THE TONNETZ

Negative Harmony & Neo-Riemannian Transformations

Musical Informatics

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Organisation

FIRST PART

Introduction:

1. Pitch class sets
2. Triads and Transpositions
3. PLR and Riemann

The Tonnetz 1:

4. Configurations
5. Demonstration

Python Implementation

SECOND PART

The Tonnetz 2:

6. Negative Harmony
7. Hamiltonian Paths
8. The Torus Model - Tension

What can we do further ?

9. Infinite Tonnetze, Trajectories
10. Tonnetz Topologies

Python Implementation

PART I

GENTLE INTRODUCTION

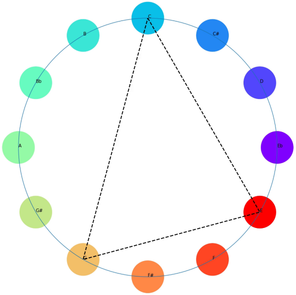
Pitch Class Sets and Circle of Fifths

Equal temperament & division of the octave

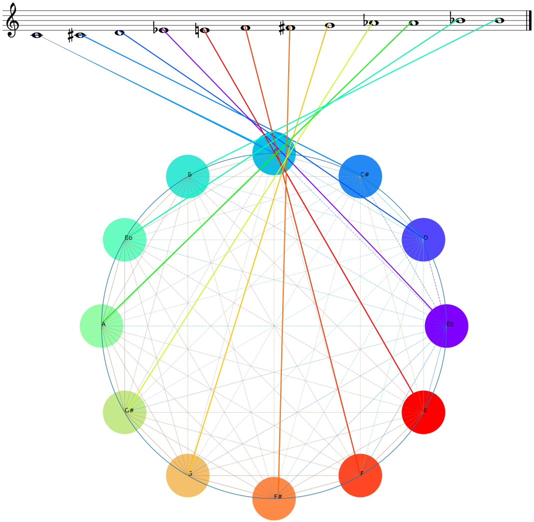
Pitch Class	Tonal	Solfege
0	C	do
1	C# or Db	
2	D	re
3	D# or Eb	
4	E	mi
5	F	fa
6	F# or Gb	
7	G	sol
8	G# or Ab	
9	A	la
10	t or A# or Bb	
11	e or B	si

$$\Rightarrow$$

$$\{0,\ 4,\ 7\} = \{C,\ E,\ G\} = C_{\mathrm{maj}}$$



Circle Representation



Circle of fifths

To build the circle of fifths we can think of the chromatic sequence as a Pitch Class Set :

$$\{\mathbf{0},\ \mathbf{1},\ \mathbf{2},\ \mathbf{3},\ \mathbf{4},\ \mathbf{5},\ \mathbf{6},\ \mathbf{7},\ \mathbf{8},\ \mathbf{9},\ \mathbf{10},\ \mathbf{11}\}$$

Multiple the entire sequence x7:

$$\{\mathbf{0},\ \mathbf{7},\ \mathbf{14},\ \mathbf{21},\ \mathbf{28},\ \mathbf{35},\ \mathbf{42},\ \mathbf{49},\ \mathbf{56},\ \mathbf{63},\ \mathbf{70},\ \mathbf{77}\}$$

Then apply modulo 12:

$$\{\mathbf{0},\ \mathbf{7},\ \mathbf{2},\ \mathbf{9},\ \mathbf{4},\ \mathbf{11},\ \mathbf{6},\ \mathbf{1},\ \mathbf{8},\ \mathbf{3},\ \mathbf{10},\ \mathbf{5}\}$$

Now rebuild the circle with these Pitch Classes.

Circle of fifths

The circle of fifths has been used as a representation of closeness between scales, notes and chords.
The fifth is an interval of particular importance and consonance as it is the first to appear in the harmonic series after the octave.

Sidenote: Many instruments (mainly wind) that work on the harmonic series skip the first harmonic and jump to the 5th such as the trumpet.

