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Introduction to Music Computing

Models of Tonal Space

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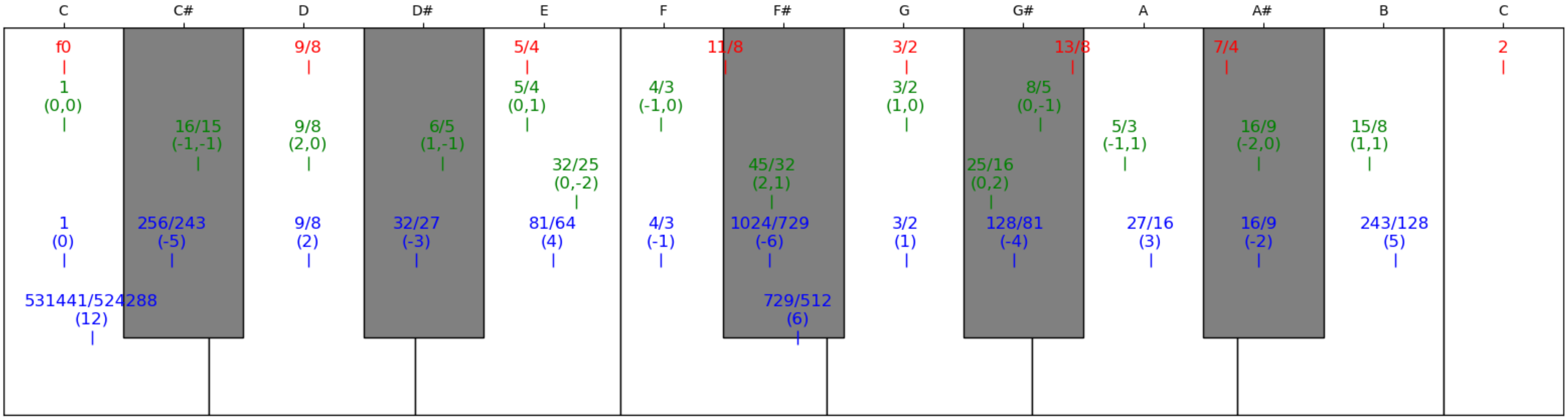
Tonnetz

Tuning Systems

Equal Temperament
Overtones
Just Intonation: (Fifths, Thirds)
Pythagorean Tuning: (Fifths)



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Just Intonation

- Tones are tuned in
  - **perfect fifths:** third harmonic / factor 3
  - **major thirds:** fifth harmonic / factor 5

  above the fundamental.
- Each note can be specified by a triplet of integers

$$\textbf{(octaves, fifths, thirds)} \rightarrow f_0 \cdot 2^{\text{octaves}} \cdot 3^{\text{fifths}} \cdot 5^{\text{thirds}}$$

  specifying the number of steps of the respective intervals.
- In many cases, we ignore the octave and only consider the **pitch class**.

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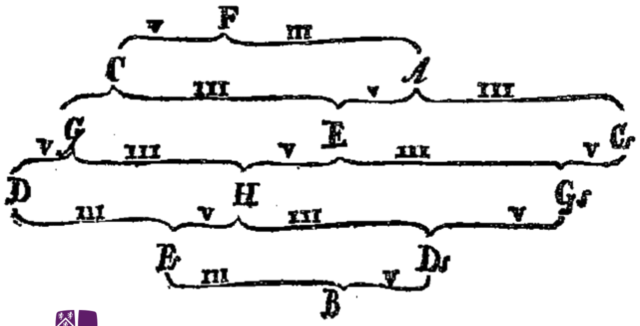
Just Intonation → Tonnetz

Major Thirds	Line of Fifths																					
	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
-2	E♭♭♭,,	B♭♭♭,,	F♭♭,,	C♭♭,,	G♭♭,,	D♭♭,,	A♭♭,,	E♭♭,,	B♭♭,,	F♭,,	C♭,,	G♭,,	D♭,,	A♭,,	E♭,,	B♭,,	F,,	C,,	G,,	D,,	A,,	E,,	B,,
-1	G♭♭,	D♭♭,	A♭♭,	E♭♭,	B♭♭,	F♭,	C♭,	G♭,	D♭,	A♭,	E♭,	B♭,	F,	C,	G,	D,	A,	E,	B,	F♯,	C♯,	G♯,	D♯,
0	**B♭♭**	**F♭**	**C♭**	**G♭**	**D♭**	**A♭**	**E♭**	**B♭**	**F**	**C**	**G**	**D**	**A**	**E**	**B**	**F♯**	**C♯**	**G♯**	**D♯**	**A♯**	**E♯**	**B♯**	**F♯♯**
1	D♭'	A♭'	E♭'	B♭'	F'	C'	G'	D'	A'	E'	B'	F♯'	C♯'	G♯'	D♯'	A♯'	E♯'	B♯'	F♯♯'	C♯♯'	G♯♯'	D♯♯'	A♯♯'
2	F''	C''	G''	D''	A''	E''	B''	F♯''	C♯''	G♯''	D♯''	A♯''	E♯''	B♯''	F♯♯''	C♯♯''	G♯♯''	D♯♯''	A♯♯''	E♯♯''	B♯♯''	F♯♯♯''	C♯♯♯''
3	A'''	E'''	B'''	F♯'''	C♯'''	G♯'''	D♯'''	A♯'''	E♯'''	B♯'''	F♯♯'''	C♯♯'''	G♯♯'''	D♯♯'''	A♯♯'''	E♯♯'''	B♯♯'''	F♯♯♯'''	C♯♯♯'''	G♯♯♯'''	D♯♯♯'''	A♯♯♯'''	E♯♯♯'''



