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Introduction to Music Computing

From Overtones to Tuning Systems & Harmonies

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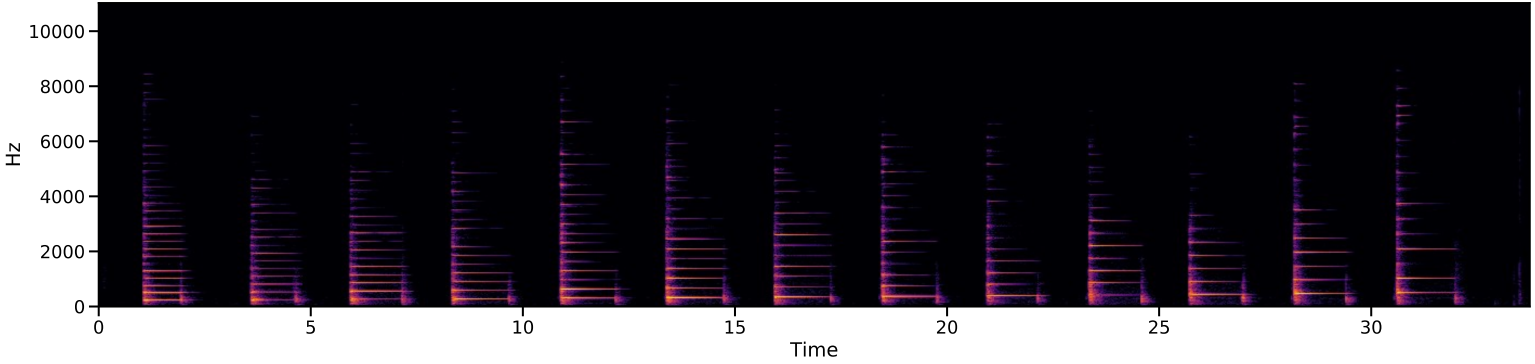
Spectrograms & Overtones

Natural Tones

- A natural tone does not correspond to a single frequency but has **overtones** on top of a fundamental frequency f_0.
- We can look at the spectrum of a sound using the Fourier transform and show this over time in a **spectrogram**.



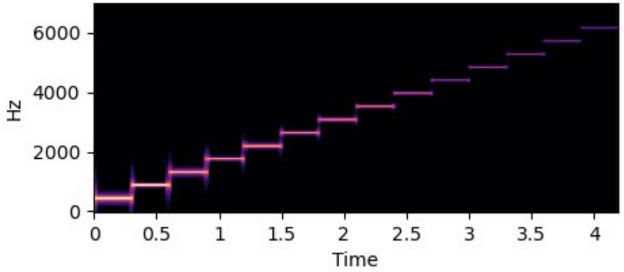
Overtones

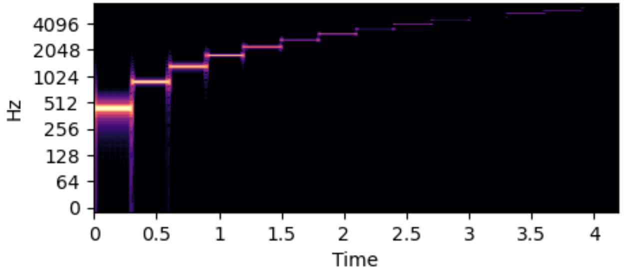
Overtones are **integer multiples of f_0**

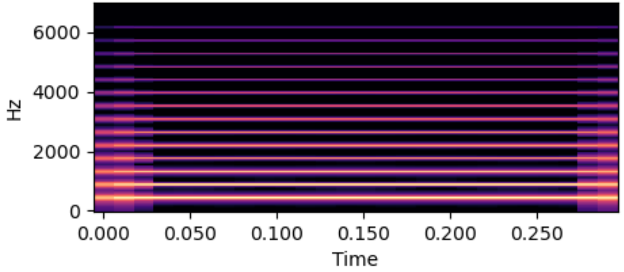
- equally spaced in frequency space
- typically with decaying amplitude