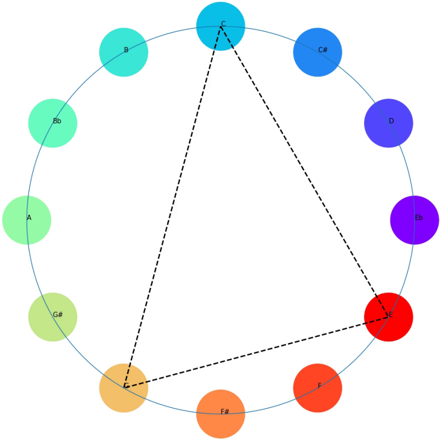
Triads

Definition

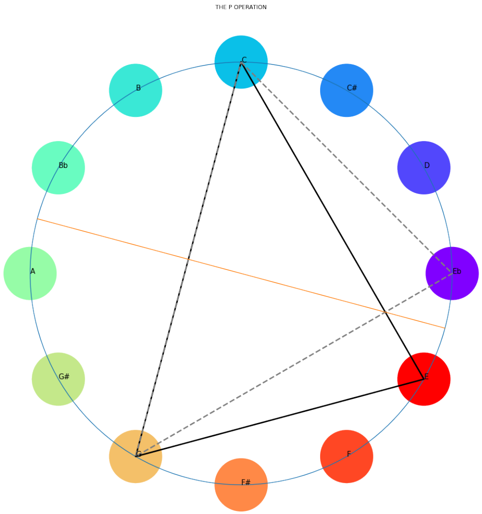
Triad is any combination of any three distinct Pitch Classes. In the Neo-Riemannian theory the major and the minor triads are of interest that are equivalent to the groups $\{n,\ n+4,\ n+7\}$ & $\{n,\ n+3,\ n+7\}$ accordingly, where $n \in \mathbb{Z}/12\mathbb{Z}$.

Transpositions of any group of triads can be seen as rotations in the Circle representation.

Transposition of Triads



PLR Operations - Mirror Symmetries



PLR Operations - Mirror Symmetries

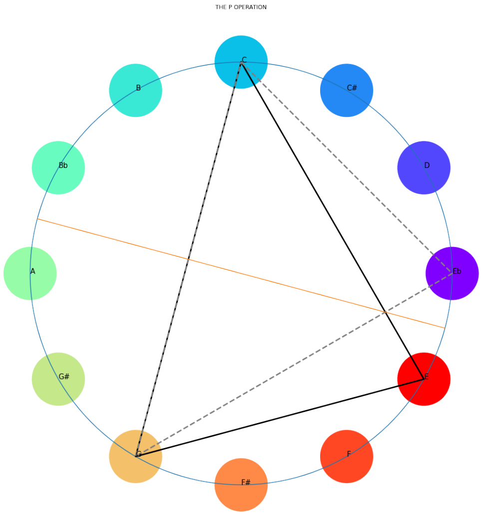
The **PLR operations** stand for :

- **P :** Parallel.
- **L :** Leading-Tone Exchange.
- **R :** Relative.

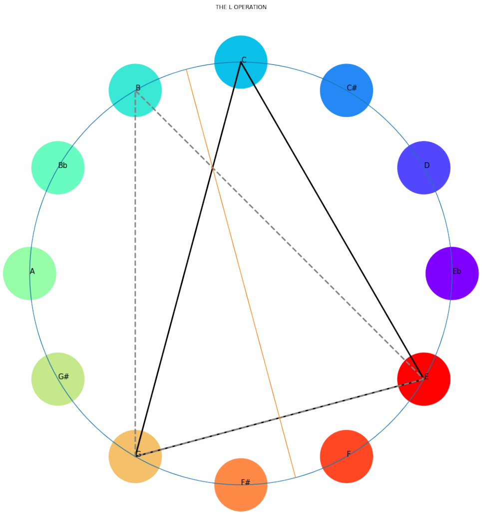
These operations/ transformations are the principal transformations of the Neo-Riemmanian theory which was mainly conceived by David Lewin (1933–2003).

