Musical Informatics: Music Alignment

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Institute of Computational Perception

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Outline

- Music Alignment
  - Applications and Examples
  - Music Representation and Features
  - Dynamic Time Warping
- Automatic Accompaniment demo

Music Alignment

Same music, different interpretations

- Different versions of the same piece/song that correspond to the same musical work!

Symbolic Recordings of Piano Performances

- Late 19th Century: Player Pianos!
  - Pianolas and piano rolls
  - Pneumatic mechanism
  - Music recorded on perforated paper: Piano-rolls
- 21st Century: Computer Controlled Grand Pianos
  - MIDI and derived formats
  - Yamaha, Bösendorfer, Steinway!
  - Competitions
  - Datasets

A player piano roll being played by Draconichiaro - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=82604752

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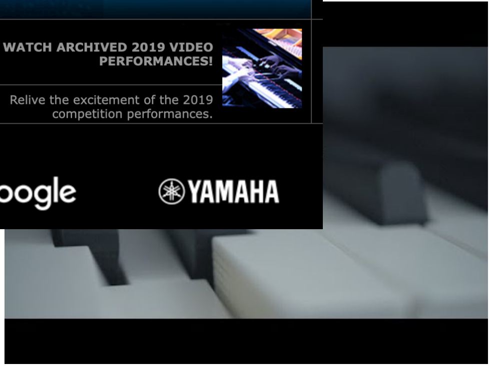

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Symbolic Recordings

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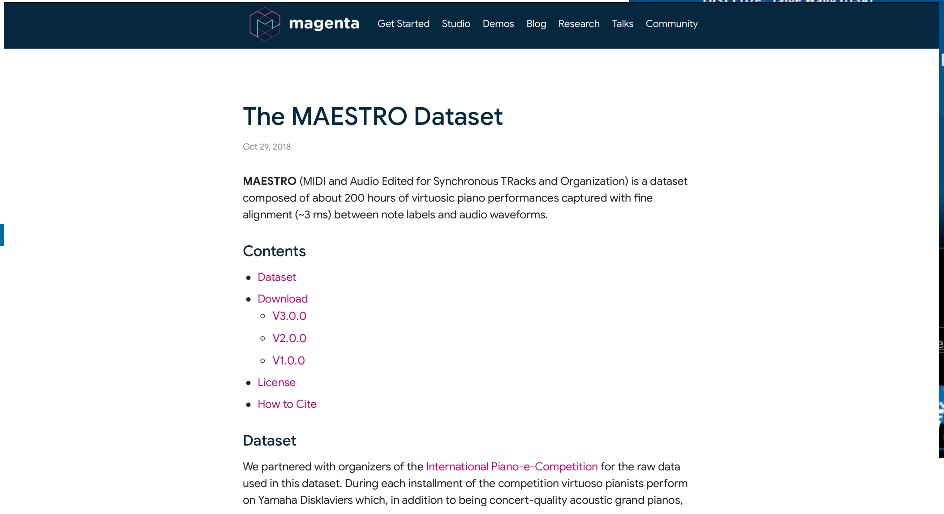
-


magenta Get Started Studio Demos Blog Research Talks Community

The MAESTRO Dataset

Oct 29, 2018

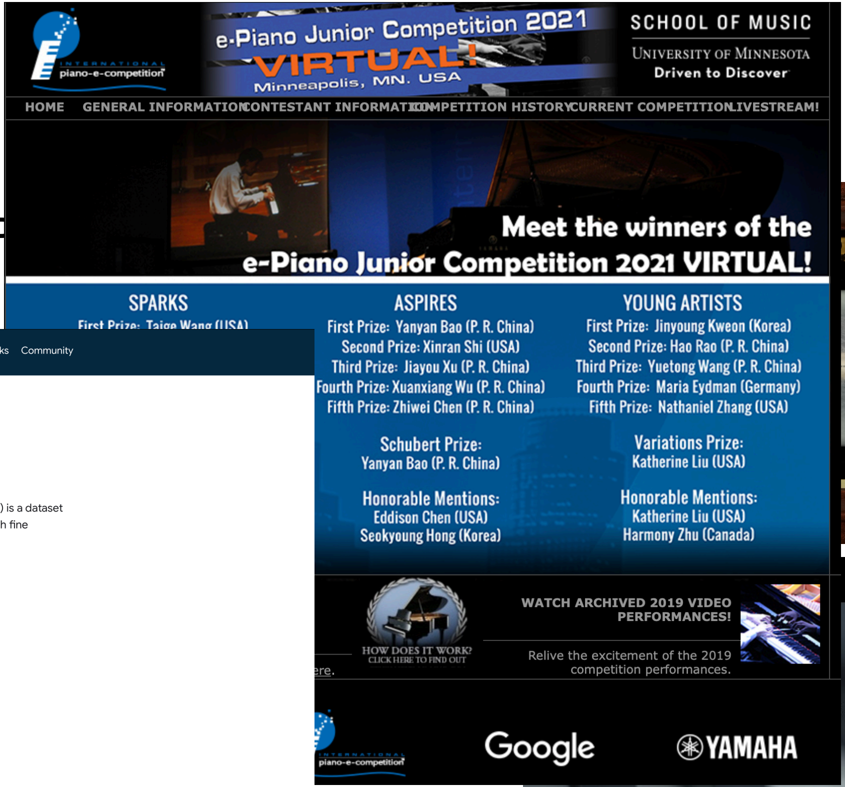
MAESTRO (MIDI and Audio Edited for Synchronous TRacks and Organization) is a dataset composed of about 200 hours of virtuosic piano performances captured with fine alignment (~3 ms) between note labels and audio waveforms.

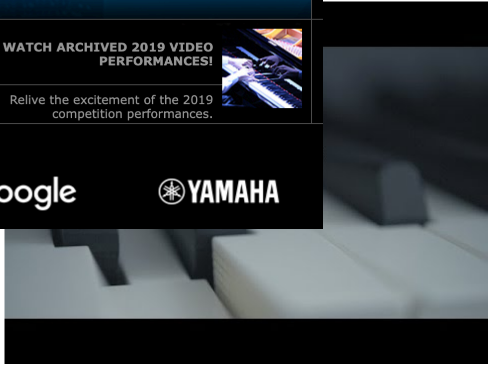
Contents

- Dataset
- Download
  - V3.0.0
  - V2.0.0
  - V1.0.0
- License
- How to Cite

Dataset

We partnered with organizers of the International Piano-e-Competition for the raw data used in this dataset. During each installment of the competition virtuoso pianists perform on Yamaha Disklaviers which, in addition to being concert-quality acoustic grand pianos,





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How do machines listen?: Humans vs. Computers

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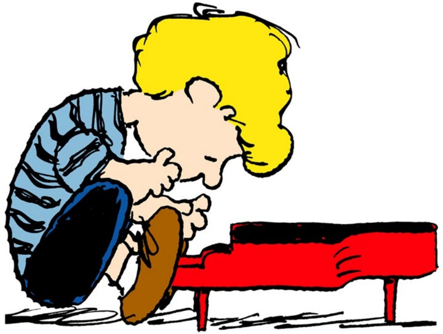
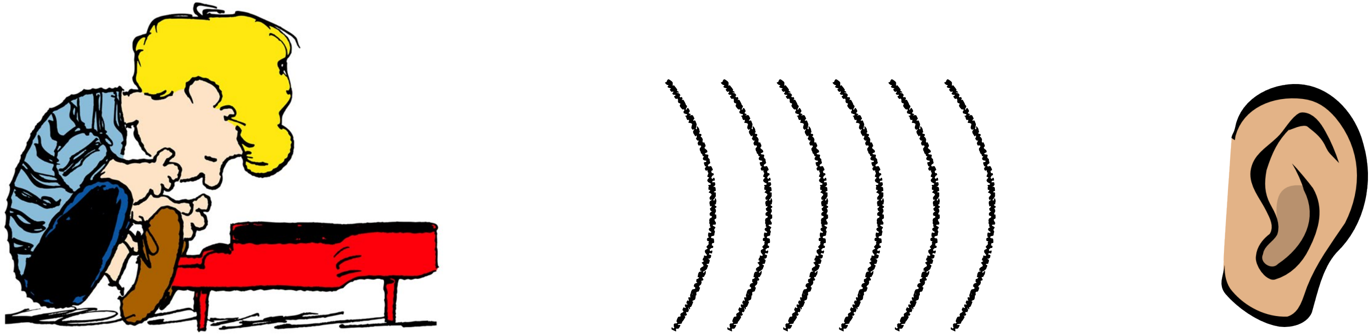
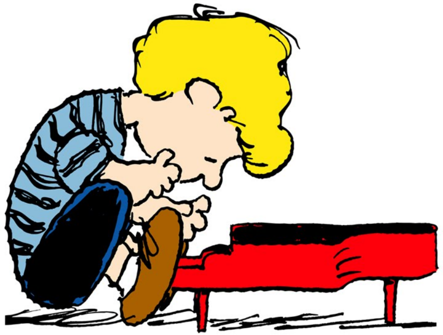
How do machines listen?: Humans vs. Computers

- Computers “perceive” audio signals in a different way than humans
  - Human brains are marvelous information processing machines that allow us to **make sense** of auditory signals (almost) effortlessly
- Computers struggle performing basic tasks!



