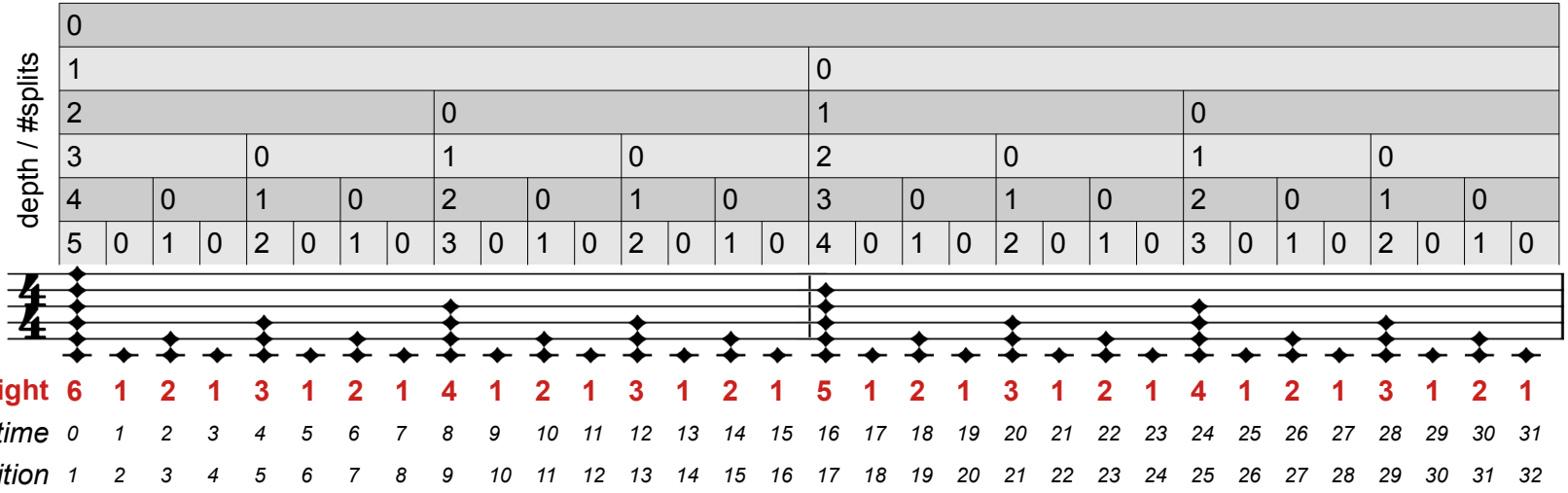
https://www.youtube.com/watch?v=frxT2qB1POQ

Metrical Grid

How is time organised in music?

- Any time interval can be repeatedly split up into two (or more) parts.
- This creates a hierarchical temporal structure: the metrical grid.
- The metrical weight of an event at time *t* corresponds to the number of possible ways it can be generated.