Note : The mirror process is relative to the chord intervalic relations not the position of the shape in the circle.

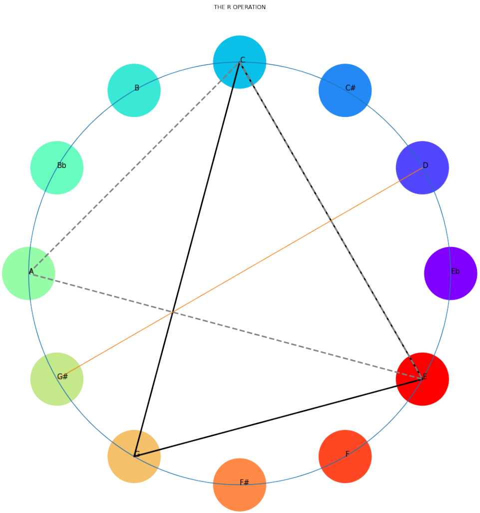
PLR Operations - Mirror Symmetries



PLR Operations - Mirror Symmetries



PLR Operations - Mirror Symmetries



PLR as Group Operations

The PLR group acts simply transitively on the set $\{n_M, n = 0 \ldots 11\} \cup \{n_m, n = 0 \ldots 11\}$ of the 24 major and minor triads, where n_M (resp. n_m) represents a major (resp. minor) triad with root n in the usual semi-tone encoding of pitch classes.

It is isomorphic to the dihedral group D24 of order 24, and is generated by the following two transformations.

The transformation $L : \mathbb{Z}_{24} \to \mathbb{Z}_{24}$ is called the leading-tone operation, and is such that:

$$L(n_M) = (n+4)_m \quad \text{and the complementary} \qquad L(n_m) = (n+8)_M$$

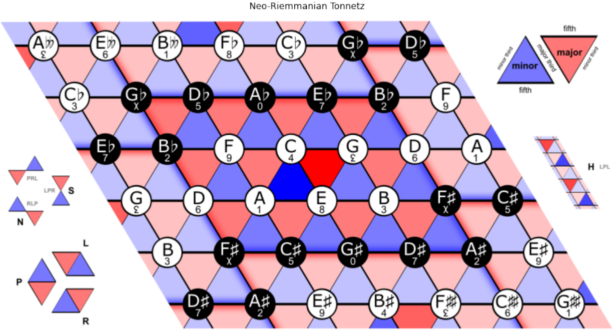
The transformation $R : \mathbb{Z}_{24} \to \mathbb{Z}_{24}$ is called the relative operation, and is such that:

$$R(n_M) = (n+9)_m \quad \text{and the similarly the complementary}$$

Though not a generator, the operation $P = (RL)3R$, called the parallel operation, is often considered, and is such that $P(n_M) = n_m$.

The Tonnetz

Stacking PLR Operation results to the Tonnetz space:



[1]

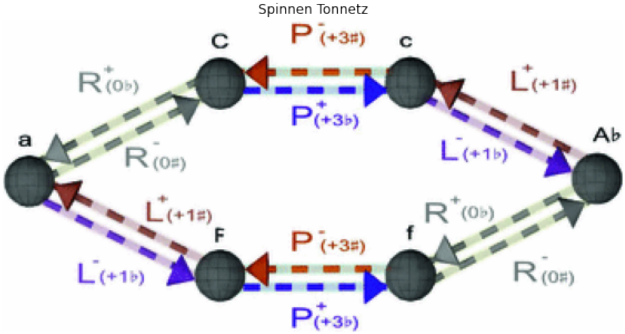
This representation is called Neo-Riemannian Tonnetz.

Nodes in this graph are **Notes**.

[1] Watchduck (a.k.a. Tilman Piesk), CC0, via Wikimedia Commons

The Tonnetz

Stacking PLR Operation results to the Tonnetz space:



[1]

This representation is called Spinnen Tonnetz

Nodes in this graph are **Chords** (triads).