Pressure Wave





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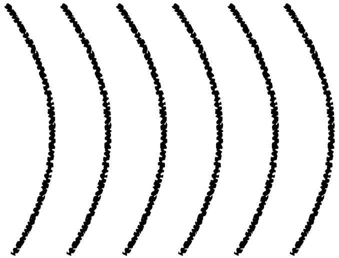
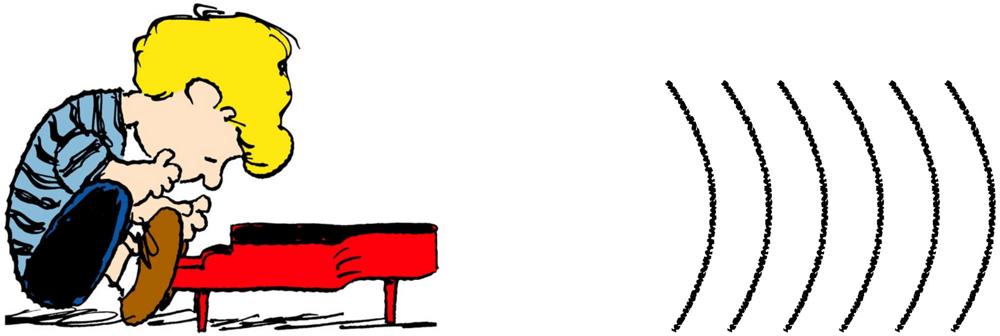
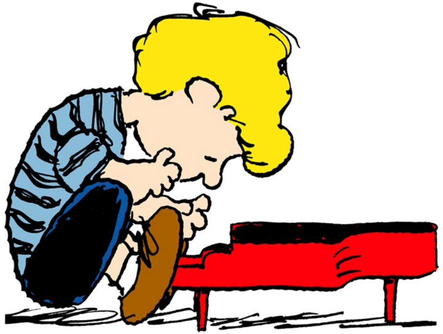


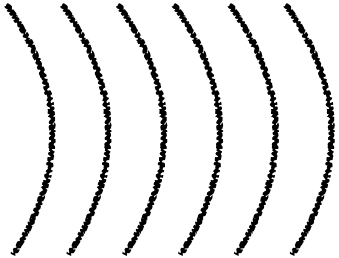
Music

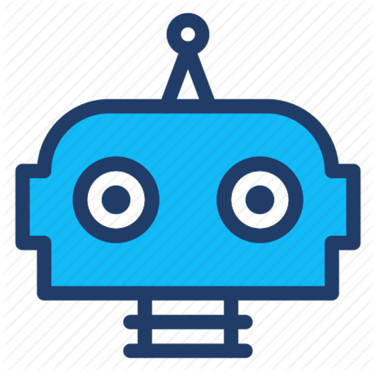
Pressure Wave



Pressure Wave







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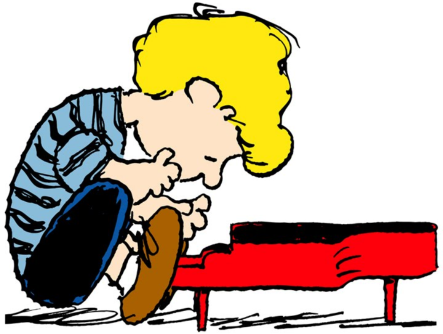

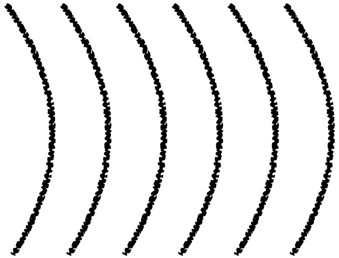

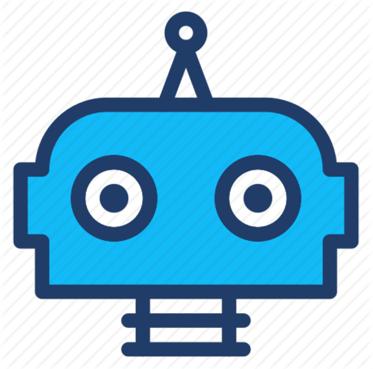

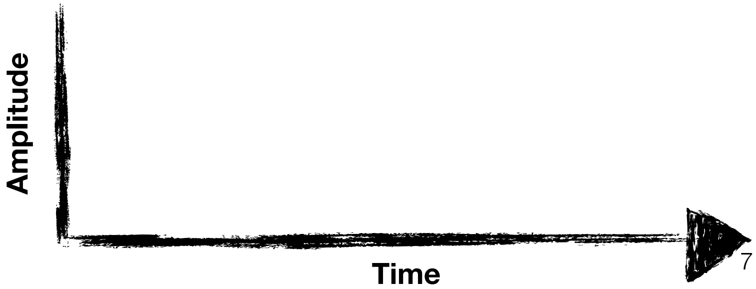

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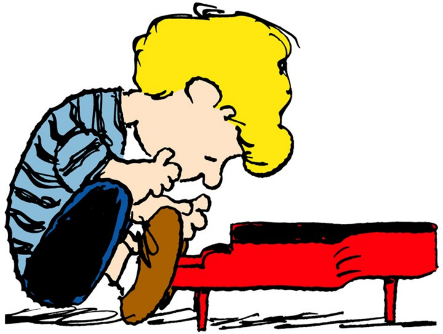

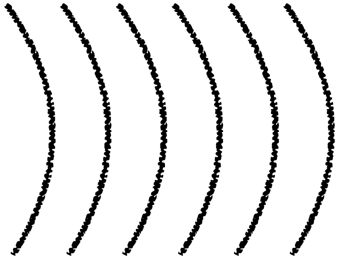

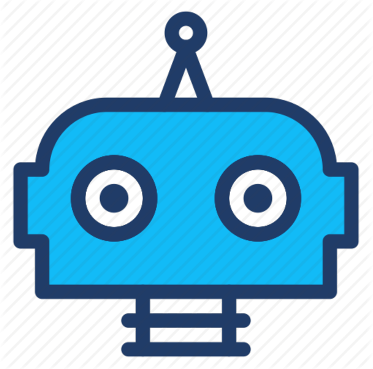

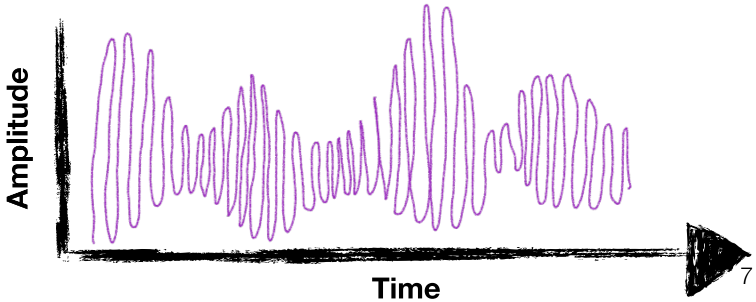

Machine Listening

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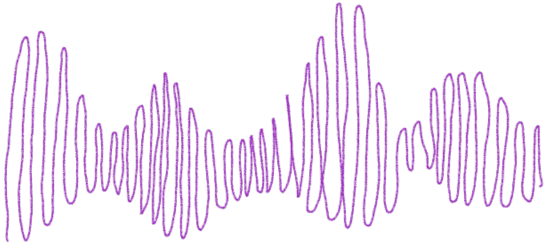
  - Developing such models will require us to understand more about **human music perception.**
  - [Herrera et al. 2009]: “We will only develop music understanding systems by means of understanding music understanding”.

Automatic Music Alignment

Score

Performance

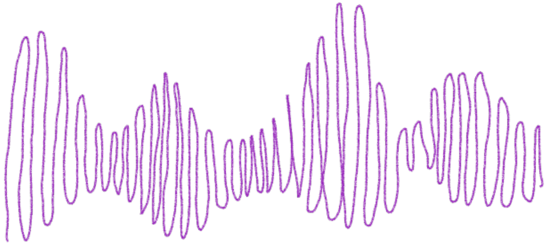


Automatic Music Alignment

- Matching **musical signals** of the **same piece**

Score

Performance



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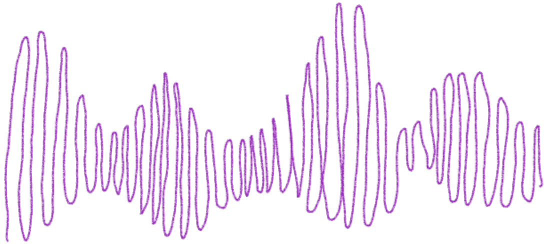
Automatic Music Alignment

- Matching **musical signals** of the **same piece**
  - Different performances of the same piece

Score

Performance



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Score

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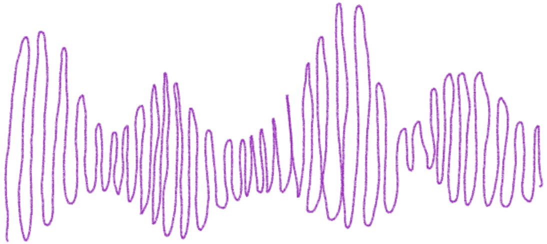

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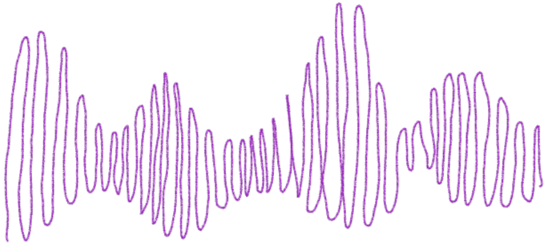

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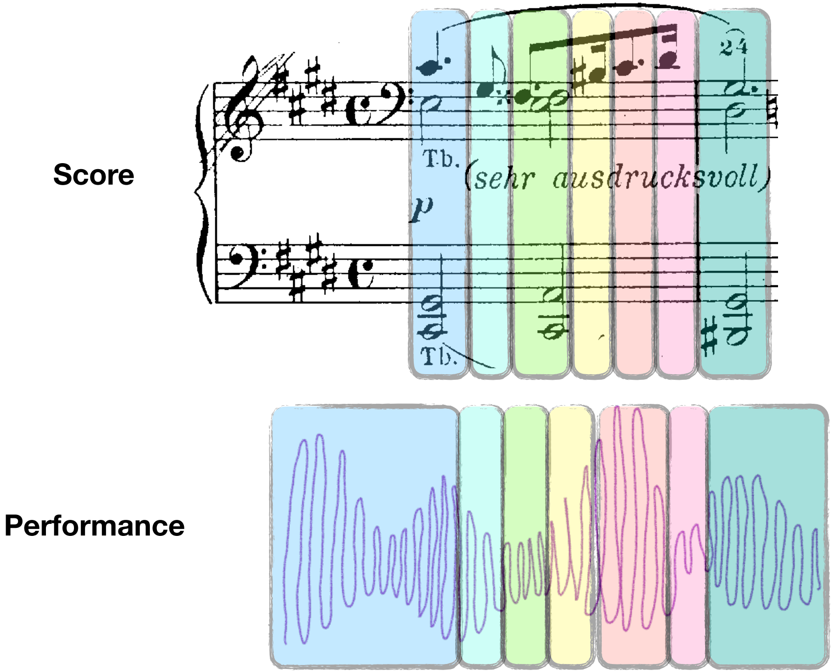
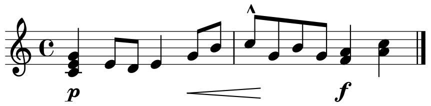
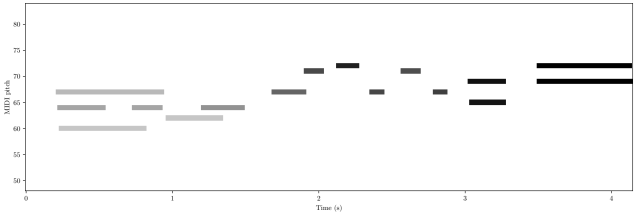
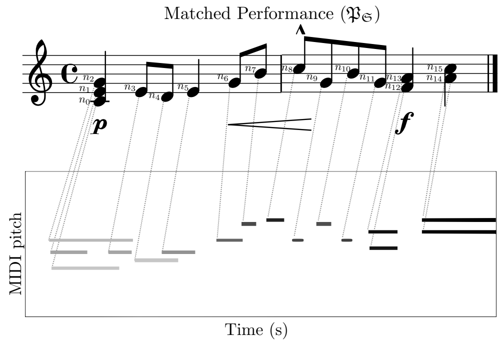


