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Abstract

- This document contains the explanation of question 9.12 of Papoulis Pillai Probability book of chapter sequence of random variables.

ASSIGNMENT-7

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Show that :- If $x(t)$ is a process with zero mean and auto correlation $f(t_1)f(t_2)w(t_1 - t_2)$, then the process $y(t) = \frac{x(t)}{f(t)}$ is WSS with auto correlation $w(t_1 - t_2)$. If $x(t)$ is white noise with auto correlation $q(t_1) \times \delta(t_1 - t_2)$ then the process $z(t) = \frac{x(t)}{q(t)}$ is WSS white noise with auto correlation $\delta(t_1 - t_2)$.

1st part:-

According to question , $x(t)$ is a zero mean process. So, $E(x(T))=0$ and

$$u_y = u_x \int_{-\infty}^{\infty} f(\alpha) d\alpha$$

$$y(t) = \frac{x(t)}{f(t)}$$

$$\begin{aligned} R_{yy}(t_1, t_2) &= \frac{R_{xx}(t_1, t_2)}{f(t_1)f(t_2)} \\ &= f(t_1)f(t_2)w(t_1 - t_2) \frac{1}{f(t_1)f(t_2)} \\ &= w(t_1 - t_2) \end{aligned}$$

2nd part:-

$x(t)$ is a white noise process which means

$$R_x(t_1 - t_2) = F^{-1}(N_0/2) = N_0\delta(t_1 - t_2)/2$$

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases}$$

This shows that $X(t_1)$ and $x(t_2)$ are correlated only when $t_1 = t_2$ and uncorrelated when $t_1 \neq t_2$. Therefore, $R_{xx}(t_1, t_2) = 0$, when t_1 is not equal to t_2

$$R_{xx}(t_1, t_2) = q(t_1) \times \delta(t_1 - t_2), \text{ when } t_1 = t_2$$

Hence, we can write $R_{xx}(t_1, t_2)$ as

$$(q(t_1))^{1/2}(q(t_2))^{1/2} \times \delta(t_1 - t_2) \quad (q(t_1) = q(t_2))$$

$$z(t) = x(t) \overline{q(t)}$$

$$R_{zz} = \frac{q(t_1)q(t_2)\delta(t_1 - t_2)}{q(t_1)q(t_2)}$$

$$R_{zz} = \delta(t_1 - t_2)$$

Hence, proved