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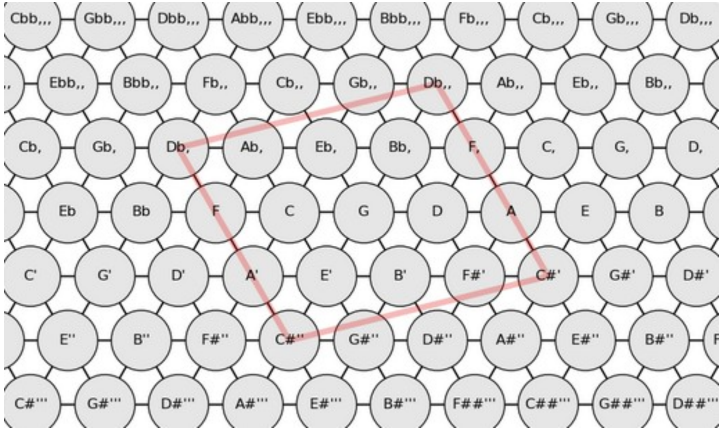
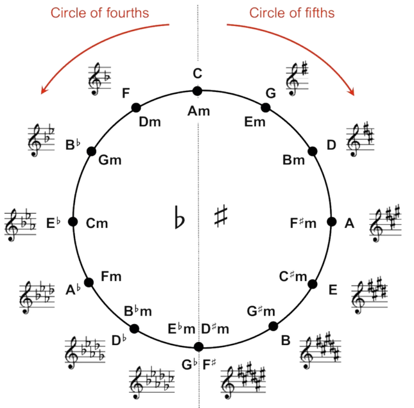
Euler (1739)





Just Intonation → Tonnetz

- Diatonic Scales
  - segments of length 7 along the line of fifths
  - different tonics possible (church modes: Ionian/major, Dorian, Phrygian, Lydian, Mixolydian, Aeolian/minor, Locrian)
- Spelled Pitch / Western Pitch Notation
  - basic names alphabetic according to Aeolian/minor
  - when shifting, cycle basic names and add ♯/♭
- Pitches in just intonation are compact region around fundamental
  - but differen “unit cells” containing all 12 enharmonically equivalent pitches are possible
- Grid can be made more compact by going from rectangular to triangular
  - two possibilities, but minor third is more consonant than minor second