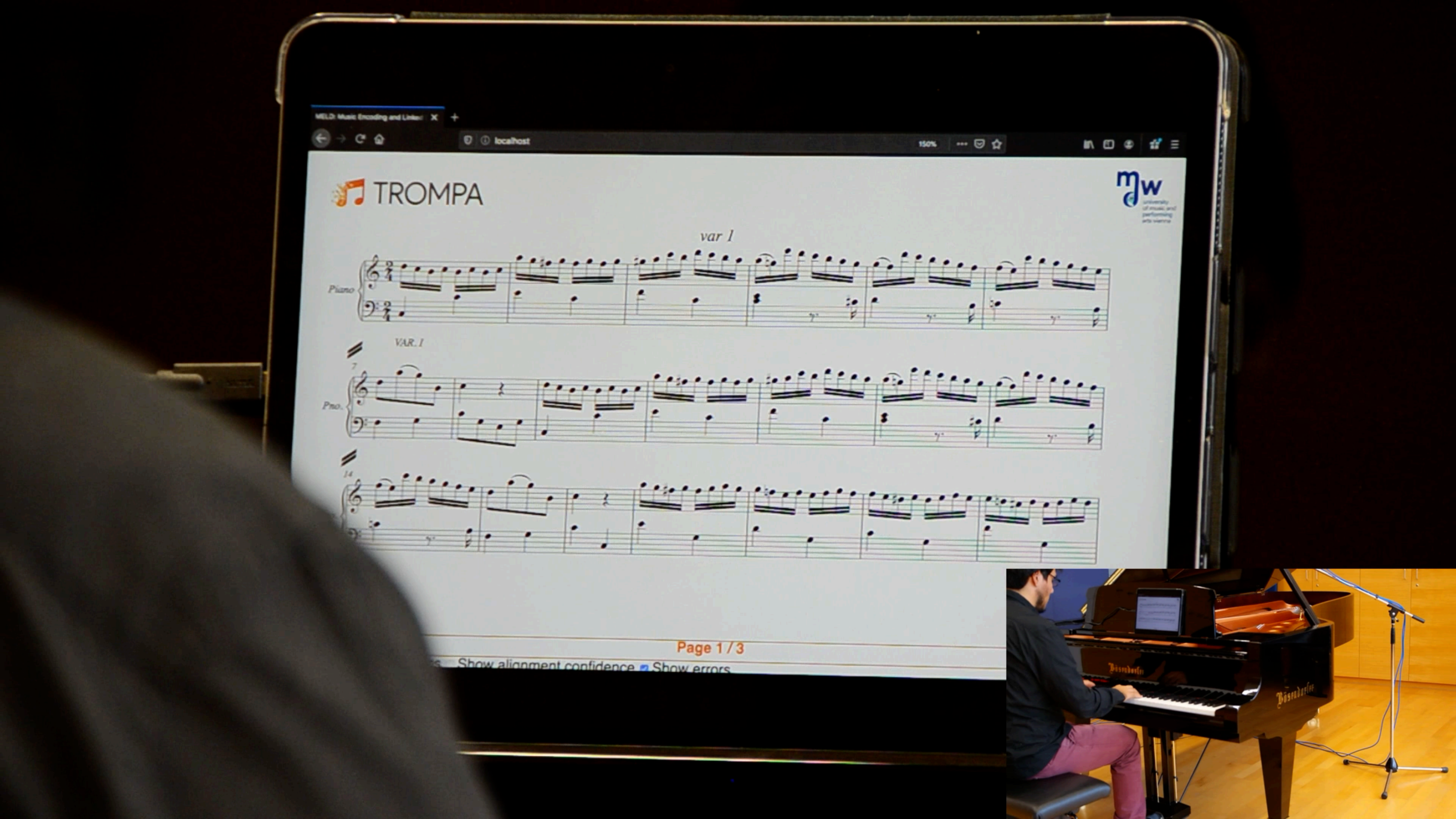
Music Alignment Pipeline

Music Representations → Extraction (Piano rolls, ANNs) → Feature Representations → Alignment Methods (DTW, HMMs…) → Alignment





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Piano
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Pno.
Page 1 / 3
Show alignment confidence
Show errors

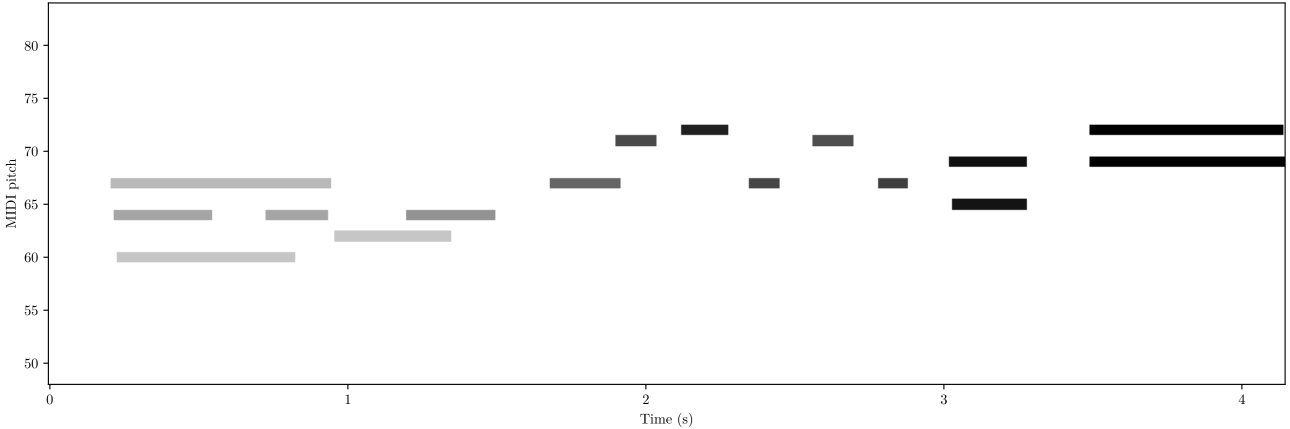
Music Representation and Features

Features

- To make data comparable and algorithmically accessible, the first step is to extract suitable features that capture relevant aspects while suppressing irrelevant details

The Piano Roll

- A 2D representation of (MIDI) pitch and time
- We can extract piano rolls from MIDI and MusicXML files with Partitura!



Other Features

- Piano rolls are not the only (or the best) features for alignment
  - Neural networks: Features learned from Autoencoders (Lattner et al., 2018) that are transposition invariant
  - Pitch Class distribution
  - and many more!

Dynamic Time Warping

Dynamic Time Warping

- Dynamic Programming Approach
- Find an **optimal** alignment between two given (time-dependent) sequences (under certain restrictions)
- **Sequences**: $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$ and $\mathbf{Y} = \{\mathbf{y}_1, \ldots, \mathbf{y}_M\}$
  - for other applications, sequences could be discrete signals, feature sequences, sequences of characters
- **Feature Space:** $\mathbf{x}_n, \mathbf{y}_m \in \mathscr{F}$ for $n \in [1:N]$ and $m \in [1:M]$
- **Local cost measure:** $c : \mathscr{F} \times \mathscr{F} \mapsto \mathbb{R}$
  - Basic intuition: $c(\mathbf{x}, \mathbf{y})$ is small if $\mathbf{x}$ and $\mathbf{y}$ are similar

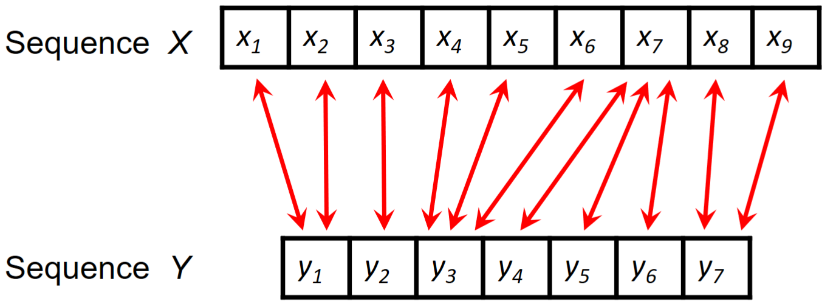
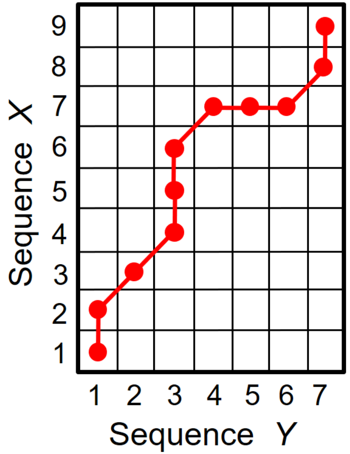


Figure 3.12 from [Müller, FMP, Springer 2015]



Cost Matrix

- The cost matrix $\mathbf{C} \in \mathbb{R}^{N \times M}$ evaluates the local cost

$$\mathbf{C}[i, j] = c(\mathbf{x}_i, \mathbf{y}_j)$$

- The **choice** of the local cost measure **matters**!
  - Euclidean Distance: $c(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_i (x_i - y_i)^2}$
  - Manhattan Distance: $c(\mathbf{x}, \mathbf{y}) = \sum_i |x_i - y_i|$
  - Cosine Distance: $c(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x}^T \mathbf{y}}{||\mathbf{x}|| \cdot ||\mathbf{y}||}$
  - Others?

