

Musical Informatics: Music Alignment



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Outline

- Music Alignment
 - Applications and Examples
 - Music Representation and Features
 - Dynamic Time Warping
- Automatic Accompaniment demo

Music Alignment

Same music, different interpretations

- Different versions of the same piece/song that correspond to the same musical work!



Symbolic Recordings of Piano Performances

- Late 19th Century: Player Pianos!
 - Pianolas and piano rolls
 - Pneumatic mechanism
 - Music recorded on perforated paper: Piano-rolls
- 21st Century: Computer Controlled Grand Pianos
 - MIDI and derived formats
 - Yamaha, Bösendorfer, Steinway!
 - Competitions
 - Datasets



A player piano roll being played by Draconichiaro - Own work, CC BY-SA 4.0, <https://commons.wikimedia.org/w/index.php?curid=82604752>

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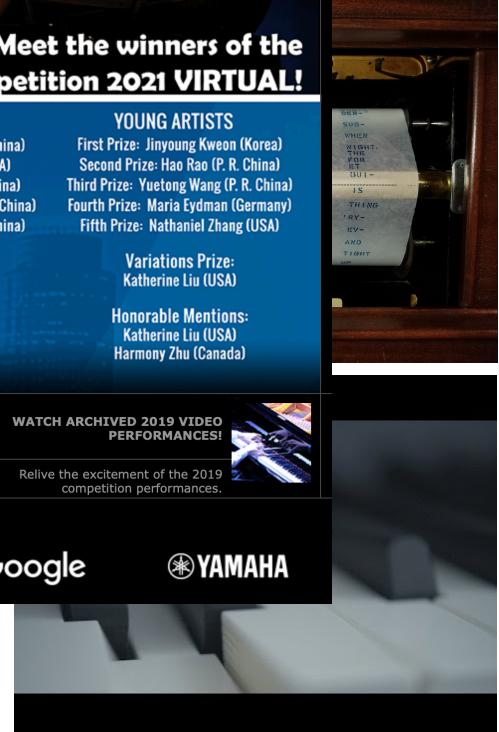
Symbolic Recordings

- Late 19th Century: Player Pianos
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The screenshot shows the homepage of the e-Piano Junior Competition 2021 VIRTUAL website. At the top, there's a banner for the competition with the text "e-Piano Junior Competition 2021 VIRTUAL! Minneapolis, MN. USA". To the right, it says "SCHOOL OF MUSIC UNIVERSITY OF MINNESOTA Driven to Discover". Below the banner, there are navigation links: HOME, GENERAL INFORMATION, CONTESTANT INFORMATION, COMPETITION HISTORY, CURRENT COMPETITION, and LIVESTREAM!. A video player in the center displays a young pianist performing. To the right of the video, the text "Meet the winners of the e-Piano Junior Competition 2021 VIRTUAL!" is displayed. The page is divided into several sections listing award winners:

- SPARKS**
 - First Prize: Taige Wang (USA)
 - Second Prize: Thannapas Luangpitpong (Thailand)
 - Third Prize: Sihong Li (P. R. China)
 - Fourth Prize: Isabel Feng (USA)
 - Fifth Prize: Masanobu Pires (USA)
- ASPIRES**
 - First Prize: Yanyan Bao (P. R. China)
 - Second Prize: Xinran Shi (USA)
 - Third Prize: Jiayou Xu (P. R. China)
 - Fourth Prize: Xuanxiang Wu (P. R. China)
 - Fifth Prize: Zhiwei Chen (P. R. China)
- YOUNG ARTISTS**
 - First Prize: Jinyoung Kweon (Korea)
 - Second Prize: Hao Rao (P. R. China)
 - Third Prize: Yuetong Wang (P. R. China)
 - Fourth Prize: Maria Eydmann (Germany)
 - Fifth Prize: Nathaniel Zhang (USA)
- Mendelssohn Prize:** Taige Wang (USA)
- Schubert Prize:** Yanyan Bao (P. R. China)
- Honorable Mentions:** Zhiyu Fang (P. R. China), Papitcha Vejmongkolkorn (Thailand)
- Honorable Mentions:** Edison Chen (USA), Seokyung Hong (Korea)
- Variations Prize:** Katherine Liu (USA)
- Honorable Mentions:** Katherine Liu (USA), Harmony Zhu (Canada)

Below these sections, there are three call-to-action buttons: "CONGRATULATIONS TO THE WINNERS OF THE 2021 COMPETITION!", "WATCH ARCHIVED 2019 VIDEO PERFORMANCES!", and "See the entire list of participants here." Logos for the School of Music (University of Minnesota) and the International piano-e-competition are at the bottom, along with Google and Yamaha logos.



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Symbolic Recordings

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How do machines listen?: Humans vs. Computers

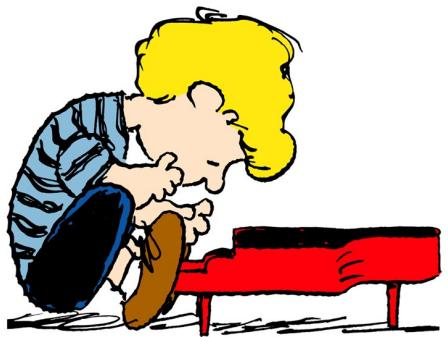
- Computers “perceive” audio signals in a different way than humans

How do machines listen?: Humans vs. Computers

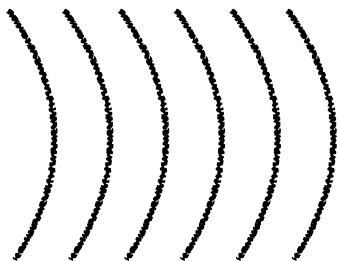
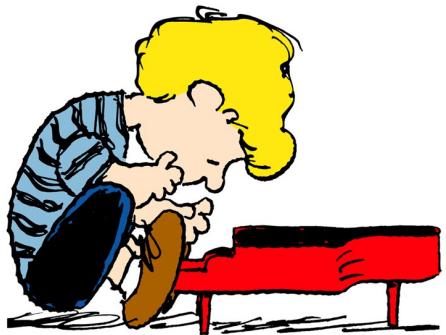
- Computers “perceive” audio signals in a different way than humans
 - Human brains are marvelous information processing machines that allow us to **make sense** of auditory signals (almost) effortlessly

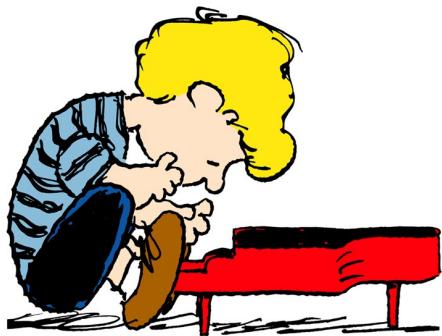
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- Computers “perceive” audio signals in a different way than humans
 - Human brains are marvelous information processing machines that allow us to **make sense** of auditory signals (almost) effortlessly
- Computers struggle performing basic tasks!

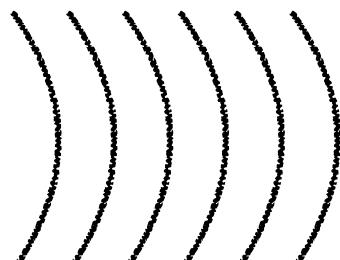


Pressure Wave





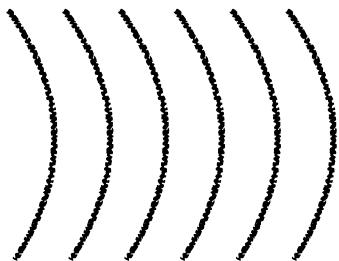
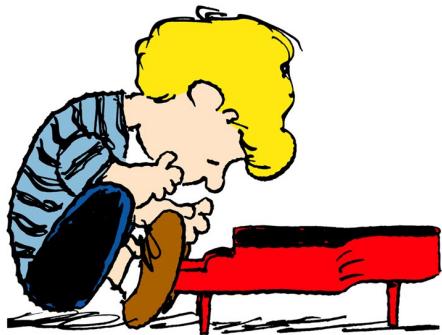
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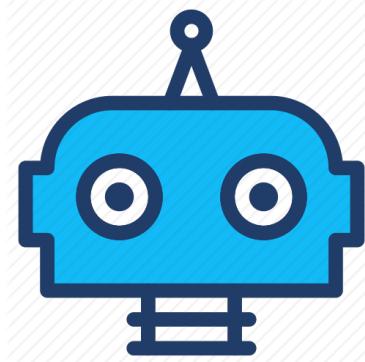
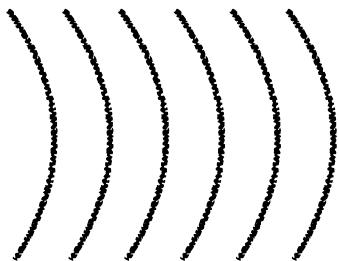
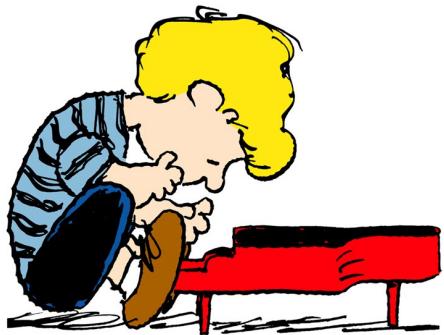
Music