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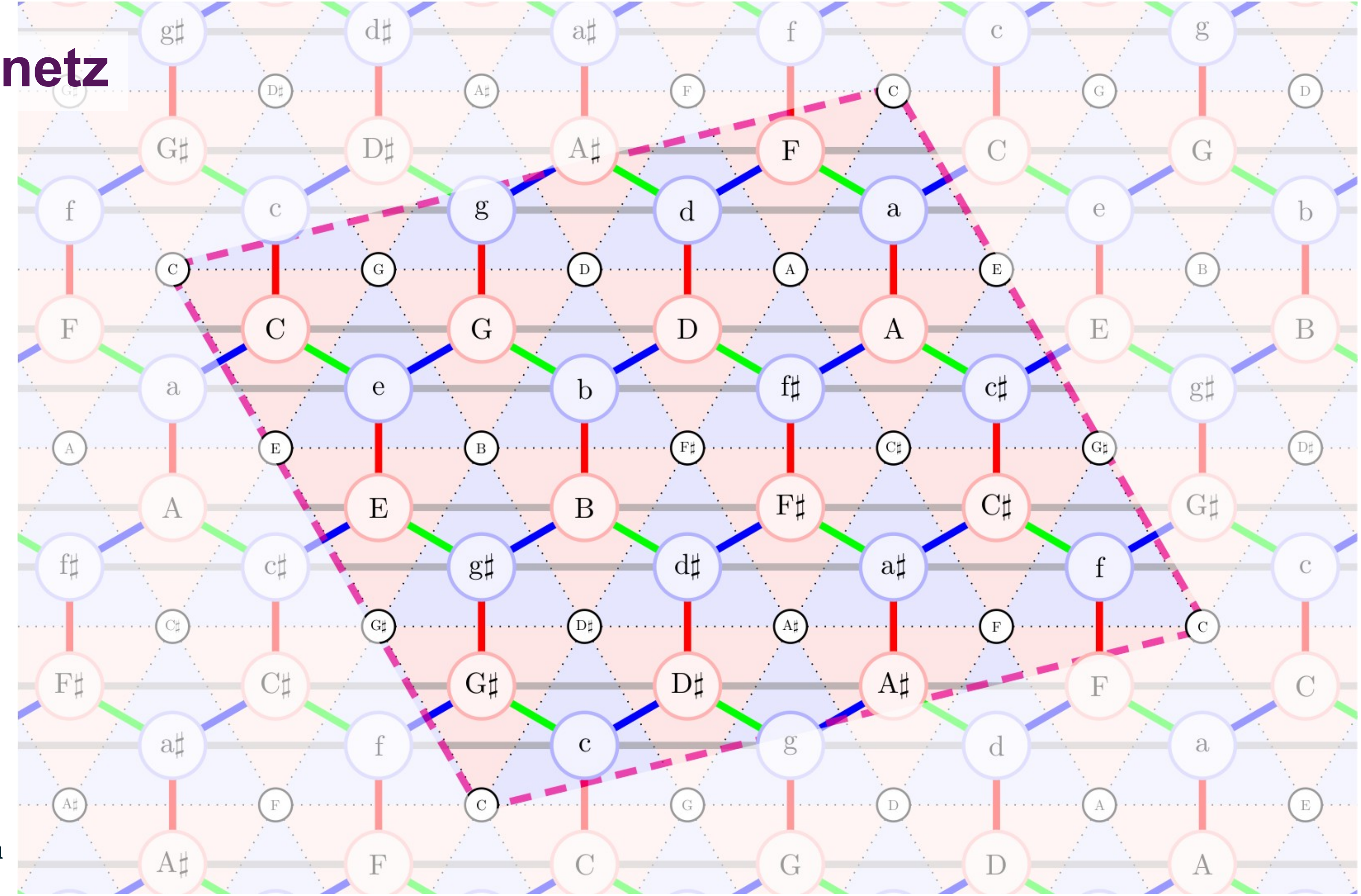
Tonnetz



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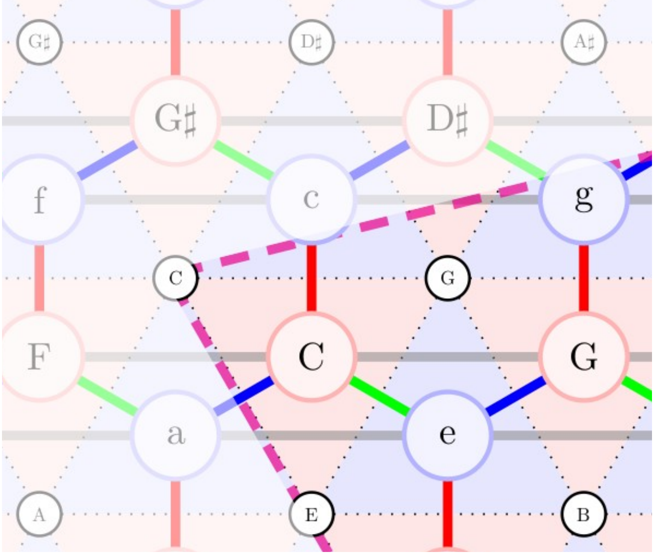
Tonnetz

- Two versions (equivalent/dual)
  - **pitch-based:** pitches as hexagonal faces (triads at corners)
  - **triad-based:** triads as triangular faces (pitches at corners)
- Neo-Riemannian operations with minimal voice-leading
  - Relative major/minor
    C major (C, E, G) ↔ a minor (A, C, E)
  - Parallel major/minor
    C major (C, E, G) ↔ c minor (C, Eb, G)
  - Leading-Tone Exchange
    C major (C, E, G) ↔ e minor (E, G, B)



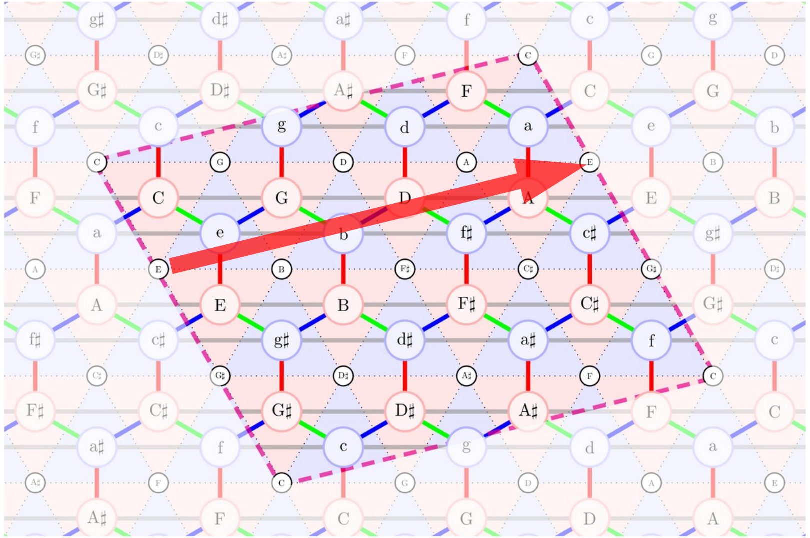
Pythagorean Tuning

- Tones are tuned in **perfect fifths** above/below the fundamental.
- Each note can be specified by a pair of integers