[1] Watchduck (a.k.a. Tilman Piesk), CC0, via Wikimedia Commons

Tonnetz - a beehive

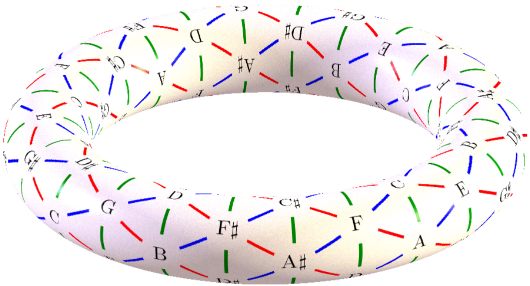
The Tonnetz Creates a beehive grid (hexagonal). So it results to three axes:

- The axis of third minors
- The axis of third majors
- The axis of fifths

Therefore it is also called the Tonnetz $T(3,4,5)$ for minor third, major third and fourth the complementary of fifth correspondingly.

Tonnetz a mathematical Playground

The Neo-Riemannian Tonnetz creates two main loops along its axes. Therefore, topologically it is equivallent to the torus :



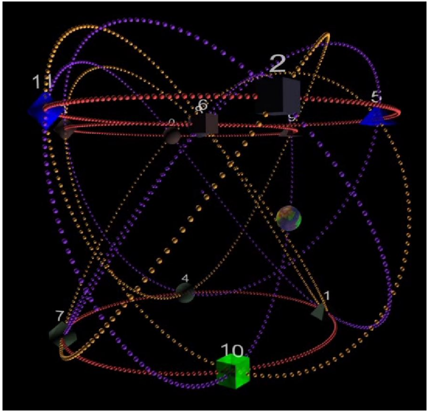
[2]

It didn't take long until people started to play around with different less conventional and less tonal configurations of the Tonnetz.

[2] Davidwbulger, Public domain, via Wikimedia Commons

Tonnetz as a Hypersphere

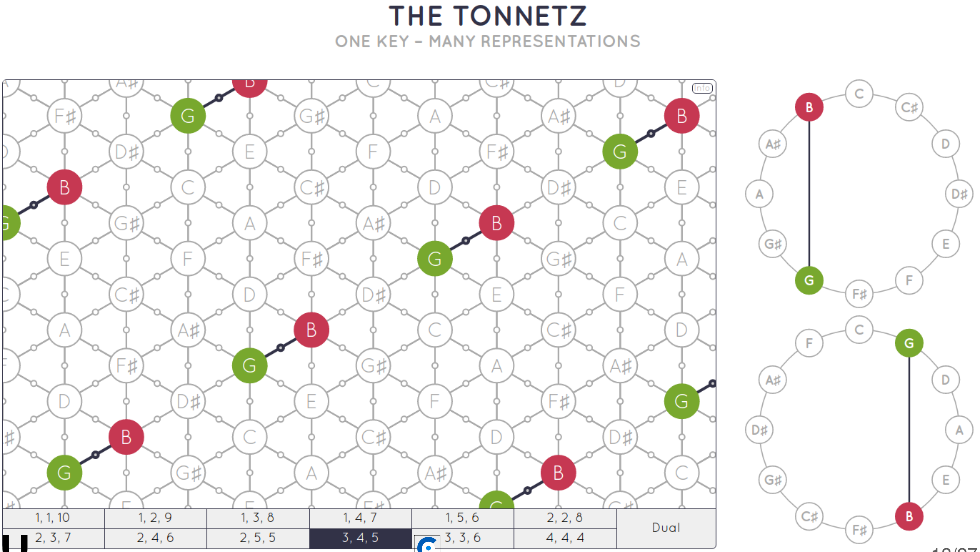
Another representation of the Tonnetz is one in 4 dimensions that manages to close the infinite grid in a compact representation.