Warping Path

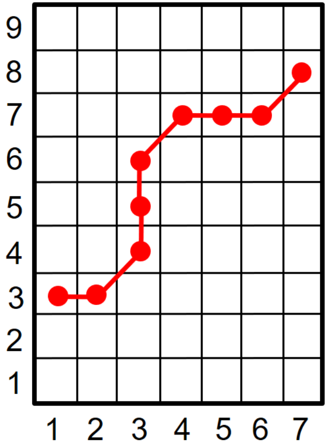
- An (N, M)-**warping path** of length $L \in \mathbb{N}$ is a sequence

$$P = (p_1, \dots, p_l)$$

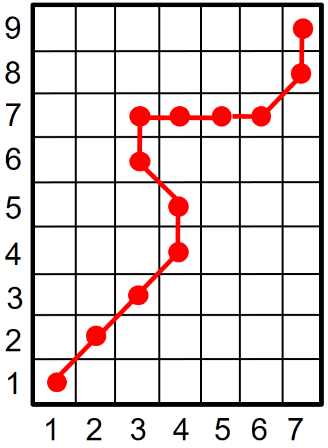
with $p_l = (n_l, m_l) \in [1 : N] \times [1 : M]$ that defines an alignment between sequences $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}$ and $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_M\}$
- Three conditions:
  - **Boundary condition**: $p_1 = (1,1)$ and $p_L = (N, M)$
  - **Monotonicity condition**: $n_1 \leq n_2 \leq \dots \leq n_L$ and $m_1 \leq m_2 \leq \dots \leq m_L$
  - **Step size condition**: $p_{l-1} - p_l \in \{(1,0), (0,1), (1,1)\}$

Warping Path

Violates Boundary condition



Violates Monotonicity



Violates Step Size Condition

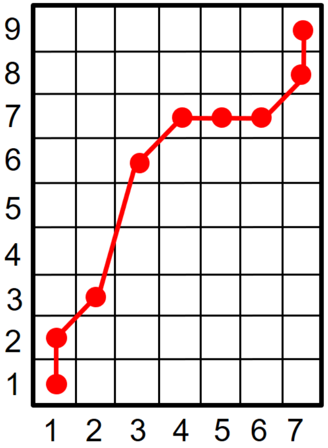


Figure 3.13 from [Müller, FMP, Springer 2015]

Optimal Warping Path

- Total cost of a warping path:
- $$c_P(\mathbf{X}, \mathbf{Y}) = \sum_{l=1}^{L} \mathbf{C}(n_l, m_l)$$
- **Optimal warping path:** minimal total cost!
  DTW$(\mathbf{X}, \mathbf{Y}) = c_{P*}(\mathbf{X}, \mathbf{Y})$
- $$= \min\{c_P(\mathbf{X}, \mathbf{Y}) \mid \ P \text{ is an } (N, M)\text{-warping path}\}$$
- DTW$(\mathbf{X}, \mathbf{Y})$: **DTW distance**
  - Not a real **distance**: is symmetric, but not positive definite
    - DTW$(\mathbf{X}, \mathbf{Y}) = 0$ does not necessarily mean that $\mathbf{X} = \mathbf{Y}$!

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Dynamic Programming Algorithm

- Dynamic Programming: Break down a problem into simpler subproblems and then combine the solution!
- Idea: Find optimal warping paths for subsequences of $\mathbf{X}$ and $\mathbf{Y}$, and then combine the solutions recursively!
- Accumulated Cost Matrix
  - $\mathbf{D}(n, m) = \text{DTW}(\mathbf{X}(1:n), \mathbf{Y}(1:m))$

Algorithm

Table 3.2 from [Müller, FMP, Springer 2015]

Algorithm: DTW

Input: Cost matrix $\mathbf{C}$ of size $N \times M$
Output: Accumulated cost matrix $\mathbf{D}$
Optimal warping path P^*

Procedure: Initialize $(N \times M)$ matrix $\mathbf{D}$ by $\mathbf{D}(n,1) = \sum_{k=1}^{n} \mathbf{C}(k,1)$ for $n \in [1:N]$ and $\mathbf{D}(1,m) = \sum_{k=1}^{m} \mathbf{C}(1,k)$ for $m \in [1:M]$. Then compute in a nested loop for $n = 2,\ldots,N$ and $m = 2,\ldots,M$:

$$\mathbf{D}(n,m) = \mathbf{C}(n,m) + \min\{\mathbf{D}(n-1,m-1), \mathbf{D}(n-1,m), \mathbf{D}(n,m-1)\}.$$

Set $\ell = 1$ and $q_\ell = (N,M)$. Then repeat the following steps until $q_\ell = (1,1)$:

Increase ℓ by one and let $(n,m) = q_{\ell-1}$.
If $n = 1$, then $q_\ell = (1,m-1)$,
else if $m = 1$, then $q_\ell = (n-1,m)$,
else $q_\ell = \operatorname{argmin}\{\mathbf{D}(n-1,m-1), \mathbf{D}(n-1,m), \mathbf{D}(n,m-1)\}$.
(If 'argmin' is not unique, take lexicographically smallest cell.)

Set $L = \ell$ and return $P^* = (q_L, q_{L-1}, \ldots, q_1)$ as well as $\mathbf{D}$.

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$\mathbf{C} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

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Pen and Paper Example!

Inputs

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$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$

$$\mathbf{C} = \begin{bmatrix} 0 & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Pen and Paper Example!

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$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$
$$C_{12} = |x_1 - y_2| = |3 - 1| = 2$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

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$$C_{13} = |x_1 - y_3| = |3 - 9| = 6$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

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$$C_{13} = |x_1 - y_3| = |3 - 9| = 6$$
$$C_{14} = |x_1 - y_4| = |3 - 9| = 6$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$C_{11} = |x_1 - y_1| = |3 - 3| = 0$$
$$C_{12} = |x_1 - y_2| = |3 - 1| = 2$$
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$$C_{14} = |x_1 - y_4| = |3 - 9| = 6$$
$$C_{15} = |x_1 - y_5| = |3 - 7| = 4$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & \\ & & & & & \\ & & & & & \end{bmatrix}$$

Pen and Paper Example!

Inputs

$$\mathbf{X} = \{3,9,8\}$$

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ & & & & & \\ & & & & & \end{bmatrix}$$

Pen and Paper Example!

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$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ & & & & & \end{bmatrix}$$

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Pen and Paper Example!

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$$C_{14} = |x_1 - y_6| = |3 - 8| = 5$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

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Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

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Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

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Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

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$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & & & & & \\ 6 & & & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & & & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0,2,6) = 8$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0,2,6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2,8,8) = 2$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0,2,6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2,8,8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8,14,2) = 2$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

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Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

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$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2,8,8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8,14,2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14,18,2) = 4$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3{,}9{,}8\}$$

$$\mathbf{Y} = \{3{,}1{,}9{,}9{,}7{,}8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

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$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8,14,2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14,18,2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18,23,4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & & & & & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

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Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

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$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & & & & \end{bmatrix}$$

Pen and Paper! Example II

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$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18,23,4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & & & \end{bmatrix}$$

Pen and Paper! Example II

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$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & & \end{bmatrix}$$

Pen and Paper! Example II

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$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0,2,6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2,8,8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8,14,2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14,18,2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18,23,4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & \end{bmatrix}$$

Pen and Paper! Example II

Inputs

$$\mathbf{X} = \{3,9,8\}$$

$$\mathbf{Y} = \{3,1,9,9,7,8\}$$

Initialization

$$D_{i,1} = \sum_{k=1}^{i} C_{k,1} \qquad D_{1,i} = \sum_{k=1}^{i} C_{1,k}$$

Iteration

$$D_{ij} = C_{ij} + \min(D_{j-1,i-1}, D_{i-1,j}, D_{i,j-1})$$

$$\mathbf{C} = \begin{bmatrix} 0 & 2 & 6 & 6 & 4 & 5 \\ 6 & 8 & 0 & 0 & 2 & 1 \\ 5 & 7 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D_{22} = C_{22} + \min(D_{2-1,2-1}, D_{2-1,2}, D_{2,2-1}) = 8 + \min(0,2,6) = 8$$

$$D_{23} = C_{23} + \min(D_{2-1,3-1}, D_{2-1,3}, D_{2,3-1}) = 0 + \min(2,8,8) = 2$$

$$D_{24} = C_{24} + \min(D_{2-1,4-1}, D_{2-1,4}, D_{2,4-1}) = 0 + \min(8,14,2) = 2$$

$$D_{25} = C_{25} + \min(D_{2-1,5-1}, D_{2-1,5}, D_{2,5-1}) = 2 + \min(14,18,2) = 4$$

$$D_{26} = C_{26} + \min(D_{2-1,6-1}, D_{2-1,6}, D_{2,6-1}) = 1 + \min(18,23,4) = 5$$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$q_1 = (3,6)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$q_1 = (3,6)$

$q_2 = \arg\min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

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Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

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$q_1 = (3,6)$

$q_2 = \arg\min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$

$q_3 = \arg\min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

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Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

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$q_1 = (3,6)$

$q_2 = \arg\min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$

$q_3 = \arg\min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$

$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & 2 & 8 & 14 & 18 & 23 \\ 6 & 8 & 2 & 2 & 4 & 5 \\ 11 & 13 & 3 & 3 & 3 & 3 \end{bmatrix}$$

$q_1 = (3,6)$

$q_2 = \arg\min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$

$q_3 = \arg\min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$

$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

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$q_3 = \arg\min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2{,}4)$

$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2{,}3)$

$q_5 = \arg\min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1{,}2)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

$$\mathbf{D} = \begin{bmatrix} 0 & \boxed{2} & 8 & 14 & 18 & 23 \\ 6 & 8 & \boxed{2} & \boxed{2} & 4 & 5 \\ 11 & 13 & 3 & 3 & \boxed{3} & \boxed{3} \end{bmatrix}$$

$q_1 = (3,6)$

$q_2 = \arg\min(D_{3-1,6-1}, D_{3,6-1}, D_{3-1,6}) = (3,5)$

$q_3 = \arg\min(D_{3-1,5-1}, D_{3,5-1}, D_{3-1,5}) = (2,4)$

$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$

$q_5 = \arg\min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

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$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$

$q_5 = \arg\min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$

$q_6 = (1, q_5[1] - 1) = (1,1)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

$\mathbf{Y} = \{3,1,9,9,7,8\}$

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$q_4 = \arg\min(D_{2-1,4-1}, D_{2,4-1}, D_{2-1,4}) = (2,3)$