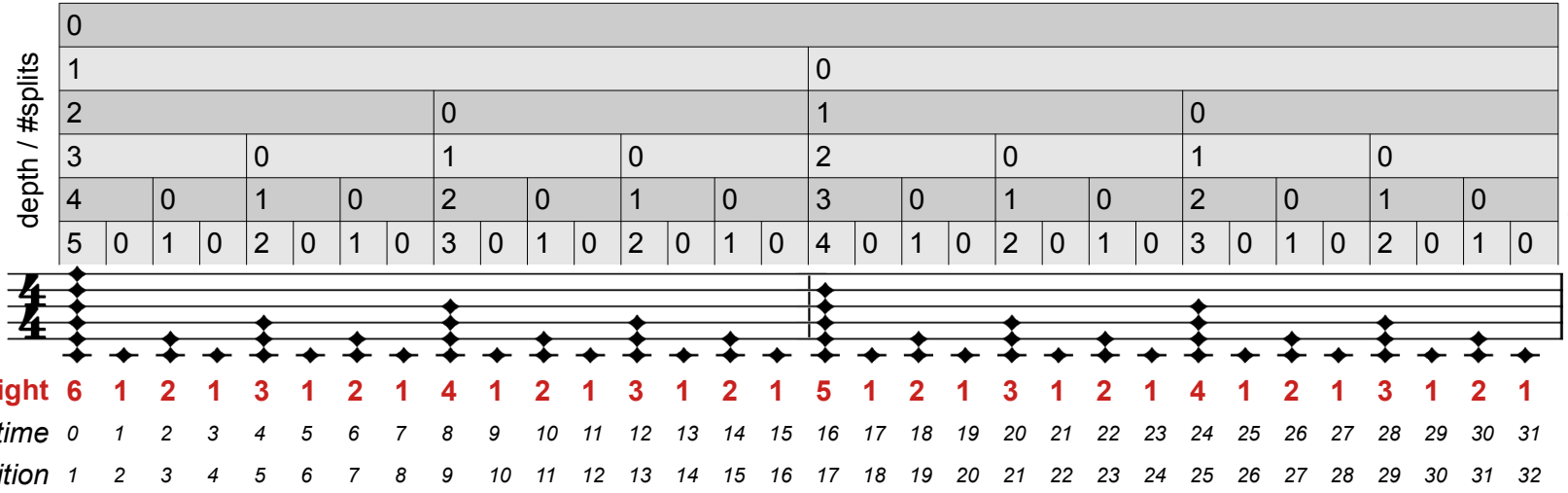


Metrical Grid

Generating Weights Top-Down

- Start with a time interval *T* and split it up recursively (*T* → *T T*).
- Count the number of times an interval was split.
- Instead of splitting, we *could have* created an even.

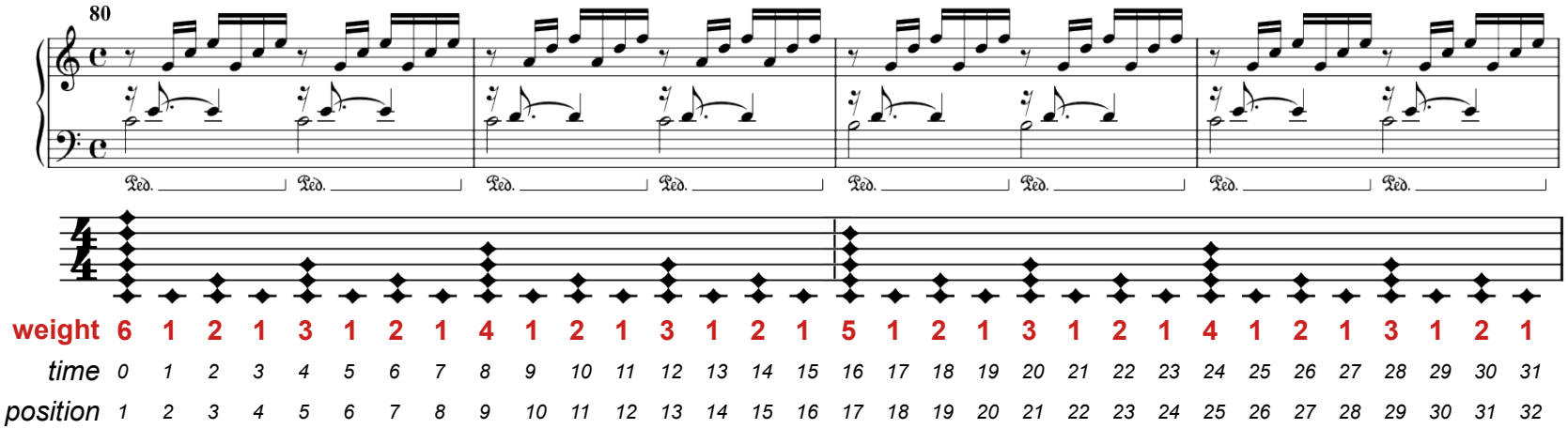
→ **#splits + 1 = #opportunities** (to create an event) ≈ w_{max} **– depth** (where event can be first created)



Metrical Grid: Rhythm versus Meter

How is time organised in music?

- The metrical grid is the “canvas” that rhythms are “painted” on.
- Not every position of the metrical grid is filled and/or the different events have different importance (e.g. when does something new happen?)
- How do we generate the actual events in a **rhythm**?