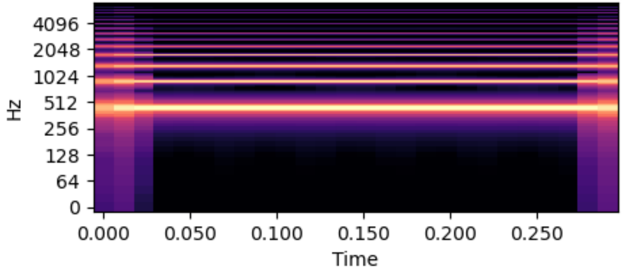
Overtones **do not sound equally spaced**

- pitch is logarithmic in frequency
- equal intervals correspond to **equal factors** in frequency space









Pitch as Log-Frequency

- pitch ≡ log-frequency
- pitch difference / step / interval ⟶ frequency ratio
- negative steps/intervals ⟶ inverse frequency ratio
- *n* equally spaced steps/intervals ⟶ n^{th} root

$$
\begin{aligned}
p &= \log(f) \\
p - p' &= \log(f/f') \\
i &= \log(r) \\
-i &= \log(1/r) \\
i/n &= \log(\sqrt[n]{r})
\end{aligned}
$$

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Overtones

Overtone	Partial	Frequency	Frequency Ratio	Interval
none/fundamental	1st	f_0	1	prime
1st	2nd	$f_1 = 2f_0$	**2**	octave
2nd	3rd	$f_2 = 3f_0$	**3**	fifth (+ octave)
3rd	4th	$f_3 = 4f_0$	2·2	2 octaves
4th	5th	$f_4 = 5f_0$	**5**	major third (+ 2 octaves)
5th	6th	$f_5 = 6f_0$	2·3	fifth (+ 2 octaves)
6th	7th	$f_6 = 7f_0$	**7**	minor seventh (+ 2 octaves)
…	…	…	…	…

Interval	Ratio
prime	1
octave	2
fifth	3/2 = 1.5
major third	5/4 = 1.25
minor seventh	7/4 = 1.75

(modulo octaves)

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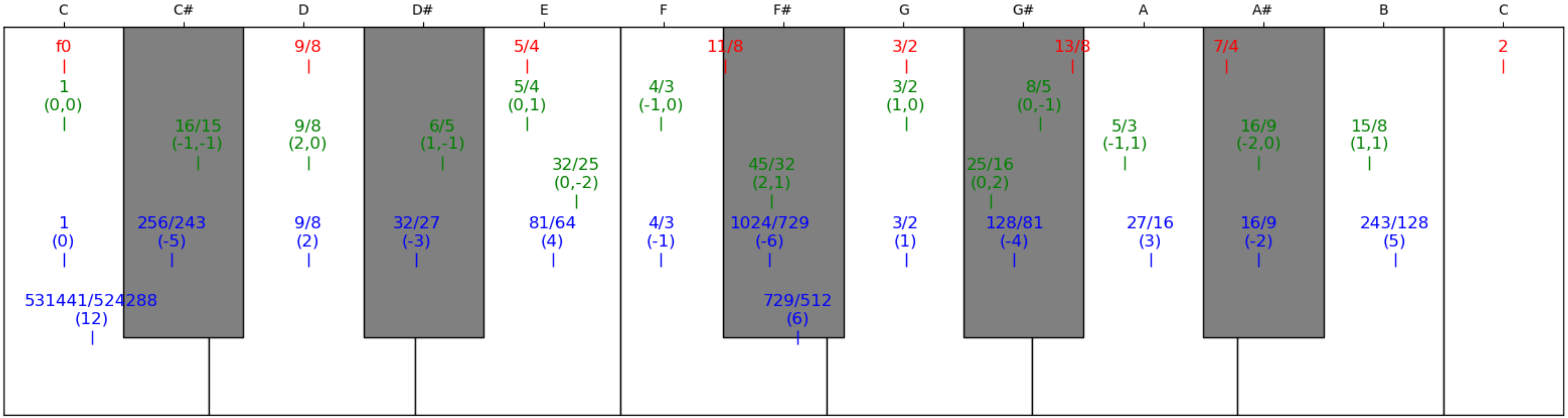
Stacked Overtones (Octaves, Fifths & Thirds)