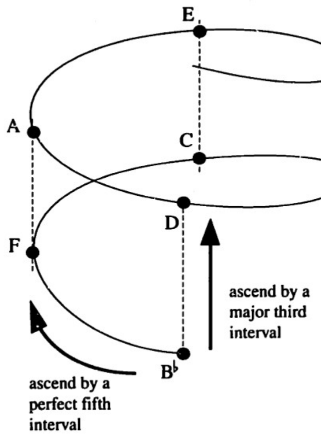
$$\textbf{(octaves, fifths)} \rightarrow \mathbf{f_0 \cdot 2^{octaves} \cdot 3^{fifths}}$$

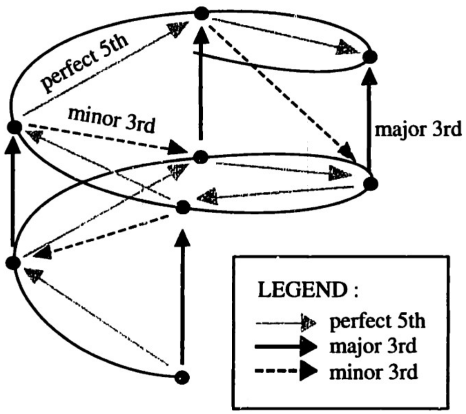
- **Pitch classes** are defined by the number of steps along the line of fifths.
- **Ignore syntonic comma**

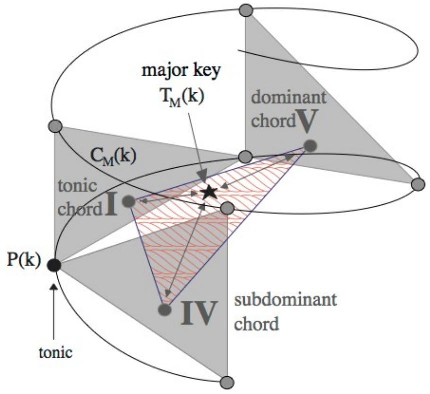
  → roll up to match pitches that differ by syntonic comma

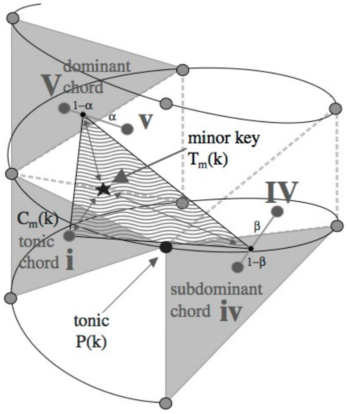


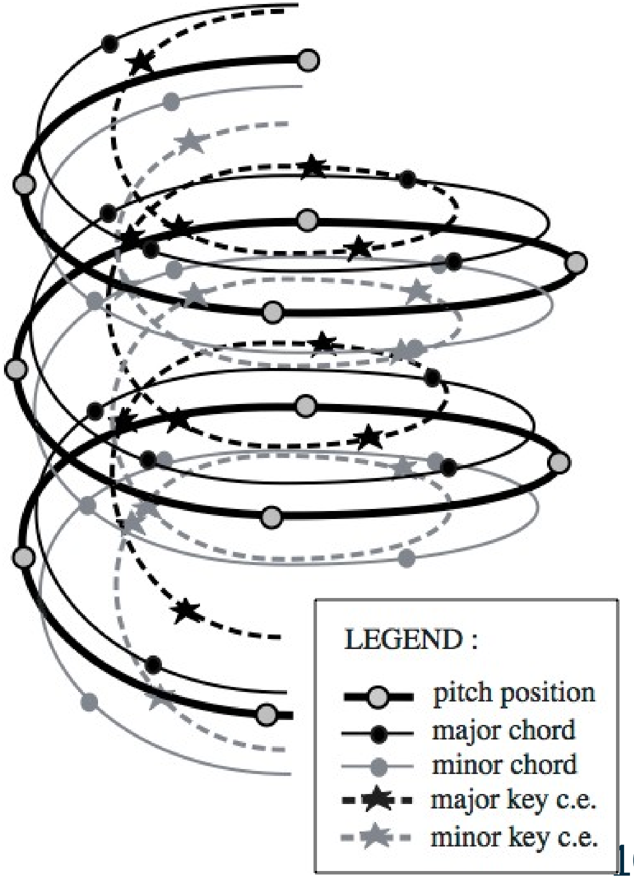
Spiral Array (Chew 2000)











Spiral Array (Chew 2000)