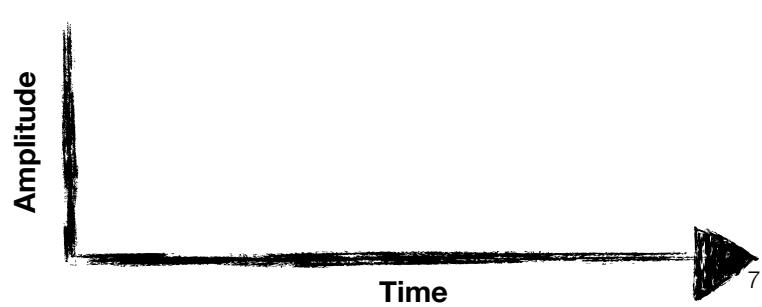
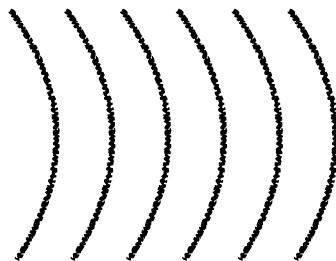
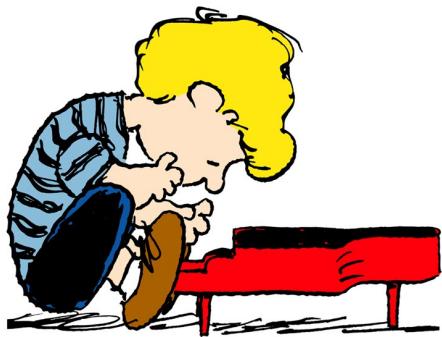
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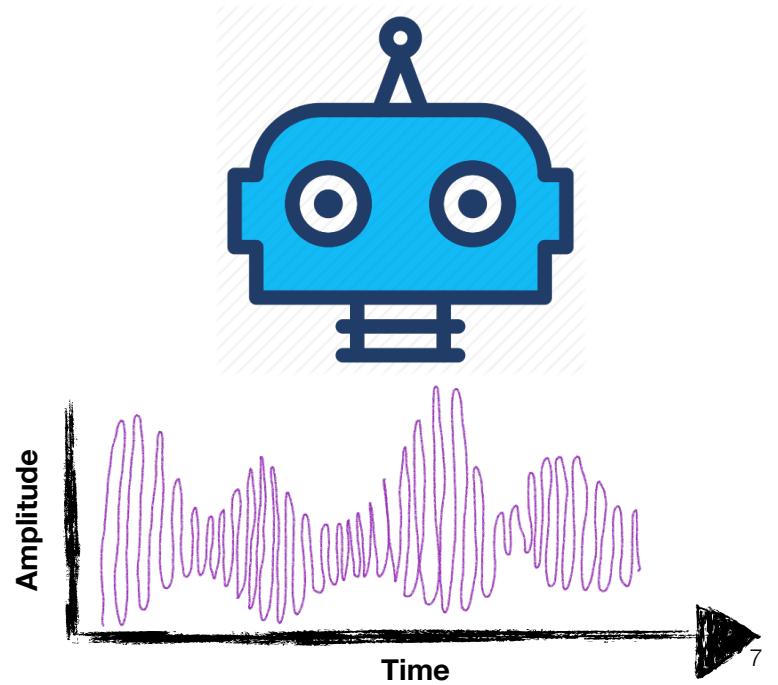
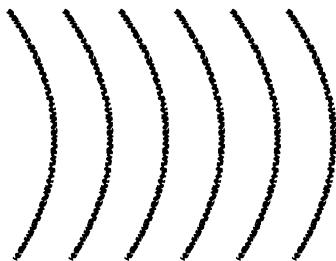
Pressure Wave



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 - Ideally, we would like to develop **computational models** that can identify or **predict some of the same kinds of patterns** in music that (expert) **human listeners** would perceive.

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 - Developing such models will require us to understand more about **human music perception**.

Machine Listening

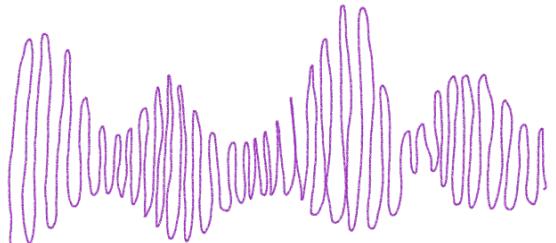
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 - Ideally, we would like to develop **computational models** that can identify or **predict some of the same kinds of patterns** in music that (expert) **human listeners** would perceive.
 - Developing such models will require us to understand more about **human music perception**.
 - [Herrera et al. 2009]: “We will only develop music understanding systems by means of understanding music understanding”.

Automatic Music Alignment

Score

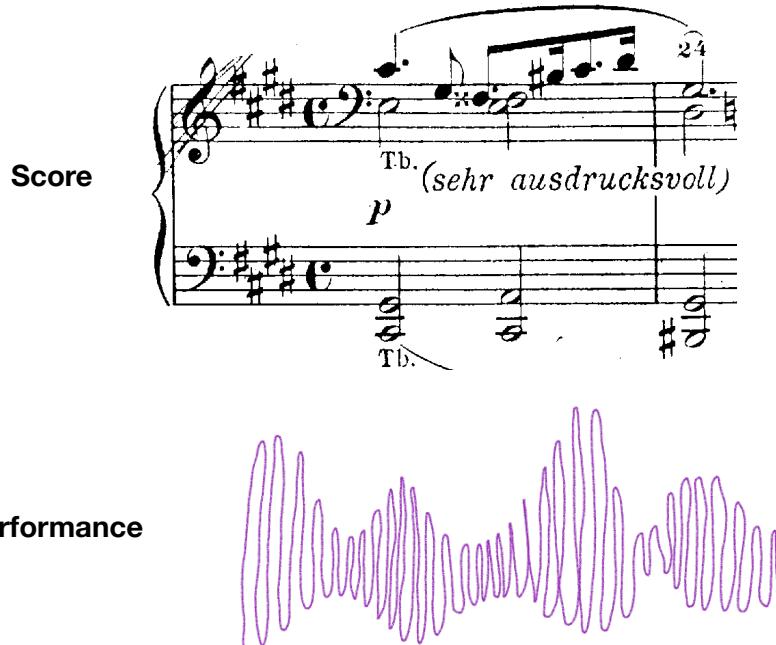


Performance



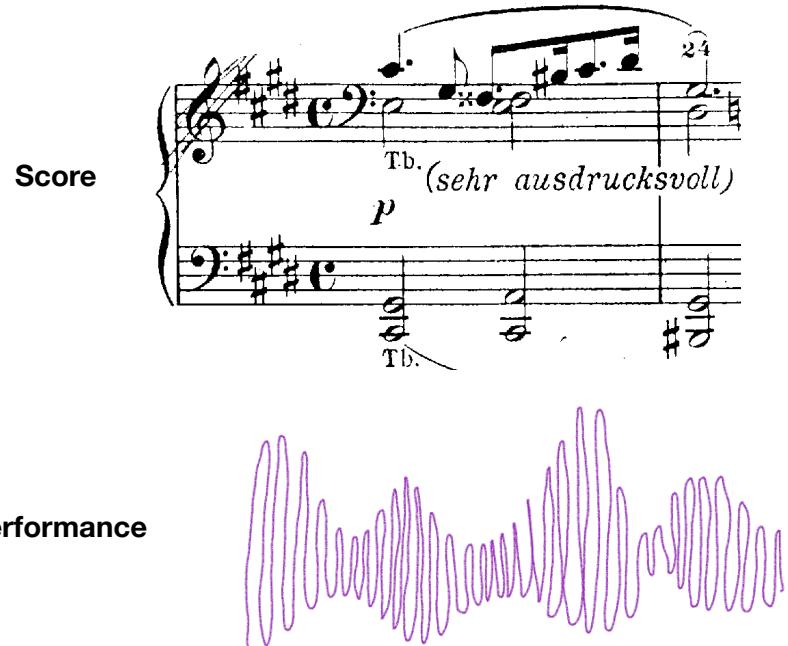
Automatic Music Alignment

- Matching **musical signals** of the **same piece**



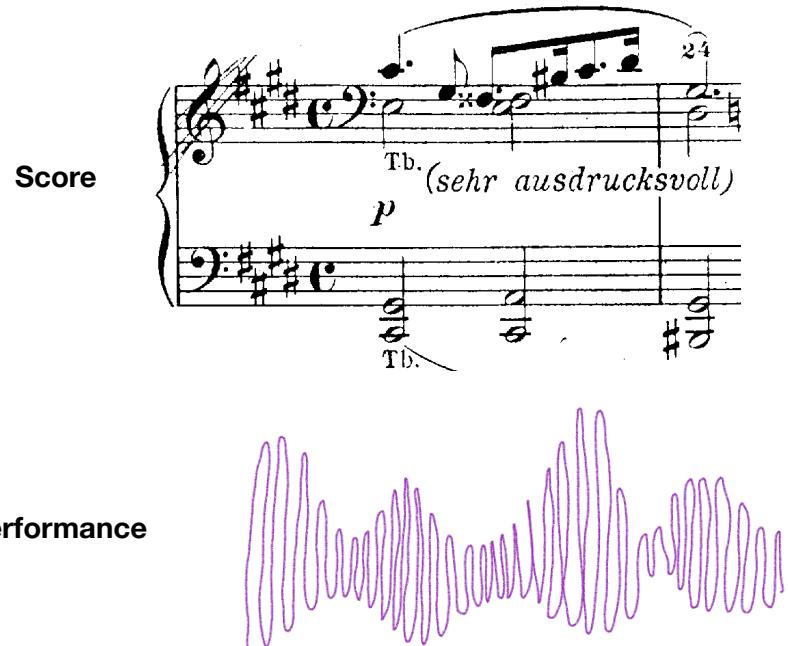
Automatic Music Alignment

- Matching **musical signals** of the **same piece**
 - Different performances of the same piece



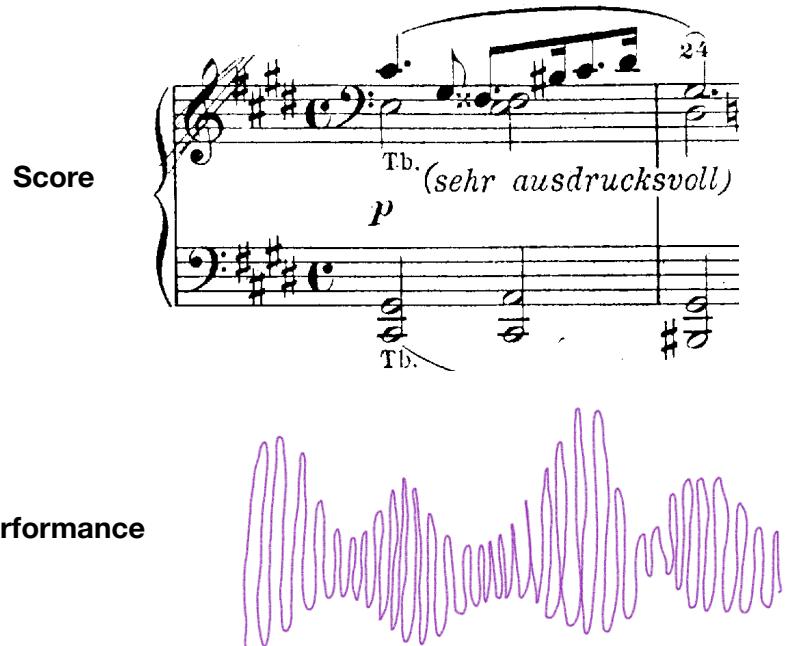
Automatic Music Alignment

- Matching **musical signals** of the **same piece**
 - Different performances of the same piece
 - A performance with its score