Picture : Personal work of Gilles Baroin

DEMONSTRATION

https://guichaoua.gitlab.io/web-hexachord/



EXERCISE

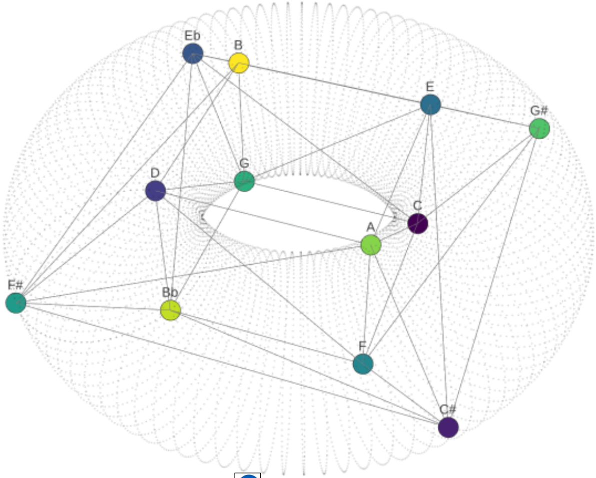
Exercise

1. Parse the Midi File and output a list of Chords.
2. Code a *bijective* function/object that takes a list of notes and returns their respective pitch classes.
3. Code a function that performs transposition on triads.
4. Code the PLR operations.

PART II

Tonnetz 2.

The Tonnetz has may possibilities.



Negative Harmony

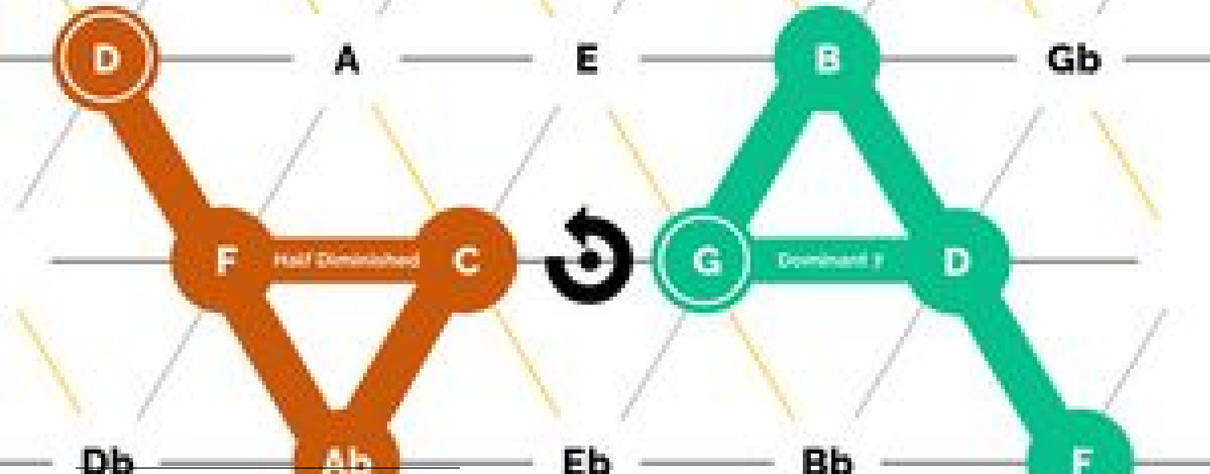
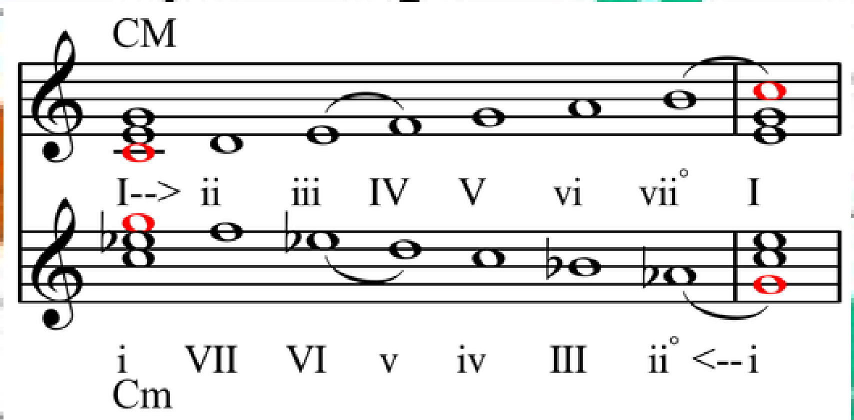