Overtones
(Fifths, Thirds)
(Fifths)



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Consonance & Pitch Similarity

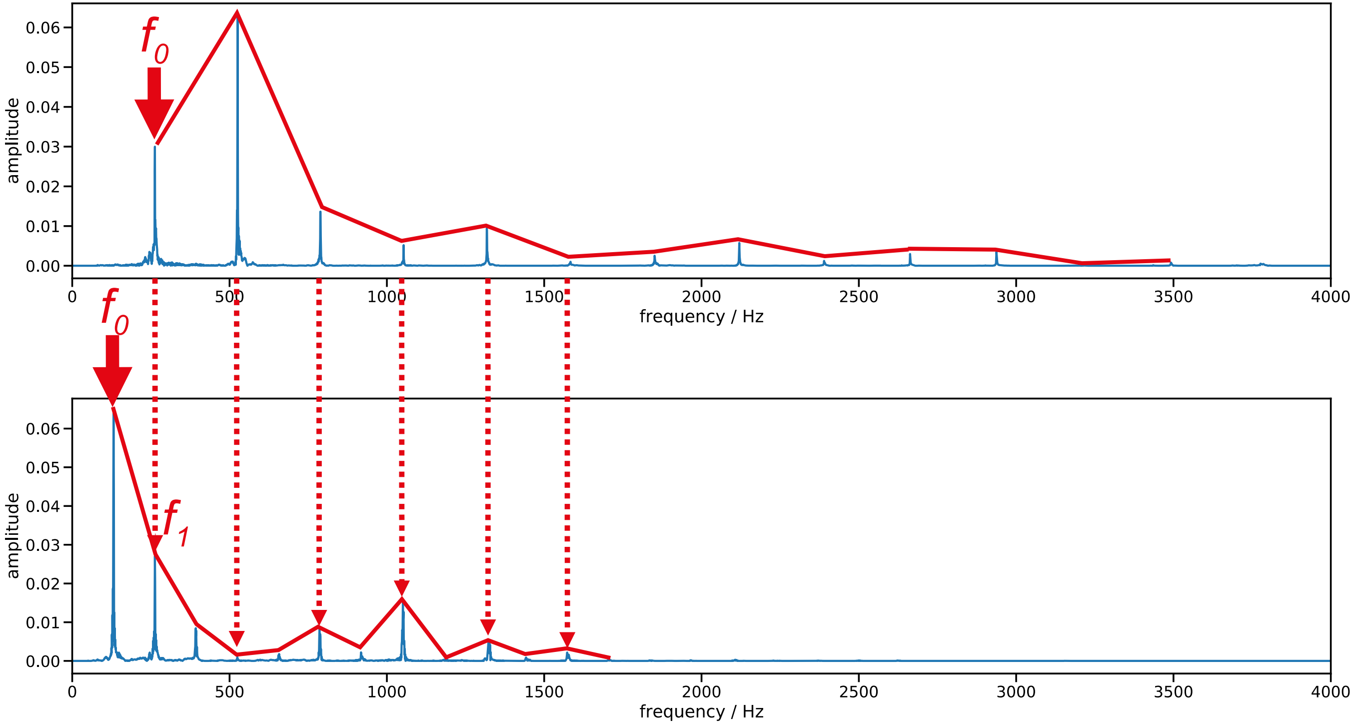
Consonance & Pitch Similarity

In music, we often want tones that **“sound good together”**.

- What does it mean to “sound good”? What does “together” mean?
- Example: octave equivalence versus similar fundamental
  - Why do tones that are an **octave apart sound similar** even though their **fundamentals differ by a factor of 2**?
  - Why does a minor second **sound dissonant** even though the two tones have relatively **close fundamental frequencies**?