- Line of fifths is a spiral around the tube’s axis.
- Pitch classes along the axis differ by a major third.
- Spiral of major/minor triads as means of the involved pitch classes.
- Spiral of major/minor keys as means of tonic, sub-dominant, and dominant chords:
  - major: (I, IV, V)
  - minor: (i, iv/IV, v/V) with parameters α and β interpolating between v/V and iv/IV, respectively

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Equal Temperament

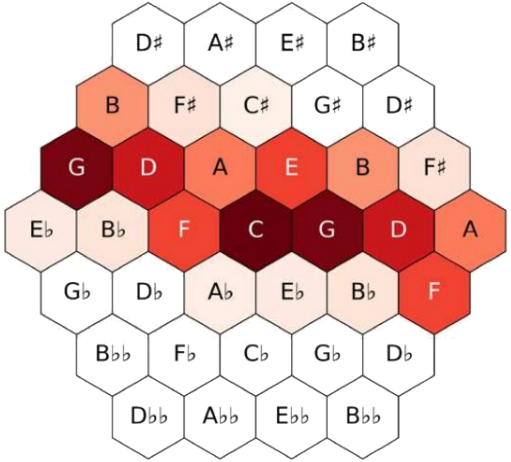
- **Ignore Pythagorean comma**
  → roll up to match pitches that differ by Pythagorean comma
- This gives us a toroidal shape:

Tonal Diffusion Model

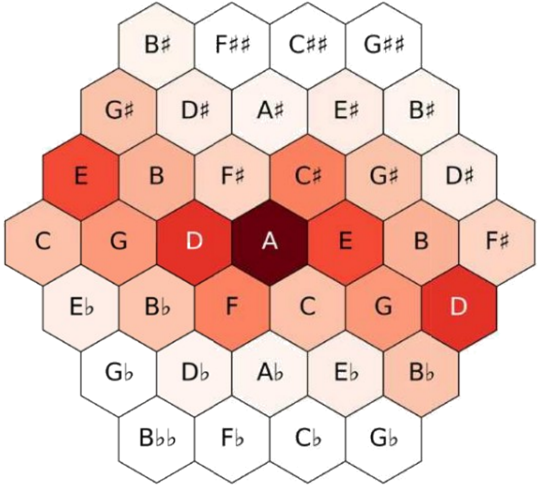
(Lieck et al. 2020, Lieck et al. 2024)

Tonal Diffusion Model

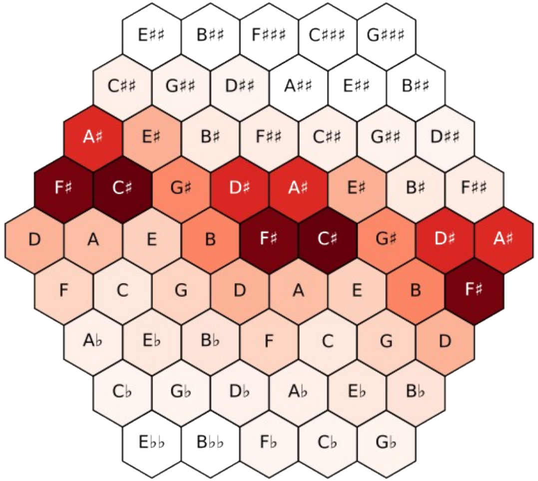
- We can use the Tonnetz to visualise (tonal/spelled) pitch class distributions
- Most pieces look like they have a **centre** from which probability **diffuses**



(a) Johann Sebastian Bach, Prelude in C major, BWV 846 (1722, Baroque).



(b) Ludwig van Beethoven, Sonata op. 31, no. 2 in D minor ‘Tempest’, 1st mov. (1802, Classical).



(c) Franz Liszt, *Bénédiction de Dieu dans la Solitude*, S. 173/3 (1853, Romantic).

