



Durham  
University

# Introduction to Music Processing

Hierarchical Models

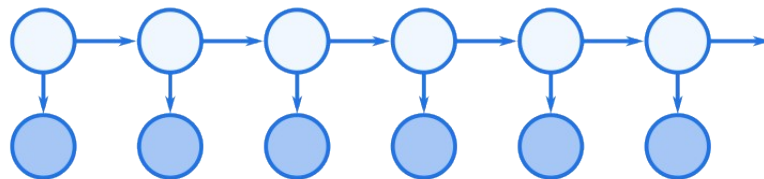
Metrical Grid and Rhythm Grammars

Dr Robert Lieck

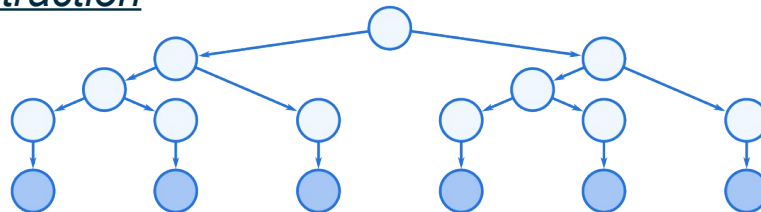
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# Computational Models of Music

- **Sequential Models** ( $n$ -gram and (hidden) Markov)
  - Look at music horizontally (left-to-right)
  - Focus on transitions



- **Hierarchical Models** (context-free grammars)
  - Look at music vertically (bottom-up/top-down)
  - Focus on abstraction



# Music as a Hierarchy

# Prélude No. 1 in C Major

from “Das Wohltemperierte Klavier” Book I  
BWV 846

Johann Sebastian Bach  
(1685 - 1750)

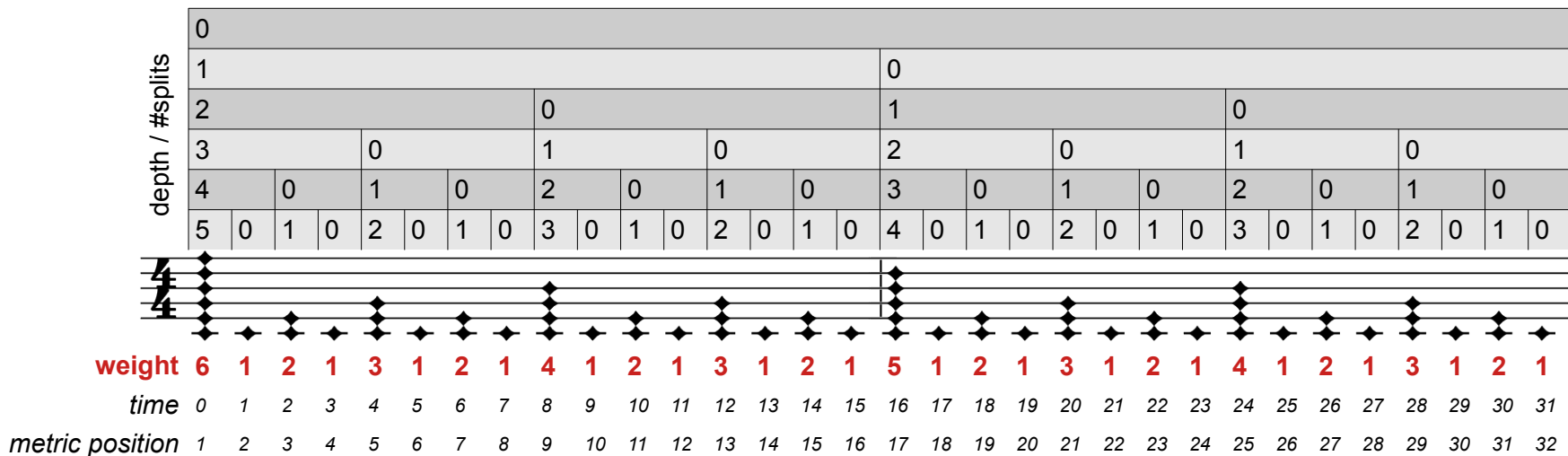
The image displays the first 9 measures of the Prélude No. 1 in C Major, BWV 846, by Johann Sebastian Bach. The score is written for piano in C major, 4/4 time. It is divided into three systems, each containing four measures. The first system starts at measure 80, the second at measure 5, and the third at measure 9. Each system features a treble and bass staff. The right hand plays a continuous eighth-note pattern, while the left hand plays a series of chords, each marked with a 'Ped.' (pedal) symbol. The key signature is one sharp (F#), indicating C major. The notation includes various musical symbols such as notes, rests, and bar lines.