Octave Equivalence

- same *pitch class*
- different *timbre*



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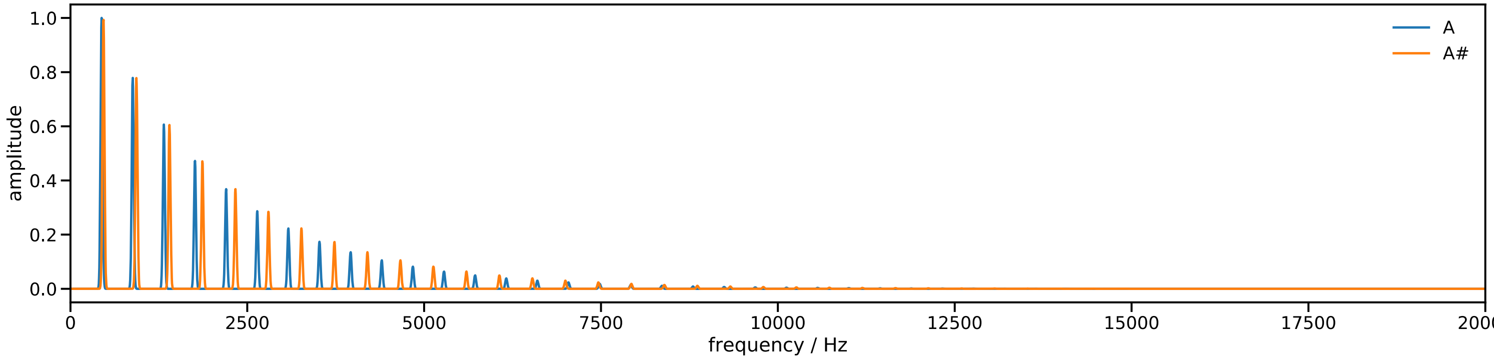
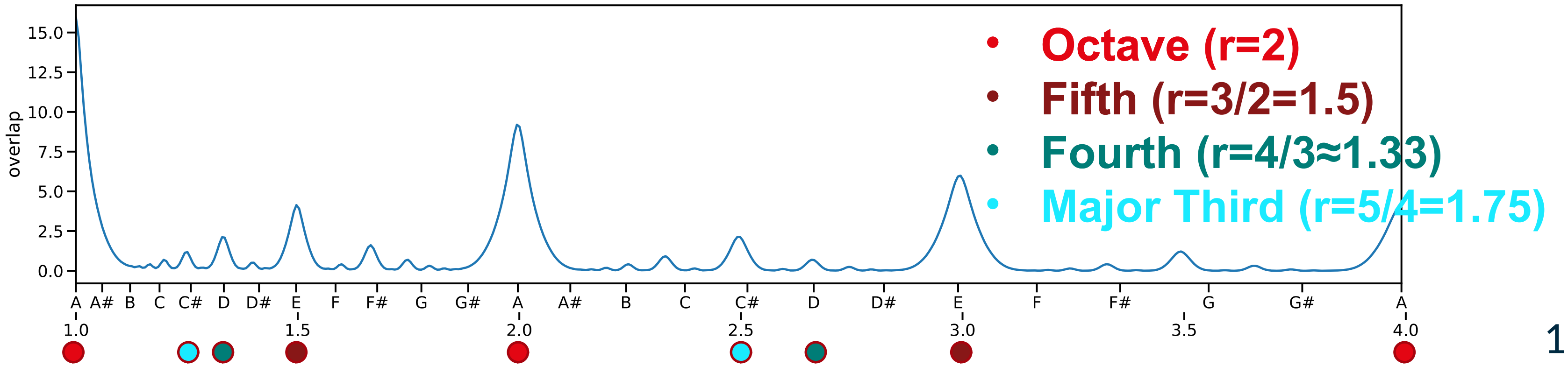
Spectral Similarity