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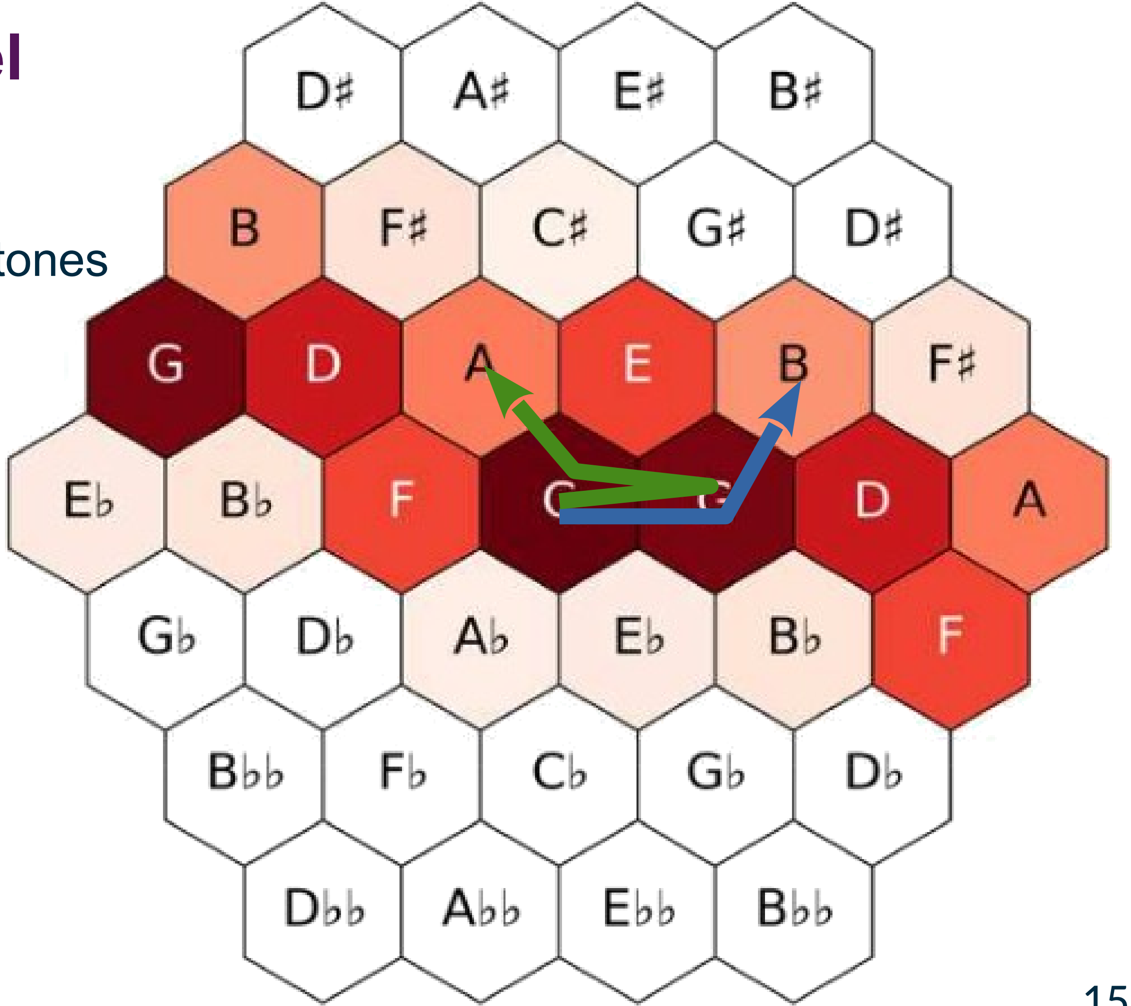
Tonal Diffusion Model

Basic Idea

- look at global distribution ("tones in a bag")
- tones are generated by starting at the centre and taking a number of steps on the Tonnetz
- this generative process defines (with specific parameters) a distribution on the Tonnetz



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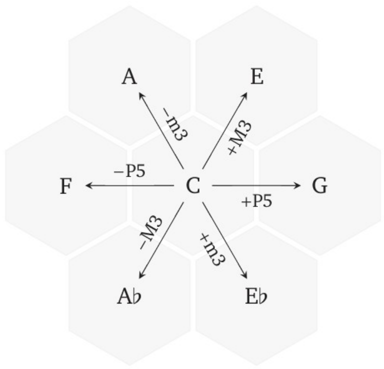
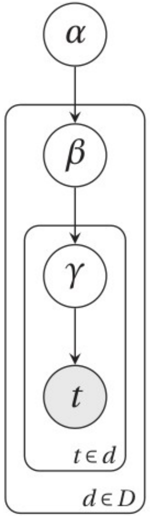
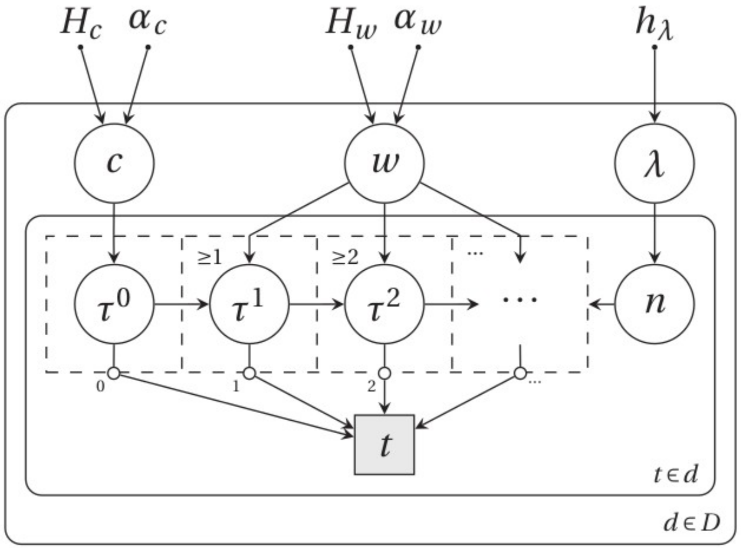
Tonal Diffusion Model – Topic Models