Metrical Grid: Probabilistic Rules

$$\begin{aligned} \text{split } (p_s): &\quad T \longrightarrow TT \\ \text{beat } (p_b): &\quad T \longrightarrow b \\ \text{rest } (p_r): &\quad T \longrightarrow r \end{aligned}$$

$$(p_s + p_b + p_r = 1)$$

This can be modelled with a set of probabilistic rules!

- Three rules that can be applied to a time interval *T.*
- Probability of generating an even at the beginning of *T*:

$$\bar{p} = p_b + p_s \bar{p} = \frac{p_b}{1 - p_s} = \frac{p_b}{p_b + p_r}$$

- Probability of generating an event at depth *d* (or deeper):

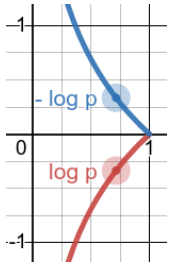
$$\bar{p}_d = p_s^d \bar{p} = p_s^d \frac{p_b}{p_b + p_r}$$

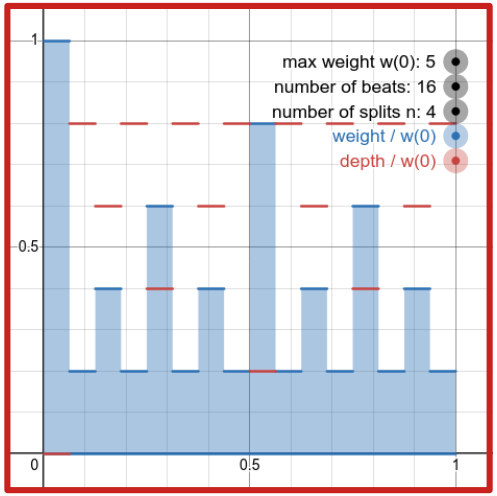
- Metrical weight corresponds to neg-log-likelihood:

$$w_d := w_0 - d$$

$$\Rightarrow \bar{p}_d = p_s^{w_0 - w_d} \frac{p_b}{p_b + p_r}$$

$$\Rightarrow \log \bar{p}_d \propto w_d + const.$$



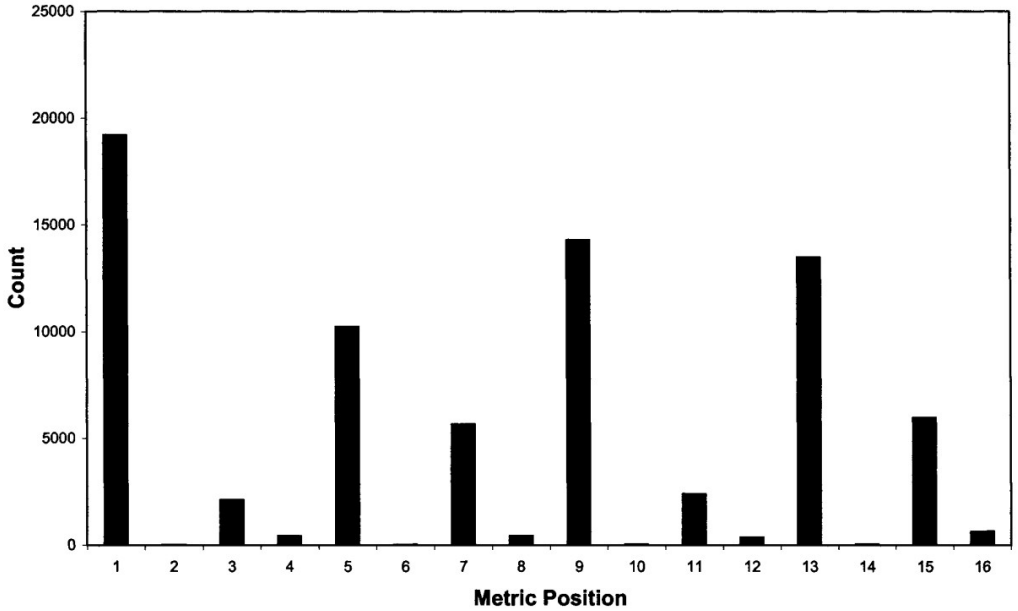


Play with the interactive [*desmos visualisation*]

→ **We generate events according to their metrical weight!**

Metrical Grid: Empirical Data

Empirical distribution of note onsets in a corpus of 1537 Germanic folksongs in 4/4 meter.