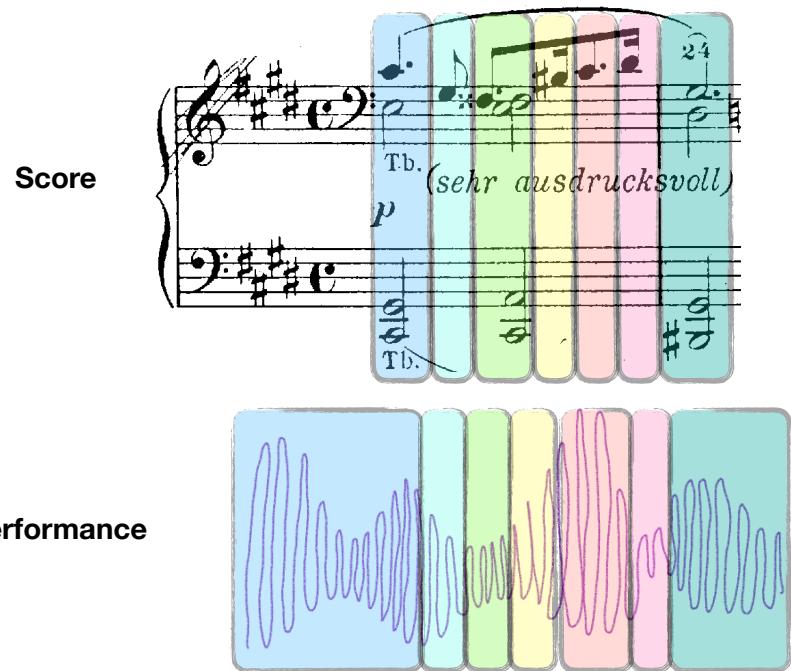
Automatic Music Alignment

- Matching **musical signals** of the **same piece**
 - Different performances of the same piece
 - A performance with its score
- If it is in real time, it is referred to as **score following**

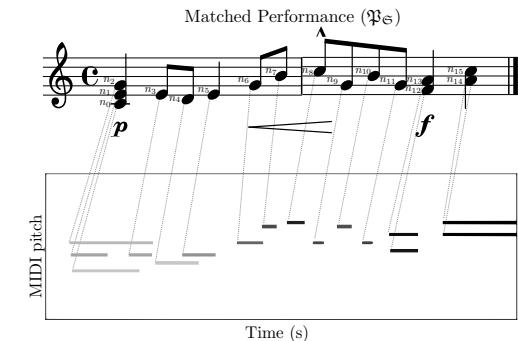
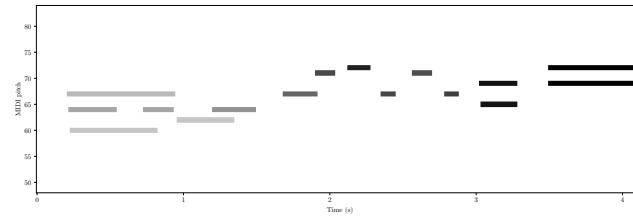
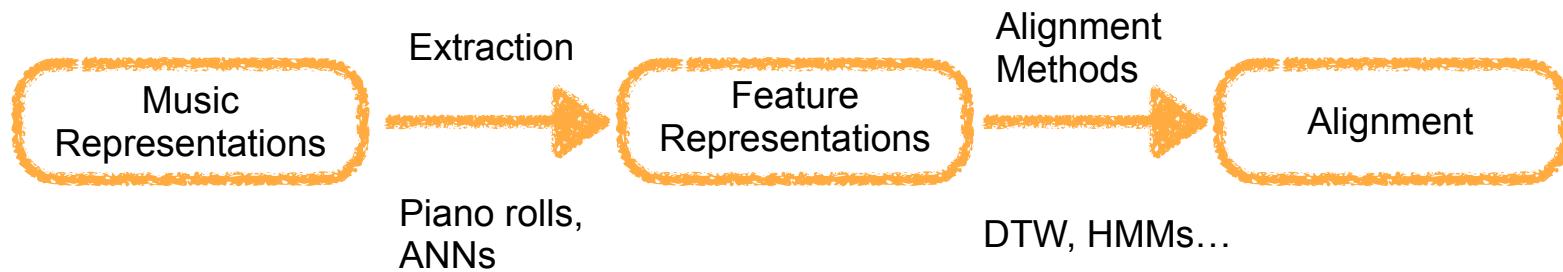


Automatic Music Alignment

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Music Alignment Pipeline



MELD: Music Encoding and Linker

TROMPA

Piano

var I

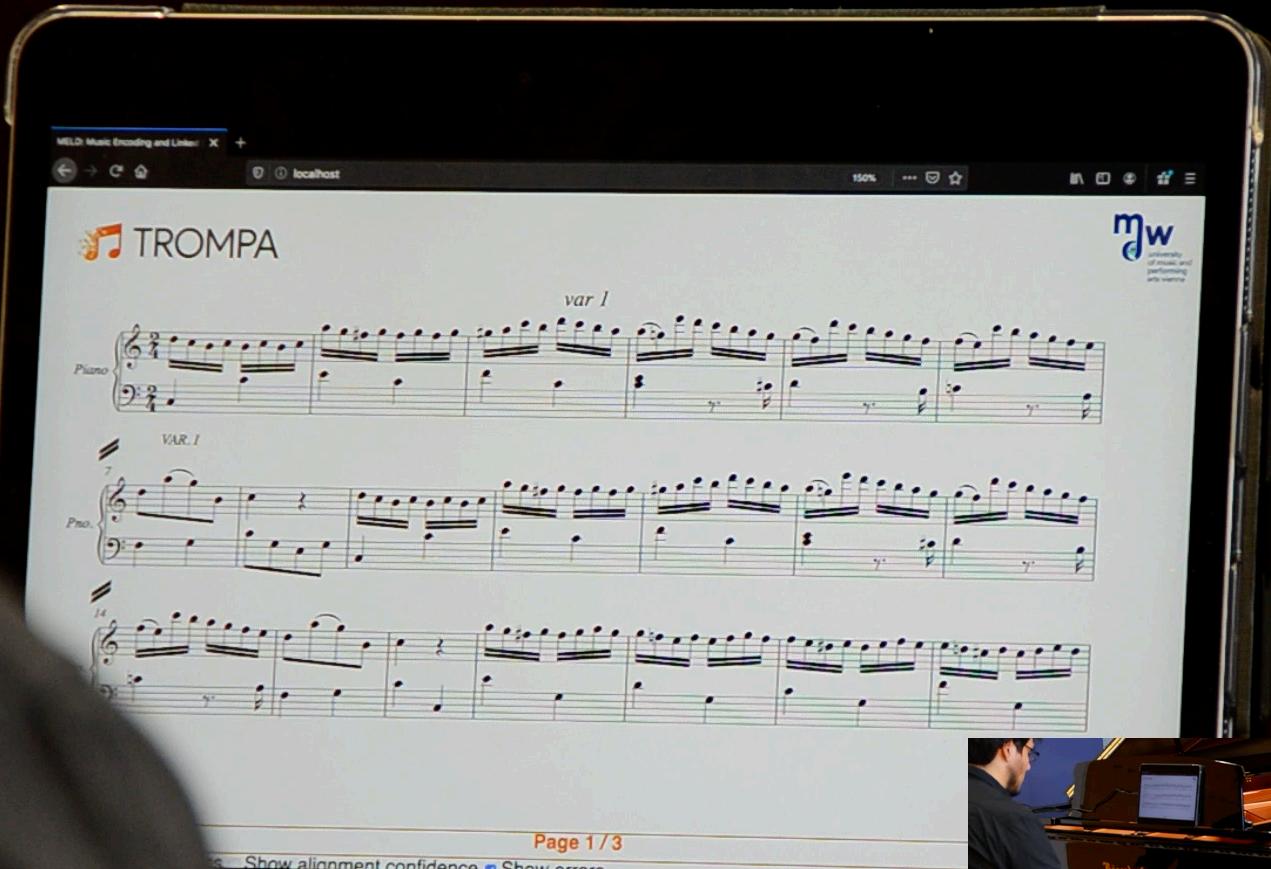
VAR. I

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14

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Show alignment confidence Show errors



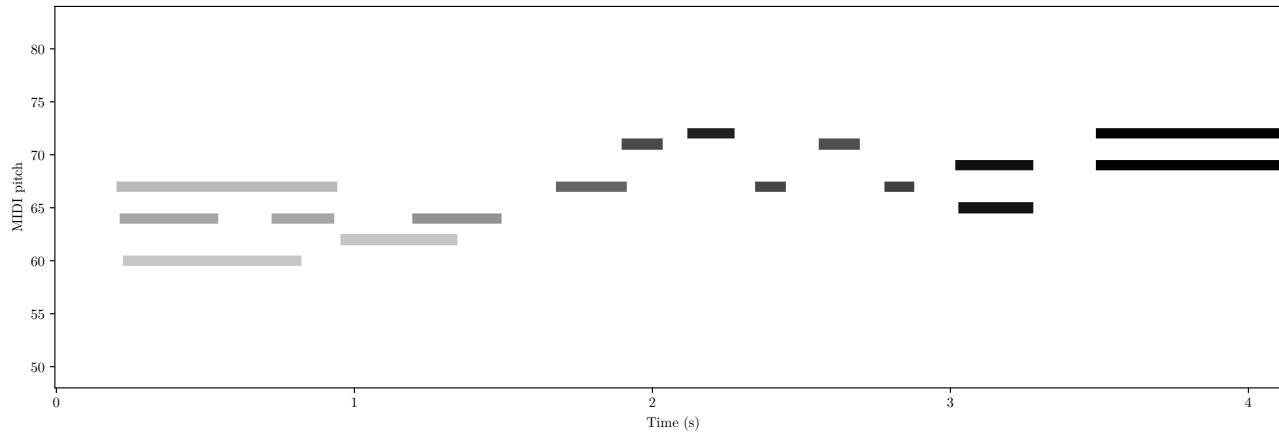
Music Representation and Features

Features

- To make data comparable and algorithmically accessible, the first step is to extract suitable features that capture relevant aspects while suppressing irrelevant details

The Piano Roll

- A 2D representation of (MIDI) pitch and time
- We can extract piano rolls from MIDI and MusicXML files with Partitura!