- Parameters/variables on different levels
  → corpus (α), piece (β), event (γ)
  - learn: corpus/piece parameters (α/β)
  - marginalise out: event variables (γ)



corpus-level parameters (α)
  $H_c, \alpha_c, H_w, \alpha_w, h_\lambda$
piece-level variables (β)
  c : distribution over tonal centers
  w : distribution over intervals
  λ : distribution over path lengths
latent variables (γ)
  τ^i : intermediate tones
  n : number of steps
observed variables
  t : tones in a piece



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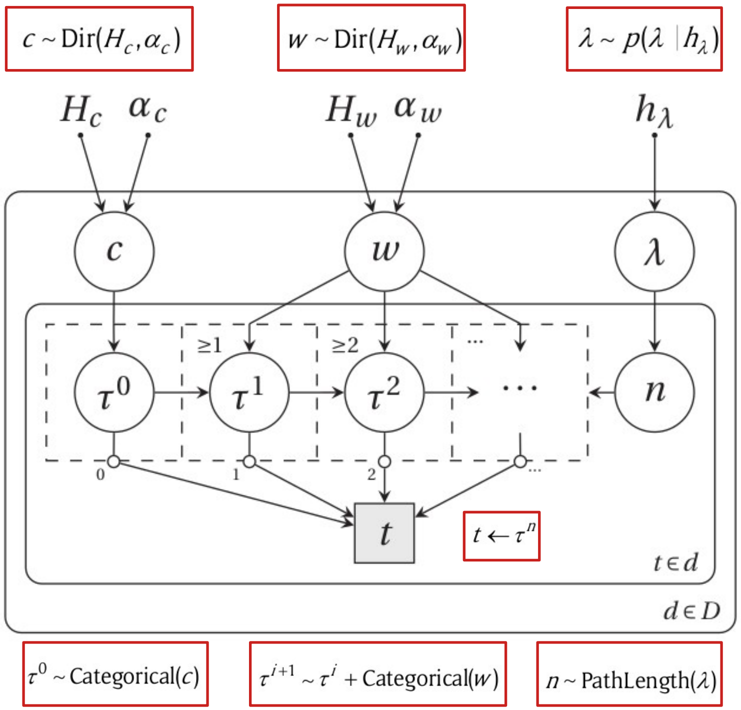
Tonal Diffusion Model



Compute pitch-class distribution as follows

- for fixed parameters *c*, *w*, *γ*
  - get initial distribution over τ^0
- iteratively compute distribution for k^{th} step
  - start at τ^0
  - compute transition probabilities using *w*
  - iterate using dynamic programming
- simultaneously sum over *n*.

→ Implement in PyTorch and backprop to optimise parameters

$$p(t \mid c,w,\lambda) = \sum_{n=0}^{\infty} p(n \mid \lambda) \underbrace{\sum_{\tau^{n-1}} p(\tau^n \mid \tau^{n-1},w) \overbrace{\sum_{\tau^{n-2}} p(\tau^{n-1} \mid \tau^{n-2},w) \cdots \sum_{\tau^0} p(\tau^1 \mid \tau^0,w)\, p(\tau^0 \mid c)}^{p(\tau^{n-1} \mid c,w)}}_{p(\tau^n \mid c,w)}$$

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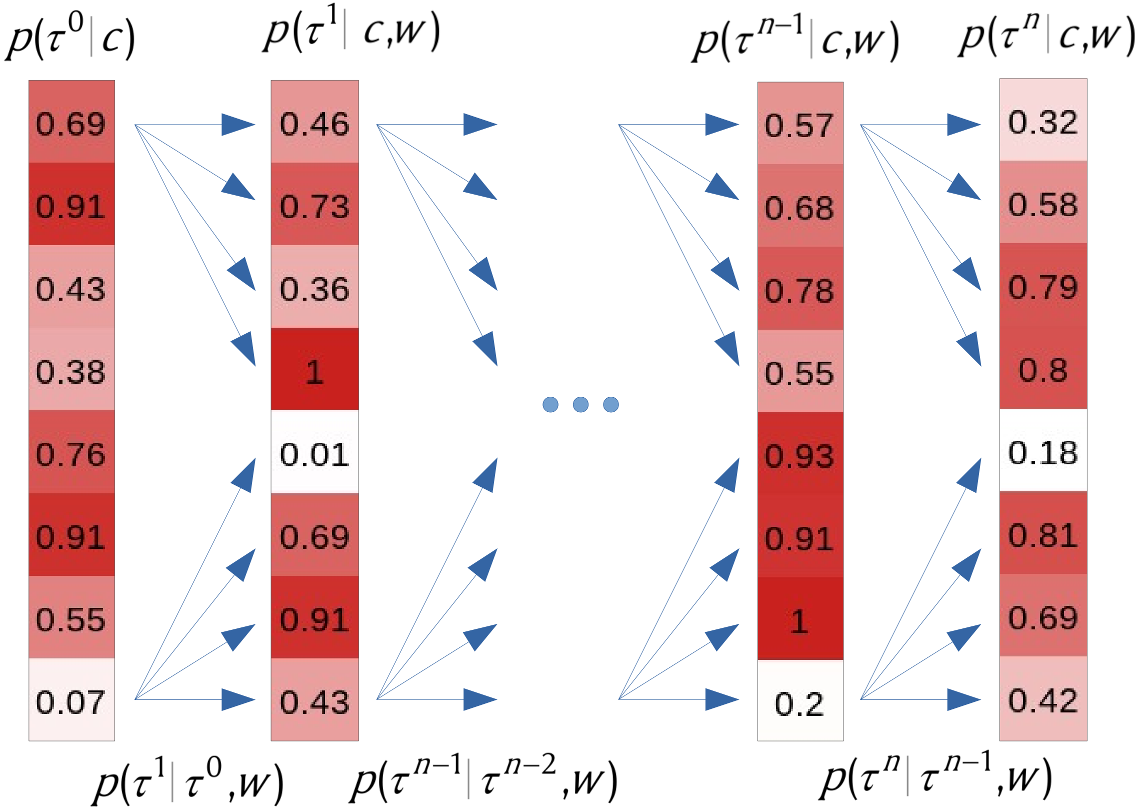
Tonal Diffusion Model