Huron D, Ommen A (2006) An empirical study of syncopation in American popular music, 1890–1939. Music Theory Spectrum 28:211–231

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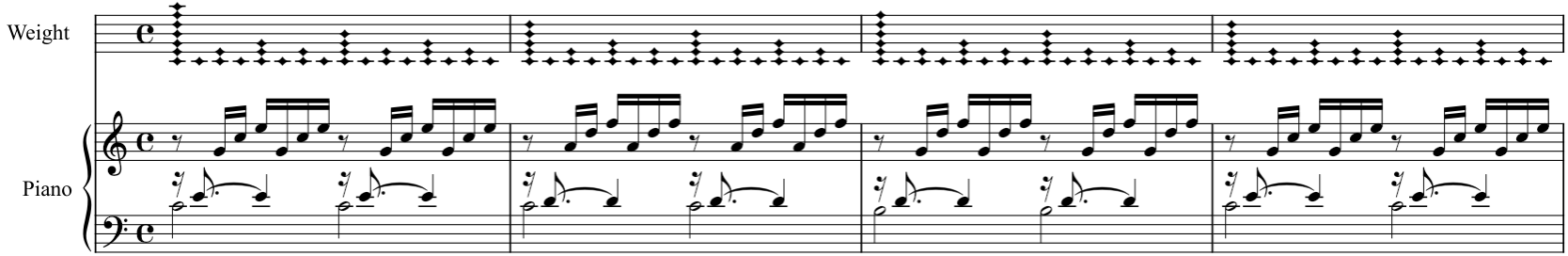
Metrical Grid: Examples

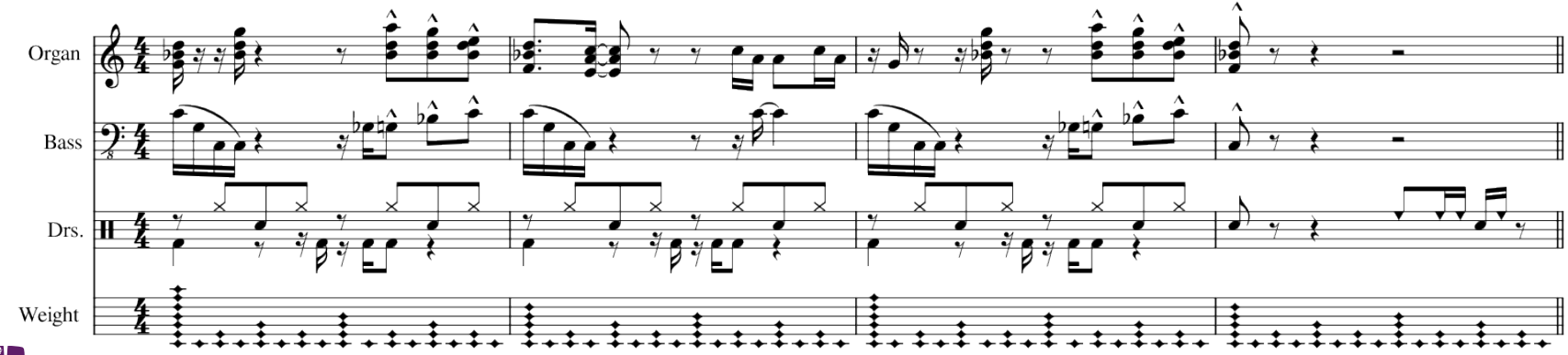
C Major Prelude (JSB)

https://www.youtube.com/watch?v=frxT2qB1POQ



Elephant‘s Foot (Maceo Parker)