$q_5 = \arg\min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$

$q_6 = (1, q_5[1] - 1) = (1,1)$

Pen and Paper Example III

Inputs

$\mathbf{X} = \{3,9,8\}$

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$q_5 = \arg\min(D_{2-1,3-1}, D_{2,3-1}, D_{2-1,3}) = (1,2)$

$q_6 = (1, q_5[1] - 1) = (1,1)$

$$\mathbf{P}^* = \{(1,1), (1,2), (2,3), (2,4), (3,5), (3,6)\}$$

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Automatic Accompaniment

Human—Computer Interaction in Music

Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning

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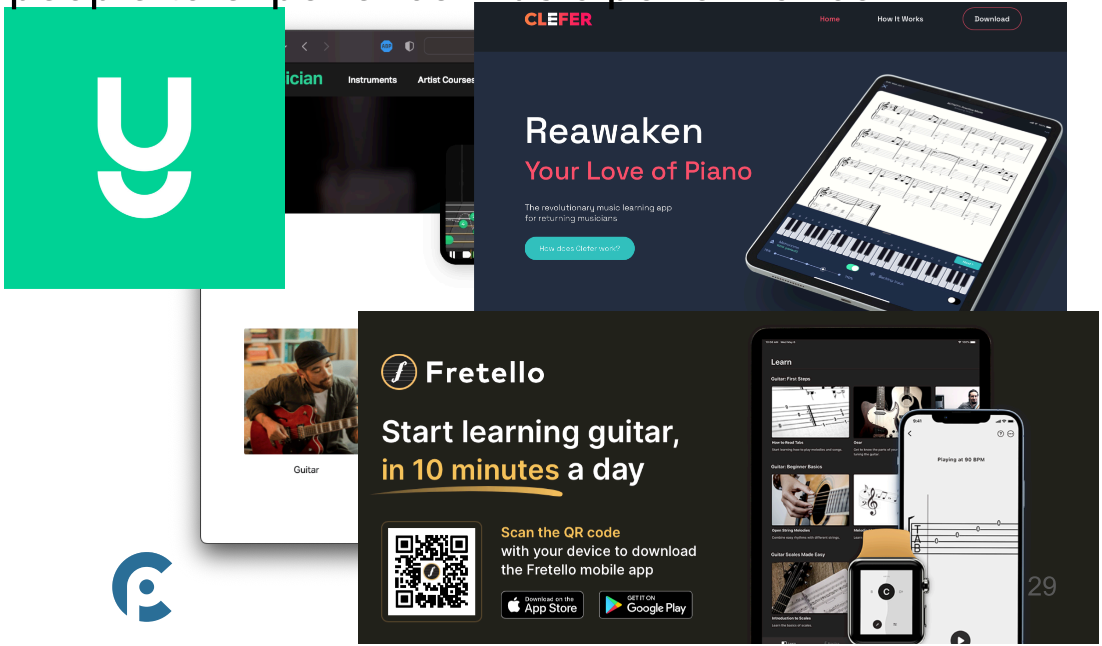
  - Learning apps



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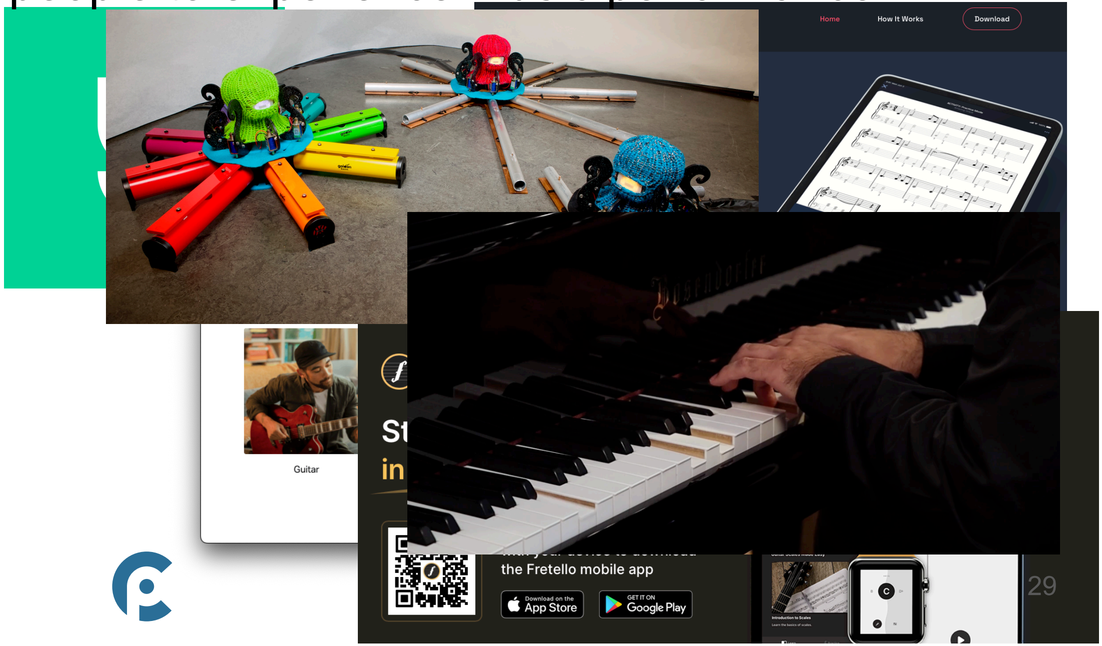
Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance
    - Learning apps
    - Interactive performances



Human—Computer Interaction in Music

- Interest in human—computer interaction (HCI) thanks to advancement in artificial intelligence (AI) and machine learning
- In music: Interfaces that allow people to experience music performance
  - Learning apps
  - Interactive performances
  - NIME

Automatic Accompaniment Systems

Automatic Accompaniment Systems

- Artificial co-performer

Automatic Accompaniment Systems

- Artificial co-performer
- (for this work) ensemble performance in there are two (or more) parts, which are **notated** (e.g., Western classical music)

Automatic Accompaniment Systems

- Artificial co-performer
- (for this work) ensemble performance in there are two (or more) parts, which are **notated** (e.g., Western classical music)
- The goal: **computational agent** that performs with a **human soloist**, and **adapts its performance** to the performance style of the soloist

Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Automatic Accompaniment Systems

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Detecting the solo part

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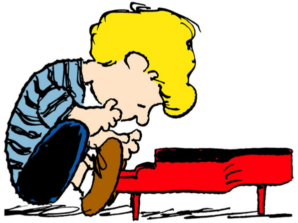

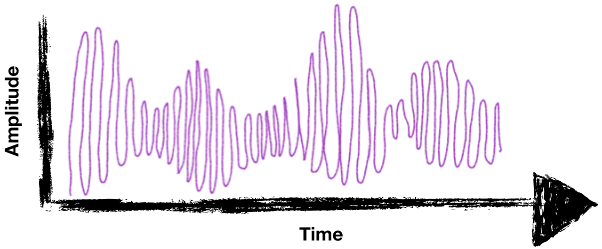

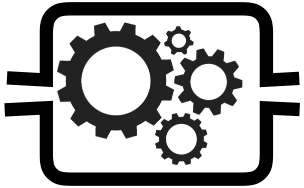

Automatic Accompaniment Systems

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Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

Detecting the solo part

Score Following

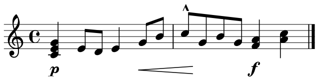
Automatic Accompaniment Systems

Dannenberg (1984) identifies three tasks that accompaniment systems must solve in order to successfully perform together with a human:

- Detecting the solo part
- Score Following
- Generating an expressive accompaniment part

Generating an Expressive Accompaniment Part

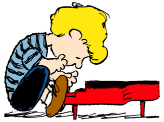
Accompaniment Score



Score Features

Note	pitch	beat	$\boldsymbol{p}$	$\boldsymbol{f}$	*crescendo*	∧
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	1	0	0.7	1
n_9	67	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

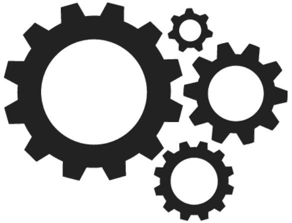
Soloist



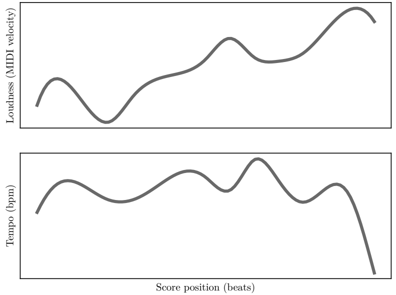
Solo Performance Features

Note	$y_{vel}(\cdot)$	$y_{\log bpr}(\cdot)$	$y_{tim}(\cdot)$	$y_{\log art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000

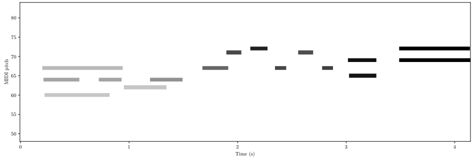
Computational Model (deep neural networks)



Expressive Parameters



Performance



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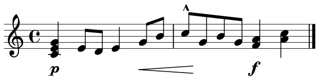
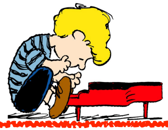
Generating an Expressive Accompaniment Part

Accompaniment Score



Soloist



Score Features

Note	pitch	beat	p	f	*crescendo*	$\wedge$
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
n_3	64	2	1	0	0.0	0
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Solo Performance Features

Note	$y_{\text{vel}}(\cdot)$	$y_{\text{log bpr}}(\cdot)$	$y_{\text{tim}}(\cdot)$	$y_{\text{log art}}(\cdot)$
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n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
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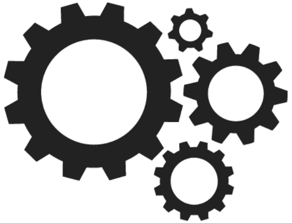
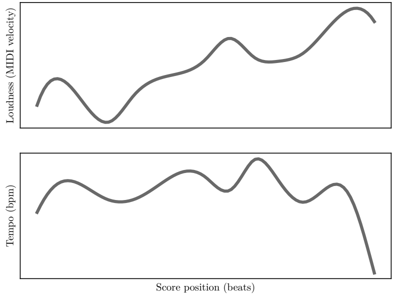
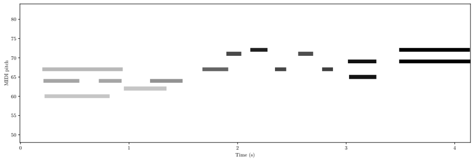