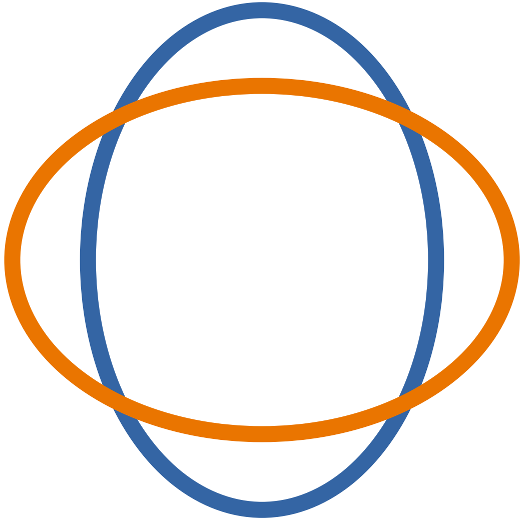
- intervals from the overtone series result in great spectral overlap
- they are experienced as “consonant” and “close” to each other



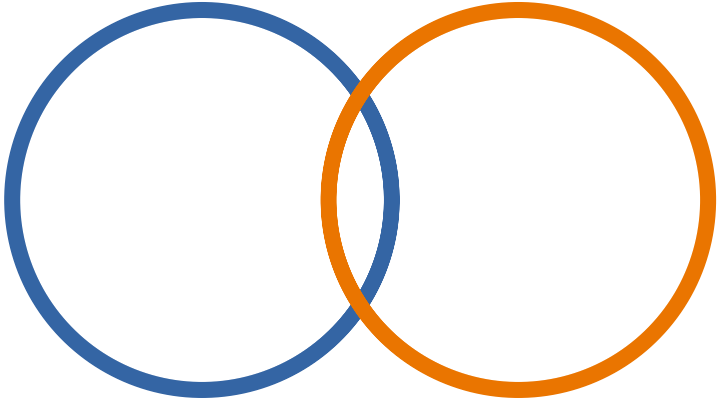


Two kinds of similarity

Which pair is more similar?



Shared content (at given time)
→ **harmony / consonance**



Can be *transformed* to each other (over time) → **voice leading**

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Tuning Systems

Tuning Systems

When building an instrument, we need:

- tones that **sound good** together, but also
- tones that **cover the octave** in small steps

→ potentially contradicting objectives!

- Octaves always work!
- Which other overtones or intervals should we use?

Tuning Systems

Overtones
(Fifths, Thirds)
(Fifths)

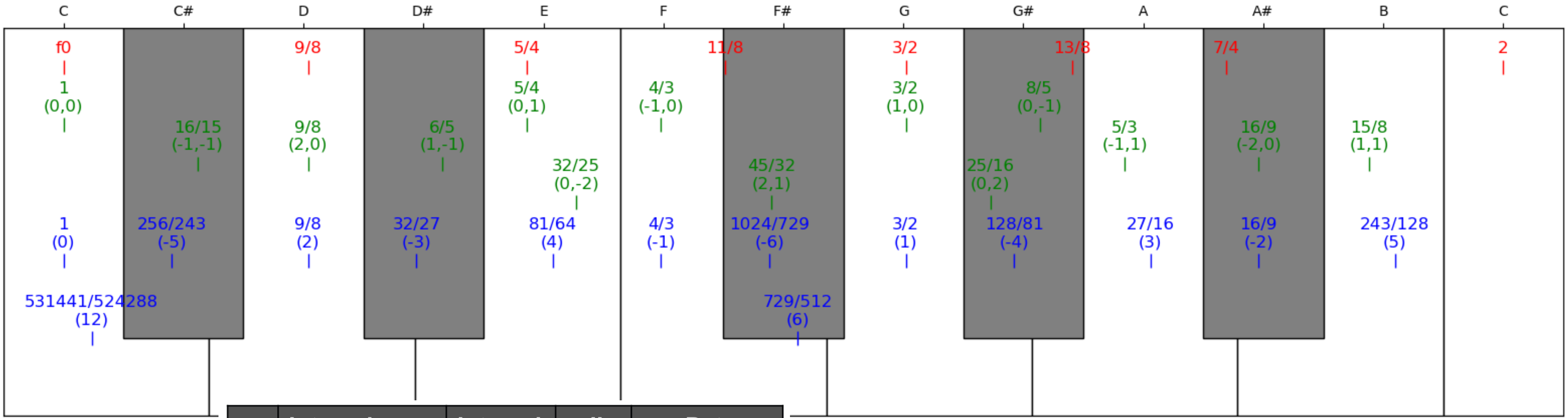

Fig 5.3 [FMP15]