# Metrical Grid



## How is time organised in music?

- Any time interval can be repeatedly split up into two (or more) parts.
- This creates a hierarchical temporal structure: the **metrical grid**.
- The **metrical weight** of an event at time  $t$  corresponds to the number of possible ways it can be generated.



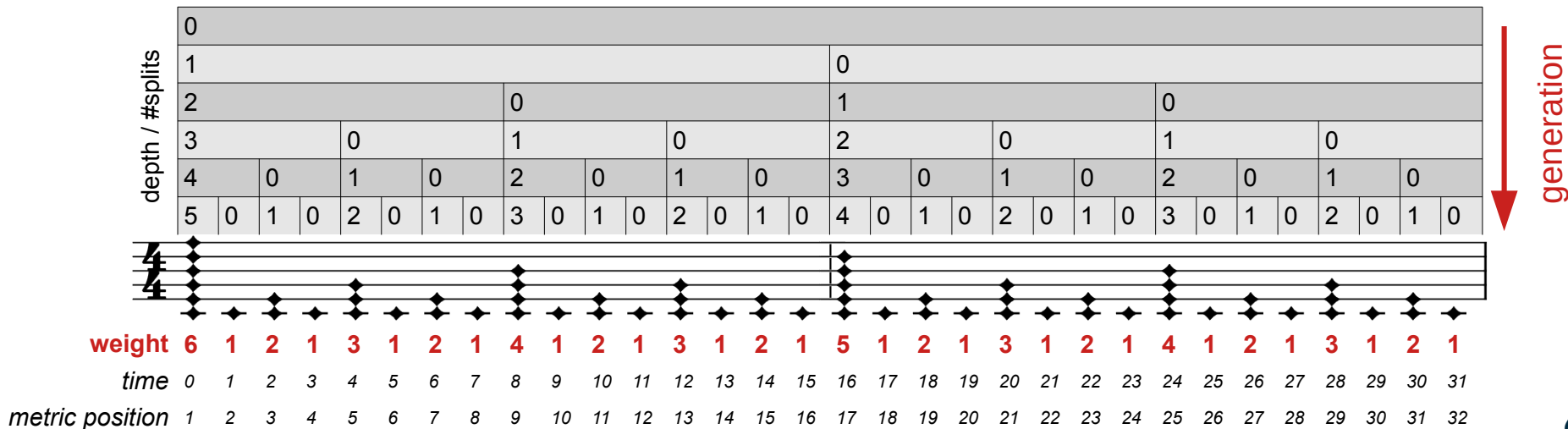
# Metrical Grid



## Generating Weights Top-Down

- Start with a time interval  $T$  and split it up recursively ( $T \rightarrow T T$ ).
- Count the number of times an interval was split.
- Instead of splitting, we *could have* created an even.