Image by Josh Walsh, Using Tonnetz Tone Mesh To Understand Jazz Harmony in jazz-library.com

Negative Harmony

Hugo Riemann's dualist system in 1880s:



The Hamiltonian Song

Definition

A **Hamiltonian** path on a graph is a walk on the graph that visits each vertex only once.
A **Hamiltonian** circle on a graph is a hamiltonian path that returns on the starting vertex.

Tonal Tension

Definition

Tonal Tension refers to the feeling of stability and instability in music. It specified as the specific sense created by melodic and harmonic motion.

Tension and Release Examples :

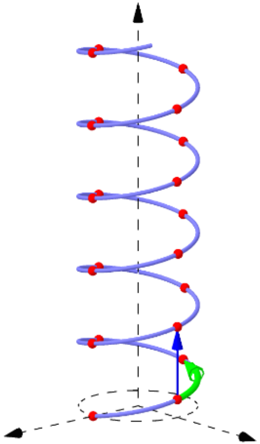
- Moving from the Dominant 7 to the Tonic
- Moving from the Major tonic to a distant key chord.

How to measure Tonal Tension?

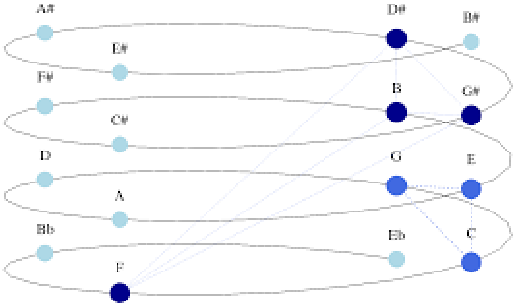
There are elaborate music theories on how to compute tension which are always distant from the computational aspect.

Tonal Tension from the Tonnetz

The spiral array. Two consecutive pitch classes lying on the helix are a perfect fifth apart (considering the orientation of the curved arrow), while the vertical arrow connects two pitch classes a major third far from each other.



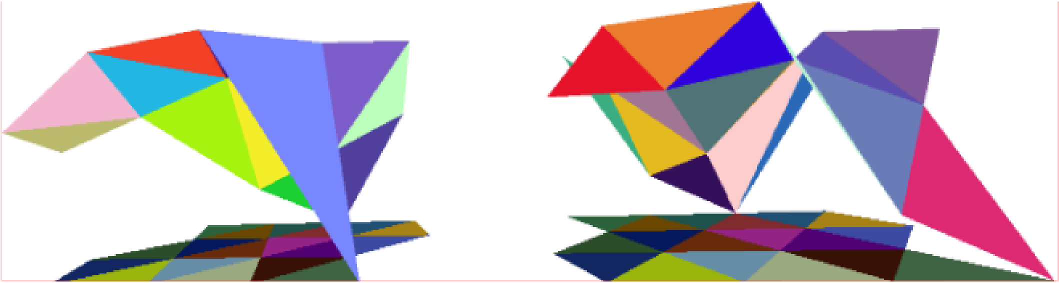
Chew’s Hellical Model



Inspired by the Torus configuration of the Tonnetz the tension of two chords is computed by considering the barycenter of each chord manifold in the spiral array.

