

# Math Major Roadmaps

This document outlines "roadmaps" of course options for undergraduates interested in particular fields and applications of mathematics. Each roadmap consists of three stages:

- Stage 1: Introductory courses with few prerequisites, accessible to a typical sophomore.
- Stage 2: More advanced classes for students who have mastered several stage-1 classes.
- Stage 3: The most advanced classes, often beginning graduate-level subjects, for students who have mastered many stage-1 and stage-2 classes. (Many math majors will never take a stage-3 course, and that's okay!)

The **stages also reflect priorities**—for example, students interested in pursuing analysis probably want to take 18.100 as early as possible, whereas a student pursuing mathematical finance might take it later to deepen their understanding.

In each specialized field or application of mathematics below, we list courses that could be **relevant for a career in that field**—not just courses narrowly in that specialization, but also useful courses in broadly related areas. On the other hand, these specific courses are **not requirements** to pursue that field.

Note that a math degree requires 18.03 and 18.06/18.700/701 (or approved substitutions thereof), but these are not necessarily listed in every roadmap below, nor do we list GIRs like 18.02.

#### **Pure Mathematics**

General comments: Below is a list of research areas. **As an undergraduate, however, you should not conceive of yourself as specializing** in one or another of these areas. A much better idea is to gain experience in several of them. You may find yourself taking more courses in one or another area, but all of these fields of study reinforce each other.

Stage 2, but not specific to any particular area: 18.821 (Project Lab in Mathematics) is a great introduction to some essential skills (research, teamwork, and communication).

# Algebra

- Stage 1: Linear Algebra+Modern Algebra, Linear Algebra I+Algebra II or Algebra I+Algebra II
- Stage 2: Seminar in Algebra, Introduction to Representation Theory, Introduction to Algebraic Geometry, Introduction to Arithmetic Geometry
- Stage 3: Commutative Algebra, Noncommutative Algebra, Algebraic Geometry I, Lie Groups and Lie Algebras I, Lie Groups and Lie Algebras II, Elliptic Curves

### Analysis & Geometry

- Stage 1: Real Analysis, Linear Algebra/Algebra I
- Stage 2: Analysis and Manifolds, Introduction to Functional Analysis, Fourier Analysis: Theory and Applications, Seminar in Analysis,
   Functions of a Complex Variable, Introduction to Partial Differential Equations, Differential Geometry, Seminar in Geometry
- Stage 3: Measure Theory and Analysis, Differential Analysis I, Differential Analysis II, Theory of Differential Forms, Geometry of Manifolds I

### Logic

- Stage 1: Introduction to Mathematical Logic and Set Theory, Algebra I or Linear Algebra+Modern Algebra
- Stage 2: Automata, Computability, and Complexity, Theory of Computation, Seminar in Logic
- Stage 3: Mathematical Logic (not offered regularly)

The MIT Philosophy department also offers subjects in logic: 24.241–24.245 and 24.711. Also consider logic classes at Harvard.

# Number Theory

- Stage 1: Linear Algebra+Modern Algebra or Algebra I+Algebra II, Theory of Numbers
- Stage 2: Seminar in Algebra, Introduction to Algebraic Geometry, Introduction to Arithmetic Geometry, Seminar in Number Theory
- Stage 3: Commutative Algebra, Algebraic Geometry I, Elliptic Curves, Number Theory I

# **Probability & Statistics**

- Stage 1: Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, Real Analysis, Probability and Random Variables, Linear Algebra/Algebra I
- Stage 2: Introduction to Functional Analysis, Fourier Analysis: Theory and Applications, Functions of a Complex Variable, Principles of
  Discrete Applied Mathematics, Combinatorial Analysis, Introduction to Stochastic Processes, Topics in Math with Applications in Finance,
  Fundamentals of Statistics

• Stage 3: Measure Theory and Analysis, Theory of Probability, Stochastic Calculus, Topics in Stochastic Processes, Eigenvalues of Random Matrices, Randomized Algorithms, Seminar in Information Theory, Mathematical Statistics, Topics in Statistics

# **Topology & Geometry**

- Stage 1: Real Analysis, Functions of a Complex Variable, Algebra I+Algebra II or Linear Algebra+Modern Algebra, Geometry and Topology in the Plane, Differential Geometry
- Stage 2: Analysis and Manifolds, Introduction to Functional Analysis, Introduction to Topology, Seminar in Topology, Theory of Differential Forms, Seminar in Geometry
- Stage 3: Riemann Surfaces, Differential Analysis I, Introduction to Algebraic Geometry, Lie Groups and Lie Algebras II, Introduction to Arithmetic Geometry, Algebraic Topology I, Algebraic Topology II, Geometry of Manifolds I, Geometry of Manifolds II

# Applied Mathematics

Stage 2, but not specific to any particular area: 18.821 (Project Lab in Mathematics) is a great introduction to some essential skills (research, teamwork, and communication).