→  $\#splits + 1 = \#opportunities$  (to create an event)  $\approx w_{max} - depth$  (where event can be first created)

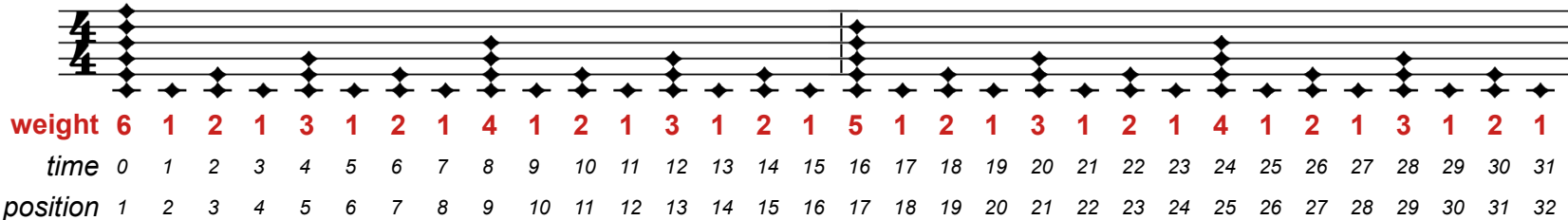


# Metrical Grid: Rhythm versus Meter



## How is time organised in music?

- The metrical grid is the “canvas” that rhythms are “painted” on.
- Not every position of the metrical grid is filled and/or the different events have different importance (e.g. when does something new happen?)
- How do we generate the actual events in a **rhythm**?



# Metrical Grid: Probabilistic Rules

💡 This can be modelled with a set of probabilistic rules!

- Three rules that can be applied to a time interval  $T$ .
- Probability of generating an even at the beginning of  $T$ :

$$\bar{p} = p_b + p_s \bar{p} = \frac{p_b}{1 - p_s} = \frac{p_b}{p_b + p_r}$$

- Probability of generating an event at depth  $d$  (or deeper):

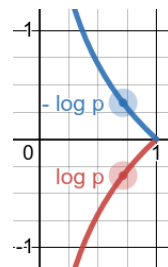
$$\bar{p}_d = p_s^d \bar{p} = p_s^d \frac{p_b}{p_b + p_r}$$

- Metrical weight corresponds to neg-log-likelihood:

$$w_d := w_0 - d$$

$$\Rightarrow \bar{p}_d = p_s^{w_0 - w_d} \frac{p_b}{p_b + p_r}$$

$$\Rightarrow \log \bar{p}_d \propto w_d + \text{const.}$$

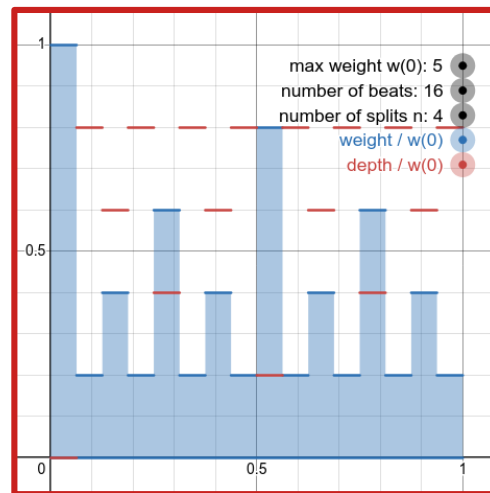


split ( $p_s$ ):  $T \longrightarrow TT$

beat ( $p_b$ ):  $T \longrightarrow b$

rest ( $p_r$ ):  $T \longrightarrow r$

$$(p_s + p_b + p_r = 1)$$



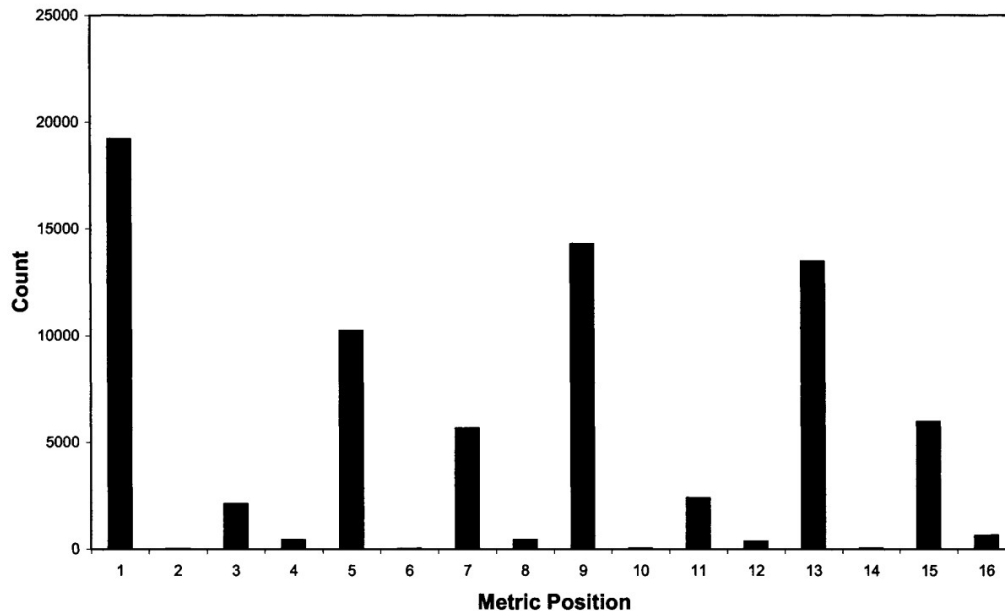
→ We generate events according to their metrical weight!

Play with the interactive  
[desmos visualisation](#)



# Metrical Grid: Empirical Data

Empirical distribution of note onsets in a corpus of 1537 Germanic folksongs in 4/4 meter.



Huron D, Ommen A (2006) An empirical study of syncopation in American popular music, 1890–1939. *Music Theory Spectrum* 28:211–231

# Metrical Grid: Examples

## C Major Prelude (JSB)

<https://www.youtube.com/watch?v=frxT2qB1POQ>

Weight

Piano

The image shows a musical score for the C Major Prelude by Johann Sebastian Bach. It features two staves: a top staff labeled 'Weight' and a bottom staff labeled 'Piano'. The 'Weight' staff is a single line with a common time signature 'C' and contains a series of vertical stems with dots, representing a metrical grid. The 'Piano' staff is a grand staff (treble and bass clefs) with a common time signature 'C'. It contains a continuous melodic line in the right hand and a harmonic accompaniment in the left hand, consisting of eighth and sixteenth notes.

## Elephant's Foot (Maceo Parker)

<https://youtu.be/PcW75oIJSVM?si=UIX-4DA9rAkbZ9eZ&t=20>

Organ

Bass

Drs.

