Problem 2

Same number of "1"s and "-1"s means that sum of all elements in Bn should be zero.

Let's denote such sums with S_n and $|A_n|$ with F_n (they are fibonacci numbers, too).

$$S_n = \sum_{i=0}^{n/2} A_{n,2i} - A_{n,2i+1} = \sum_{i=0}^{n-1} A_{n,i} \cdot (-1)^i$$

Now we can obtain a recurrence for S_n:

$$S_n = \begin{cases} S_{n-1} + S_{n-2}, & \text{if } F_{n-1} \text{ is even} \\ S_{n-1} - S_{n-2}, & \text{if } F_{n-1} \text{ is odd} \end{cases}$$ (because $S_{n,F_{n-1}}$ must have -1 as it's coefficient)

$F_n \equiv 0 \pmod 2 \Leftrightarrow n \equiv 0 \pmod 3$

Base: $n = 1 \dots 3$

Step: if $n \equiv 0 \pmod 3$

$F_{n-1}, F_{n-2} \equiv 1 \pmod 2$ by induction

$F_n \equiv 1+1 \equiv 0 \pmod 2$

if $n \equiv 1 \pmod 3$

$F_{n-1} \equiv 0$, $F_{n-2} \equiv 1 \pmod 2$ by induction

$F_n \equiv 0+1 \equiv 1 \pmod 2$

and if $n \equiv 2 \pmod 3$

similary $F_n \equiv 1+0 \equiv 1 \pmod 2$

Then $$S_n = \begin{cases} S_{n-1} + S_{n-2}, & \text{if } n \,\vdots\, 3 \\ S_{n-1} - S_{n-2}, & \text{else} \end{cases}$$

Let's look at first few values of S_n:

n	1	2	3	4	5	6	7	8	9	
S_n	0	1	1	2	1	-1	0	1	1	...

Seems like there is a cycle of length 6, let's prove it.

$S_n = S_{n \bmod 6}$ (assume $S_0 = -1$ for convinience)

Base: $0 \leq n < 6$

Step: if $n \,\vdots\, 3$

$S_n = S_{n-1} + S_{n-2} = S_{(n-1) \bmod 6} + S_{(n-2) \bmod 6} = S_{n \bmod 6}$, because $n \bmod 6 \equiv n \pmod 3$

else $S_n = S_{n-1} - S_{n-2} = S_{n \bmod 6}$, for the same reason

So, $S_n = 0$ for $n = 6k+1$, $k \in \mathbb{Z}_{\geq 0}$