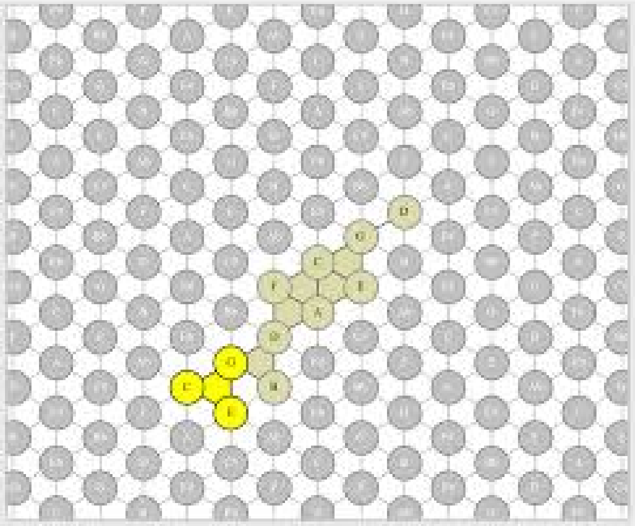
Anisotropic Tonnetz

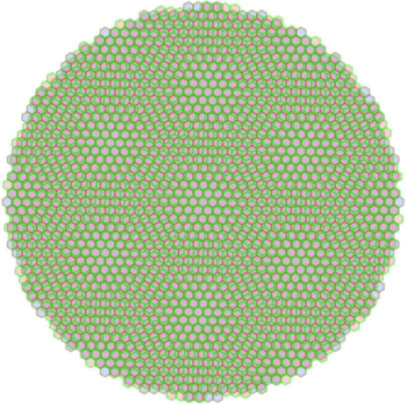
Another use of the Tonnetz was to identify compositional trends by measuring strength of pitch classes on a Tonnetz manifold. This work was presented on the thesis of Mattia Bergomi.



Deformed geometries generated from the Tonnetz.

Tonnetz as an Infinite Grid

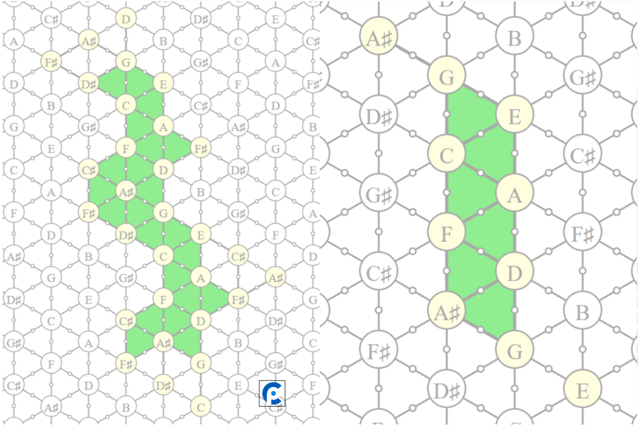




Harmonic Trajectories

Definition

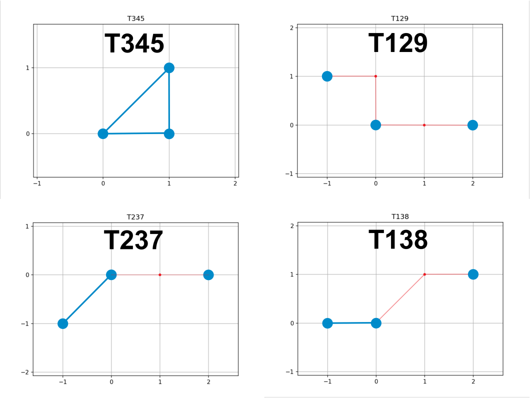
The trajectory is defined as a path X in the Tonnetz T, i.e. an ordered list ofpositions in the space T.



