ASSIGNMENT -2

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May 2022

Problem 1: Using matrices,solve the following system of equations:-

$$\begin{aligned} 2x-3y+5z &= 11 \\ 3x+2y-4z &= -5 \\ x+y-2z &= -3 \end{aligned}$$

Solution:-

1. Three equations provided in the question are:-

$$2x-3y+5z = 11 \tag{1}$$
$$3x+2y-4z = -5 \tag{2}$$
$$x+y-2z = -3 \tag{3}$$

The above equations can be represented in the form:-

$$\mathbf{AX} = \mathbf{B} \tag{4}$$

where,

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{5}$$

$$So, \mathbf{X} = \mathbf{BA}^{-1} \tag{6}$$

$$\mathbf{A}^{-1} = \frac{adj\mathbf{A}}{|\mathbf{A}|} \tag{7}$$

$$|\mathbf{A}| = 2\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix} + 3\begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix} + 5\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

$|\mathbf{A}|$=2(-4+4) +3(-6+4)+5(3-2)
=-6+5=-1

$$adj\mathbf{A} = \begin{pmatrix} \mathbf{A_{11}} & \mathbf{A_{21}} & \mathbf{A_{31}} \\ \mathbf{A_{12}} & \mathbf{A_{22}} & \mathbf{A_{32}} \\ \mathbf{A_{13}} & \mathbf{A_{23}} & \mathbf{A_{33}} \end{pmatrix} \tag{8}$$

$$\mathbf{A_{11}} = \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} \tag{9}$$
$$\mathbf{A_{11}} = -4+4 = 0 \tag{10}$$
$$\mathbf{A_{12}} = -\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} \tag{11}$$
$$\mathbf{A_{12}} = -(-6+4) = 2 \tag{12}$$
$$\mathbf{A_{13}} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} \tag{13}$$
$$\mathbf{A_{13}} = 3-2 = 1 \tag{14}$$
$$\mathbf{A_{21}} = -\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} \tag{15}$$
$$\mathbf{A_{21}} = -(6-5) = -1 \tag{16}$$
$$\mathbf{A_{22}} = \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} \tag{17}$$
$$\mathbf{A_{22}} = -4-5 = -9 \tag{18}$$
$$\mathbf{A_{23}} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} \tag{19}$$
$$\mathbf{A_{23}} = -(2+3) = -5 \tag{20}$$
$$\mathbf{A_{31}} = \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} \tag{21}$$
$$\mathbf{A_{31}} = 12-10 = 2 \tag{22}$$
$$\mathbf{A_{32}} = -\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} \tag{23}$$
$$\mathbf{A_{32}} = -(-8-15) = 23 \tag{24}$$
$$\mathbf{A_{33}} = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} \tag{25}$$
$$\mathbf{A_{33}} = 4+9 = 13 \tag{26}$$
$$\tag{27}$$

From equation 10,we get

$$adj\mathbf{A} = \begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix} \tag{28}$$

From equation 9 , we get

$$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \tag{29}$$

$$\tag{30}$$

From equation 8, we can say that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} \tag{31}$$

$$= \begin{pmatrix} 6-5 \\ 69-22-45 \\ 39-11-25 \end{pmatrix} \tag{32}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{33}$$

$$\implies x = 1, y = 2 and z = 3 \tag{34}$$

$$\tag{35}$$

Solutions of the given equations are $x = 1$,$y = 2$and $z = 3$.