

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

⇒ Here, $a = 3$, $b = 2$

$$\log_b a = \log_2 3 = 1.58$$

$$n^{\log_b a} = n^{1.58}$$

Here, $f(n) = n^2$

$$\therefore n^2 > n^{1.58}$$

$$\therefore T.C = \Theta(n^2)$$

2. $T(n) = 4(T(n/2)) + n^2$

⇒ $a = 4$, $b = 2$

$$\log_b a = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$$

$$\therefore n^{\log_b a} = n^2$$

$$\therefore f(n) = n^2$$

$$\therefore n^2 = n^2$$

$$\therefore T.C = \Theta(n^2 \log n)$$

3. $T(n) = T(n/2) + 2^n$

Here $a = 1$, $b = 2$

$$\log_b a = \log_2 1 = 0$$

$$n^c = n^{\log_b a} = n^0 \Rightarrow 1$$

$$\therefore 2^n > 1$$

$$\therefore T.C = \Theta(2^n)$$

4. $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow a = 2^n$

which cannot be possible

Since $a > 1$ & a cannot ^{is not} be constant

Thus, master's method do not apply.

5. $T(n) = 16 T(\frac{n}{4}) + n$

$\Rightarrow a = 16 \quad b = 4$

$\log_b a = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4 = 2$

$n^{\log_b a} = n^2$

$f(n) = n \therefore n^{\log_b a} > f(n)$

$\therefore T.C = \Theta(n^2)$

6. $T(n) = 2 T(n/2) + n \log n$

$\Rightarrow a = 2 \quad b = 2$

$\log_b a = \log_2 2 = 1$

$f(n) = n \log n$

$\therefore n^c = n^{\log_b a} = n^1 = n$

$\therefore f(n) > n^c$

$\therefore T.C = \Theta(n \log^2 n)$

7. $T(n) = 2 T(n/2) + \frac{n}{\log n}$

$a = 2 \quad b = 2$

$n^c = n^{\log_b a} = n^{\log_2 2} = n^1$

$f(n) = \frac{n}{\log n}$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

$$8. T(n) = 2T(n/4) + n^{0.51}$$

$$\Rightarrow a=2 \quad b=4$$

$$c = \log_b a = \log_4 2 = 0.5$$

$$n^c = n^{0.5}$$

$$f(n) = n^{0.51}$$

$$\therefore f(n) > n^c$$

$$T(n) = \Theta(n^{0.51})$$

$$9. T(n) = 0.5T(n/2) + 1/n$$

$$\Rightarrow a=0.5 \quad b=2$$

$a < 1$ Therefore we cannot apply Master's Theorem.

$$10. T(n) = 16T(n/4) + n^6$$

$$\Rightarrow a=16 \quad b=4$$

$$c = \log_b a = \log_4 16 = 2 \log_4 4 = 2$$

$$n^c = n^2$$

$$\text{As } n^6 > n^2$$

$$\therefore T(n) = \Theta(n^6)$$

$$11. 4T(n/2) + \log n$$

$$\Rightarrow a=4 \quad b=2$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = \log n$$

$$\therefore \log n < n^2$$

$$T(n) = \Theta(n^2)$$

$$12. T(n) = \sqrt{n} T(n/2) + \log n$$

$$a = \sqrt{n} \quad b = 2$$

$$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$$

$$\frac{1}{2} \log_2 n < \log(n) \quad \therefore T(n) = \Theta(\log(n))$$

13. $T(n) = 3T(n/2) + n$

$\Rightarrow a=3 \quad b=2$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$f(n) = n$

$\therefore n < n^{1.58}$

$\therefore T(n) = O(n^{1.5849})$

Q14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$\Rightarrow a=3 \quad b=3$

$c = \log_3 3 = 1$

$n^c = \underline{1} \quad n^1 = n$

$\text{sqrt}(n) < 1$

$f(n) < n^c$

$\therefore T(n) = O(n)$

Q15. $T(n) = 4T(n/2) + n$

$a=4 \quad b=2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n$

$\therefore n^2 > n$

$n^c > f(n)$

$T(n) = O(n^2)$

Q16. $T(n) = 3T(n/4) + n \log n$

$a=3 \quad b=4 \quad f(n) = n \log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n \log n$

$T(n) = O(n \log n)$

Q17. $T(n) = 3T(n/3) + n/2$

$\Rightarrow a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) = n/2$

$\therefore \frac{n}{2} < n$

$T(n) = O(n)$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$\Rightarrow a=6, b=3$

$c = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

$f(n) = n^2 \log n$

$\therefore T(n) = O(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow a=4, b=2$

$c = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \frac{n}{\log n}$

$\therefore \frac{n}{\log n} < n^2 \therefore T(n) = O(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

$a=64, b=8$

$c = \log_8 64 = 2$

$n^c = n^2, f(n) = n^2 \log n$

$T(n) = O(n^2 \log n)$

Q1. $T(n) = 7T(n/3) + n^2$

$\Rightarrow a=7, b=3$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$f(n) = n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

Q2. $T(n) = T(n/2) + n(2 - \cos n)$
 $a=1, b=2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) = n(2 - \cos n)$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$