Other Features

- Piano rolls are not the only (or the best) features for alignment
 - Neural networks: Features learned from Autoencoders (Lattner et al., 2018) that are transposition invariant
 - Pitch Class distribution
 - and many more!

Dynamic Time Warping

Dynamic Time Warping

- Dynamic Programming Approach
- Find an **optimal** alignment between two given (time-dependent) sequences (under certain restrictions)
- **Sequences:** $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ and $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_M\}$
 - for other applications, sequences could be discrete signals, feature sequences, sequences of characters
- **Feature Space:** $\mathbf{x}_n, \mathbf{y}_m \in \mathcal{F}$ for $n \in [1 : N]$ and $m \in [1 : M]$
- **Local cost measure:** $c : \mathcal{F} \times \mathcal{F} \mapsto \mathbb{R}$
 - Basic intuition: $c(\mathbf{x}, \mathbf{y})$ is small if \mathbf{x} and \mathbf{y} are similar

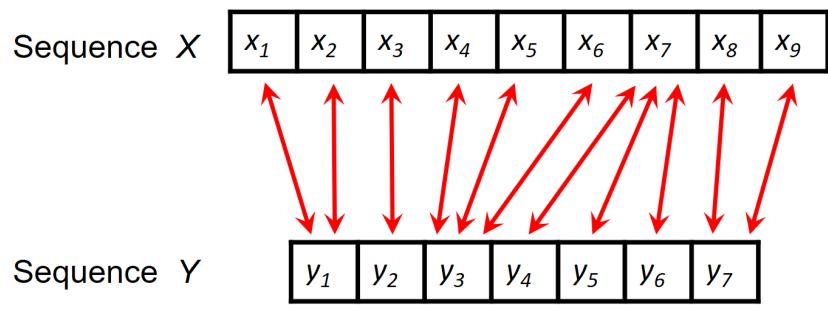
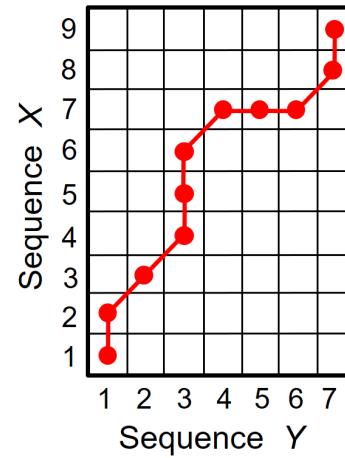


Figure 3.12 from [Müller, FMP, Springer 2015]



Cost Matrix

- The cost matrix $\mathbf{C} \in \mathbb{R}^{N \times M}$ evaluates the local cost

$$\mathbf{C}[i, j] = c(\mathbf{x}_i, \mathbf{y}_j)$$

- The **choice** of the local cost measure **matters!**

- Euclidean Distance: $c(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_i (x_i - y_i)^2}$
- Manhattan Distance: $c(\mathbf{x}, \mathbf{y}) = \sum_i |x_i - y_i|$
- Cosine Distance: $c(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x}^T \mathbf{y}}{||\mathbf{x}|| \cdot ||\mathbf{y}||}$
- Others?

Warping Path

- An (N, M) -**warping path** of length $L \in \mathbb{N}$ is a sequence

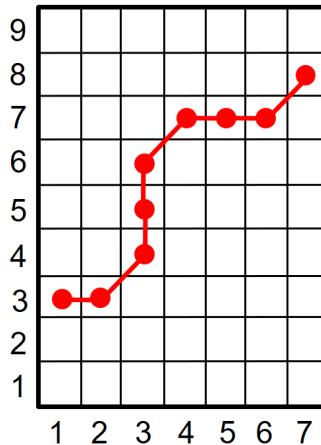
$$P = (p_1, \dots, p_l)$$

with $p_l = (n_l, m_l) \in [1 : N] \times [1 : M]$ that defines an alignment between sequences $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ and $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_M\}$

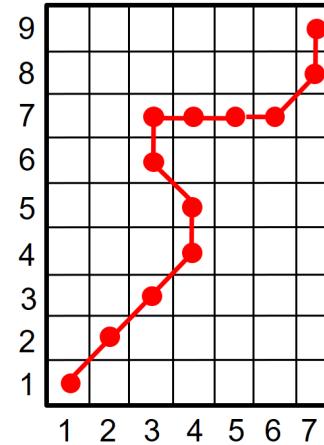
- Three conditions:
 - **Boundary condition:** $p_1 = (1,1)$ and $p_L = (N, M)$
 - **Monotonicity condition:** $n_1 \leq n_2 \leq \dots \leq n_L$ and $m_1 \leq m_2 \leq \dots \leq m_L$
 - **Step size condition:** $p_{l-1} - p_l \in \{(1,0), (0,1), (1,1)\}$

Warping Path

Violates Boundary condition



Violates Monotonicity



Violates Step Size Condition

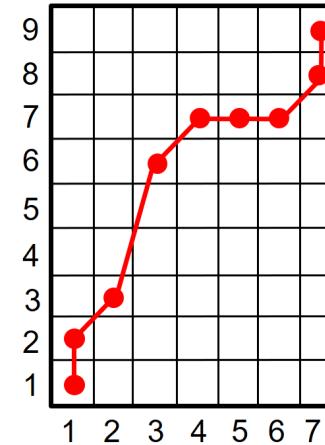


Figure 3.13 from [Müller, FMP, Springer 2015]

Optimal Warping Path

- Total cost of a warping path:

$$c_P(\mathbf{X}, \mathbf{Y}) = \sum_{l=1}^L C(n_l, m_l)$$

- **Optimal warping path:** minimal total cost!

$$\text{DTW}(\mathbf{X}, \mathbf{Y}) = c_{P^*}(\mathbf{X}, \mathbf{Y})$$

- $= \min\{c_P(\mathbf{X}, \mathbf{Y}) \mid P \text{ is an } (N, M)\text{-warping path}\}$

- **DTW(\mathbf{X}, \mathbf{Y}): DTW distance**

- Not a real **distance**: is symmetric, but not positive definite

- $\text{DTW}(\mathbf{X}, \mathbf{Y}) = 0$ does not necessarily mean that $\mathbf{X} = \mathbf{Y}$!

Dynamic Programming Algorithm

- Dynamic Programming: Break down a problem into simpler subproblems and then combine the solution!
- Idea: Find optimal warping paths for subsequences of \mathbf{X} and \mathbf{Y} , and then combine the solutions recursively!
- Accumulated Cost Matrix
 - $\mathbf{D}(n, m) = \text{DTW}(\mathbf{X}(1 : n), \mathbf{Y}(1 : m))$

Algorithm

Algorithm: DTW

Table 3.2 from [Müller, FMP, Springer 2015]

Input: Cost matrix \mathbf{C} of size $N \times M$

Output: Accumulated cost matrix \mathbf{D}
Optimal warping path P^*

Procedure: Initialize $(N \times M)$ matrix \mathbf{D} by $\mathbf{D}(n, 1) = \sum_{k=1}^n \mathbf{C}(k, 1)$ for $n \in [1 : N]$ and $\mathbf{D}(1, m) = \sum_{k=1}^m \mathbf{C}(1, k)$ for $m \in [1 : M]$. Then compute in a nested loop for $n = 2, \dots, N$ and $m = 2, \dots, M$:

$$\mathbf{D}(n, m) = \mathbf{C}(n, m) + \min \{\mathbf{D}(n - 1, m - 1), \mathbf{D}(n - 1, m), \mathbf{D}(n, m - 1)\}.$$

Set $\ell = 1$ and $q_\ell = (N, M)$. Then repeat the following steps until $q_\ell = (1, 1)$:

Increase ℓ by one and let $(n, m) = q_{\ell-1}$.

If $n = 1$, then $q_\ell = (1, m - 1)$,

else if $m = 1$, then $q_\ell = (n - 1, m)$,

else $q_\ell = \operatorname{argmin} \{\mathbf{D}(n - 1, m - 1), \mathbf{D}(n - 1, m), \mathbf{D}(n, m - 1)\}$.

(If ‘ argmin ’ is not unique, take lexicographically smallest cell.)

Set $L = \ell$ and return $P^* = (q_L, q_{L-1}, \dots, q_1)$ as well as \mathbf{D} .

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{C} = \left[\begin{array}{c} \\ \\ \\ \\ \\ \end{array} \right]$$

Pen and Paper Example!

Inputs

$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$\mathbf{C} = \begin{bmatrix} 0 \\ \vdots \end{bmatrix}$$

Pen and Paper Example!

Inputs

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$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

$$C_{12} = |x_1 - y_2| = |3 - 1| = 2$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 \\ \end{bmatrix}$$

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$$\mathbf{X} = \{3, 9, 8\}$$

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$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

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$$C_{13} = |x_1 - y_3| = |3 - 9| = 6$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 \\ \end{bmatrix}$$

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$$C_{14} = |x_1 - y_4| = |3 - 9| = 6$$

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$$C_{15} = |x_1 - y_5| = |3 - 7| = 4$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 \end{bmatrix}$$

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$$C_{16} = |x_1 - y_6| = |3 - 8| = 5$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \end{bmatrix}$$

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$$C_{15} = |x_1 - y_5| = |3 - 7| = 4$$

$$C_{16} = |x_1 - y_6| = |3 - 8| = 5$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & & & & & \end{bmatrix}$$

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

$$C_{12} = |x_1 - y_2| = |3 - 1| = 2$$

$$C_{13} = |x_1 - y_3| = |3 - 9| = 6$$

$$C_{14} = |x_1 - y_4| = |3 - 9| = 6$$

$$C_{15} = |x_1 - y_5| = |3 - 7| = 4$$

$$C_{16} = |x_1 - y_6| = |3 - 8| = 5$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & & & & \end{bmatrix}$$

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & & & \end{bmatrix}$$

Pen and Paper Example!