https://youtu.be/PcW75olJSVM?si=UIX-4DA9rAkbZ9eZ&t=20



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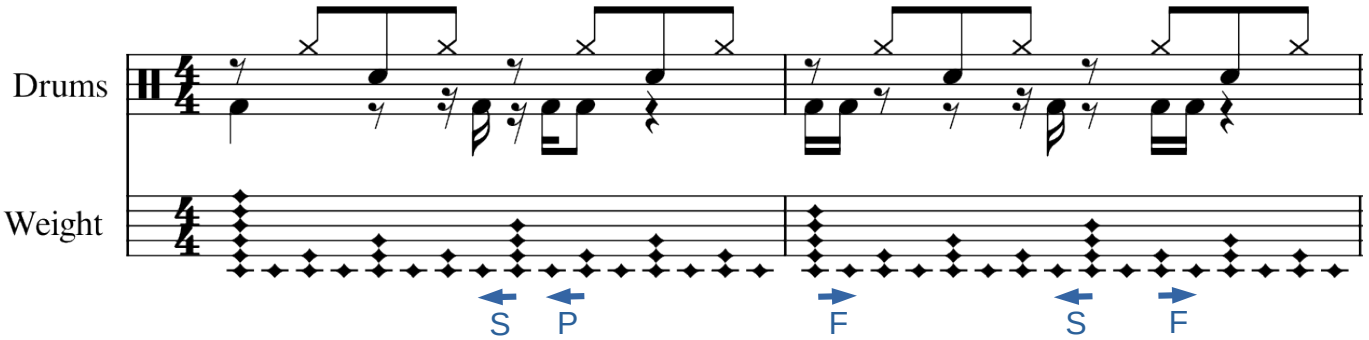
Metrical Grid: Syncopation

(irregular/off-beat events)

- **Need to differentiate between real and conceptual (or expected) location of events.**
- **Three more "time stealing" rules to account for syncopation:**
  - **shift ($\leftarrow_s$):** Shift an existing event to an earlier position with smaller metrical weight (the event keeps its original weight).
  - **preparation ($\leftarrow_P$):** For an existing event, generate an additional event at an earlier position with smaller metrical weight (the new event has a smaller weight).
  - **follow-up ($\rightarrow_F$):** For an existing event, generate an additional event at a later position with smaller metrical weight (the new event has a smaller weight).



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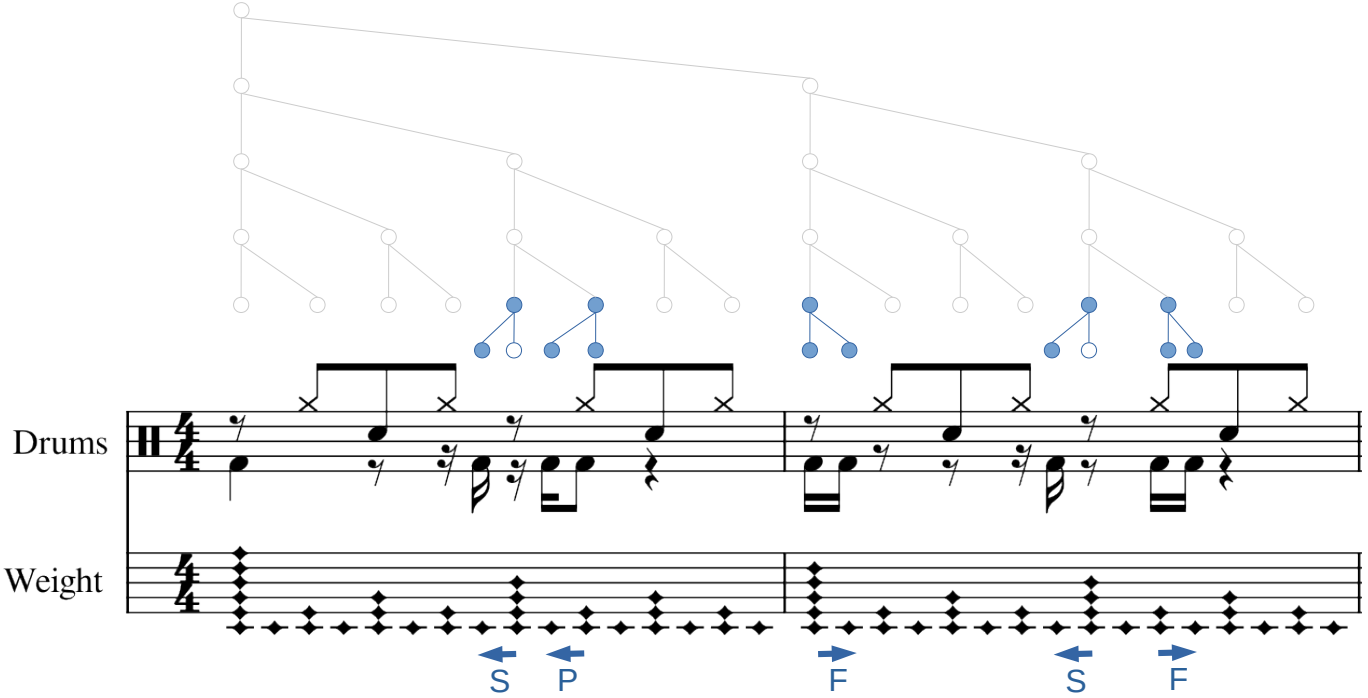
Rohrmeier M (2020) Towards a formalisation of musical rhythm. In: ISMIR