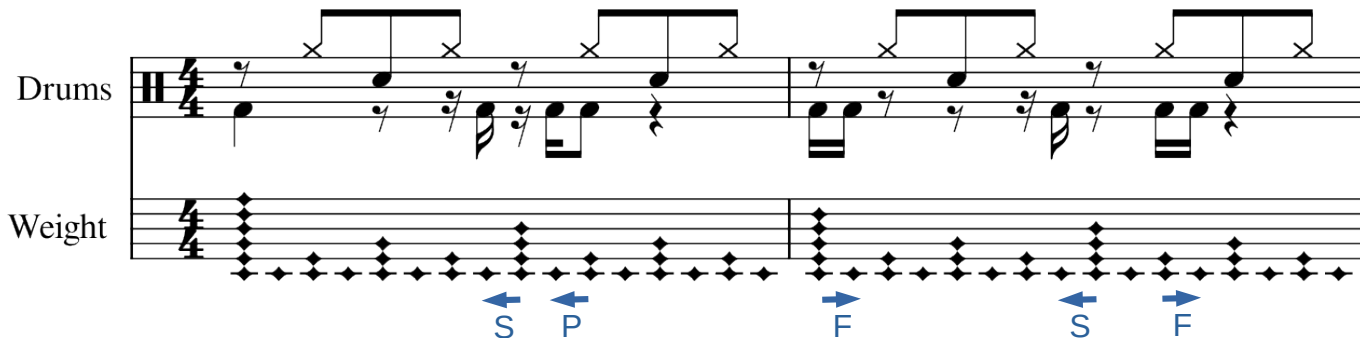
Weight

The image shows a musical score for 'Elephant's Foot' by Maceo Parker. It features four staves: 'Organ', 'Bass', 'Drs.', and 'Weight'. The 'Organ' staff is in 4/4 time and contains a melodic line with many accidentals. The 'Bass' staff is in 4/4 time and contains a melodic line with many accidentals. The 'Drs.' staff is in 4/4 time and contains a rhythmic line with many accidentals. The 'Weight' staff is in 4/4 time and contains a series of vertical stems with dots, representing a metrical grid.

# Metrical Grid: Syncopation

(irregular/off-beat events)

- Need to differentiate between **real** and **conceptual** (or **expected**) location of events.
- Three more “time stealing” rules to account for syncopation:
  - **shift** ( $\leftarrow_s$ ): Shift an existing event to an earlier position with smaller metrical weight (the event keeps its original weight).
  - **preparation** ( $\leftarrow_p$ ): For an existing event, generate an additional event at an earlier position with smaller metrical weight (the new event has a smaller weight).
  - **follow-up** ( $\rightarrow_f$ ): For an existing event, generate an additional event at a later position with smaller metrical weight (the new event has a smaller weight).



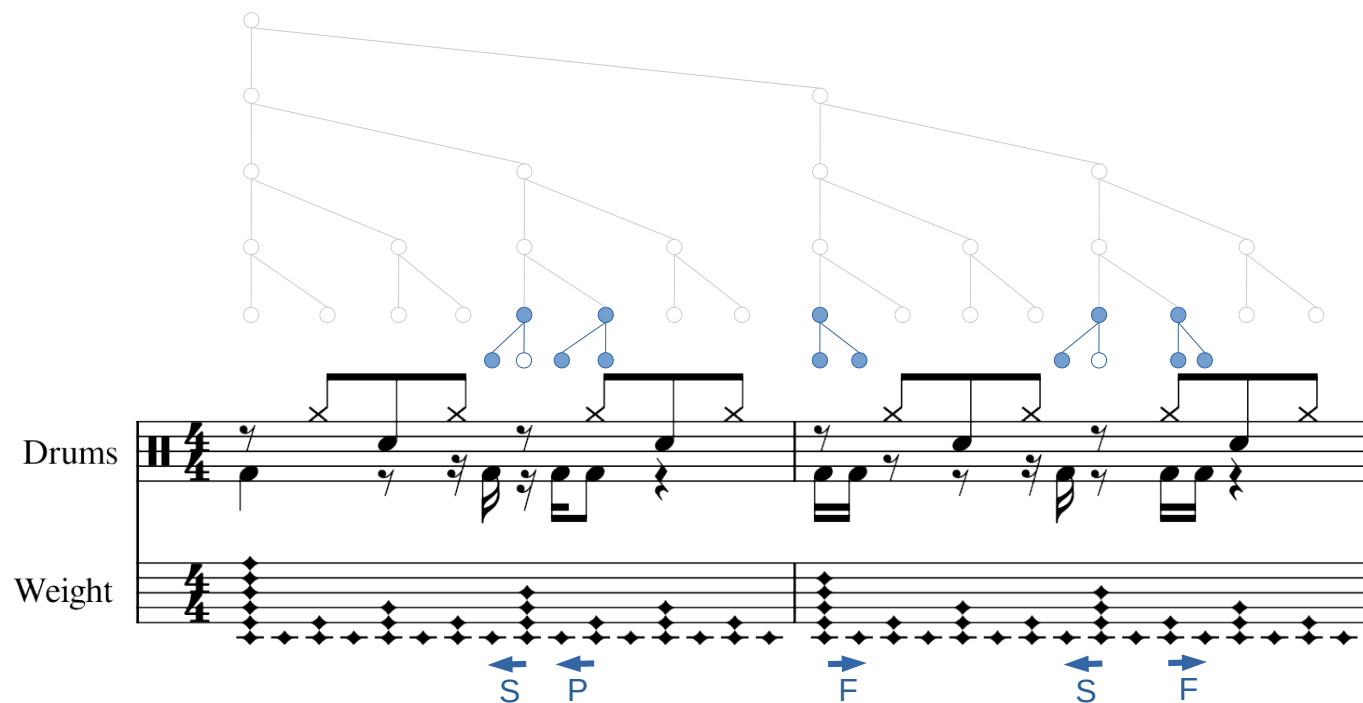
The diagram illustrates the concept of syncopation using a musical notation system. It consists of two staves: "Drums" and "Weight".