Inputs

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & & \end{bmatrix}$$

Pen and Paper Example!

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 \end{bmatrix}$$

Pen and Paper Example!

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$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \end{bmatrix}$$

Pen and Paper Example!

Inputs

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$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Pen and Paper Example!

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$$C_{16} = |x_1 - y_6| = |3 - 8| = 5$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1} \quad D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \left[\begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \right]$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1} \quad D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 \\ 6 \\ 11 \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & & & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2, 8, 8) = 2$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2, 8, 8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8, 14, 2) = 2$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

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$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2, 8, 8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8, 14, 2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2, 8, 8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8, 14, 2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

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$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

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$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

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Initialization

$$D_{i,1} = \sum_{k=1}^i C_{k,1}$$

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$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

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$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

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$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

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Initialization

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$$D_{1,i} = \sum_{k=1}^i C_{1,k}$$

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$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0, 2, 6) = 8$$

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$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8, 14, 2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14, 18, 2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18, 23, 4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3, 9, 8\}$$

$$\mathbf{Y} = \{3, 1, 9, 9, 7, 8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3, 6)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2, 3)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3, 5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2, 4)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_5 = \arg \min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_5 = \arg \min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_5 = \arg \min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$$

$$q_6 = (1, q_5[1] - 1) = (1,1)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_5 = \arg \min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$$

$$q_6 = (1, q_5[1] - 1) = (1,1)$$

Pen and Paper Example III

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$$q_1 = (3,6)$$

$$q_2 = \arg \min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$$

$$q_3 = \arg \min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$$

$$q_4 = \arg \min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$$

$$q_5 = \arg \min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$$

$$q_6 = (1, q_5[1] - 1) = (1,1)$$

$$\mathbf{P}^* = \{(1,1), (1,2), (2,3), (2,4), (3,5), (3,6)\}$$

Automatic Accompaniment

Human—Computer Interaction in Music

Human—Computer Interaction in Music

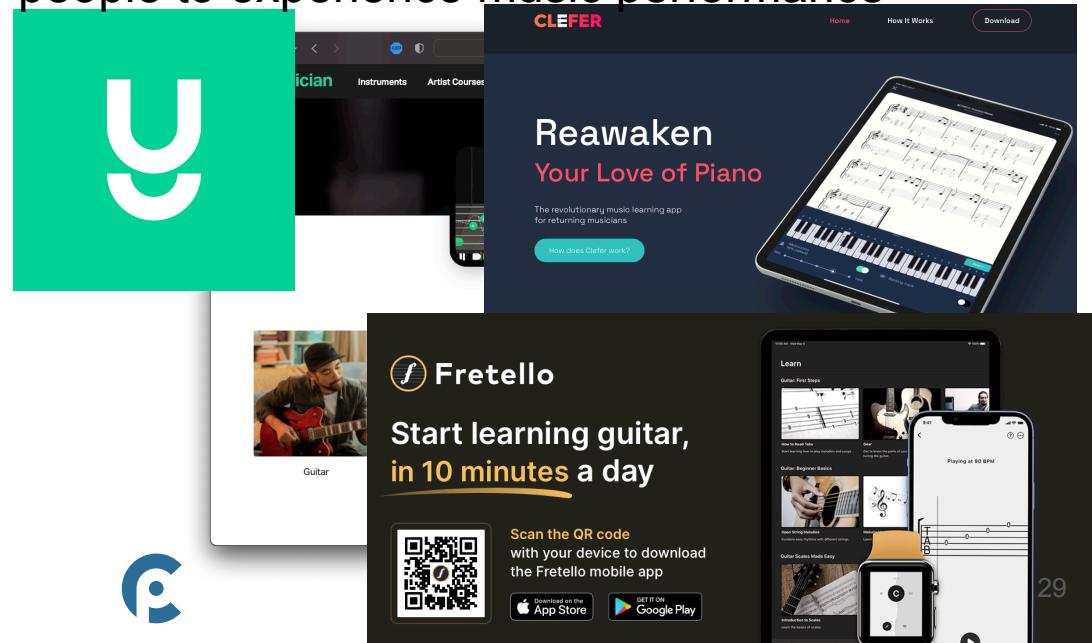
- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning

Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance

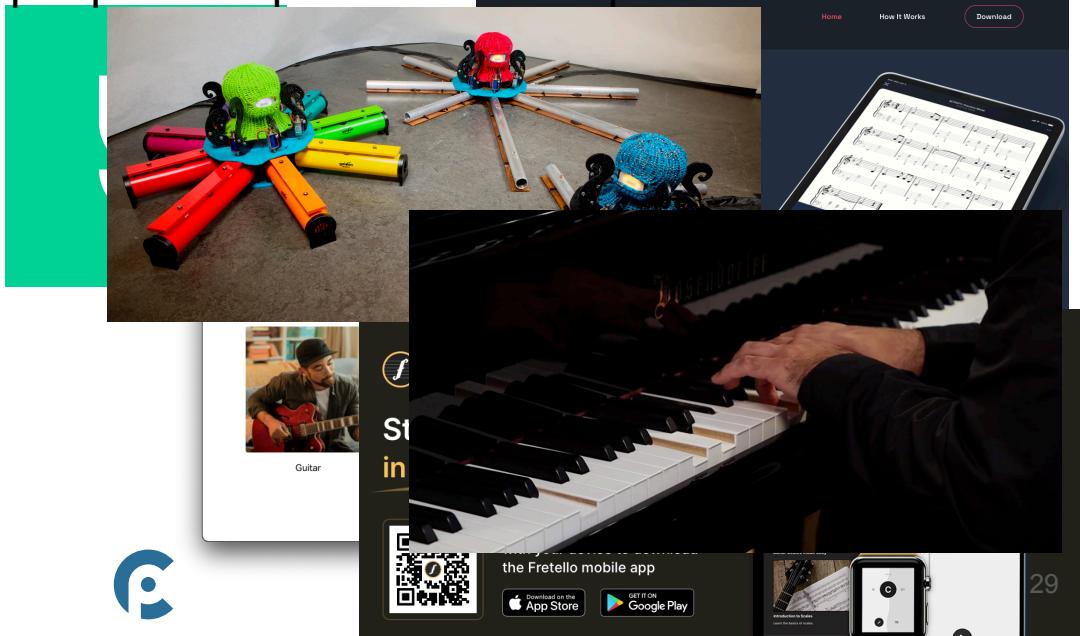
Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance
 - Learning apps



Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance
 - Learning apps
 - Interactive performances



Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance
 - Learning apps
 - Interactive performances
 - NIME

The collage illustrates various applications of Human-Computer Interaction in music:

- A robotic arm with multiple grippers is positioned over a circular array of metallic tubes, suggesting a performance or experimental setup.
- A screenshot of the **Magenta Studio (v1.0)** interface. The page header includes "Home", "How It Works", and "Download". The main content area features the text "Magenta Studio (v1.0)" and "Magenta is a collection of music plugins built on Magenta's open source tools and models. They use cutting-edge machine learning techniques for music generation." Below this, it says "These tools are available both as standalone applications and as plugins for Ableton Live. To find out more information, choose one of the links below:" followed by "Ableton Live Plugins" and "Standalone Applications". At the bottom, five small plugin interface cards are shown: "CONTINUOUSSESS", "GENERATE 4 BARS", "DRUMIFY", "INTERPOLATE", and "GROOVE".
- A graphic for the **2021 New Interfaces for Music and the Arts** conference. It features a stylized orange waveform logo on the left, the year "2021" in large orange digits, and the text "New Interface" and "Online and NYU Shanghai" below it. The date "June 14–18, 2021" is also mentioned.

Automatic Accompaniment Systems

2 (120)

S O N A T E
für zwei Pianoforte
von
W. A. M O Z A R T.
Köch. Verz. № 448.
Serie 19 № 8.

Mozarts Werke.

Allergo con spirto.

Composiet 1784 in Wien.

Pianoforte I.

Pianoforte II.



Automatic Accompaniment Systems

- Artificial co-performer

2 (120)

S O N A T E
für zwei Pianoforte
von
W. A. M O Z A R T.
Köch. Verz. № 448.
Serie 19 № 8.

Allergo con spirto.
Componirt 1784 in Wien.

Pianoforte I.

Pianoforte II.

A musical score for two pianos (Pianoforte I and Pianoforte II) in G major (two sharps). The score consists of two staves of music. Staff 1 (Pianoforte I) starts with a forte dynamic (f) and includes various rhythmic patterns like eighth-note pairs and sixteenth-note chords. Staff 2 (Pianoforte II) also starts with a forte dynamic (f) and features eighth-note patterns. The title "Allergo con spirto." is written above the staves, and a note at the top right indicates the piece was composed in Vienna in 1784.