Metrical Grid: Analysis



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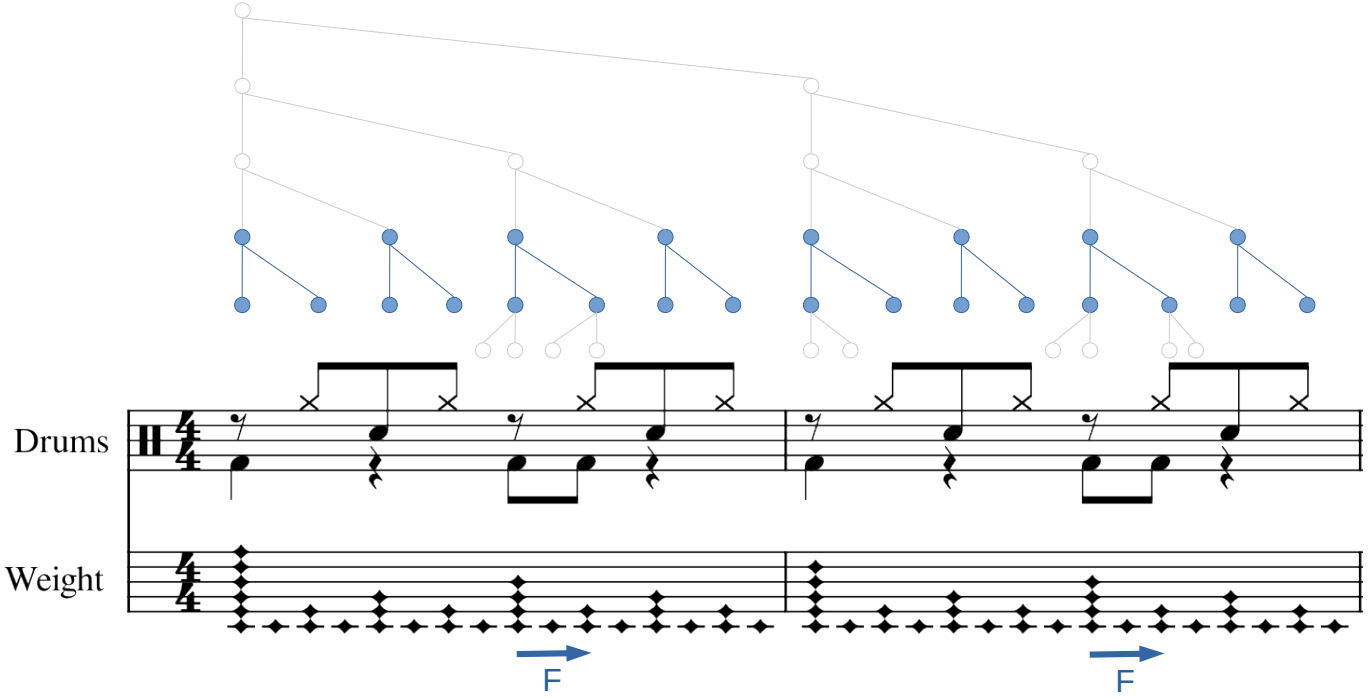
Metrical Grid: Analysis