#### Combinatorics

- Stage 1: Real Analysis, Principles of Discrete Applied Mathematics, Automata, Computability, and Complexity, Probability and Random Variables, Algebra I+Algebra II or Linear Algebra+Modern Algebra, Geometry and Topology in the Plane
- Stage 2: Undergraduate Seminar in Discrete Math, Combinatorial Analysis, Algebraic Combinatorics, Functions of a Complex Variable, Theory of Computation, Design and Analysis of Algorithms, Combinatorial Optimization, Introduction to Algebraic Geometry, Theory of Numbers, Introduction to Topology, Differential Geometry
- Stage 3: Combinatorial Theory, Topics in Combinatorics, Graph Theory and Additive Combinatorics, Probabilistic Methods in Combinatorics, Eigenvalues of Random Matrices, Advanced Combinatorial Optimization, Introduction to Stochastic Processes, Commutative Algebra, Introduction to Representation Theory, Algebraic Geometry I, Lie Groups and Lie Algebras I

# **Computer Science**

- Stage 1: 18.06/Linear Algebra, Mathematics for Computer Science, Principles of Discrete Applied Mathematics, Introduction to Numerical Analysis, Probability and Random Variables, Fundamentals of Statistics, Algebra I
- Stage 2: Undergraduate Seminar in Discrete Math, Combinatorial Analysis, Automata, Computability, and Complexity, Theory of Computation, Design and Analysis of Algorithms, Seminar in Information Theory, Seminar in Theoretical Computer Science, Combinatorial Optimization
- Stage 3: Parallel Computing and Scientific Machine Learning, Advanced Algorithms, Randomized Algorithms, Cryptography and Cryptanalysis, Quantum Computation, Distributed Algorithms, Advanced Combinatorial Optimization, Elliptic Curves

Students in this area should strongly consider supplementing their math courses with several courses in computer science; see the 18c major requirements for typical choices.

### **Economics and Finance**

- Stage 1: Differential Equations, Introduction to Probability and Statistics, Linear Algebra/Linear Algebra, Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, Principles of Continuum Applied Mathematics, Introduction to Numerical Analysis, Probability and Random Variables, Fundamentals of Statistics
- Stage 2: Real Analysis, Introduction to Partial Differential Equations, Linear PDEs: Analysis and Numerics, Combinatorial Optimization,
   Topics in Math with Applications in Finance
- Stage 3: Fourier Analysis: Theory and Applications, Measure Theory and Analysis, Theory of Probability, Stochastic Calculus, Topics in Stochastic Processes, Introduction to Numerical Methods, Parallel Computing and Scientific Machine Learning, Eigenvalues of Random Matrices, Nonlinear Dynamics: The Natural Environment, Nonlinear Dynamics: Chaos, Fluid Mechanics, Introduction to Stochastic Processes, Mathematical Statistics

Students interested in economics and finance should strongly consider supplementing their math courses with several classes in course 14 and 15, if not a minor or double major.

### Computational Science and Engineering

- Stage 1: Differential Equations, Linear Algebra/Linear Algebra, Complex Variables with Applications, Introduction to Probability and Statistics, Mathematics for Computer Science, Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, Principles of Discrete Applied Mathematics, Linear PDEs: Analysis and Numerics, Introduction to Numerical Analysis, Probability and Random Variables, Fundamentals of Statistics, Special Subject: Introduction to Computational Science and Engineering, Special Subject: Introduction to Computational Thinking for Real-World Problems
- Stage 2: Real Analysis, Theory of Computation, Seminar in Theoretical Computer Science, Design and Analysis of Algorithms, Algebra I
- Stage 3: Introduction to Numerical Methods, Fast Methods for Partial Differential and Integral Equations, Parallel Computing and Scientific Machine Learning, Eigenvalues of Random Matrices, Waves and Imaging, Advanced Algorithms, Distributed Algorithms

Students in this area should consider supplementing their math courses with several courses in computer science, both for software engineering (e.g. 6.0001, 6.009) and numerical methods or optimization (e.g. 6.215, 6.339). For students interested in computational modeling of physical systems, see also the Physical Applied Math classes below.

# **Physical Applied Math**

- Stage 1: Differential Equations, Complex Variables with Applications, Linear Algebra/Linear Algebra, Principles of Continuum Applied Mathematics, Linear PDEs: Analysis and Numerics, Introduction to Numerical Analysis, Nonlinear Dynamics: The Natural Environment, Nonlinear Dynamics: Chaos, Nonlinear Dynamics: Continuum Systems, Probability and Random Variables, Fundamentals of Statistics
- Stage 2: Real Analysis, Functions of a Complex Variable, Introduction to Partial Differential Equations, Undergraduate Seminar in Physical Mathematics, Introduction to Computational Molecular Biology
- Stage 3: Introduction to Functional Analysis, Fourier Analysis: Theory and Applications, Advanced Analytic Methods in Science and Engineering, Advanced Partial Differential Equations with Applications, Introduction to Numerical Methods, Fluid Mechanics, Interfacial Phenomena, Waves and Imaging, Mathematical Methods in Nanophotonics, Wave Propagation, Nonlinear Dynamics and Waves

Students interested in physical applied mathematics should strongly consider supplementing their math courses with at least 2–3 non-GIR courses in physics and/or engineering, depending on their field of interest, if not a minor or double major.

### Statistics and Data Science

- Stage 1: Introduction to Probability and Statistics, Linear Algebra/Linear Algebra, Mathematics for Computer Science, Matrix Methods in Data Analysis, Signal Processing, and Machine Learning, Real Analysis, Principles of Discrete Applied Mathematics, Introduction to Numerical Analysis, Probability and Random Variables, Fundamentals of Statistics, Algebra I
- Stage 2: Introduction to Functional Analysis, Functions of a Complex Variable, Undergraduate Seminar in Discrete Math, Topics in Math with Applications in Finance
- Stage 3: Measure Theory and Analysis, Theory of Probability, Stochastic Calculus, Introduction to Numerical Methods, Eigenvalues of Random Matrices, Introduction to Stochastic Processes, Topics in Statistics

Students in this area should consider supplementing their math courses with courses in computer science on machine learning and courses in economics on econometrics.