Δ	Interval name	Interval	JI ratio	Pyt. ratio
0	(Perfect) unison	C4 – C4	1:1	1:1
1	Minor second	C4 – D♭4	15:16	$3^5:2^8$
2	Major second	C4 – D4	8:9	$2^3:3^2$
3	Minor third	C4 – E♭4	5:6	$3^3:2^5$
4	Major third	C4 – E4	4:5	$2^6:3^4$
5	(Perfect) fourth	C4 – F4	3:4	$3:2^2$
6	Tritone	C4 – F♯4	32:45	$2^9:3^6$ or $3^6:2^{10}$
7	(Perfect) fifth	C4 – G4	2:3	2:3
8	Minor sixth	C4 – A♭4	5:8	$3^4:2^7$
9	Major sixth	C4 – A4	3:5	$2^4:3^3$
10	Minor seventh	C4 – B♭4	5:9	$3^2:2^4$
11	Major seventh	C4 – B4	8:15	$2^7:3^5$
12	(Perfect) octave	C4 – C5	1:2	1:2

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Just Intonation

In **just intonation** all intervals of a **scale** are tuned as (preferably small) **exact integer ratios** relative to the fundamental f_0 or **tonic**.

- tones are **exact overtones-of-overtones** (or "undertones") of the tonic
- scales are always relative to the tonic and **not translational invariant**
- diatonic scales (major/minor) need a **maximum of two <u>fifth</u> and/or <u>third</u> steps**
- more generally, each note can be specified by a triplet of integers

$$\textbf{(octaves, fifths, thirds)} \rightarrow f_0 \cdot 2^{\text{octaves}} \cdot 3^{\text{fifths}} \cdot 5^{\text{thirds}}$$

  specifying the number of steps of the respective intervals

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Pythagorean Tuning

In **Pythagorean tuning** all intervals are tuned in **multiples of perfect fifths** relative to the fundamental f_0 or **tonic**.

- tones are **exact fifths-of-fifths-of-fifths-…** of the tonic
- scales are **approximately translational invariant** (when going in fifths steps)
- each note can be specified by a pair of integers

$$\textbf{(octaves, fifths)} \rightarrow \mathbf{f_0 \cdot 2^{octaves} \cdot 3^{fifths}}$$

  specifying the number of steps of octaves and fifths
- this is where **spelled pitch** and **key signatures** come from!

Fifths	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Name	Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#	F##