Expressive Parameters



Performance



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Generating an Expressive Accompaniment Part

Accompaniment Score

Score Features

Note	pitch	beat	*p*	*f*	*crescendo*	∧
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
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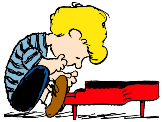
n_8	72	1	1	0	0.7	1
n_9	67	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
n_{11}	67	2	1	0	1.0	0
n_{12}	65	3	0	1	0.0	0
n_{13}	69	3	0	1	0.0	0
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

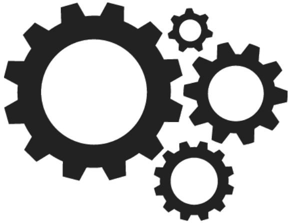
Soloist



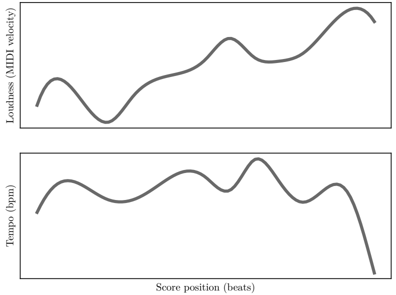
Solo Performance Features

Note	$y_{vel}(\cdot)$	$y_{\log bpr}(\cdot)$	$y_{tim}(\cdot)$	$y_{\log art}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
n_1	0.346	0.091	0.000	-0.624
n_2	0.268	0.091	0.014	0.551
n_3	0.346	-0.061	0.000	-0.152
n_4	0.220	0.016	0.000	0.697
n_5	0.417	0.013	0.000	-0.690
n_6	0.591	-0.142	0.000	0.113
n_7	0.693	-0.114	0.000	-0.619
n_8	0.850	-0.041	0.000	-0.534
n_9	0.709	-0.206	0.000	-1.037
n_{10}	0.677	-0.101	0.000	-0.675
n_{11}	0.732	0.028	0.000	-1.379
n_{12}	0.898	-0.053	-0.002	-0.889
n_{13}	0.913	-0.053	0.002	-0.841
n_{14}	0.969	0.455	-0.002	0.023
n_{15}	0.976	0.455	0.002	0.000

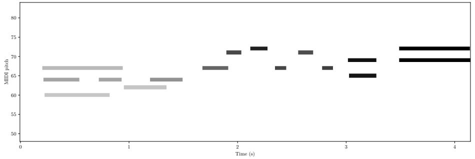
Computational Model (deep neural networks)



Expressive Parameters

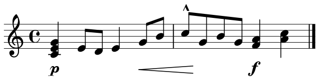


Performance



Generating an Expressive Accompaniment Part

Accompaniment Score



Score Features

Note	pitch	beat	$\boldsymbol{p}$	$\boldsymbol{f}$	*crescendo*	$\wedge$
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	1	0	0.7	1
n_9	67	2	1	0	1.0	0
n_{10}	71	2	1	0	1.0	0
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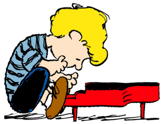
Soloist



Solo Performance Features

Note	$\mathrm{y}_{\mathrm{vel}}(\cdot)$	$\mathrm{y}_{\log \mathrm{bpr}}(\cdot)$	$\mathrm{y}_{\mathrm{tim}}(\cdot)$	$\mathrm{y}_{\log \mathrm{art}}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
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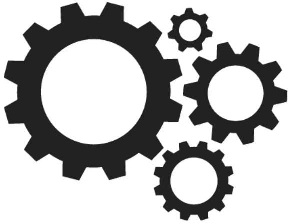

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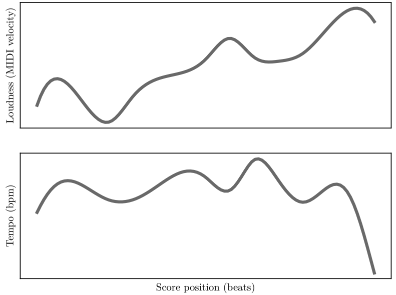

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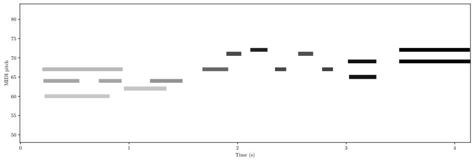

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Generating an Expressive Accompaniment Part

Accompaniment Score

Score Features

Note	pitch	beat	$\boldsymbol{p}$	$\boldsymbol{f}$	*crescendo*	$\wedge$
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
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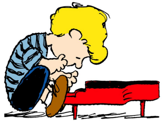

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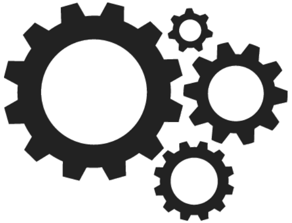

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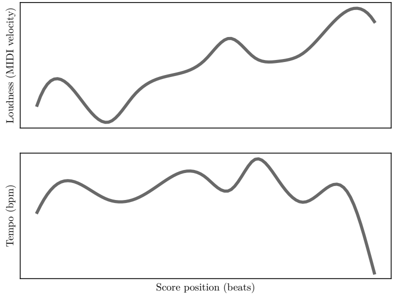
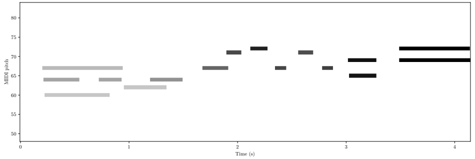


Performance



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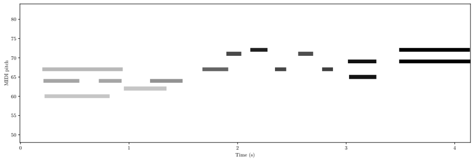
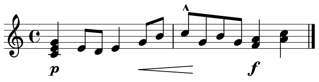
Generating an Expressive Accompaniment Part

Performance



Accompaniment Score



Score Features

Note	pitch	beat	*p*	*f*	*crescendo*	∧
n_0	60	1	1	0	0.0	0
n_1	64	1	1	0	0.0	0
n_2	67	1	1	0	0.0	0
n_3	64	2	1	0	0.0	0
n_4	62	2	1	0	0.0	0
n_5	64	3	1	0	0.0	0
n_6	67	4	1	0	0.0	0
n_7	71	4	1	0	0.3	0
n_8	72	1	1	0	0.7	1
n_9	67	2	1	0	1.0	0
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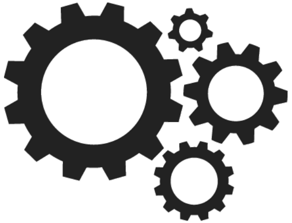
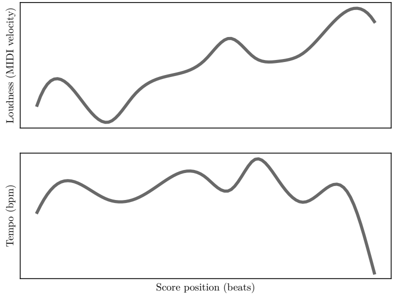
n_{14}	69	4	0	1	0.0	0
n_{15}	72	4	0	1	0.0	0

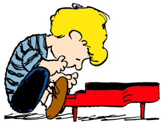
Computational Model (deep neural networks)



Expressive Parameters



Soloist



Solo Performance Features

Note	$y_{\text{vel}}(\cdot)$	$y_{\log \text{bpr}}(\cdot)$	$y_{\text{tim}}(\cdot)$	$y_{\log \text{art}}(\cdot)$
n_0	0.220	0.091	-0.014	0.230
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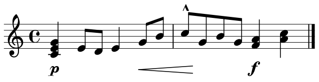
Generating an Expressive Accompaniment Part

Accompaniment Score



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n_0	60	1	1	0	0.0	0
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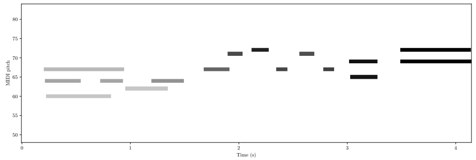
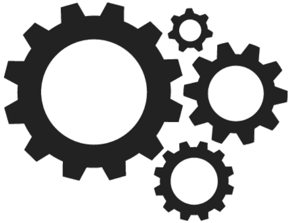
Performance



Computational Model (deep neural networks)



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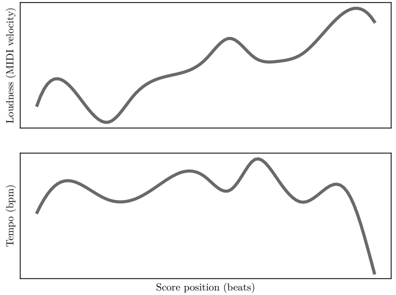

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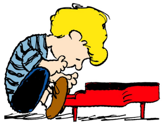

Solo Performance Features

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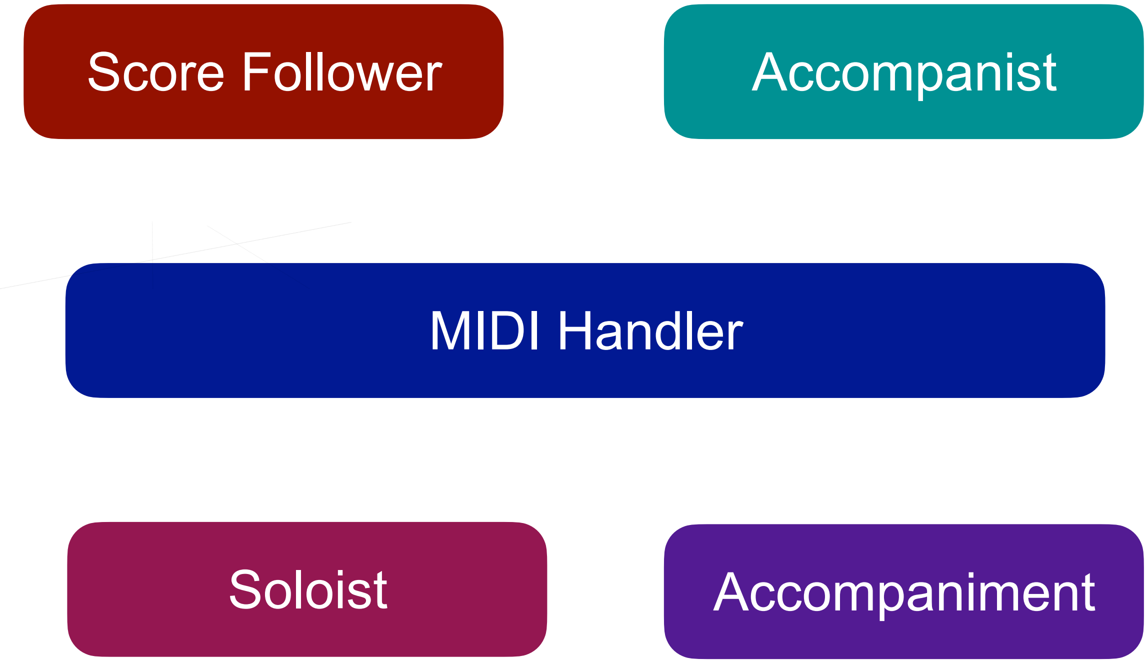
The ACCompanion