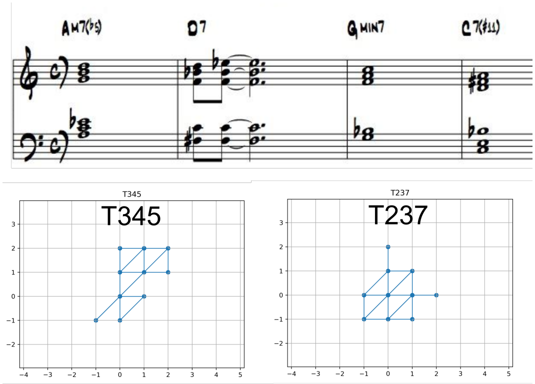
Generalized Tonnetze

$\mathbf{C(n1, n2, n3)}$	**Simplicial Complex Representative**	**Trichord**
$C(1,1,10)$	cylinder	$\{0,1,2\} = \{C, C\#, D\}$
$C(1,2,9)$	torus	$\{0,1,3\} = \{C, C\#, D\#\}$
$C(1,3,8)$	torus	$\{0,1,4\} = \{C, C\#, E\}$
$C(1,4,7)$	torus	$\{0,1,5\} = \{C, C\#, F\}$
$C(1,5,6)$	circle of 6 tetrahedra boundaries	$\{0,1,6\} = \{C, C\#, F\}$
$C(2,2,8)$	two disjoint cylinders	$\{0,2,4\} = \{C, D, E\}$
$C(2,3,7)$	torus	$\{0,2,5\} = \{C, D, F\}$
$C(2,4,6)$	two disjoint circles of 3 tetrahedra boundaries	$\{0,2,6\} = \{C, D, F\#\}$
$C(2,5,5)$	cylinder	$\{0,2,7\} = \{C, D, G\}$
$C(3,3,6)$	three disjoint tetrahedra boundaries	$\{0,3,6\} = \{C, Eb, Gb\}$
$C(3,4,5)$	torus	$\{0,3,7\} = \{C, Eb, G\}$
$C(4,4,4)$	four disjoint 2-simplices	$\{0,4,8\} = \{C, E, G\#\}$

Tonnetz Configurations

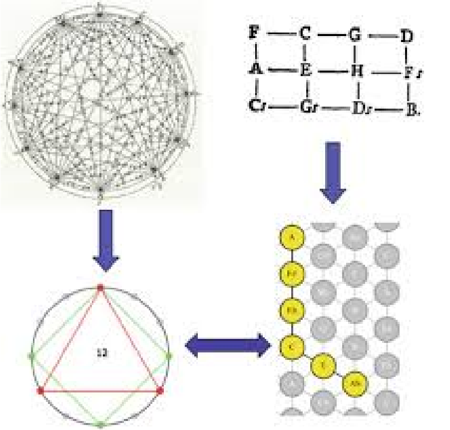


The C major chord in different Tonnetz configurations

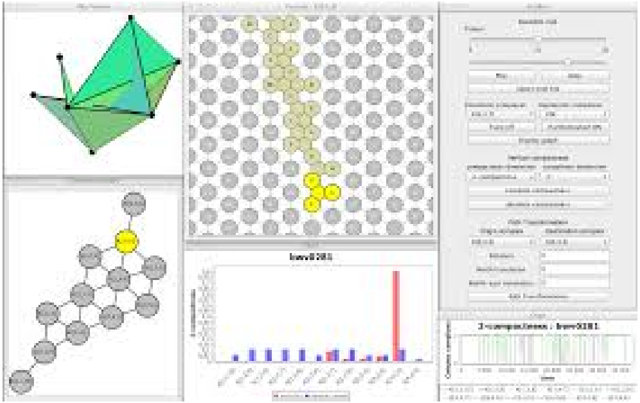


Jazz Sequence Trajectory in different Tonnetz

Combinations



ATIAM Lecture Slides by Moreno Andreatta



Hexachord Software by Louis Bigo.

EXERCISE

Exercises

Hamiltonian Paths

Find all Hamiltonian Paths in the Tonnetz given a starting triad and the Vocabulary of triads (i.e. Given by the space).

Negative Harmony

Given a Piece and a reference point Rotate everything in the Tonnetz 180 Degrees and return the midi and the audio.