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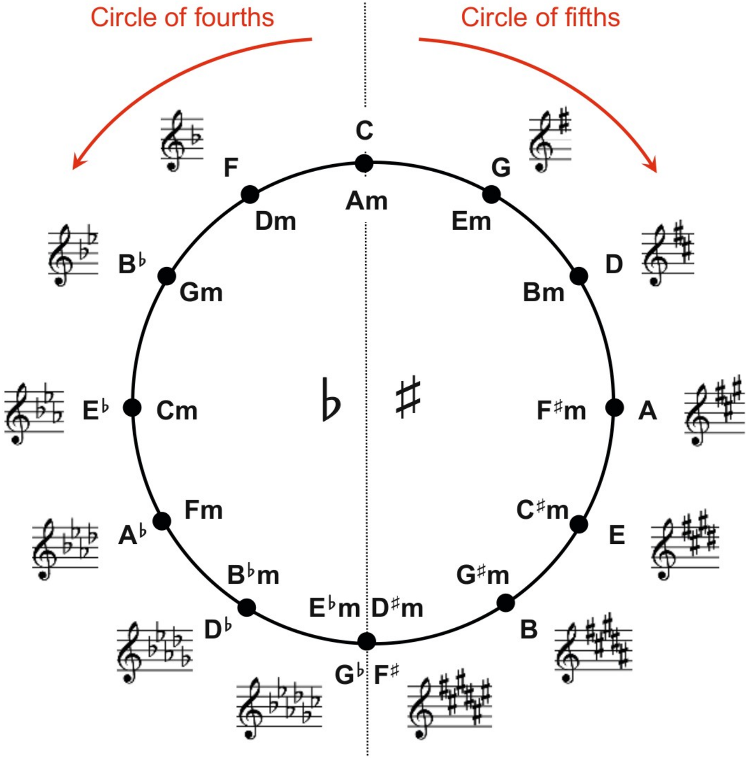
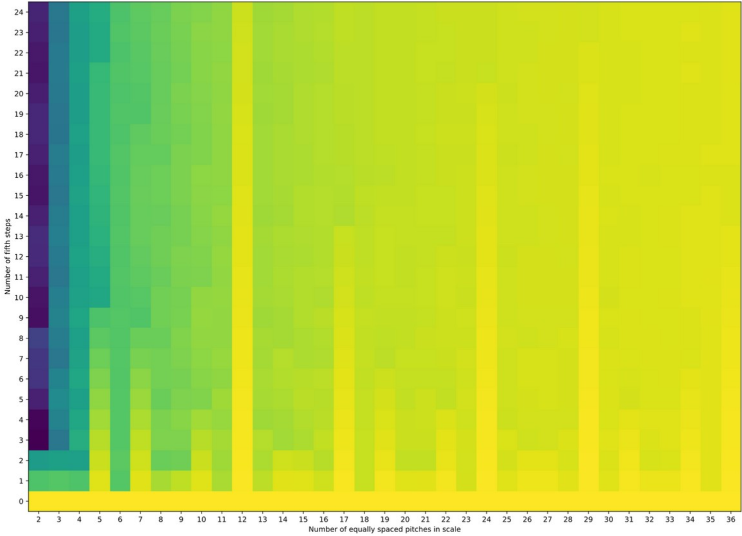
Pythagorean Tuning – Circle of Fifths



Fifths	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Name	Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#	F##

Comparison of Tuning Systems

alternative equal temperaments
(fit to Pythagorean tuning)



Just Intonation

- cleanest intervals and harmonies
- least translational invariant
- tuned only for one specific key

Pythagorean

- ignore **syntonic comma**: 81/64 vs 5/4 = 81/80 = 1.0125 for major third
- approximately translational invariant for fifth-related keys

12-Tone Equal Temperament (12TET)

- octave divided into steps of equal size (factor $\sqrt[12]{2}$ or interval 1/12 log(2))
- ignore **Pythagorean comma**: $3^{12}/2^{19}$ = 531441/524288 ≈ 1.0136 for octave
- perfectly translational invariant

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Harmony & Voice Leading

Harmony & Voice Leading

In music, we typically want some balance between

- **known, stable, static, expected**
- **novel, unstable, dynamic, surprising**

That balance may be

- different for different **cultures/styles/genres**
- different along different **musical dimensions** (e.g. rhythm vs harmony)
- **changing throughout a piece**

In Western music, **harmony** (tones sounding simultaneously) and **voice leading** (how tones move over time) contribute to maintaining this balance.

Harmony & Voice Leading

- **Chords/harmonies** must achieve a compromise between consonance/stability and diversity
- **Voice leading** can
  - “fix” dissonant harmonies by creating convincing melodic lines
  - create transient dissonances that quickly resolve
- **Changes in tension** make a piece interesting and pleasant
  - a consonance created by resolving a dissonance feels even more stable
- **Mixing and interplay** between the two kinds of similarity
  - harmony (vertical) & voice leading (horizontal)

References

- [FMP15] Meinard Müller (2015) Fundamentals of music processing: Audio, analysis, algorithms, applications. Springer
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- Dean RT, Milne AJ, Bailes F (2019) Spectral Pitch Similarity is a Predictor of Perceived Change in Sound- as Well as Note-Based Music. Music & Science 2:2059204319847351. https://doi.org/10.1177/2059204319847351