Automatic Accompaniment Systems

- Artificial co-performer
- (for this work) ensemble performance in there are two (or more) parts, which are **notated** (e.g., Western classical music)



Automatic Accompaniment Systems

- Artificial co-performer
- (for this work) ensemble performance in there are two (or more) parts, which are **notated** (e.g., Western classical music)
- The goal: **computational agent** that performs with a **human soloist**, and **adapts its performance** to the performance style of the soloist



Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

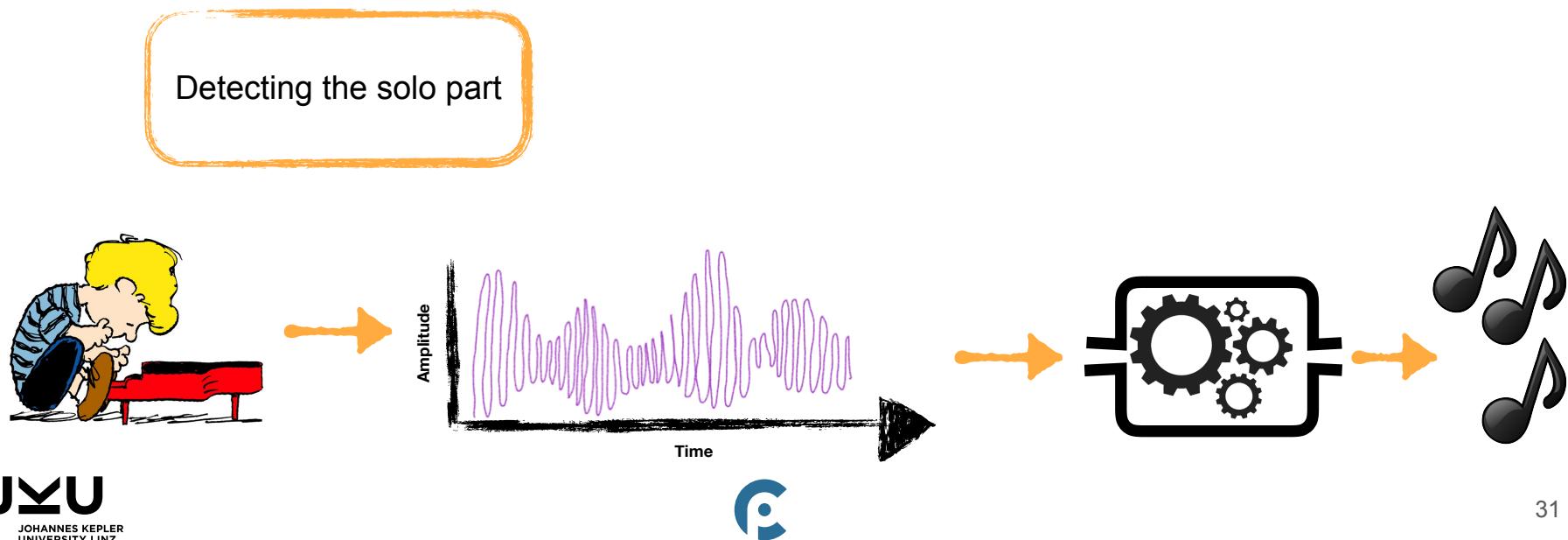
Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Detecting the solo part

Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:



Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Detecting the solo part

Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Detecting the solo part

Score Following

Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Detecting the solo part

Score Following

Generating an expressive accompaniment part

Generating an Expressive Accompaniment Part

Accompaniment Score



Score Features

Note	pitch	beat	p	f	crescendo	A
n_0	60	1	1	0	0.0	0
n_1	63	1	1	0	0.0	0
n_2	67	1	0	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	0	0	0.7	1
n_9	77	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

Soloist

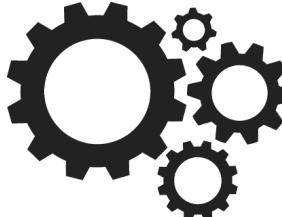


Solo Performance Features

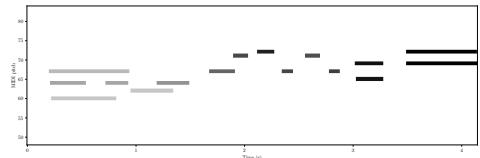
Note	$y_{vel}(\cdot)$	$y_{log\ lips}(\cdot)$	$y_{tim}(\cdot)$	$y_{log\ art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000



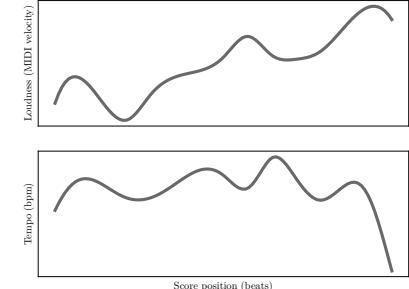
Computational Model (deep neural networks)



Performance



Expressive Parameters



Generating an Expressive Accompaniment Part



Score Features

Note	pitch	beat	p	f	crescendo	A
n_0	60	1	1	0	0.0	0
n_1	63	1	1	0	0.0	0
n_2	67	1	0	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	0	0	0.7	1
n_9	67	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

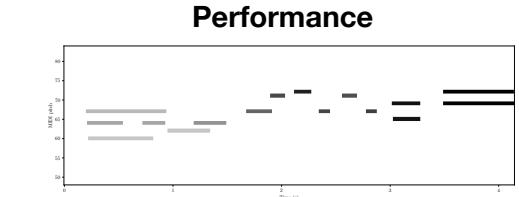
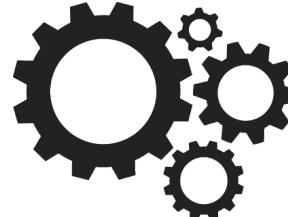


Solo Performance Features

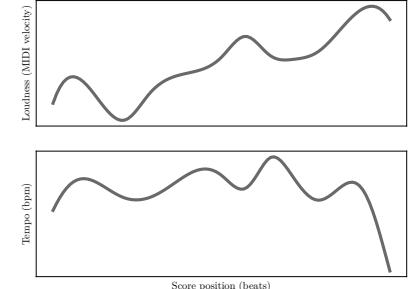
Note	$y_{vel}(\cdot)$	$y_{log\ lips}(\cdot)$	$y_{tim}(\cdot)$	$y_{log\ art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000



Computational Model
(deep neural networks)



Expressive Parameters



Generating an Expressive Accompaniment Part

Accompaniment Score



Soloist



JYU

JOHANNES KEPLER
UNIVERSITY LINZ

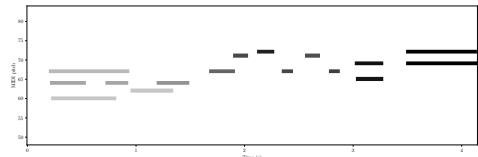
Score Features

Note	pitch	beat	p	f	crescendo	A
n_0	60	1	1	0	0.0	0
n_1	63	1	1	0	0.0	0
n_2	67	1	0	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	1	0	0.7	1
n_9	67	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

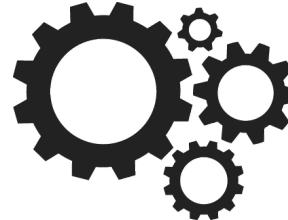
Solo Performance Features

Note	$y_{vel}(\cdot)$	$y_{log\ lips}(\cdot)$	$y_{tim}(\cdot)$	$y_{log\ art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000

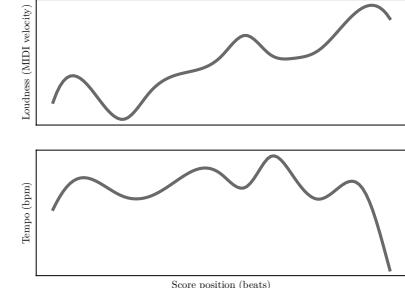
Performance



Computational Model
(deep neural networks)



Expressive Parameters



Generating an Expressive Accompaniment Part

Accompaniment Score



Score Features

Note	pitch	beat	p	f	crescendo	A
n_0	60	1	1	0	0.0	0
n_1	63	1	1	0	0.0	0
n_2	67	1	0	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	0	0	0.7	1
n_9	77	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

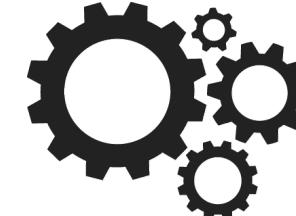
Soloist



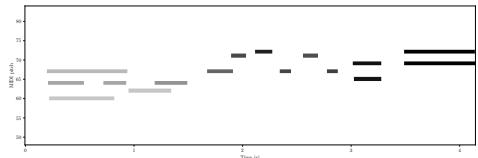
Solo Performance Features

Note	$y_{vel}(\cdot)$	$y_{log\ lips}(\cdot)$	$y_{tim}(\cdot)$	$y_{log\ art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
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n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000

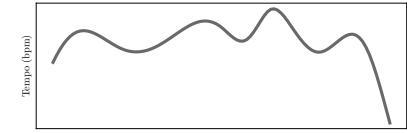
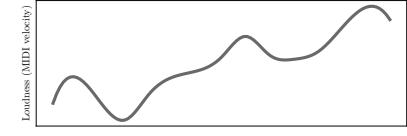
Computational Model
(deep neural networks)



Performance



Expressive Parameters



Generating an Expressive Accompaniment Part

Accompaniment Score



Soloist



JYU

JOHANNES KEPLER
UNIVERSITY LINZ

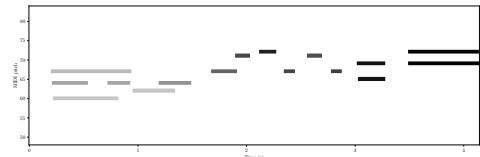
Score Features

Note	pitch	beat	p	f	crescendo	A
n_0	60	1	1	0	0.0	0
n_1	63	1	1	0	0.0	0
n_2	67	1	0	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	0	0	0.7	1
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n_{14}	69	4	0	1	0.0	0
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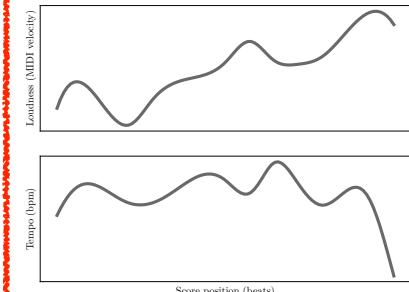
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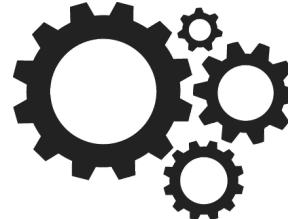
Performance



Expressive Parameters



Computational Model
(deep neural networks)