The "Drums" staff is in 4/4 time and shows a sequence of events (represented by notes and rests) over two measures. The events are marked with 'x' above them, indicating their conceptual location. The "Weight" staff shows the metrical weight of these events, represented by dots on a grid. The weight of each event is indicated by the number of dots above it.

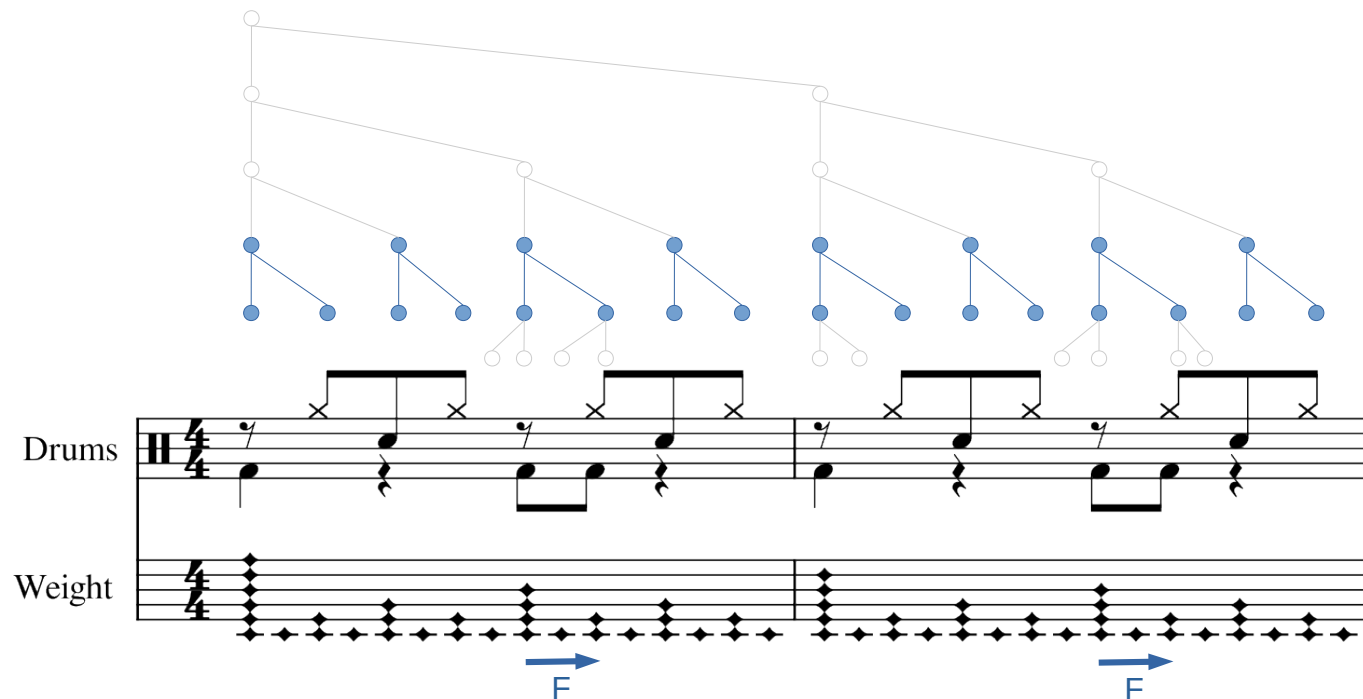
Below the "Weight" staff, five arrows indicate the application of syncopation rules:

- S** (Shift): An arrow pointing left, indicating a shift to an earlier position.
- P** (Preparation): An arrow pointing left, indicating a preparation event.
- F** (Follow-up): An arrow pointing right, indicating a follow-up event.
- S** (Shift): An arrow pointing left, indicating a shift to an earlier position.
- F** (Follow-up): An arrow pointing right, indicating a follow-up event.