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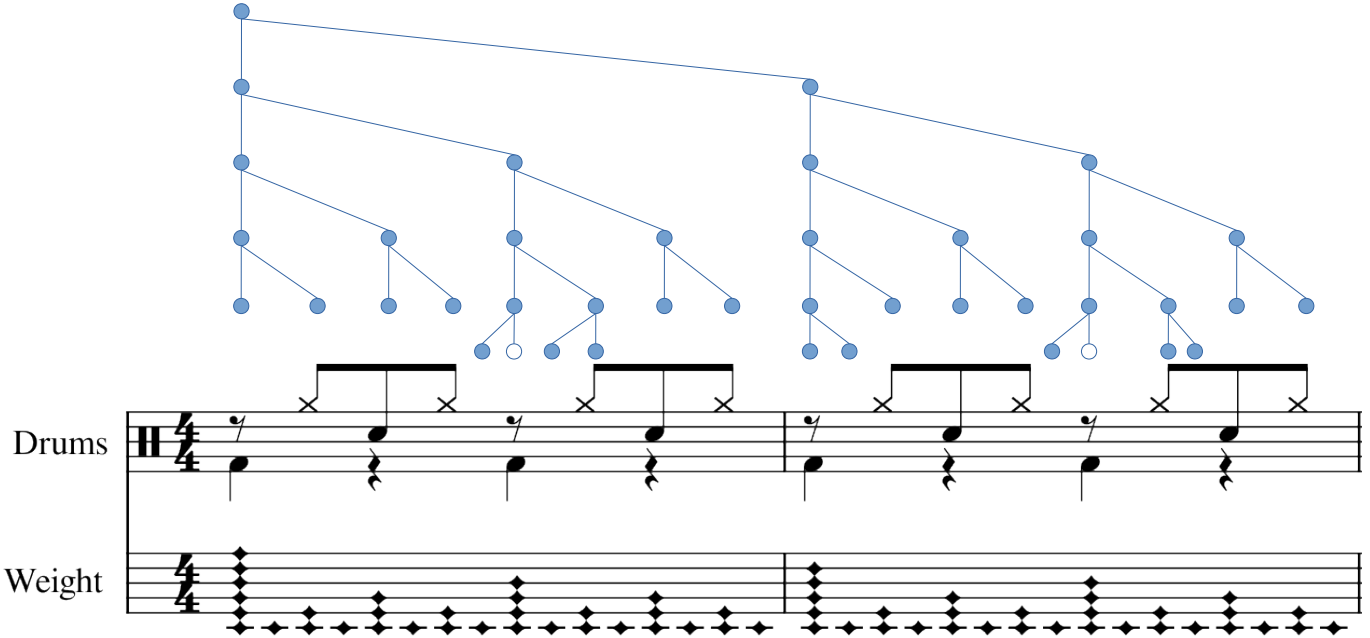
Metrical Grid: Analysis



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References

1) Lerdahl F, Jackendoff R (1983) A generative theory of tonal music. MIT press
2) Goodman J (1999) Semiring parsing. Computational Linguistics 25:573–605
3) Huron D, Ommen A (2006) An empirical study of syncopation in American popular music, 1890–1939. Music Theory Spectrum 28:211–231
4) Grune D, Jacobs CJ (2007) Parsing techniques. Monographs in Computer Science Springer
5) Rohrmeier M (2011) Towards a generative syntax of tonal harmony. Journal of Mathematics and Music 5:35–53. https://doi.org/10.1080/17459737.2011.573676
6) Rohrmeier M (2020) Towards a formalisation of musical rhythm. In: Proceedings of the 21st Int. Society for Music Information Retrieval Conf
7) Rohrmeier M (2020) The Syntax of Jazz Harmony: Diatonic Tonality, Phrase Structure, and Form. Music Theory and Analysis (MTA) 7:1–63. https://doi.org/10.11116/MTA.7.1.1
8) Rohrmeier M, Moss FC (2021) A formal model of extended tonal harmony. In: Proceedings of the 22nd International Society for Music Information Retrieval Conference. pp 569–578
9) Lieck R, Rohrmeier M (2021) Recursive Bayesian Networks: Generalising and Unifying Probabilistic Context-Free Grammars and Dynamic Bayesian Networks. In: Advances in Neural Information Processing Systems 34 (NeurIPS 2021)