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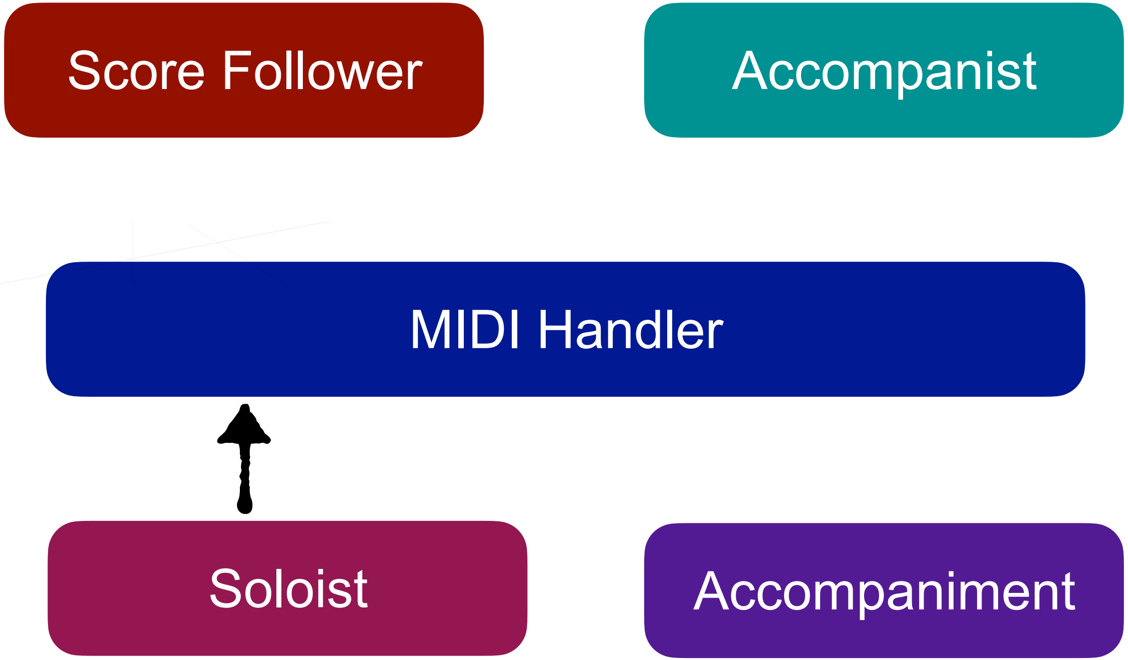

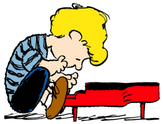

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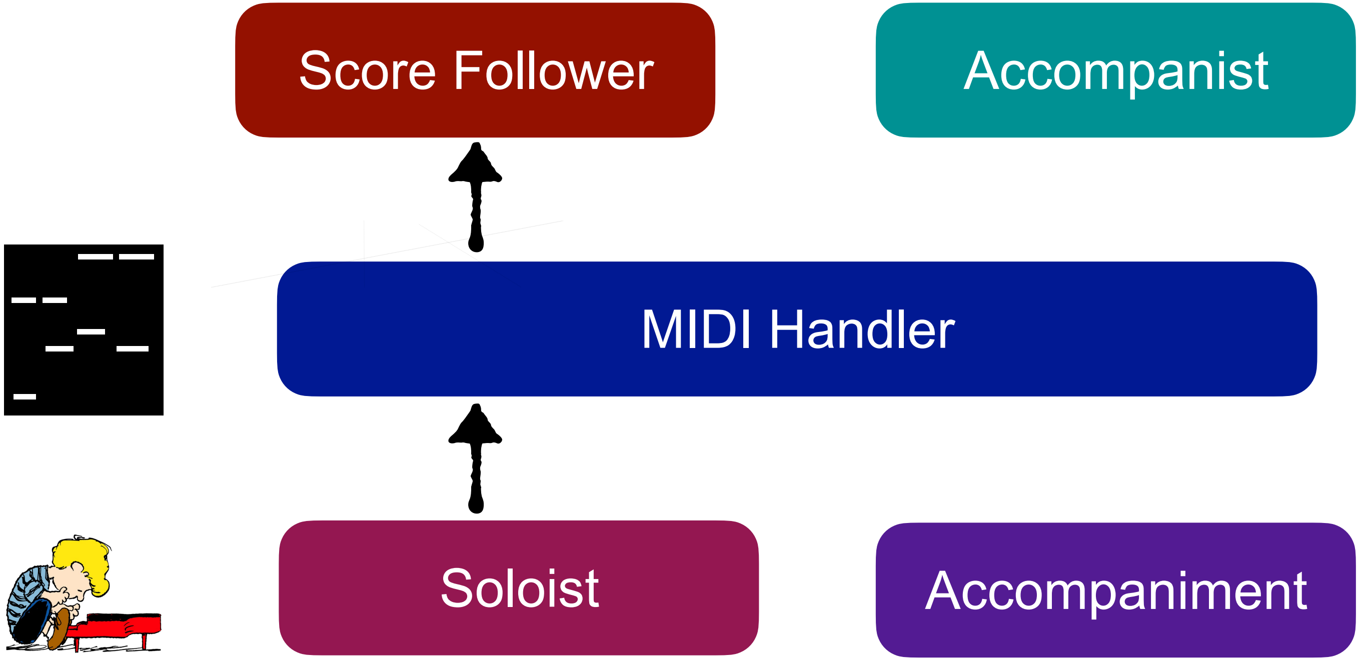

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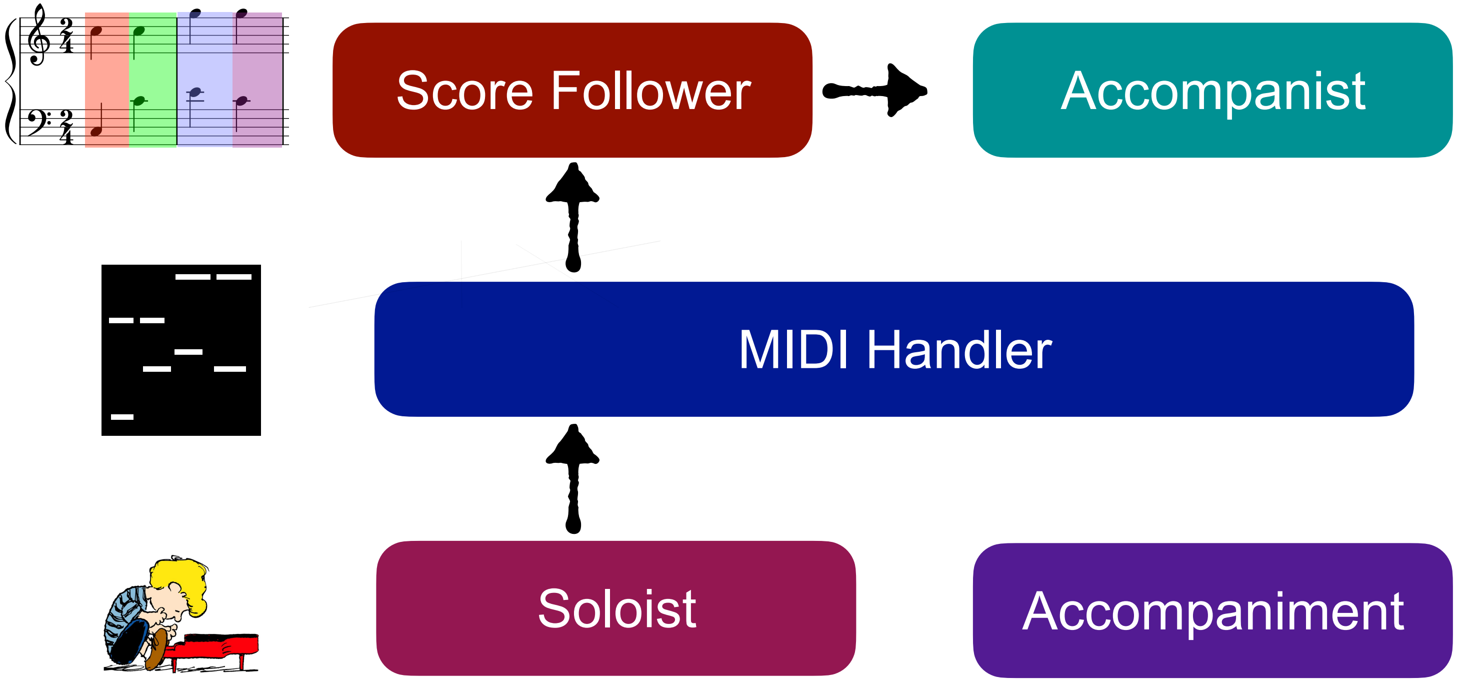