Generating an Expressive Accompaniment Part

Accompaniment Score



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Soloist

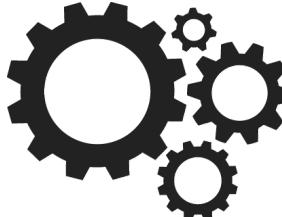


Solo Performance Features

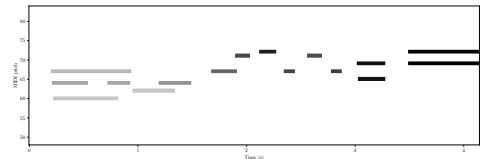
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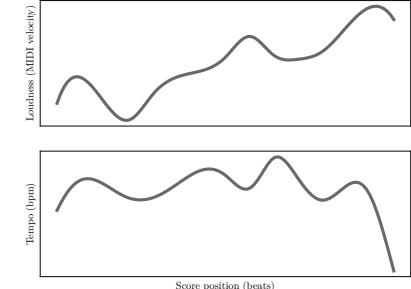
Computational Model (deep neural networks)



Performance



Expressive Parameters



Generating an Expressive Accompaniment Part

Accompaniment Score



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Note	pitch	beat	p	f	crescendo	A
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n_{15}	72	4	0	1	0.0	0

Soloist

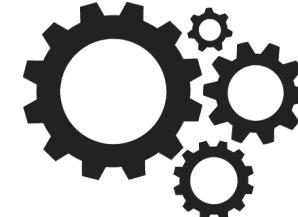


Solo Performance Features

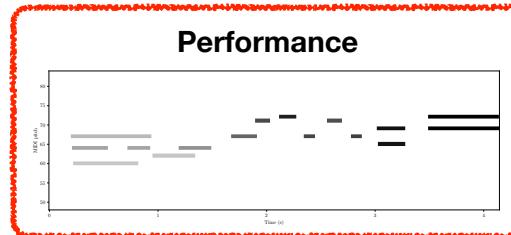
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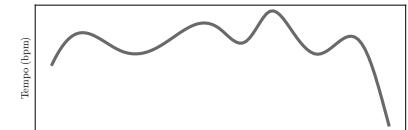
Computational Model (deep neural networks)



Performance



Expressive Parameters



Score position (beats)

