School of Engineering and Applied Science
Ahmedabad University

B. Tech. (ICT): Semester II
EMT Project: part I

The radiation emitted due to a time dependent current

Feel free to use any resources you like (e.g. www or MATLAB or gnuplot). This is a take home problem (aka project), the assessment shall be based on:

1. understanding of the background material,
2. the result: finding the solution of the problem given,
3. interpretation of the result.

In this problem, one has to use one's knowledge of Maxwell's equations to determine the electric and magnetic fields around a straight wire as the current in the wire changes in an arbitrary way.

- The wire is assumed to be infinitesimally thick,
- The wire is assumed to be infinitely long (or, equivalently, we are sufficiently close to the wire that it appears to be infinitely long),
- The wire is assumed to be exactly straight and lies along the z-axis,
- We assume a translational symmetry along the wire (so, nothing depends on z),
- We also assume complete cylindrical symmetry (so, nothing depends on ϕ).
- The current in the wire will be the same at all locations on the wire is given by a function $I(t)$ which is specified by by the user.

In this problem, it is useful to use cylindrical polar coordinates (s, ϕ, z). As the current through the wire changes in an arbitrary way, the wire acts as an antenna and emits electromagnetic waves. One can convince oneself that the resulting electric field has only a $z-$component and depends on only s and t i.e. $E_z(s,t)$, similarly, $B_\phi(s,t)$. *You are expected to write a program (e.g. in C) which takes $I(t)$ as an input and provides $E_z(s,t)$ and $B_\phi(s,t)$ as the output.*

Notice that the user specified function $I(t)$ could be specified as a data file which contains two columns, the first column will have the value of t (the independent variable) and the second column will have the value of $I(t)$. Similarly the output such as $E_z(s,t)$ could be specified as a data file. If we wish to be able to sample the output function well, we need to evaluate it at many points. Suppose we have N_s data points along $s-$axis and N_t data points along $t - axis$, then the data file for $E_z(s,t)$ will have N_s rows and N_t columns and each value will just be the value of $E_z(s,t)$. Similar remarks apply to $B_\phi(s,t)$.

1 Background

We convinced ourselves during this course that every situation involving the electromagnetic field is ultimately a solution of Maxwell's equations. One could solve Maxwell's equations to determine the fields in any given circumstance including the current problem. As we have learnt, this is best done by first evaluating the scalar potential, $V(t,\vec{r})$, and vector potential, $\vec{A}(t,\vec{r})$, and then evaluating the fields by using the relations

$$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}\,. \tag{1}$$

One could determine the potentials (in what is called Lorentz gauge) by the relation (see sec. 10.2 of Griffiths)

$$V(t,\vec{r}) = \frac{1}{4\pi\epsilon_0}\int d\tau'\,\frac{\rho(t-\frac{|\vec{r}-\vec{r'}|}{c},\vec{r'})}{|\vec{r}-\vec{r'}|}\,, \tag{2}$$

$$\vec{A}(t,\vec{r}) = \frac{\mu_0}{4\pi}\int d\tau'\,\frac{\vec{J}(t-\frac{|\vec{r}-\vec{r'}|}{c},\vec{r'})}{|\vec{r}-\vec{r'}|}\,. \tag{3}$$

Let us pause for a moment to understand the meaning of this: suppose I wished to find the scalar potential V at a desired time t and at the desired location $\vec{r}$. What the equation above tells us is that I can do that by integrating over the contributions due to all the points in space. But in order to find the contribution of the source at the location $\vec{r'}$, what I need is the charge density **NOT** at the desired time t but at a time $t-\frac{|\vec{r}-\vec{r'}|}{c}$. Notice that $\frac{|\vec{r}-\vec{r'}|}{c}$ is the time it would take for something travelling at the speed c to go from $\vec{r'}$ to $\vec{r}$. Thus, $t-\frac{|\vec{r}-\vec{r'}|}{c}$ is the moment at which something travelling at the speed c must have started from the location $\vec{r'}$ if it wanted to arrive at the desired location $\vec{r}$ at the desired time t.

Thus, though the expressions above appear similar to the expressions for potentials in electrostatic or magnetostatic situations, they are quite subtle and one must understand it carefully before proceeding. The potentials found from the above expressions are called *retarded potentials* and the time $t-\frac{|\vec{r}-\vec{r'}|}{c}$ is called the retarded time corresponding to the location $\vec{r'}$. Needless to say, the speed c is the speed of light in vacuum.

1.1 A few test cases

Example 10.2 and unsolved problem 10.9 in Griffiths illustrates, how one could use the above expressions for finding the scalar and vector potentials in a simple situation. For these cases for which the analytical calculations are available, the result of the numerical code should agree with analytical predictions.

2 Things to ponder

Are there any EM waves emitted in this situation? How do you know? What is the direction of propagation of the waves? What's the shape of the wavefront? How can you find the total power radiated per unit length of the wire? How can you generalize this for a current which builds up in more complicated ways? Can you think of any obvious limitations of this analysis? What is the amount of energy which gets radiated as we turn on the current in a typical wire we work with in a lab? Can you think of some ways of reducing this radiative loss? Can you think of a simple way to detect the emitted em waves?