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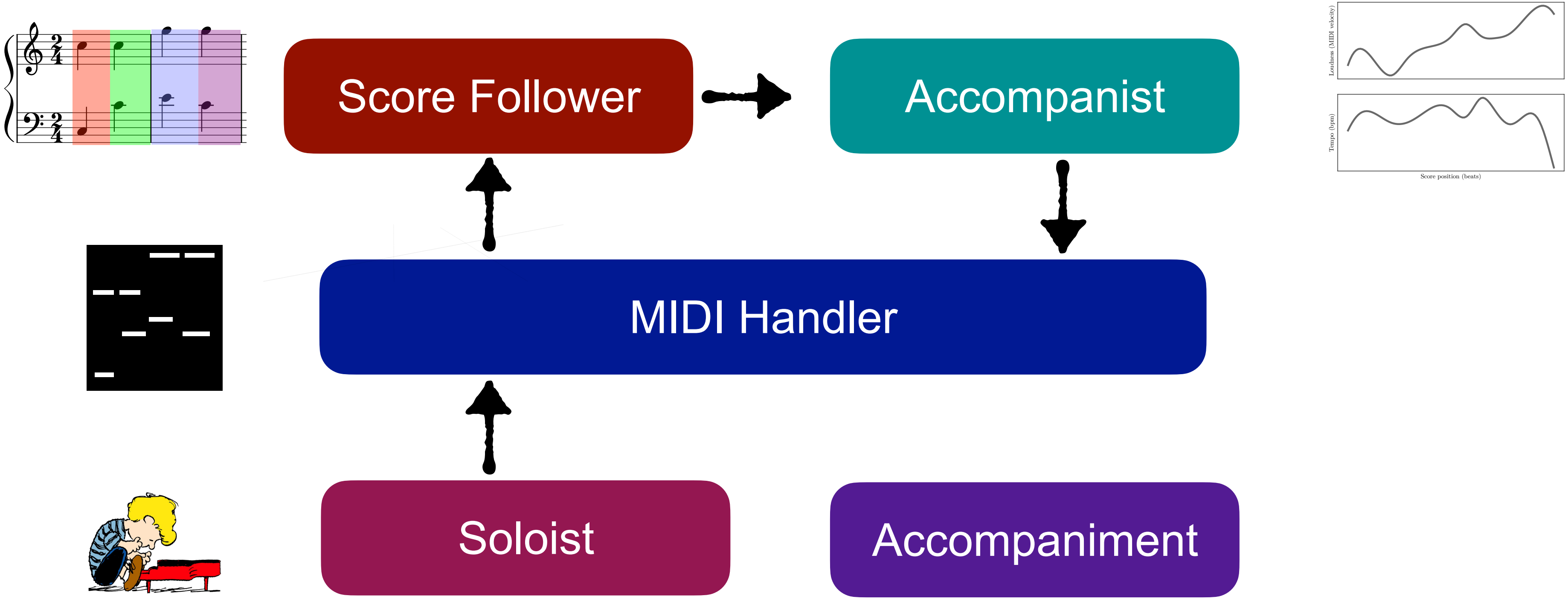

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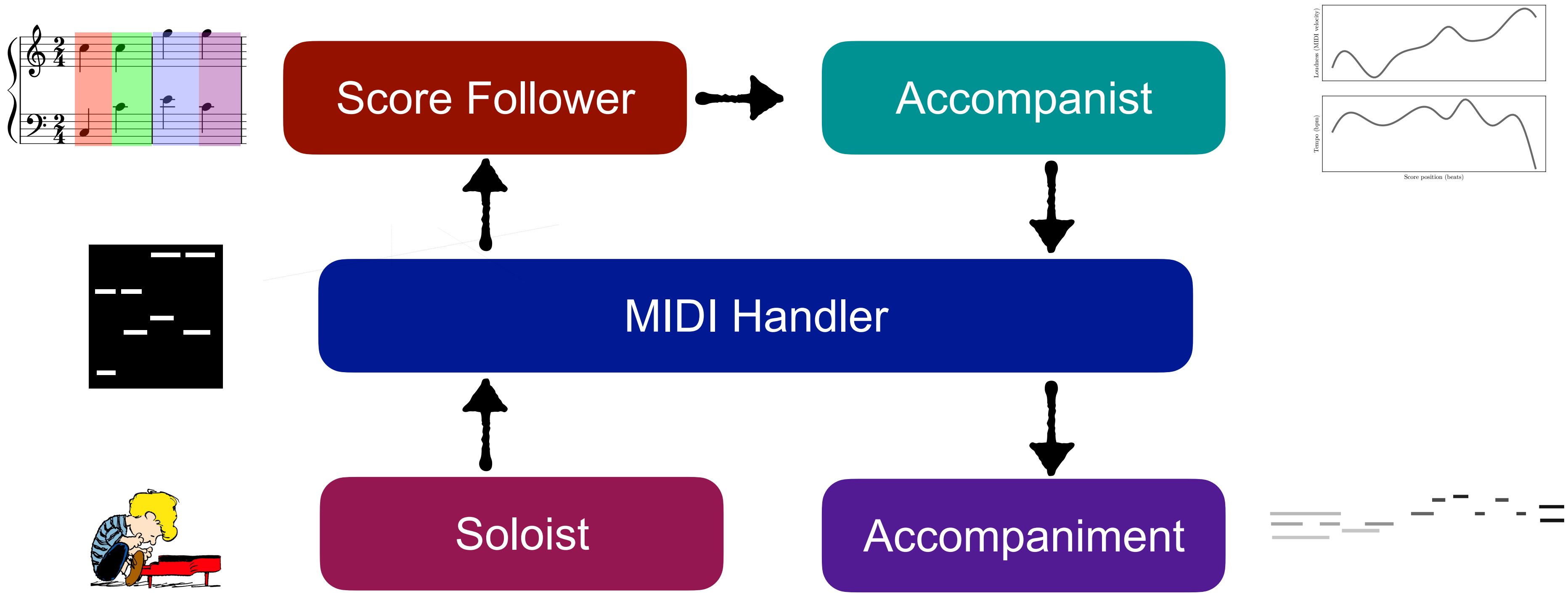

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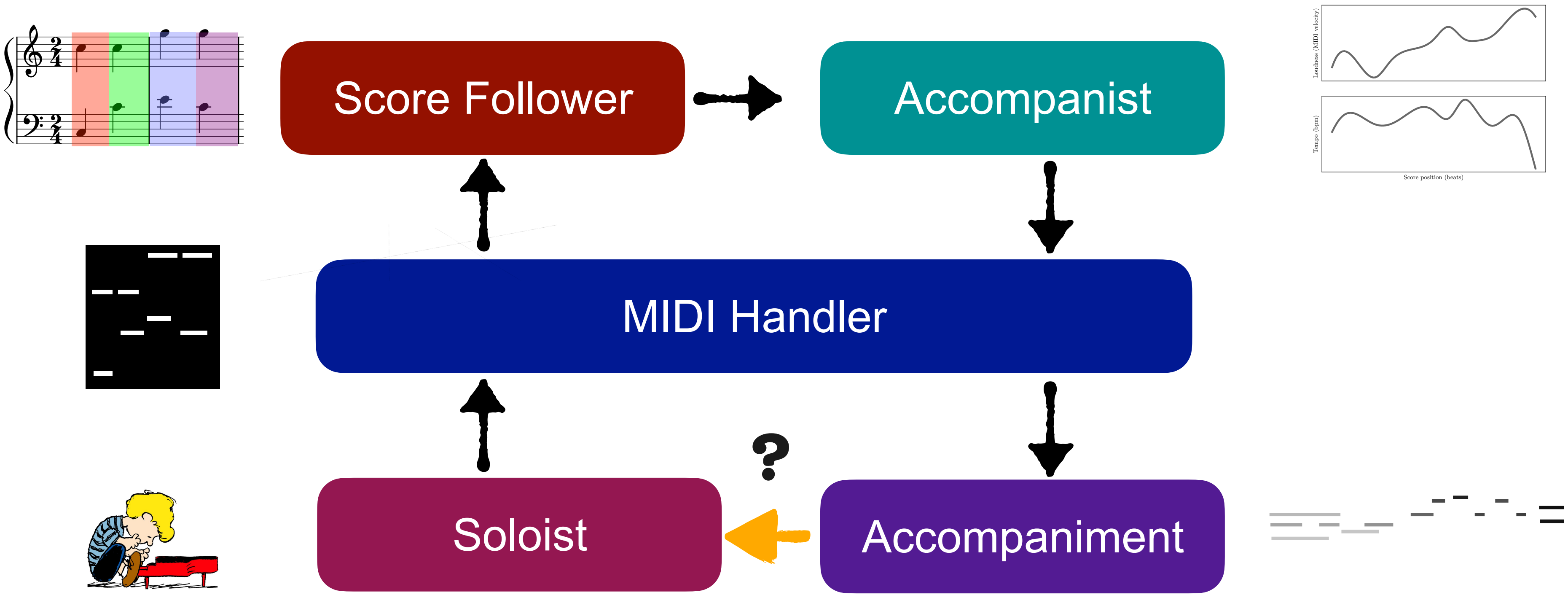

Togetherness in Musical HCI

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- Togetherness is primarily **phenomenological**

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- Togetherness is primarily **phenomenological**
  - Experience is a **complex cognitive process**
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  - No **mental states**
  - No **explicit intentions**
  - “Experience” is limited to whatever the hardware/software is **designed/ allowed to perceive**

Artificial Music Togetherness

Increasing togetherness

Cognitive-emotional alignment with others

Achieving shared goals

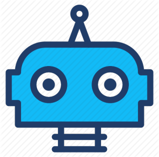
Sharing higher-order experiences with others
(e.g., goals, perceptions)

Interaction with others

Awareness of others in a shared musical setting

(Bishop, 2021)

Artificial Music Togetherness



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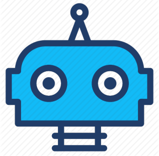

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*As long as we can extract the information from musical signals

(Bishop, 2021)

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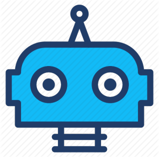
Artificial Music Togetherness



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*As long as we can extract the information from musical signals

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Artificial Music Togetherness

Increasing togetherness

Cognitive-emotional alignment with others

Achieving shared goals

Sharing higher-order experiences with others (e.g., goals, perceptions) ✗

Interaction with others ✓

Awareness of others in a shared musical setting ✓*

*As long as we can extract the information from musical signals

(Bishop, 2021)

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Artificial Music Togetherness

Increasing togetherness

Cognitive-emotional alignment with others

Achieving shared goals — (✓**)

Sharing higher-order experiences with others (e.g., goals, perceptions) — ✗

Interaction with others — ✓

Awareness of others in a shared musical setting — ✓*

(Bishop, 2021)

*As long as we can extract the information from musical signals

**As long as we can formulate the goals in terms of computable functions

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Artificial Music Togetherness

Increasing togetherness

Cognitive-emotional alignment with others	✗
Achieving shared goals	(✓**)
Sharing higher-order experiences with others (e.g., goals, perceptions)	✗
Interaction with others	✓
Awareness of others in a shared musical setting	✓*

(Bishop, 2021)

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