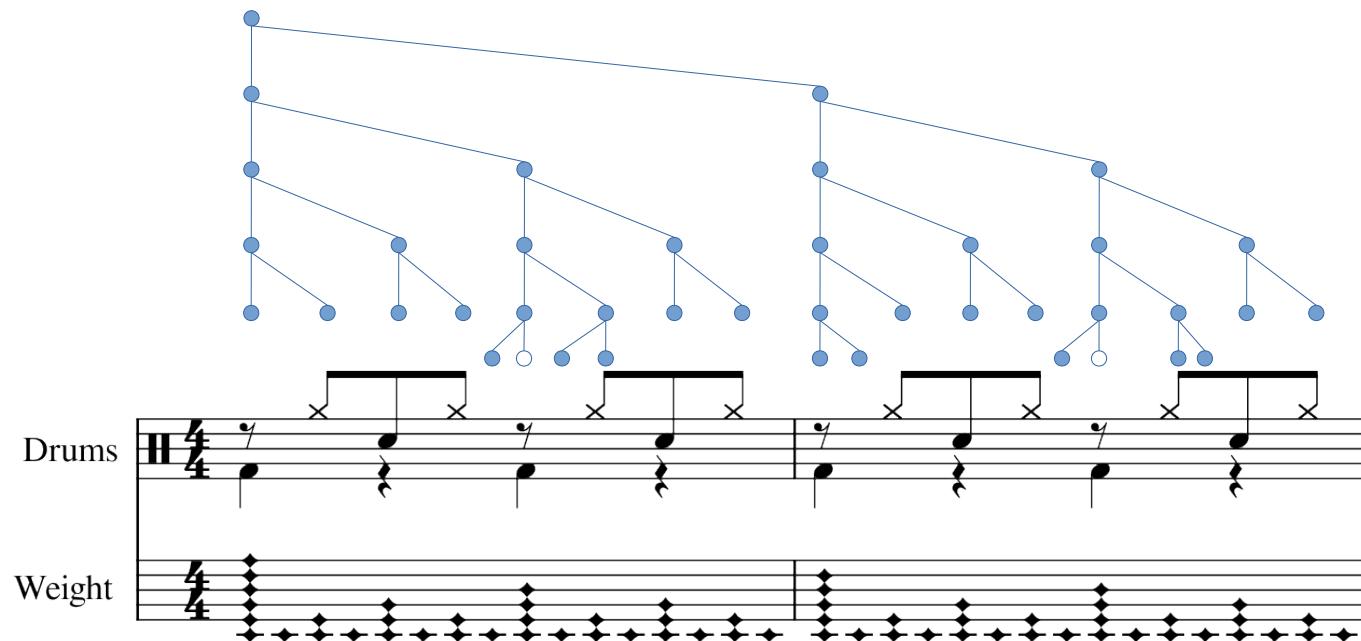
# Metrical Grid: Analysis



# Metrical Grid: Analysis



# Metrical Grid: Analysis



# References

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- 8) Rohrmeier M, Moss FC (2021) A formal model of extended tonal harmony. In: *Proceedings of the 22nd International Society for Music Information Retrieval Conference*. pp 569–578
- 9) Lieck R, Rohrmeier M (2021) Recursive Bayesian Networks: Generalising and Unifying Probabilistic Context-Free Grammars and Dynamic Bayesian Networks. In: *Advances in Neural Information Processing Systems* 34 (NeurIPS 2021)