The ACCompanion

Score Follower

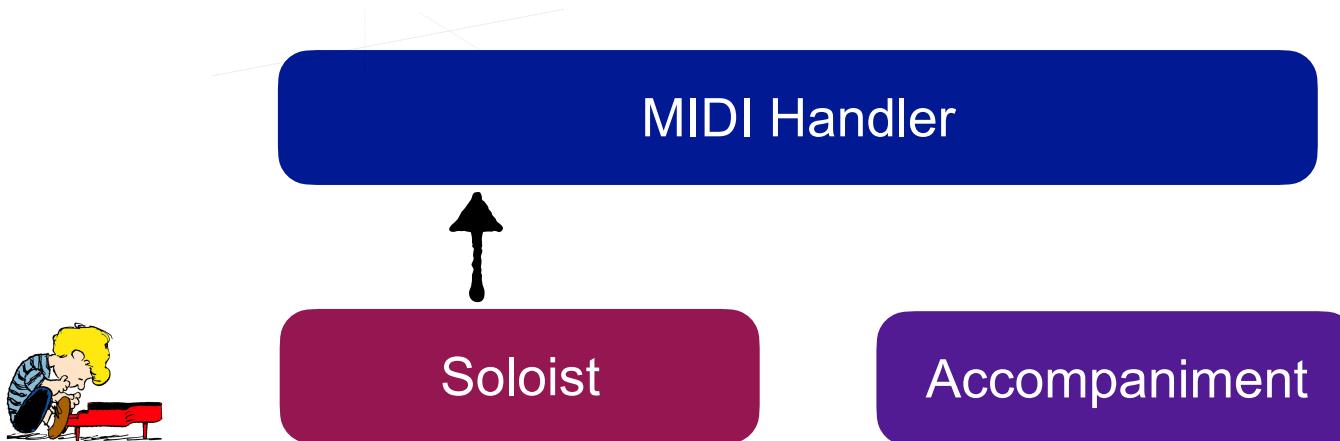
Accompanist

MIDI Handler

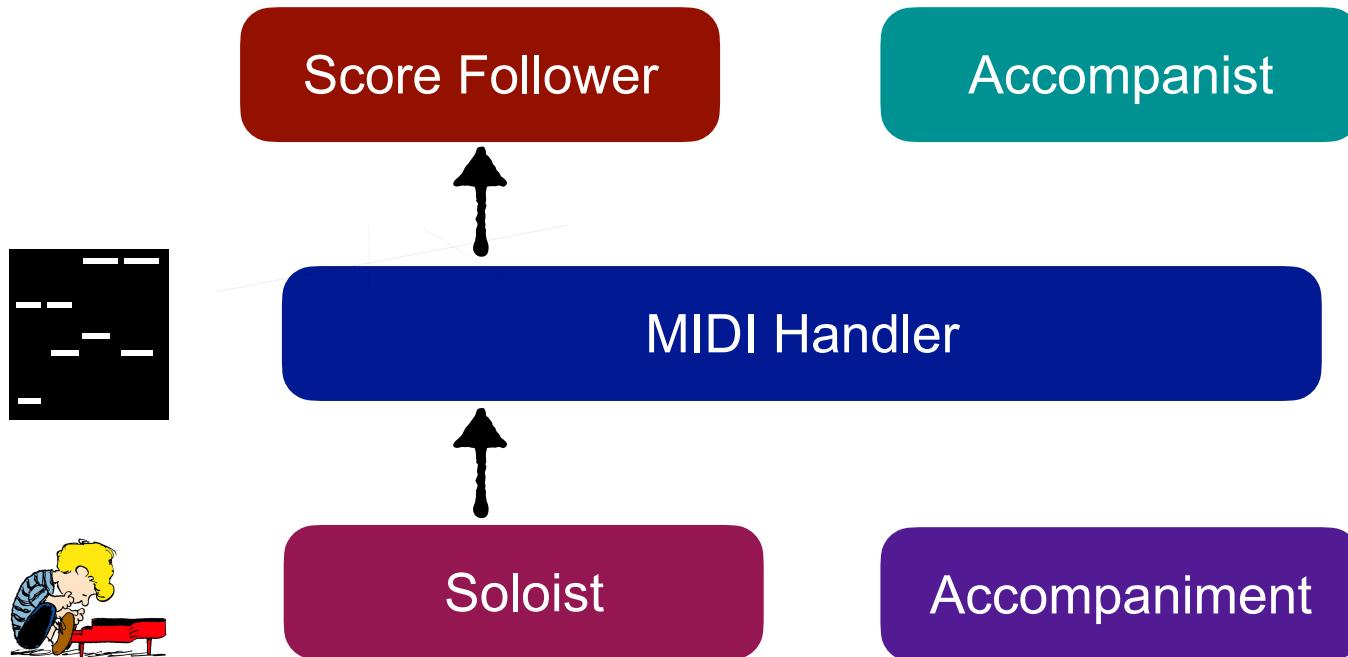
Soloist

Accompaniment

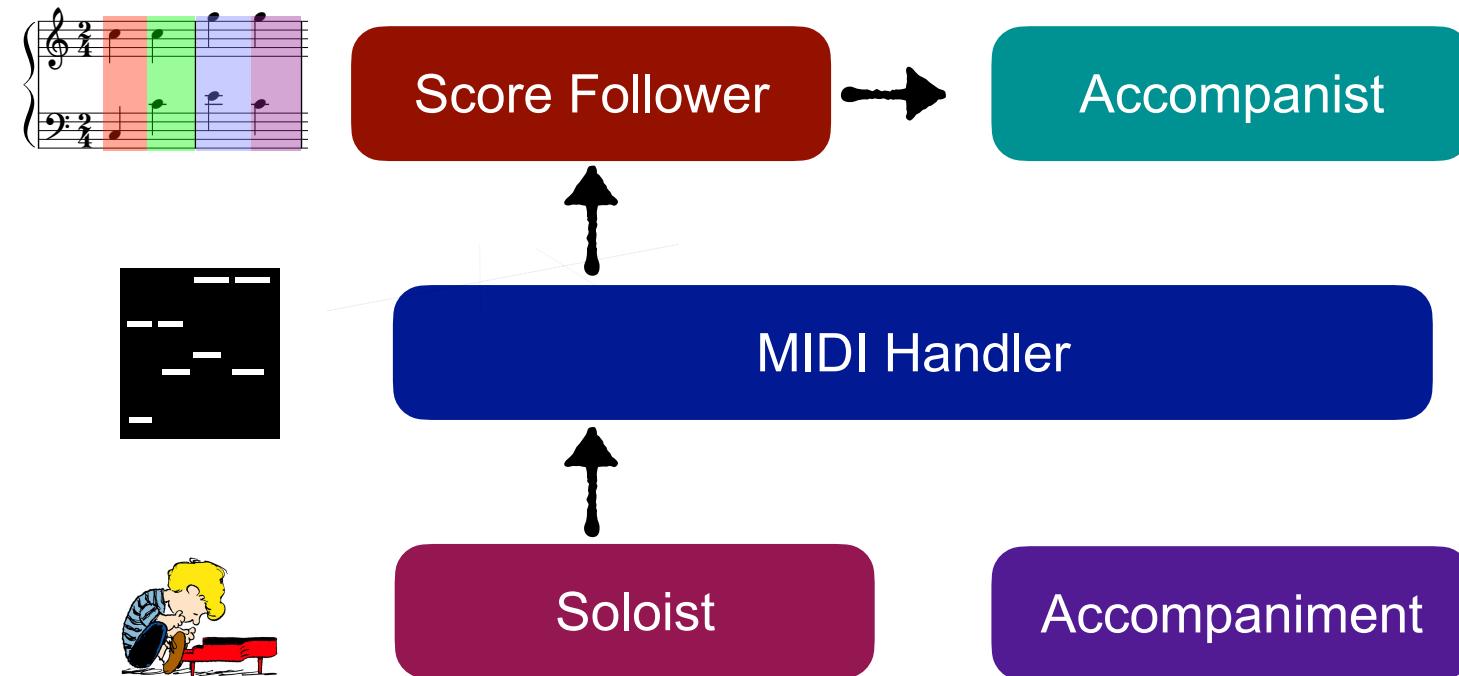
The ACCompanion



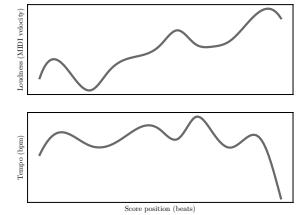
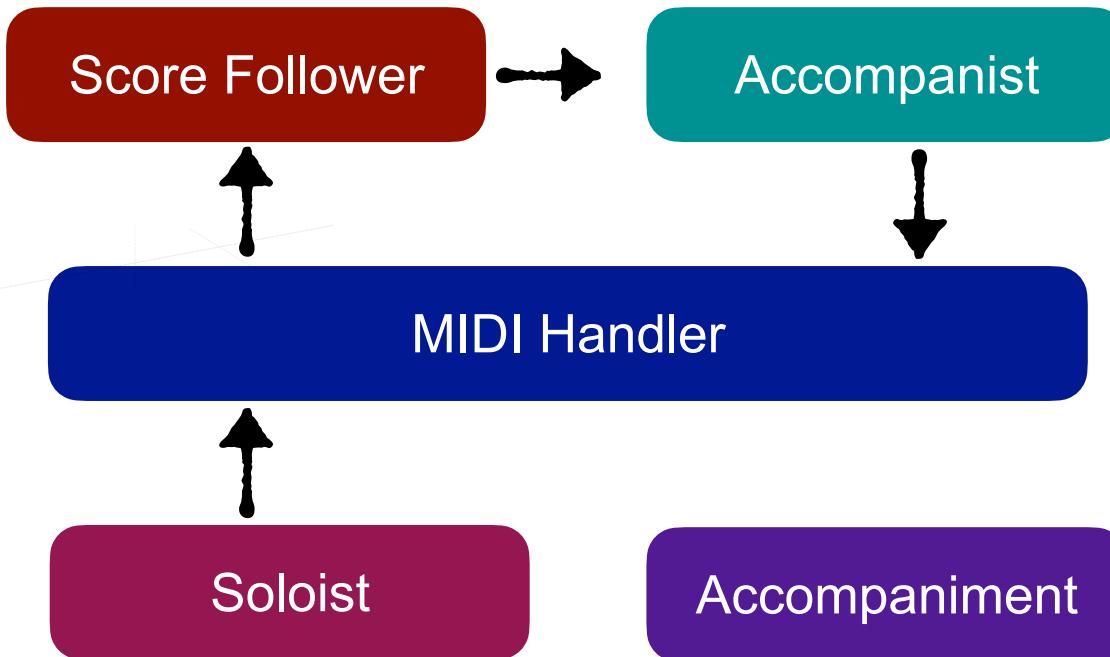
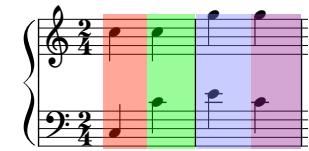
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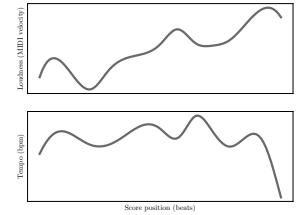
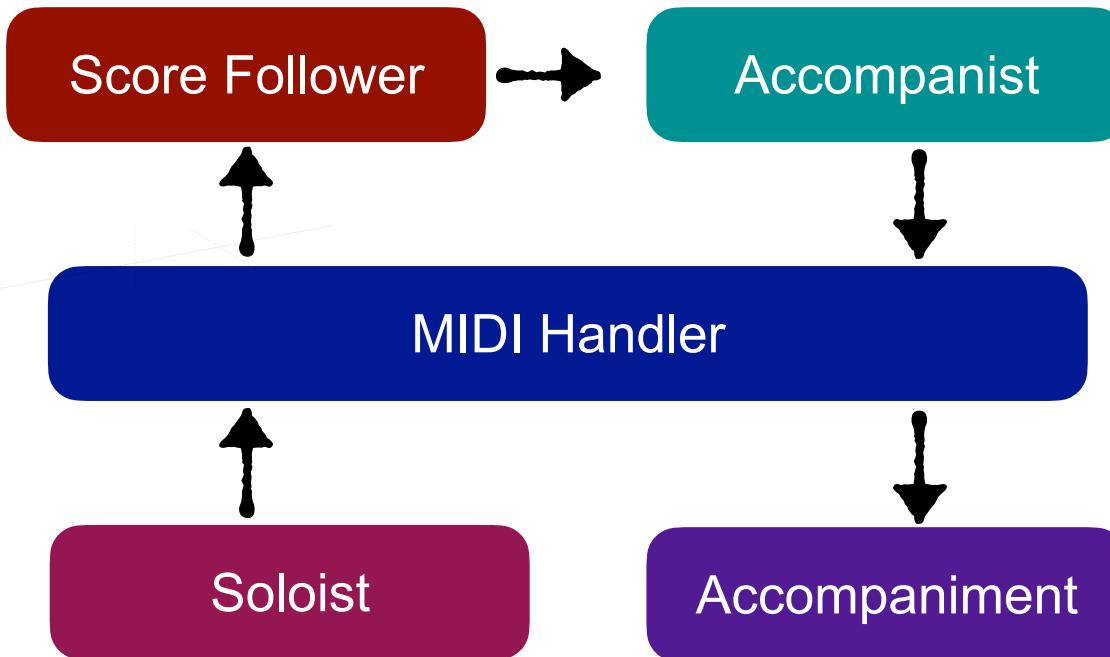
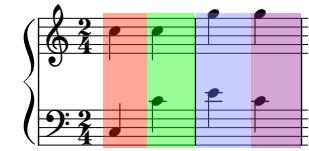
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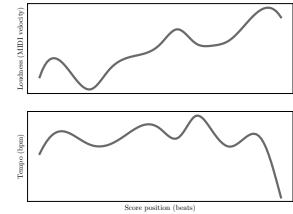
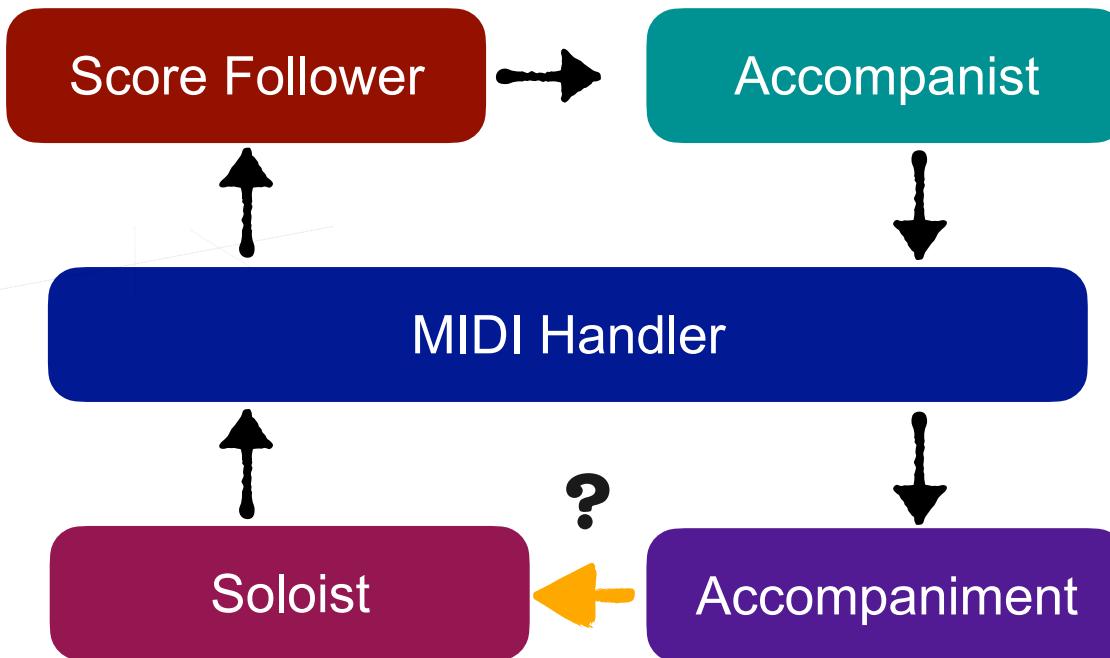
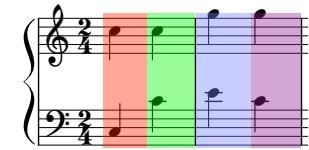
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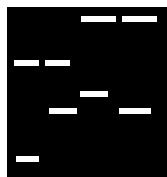
The ACCompanion



The ACCompanion



Score position (beats)



Togetherness in Musical HCI

Togetherness in Musical HCI

- Togetherness is primarily **phenomenological**

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Togetherness in Musical HCI

- Togetherness is primarily **phenomenological**
 - Experience is a **complex cognitive process**
- Computational partners/agents **cannot experience togetherness** (yet!)
 - No **mental states**
 - No **explicit intentions**
 - “Experience” is limited to whatever the hardware/software is **designed/allowed to perceive**

Artificial Music Togetherness



Increasing togetherness



Cognitive-emotional alignment with others

Achieving shared goals

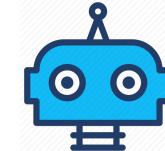
Sharing higher-order experiences with others
(e.g., goals, perceptions)

Interaction with others

Awareness of others in a shared musical setting

(Bishop, 2021)

Artificial Music Togetherness



Increasing togetherness



Cognitive-emotional alignment with others

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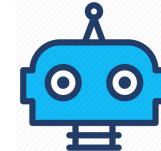
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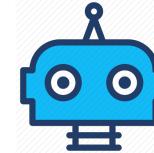
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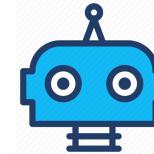
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(Bishop, 2021)

*As long as we can extract the information
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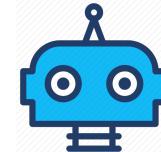
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Awareness of others in a shared musical setting



(Bishop, 2021)

*As long as we can extract the information from musical signals



**As long as we can formulate the goals in terms of computable functions

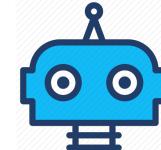
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