Outline

Abstract

- This document contains the explanation of question 9.12 of Papoulis Pillai Probability book of chapter sequence of random variables.

ASSIGNMENT-9

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Consider a general one-dimensional random walk on the possible states $e_o, e_1, e_2, \ldots.$ Let S_n represent the location of the particle at time n on a straight line such that at each interior state e_j , the particle either moves to the right to e_{j+1} with probability P_j, or to the left to e_{j-1} with probability q_j or remains where it is at e_j.

At state e_0 it can either stay at the same position with probability r_0 or move to the right to e_1 with probability P_1.
The transition matrix for the given problem is:-

$$\begin{pmatrix} r_0 & p_0 & 0 & 0 & \dots \\ q_1 & r_1 & p_1 & 0 & \dots \\ 0 & q_2 & r_2 & p_2 & \dots \\ 0 & 0 & q_3 & r_3 & p_3 \end{pmatrix}$$

Random walk on a line with $r_0 + p_0 = 1$

$q_i + r_i + p_i = 1$ i=1,2,3,......

Thus, $P_{00} = r_0$ $\mathrm{p}_{01} = p_0$ $p_{0j} = 0, j > 1$

and for i ≥ 1

p_{ij}= { $p_i > 0$ j=i+1

$r_i >= 0$j=i

$q_i > 0$ j=i-1

0 otherwise }

The model with $p_i = p, q_i = 1 - p, r_i = 0$for i$>$ 1, and $r_0$corresponds to the gambler's ruin problem.
Here, the distribution of distance d_N travelled after a given number of steps. Let n_1 be the number of steps travelled towards left and total steps be N.
$d_N = 2n_1 - N$
d=a$_1 + a_2 + ...a_n$
$< d \geq< (a_1 + a_2 + a_3 + a_4 + ...a_n) >$
$< d \geq< a_1 > + < a_2 > + < a_3 > + < a_4 > .... < a_n >$
d ≥ 0 $\{$As, $la_i >= 0$ if there is equal probability to move forward or backward and the steps taken are equal in distance.$\}$
Average of D^2
$< d^2 \geq< (a_1 + a_2 + a_3 + ...a_n)^2 >$
$< d^2 \geq< {a_1}^2 > + < {a_2}^2 > + < {a_3}^2 > +...... < a_n^2 > +2\{< a_1 >< a_2 >$
$+ < a_2 >< a_3 > + < a_4 >< a_5 > +.....\}$
$< d^2 \geq N$ $\{$ $\sqrt{< a^2 >}$ is average positive distance $\}$
$\sqrt{< d^2 >} = \sqrt{N}$
We expect that after N steps,we are $\sqrt{N}$ steps away from where we start.