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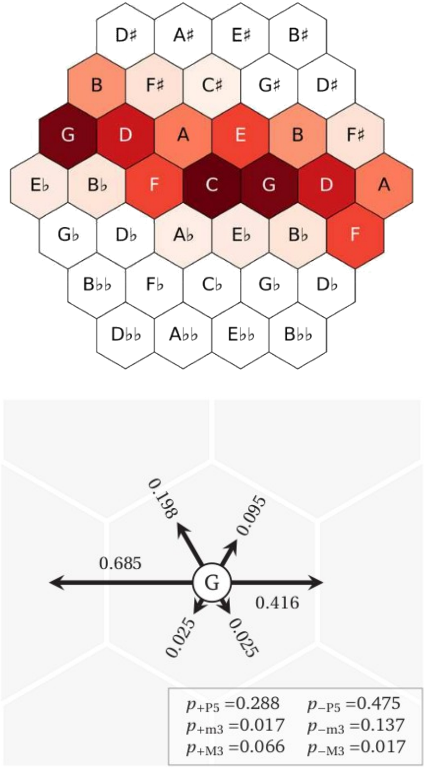
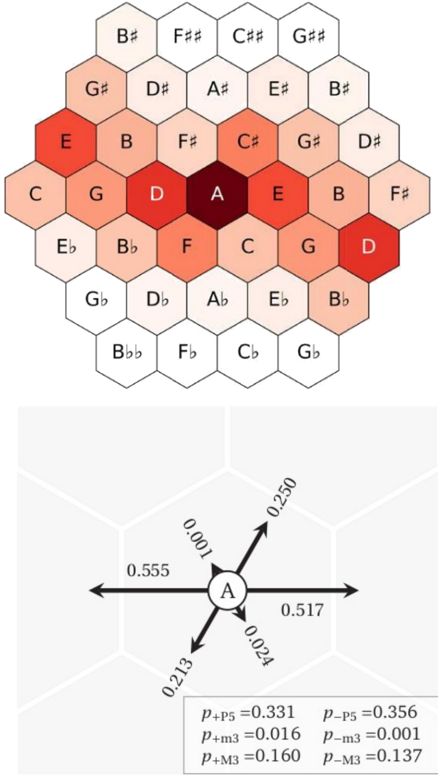
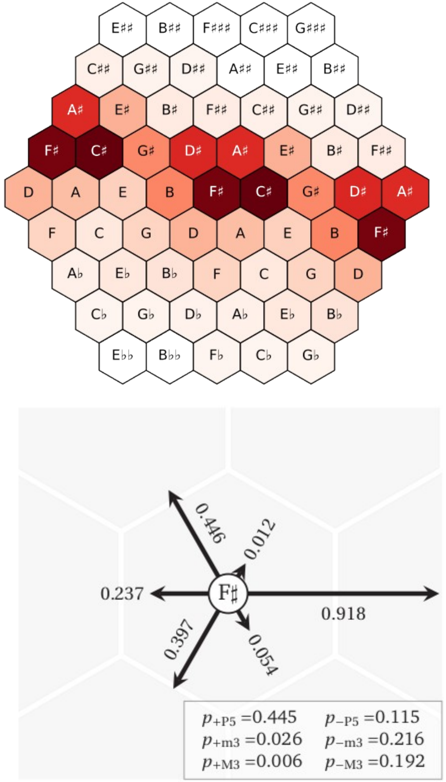
Tonal Diffusion Model



(a) Bach, C major Prelude ($\mu = 1.44, \sigma = 0.69$).



(b) Beethoven, 'Tempest' Sonata ($\mu = 1.56, \sigma = 0.76$).



(c) Liszt, *Bénédiction de Dieu dans la Solitude* ($\mu = 2.06, \sigma = 1.25$).

References

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