ASSIGNMENT -2

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Problem 1: Using matrices,solve the following system of equations:-

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

Solution:- Three equations are:-

1.

$$2x - 3y + 5z = 11 \tag{1}$$
$$3x + 2y - 4z = -5 \tag{2}$$
$$x + y - 2z = -3 \tag{3}$$

The above equations can be represented in the form:-

$$\mathbf{AX} = \mathbf{B} \tag{4}$$

where,

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}, \tag{5}$$

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \tag{6}$$

$$\mathbf{B} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{7}$$

$$So, \mathbf{X} = \mathbf{BA}^{-1} \tag{8}$$

$$\mathbf{A}^{-1} = \frac{adj\mathbf{A}}{|\mathbf{A}|} \tag{9}$$

$|\mathbf{A}|=2\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}+3\begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix}+ 5\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$
$|\mathbf{A}|$=2(-4+4) +3(-6+4)+5(3-2)
=-6+5=-1

$$adj\mathbf{A} = \begin{pmatrix} \mathbf{A_{11}} & \mathbf{A_{21}} & \mathbf{A_{31}} \\ \mathbf{A_{12}} & \mathbf{A_{22}} & \mathbf{A_{32}} \\ \mathbf{A_{13}} & \mathbf{A_{23}} & \mathbf{A_{33}} \end{pmatrix} \tag{10}$$

$$\mathbf{A_{11}} = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} \tag{11}$$

$$\mathbf{A_{11}} = -4+4 = 0 \tag{12}$$

$$\mathbf{A_{12}} = -\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} \tag{13}$$

$$\mathbf{A_{12}} = -(-6+4) = 2 \tag{14}$$

$$\mathbf{A_{13}} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \tag{15}$$

$$\mathbf{A_{13}} = 3-2 = 1 \tag{16}$$

$$\mathbf{A_{21}} = -\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} \tag{17}$$

$$\mathbf{A_{21}} = -(6-5) = -1 \tag{18}$$

$$\mathbf{A_{22}} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} \tag{19}$$

$$\mathbf{A_{22}} = -4-5 = -9 \tag{20}$$

$$\mathbf{A_{23}} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \tag{21}$$

$$\mathbf{A_{23}} = -(2+3) = -5 \tag{22}$$

$$\mathbf{A_{31}} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} \tag{23}$$

$$\mathbf{A_{31}} = 12-10 = 2 \tag{24}$$

$$\mathbf{A_{32}} = -\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} \tag{25}$$

$$\mathbf{A_{32}} = -(-8-15) = 23 \tag{26}$$

$$\mathbf{A_{33}} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} \tag{27}$$

$$\mathbf{A_{33}} = 4+9 = 13 \tag{28}$$

$$\tag{29}$$

From equation 10,we get

$$adj\mathbf{A} = \begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix} \tag{30}$$

From equation 9 , we get

$$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \tag{31}$$

$$\tag{32}$$

From equation 8, we can say that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{33}$$

$$= \begin{pmatrix} 6-5 \\ 69-22-45 \\ 39-11-25 \end{pmatrix} \tag{34}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{35}$$

$$\Rightarrow x = 1, y = 2 and z = 3 \tag{36}$$

$$\tag{37}$$

Solutions of the given equations are $x = 1$,$y = 2$and $z = 3$.