PRACTICAL 7

To perform the Taylor's series expansion of a given function f(z) around a given point z

Ques1. f [z] = Exp [z] , z = 0 , n = 6

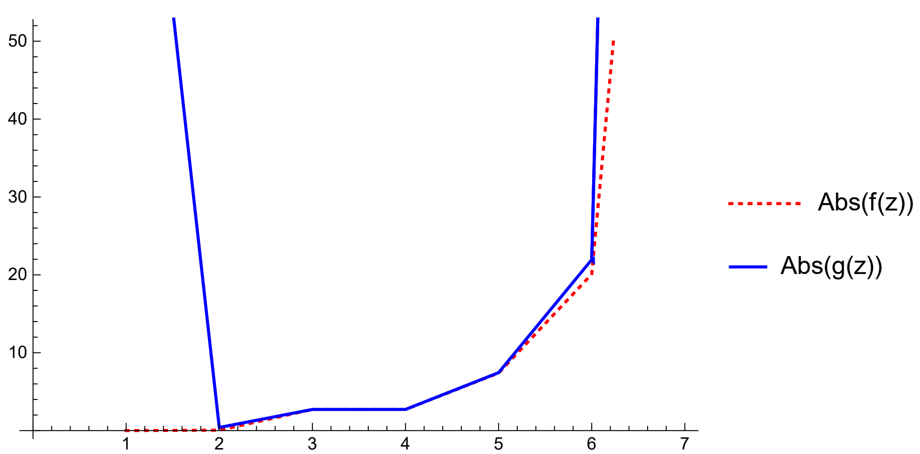
In[1]:=
```
f[z_] = Exp[z];
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 6}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=",
 z0, "is \n", g[z]]
k = ListLinePlot[Table[Abs[f[z]], {z, Z}],
   PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red, Dotted}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z) = e^z

The Taylor series Expansion of f(z) around z=0is

$1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{120} + \frac{z^6}{720}$

Out[9]=



Ques2 . f[z] = 1 / (4 - z^2) , z = 0 , n = 30

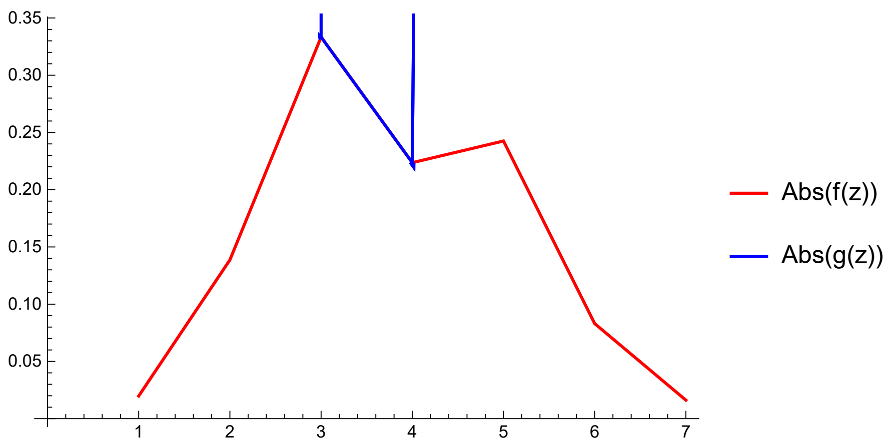
In[10]:=
```
f[z_] = 1 / (4 - z^2);
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 30}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function $f(z) = \frac{1}{4 - z^2}$

The Taylor series Expansion of f(z) around z=0is

$$\frac{1}{4} + \frac{z^2}{16} + \frac{z^4}{64} + \frac{z^6}{256} + \frac{z^8}{1024} + \frac{z^{10}}{4096} + \frac{z^{12}}{16\,384} + \frac{z^{14}}{65\,536} + \frac{z^{16}}{262\,144} + \frac{z^{18}}{1\,048\,576} + \frac{z^{20}}{4\,194\,304} + \frac{z^{22}}{16\,777\,216} + \frac{z^{24}}{67\,108\,864} + \frac{z^{26}}{268\,435\,456} + \frac{z^{28}}{1\,073\,741\,824} + \frac{z^{30}}{4\,294\,967\,296}$$

Out[18]=



Ques 3. f[z] = z / (z^4 + 9) , z = 0 , n = 10

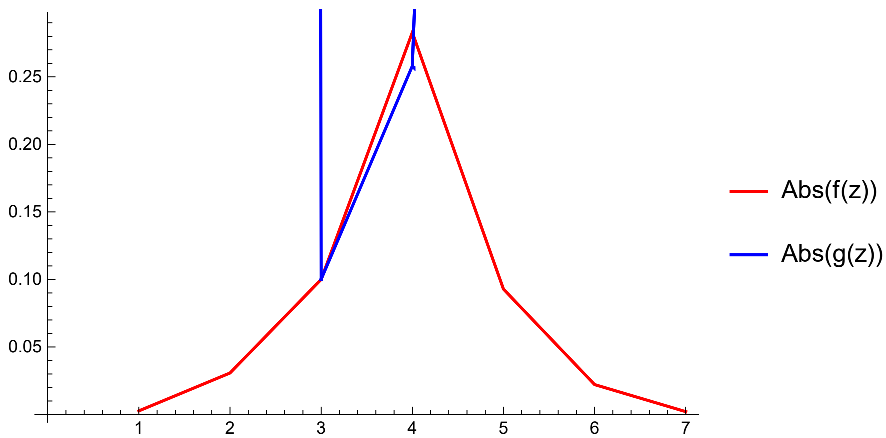
In[19]:=
```
f[z_] = z / (z^4 + 9);
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z) = $\frac{z}{9+z^4}$

The Taylor series Expansion of f(z) around z=0is

$\frac{z}{9} - \frac{z^5}{81} + \frac{z^9}{729}$

Out[27]=



Ques 4. f[z] = (1+2 z^2) / (z^3 + z^5) , z =0 , n = 10

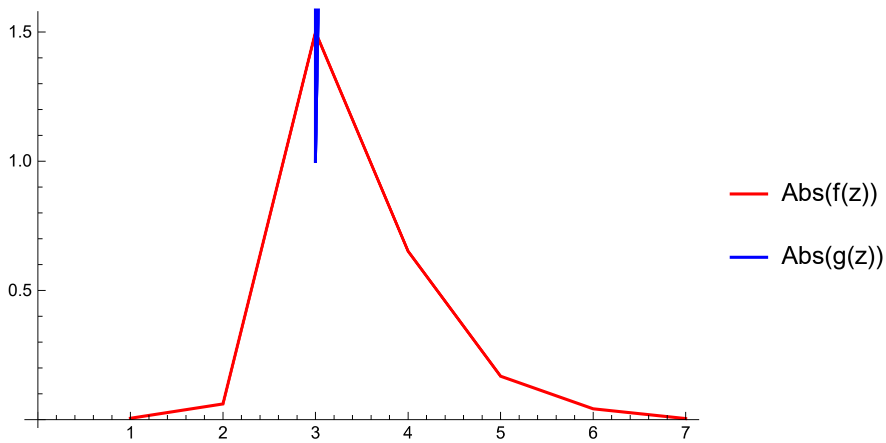
In[28]:=
```
f[z_] = (1 + 2 z^2) / (z^3 + z^5);
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function $f(z) = \frac{1 + 2 z^2}{z^3 + z^5}$

The Taylor series Expansion of $f(z)$ around z=0is

$$\frac{1}{z^3} + \frac{1}{z} - z + z^3 - z^5 + z^7 - z^9$$

Out[36]=



Ques 5. f [z] = 1 / (1 - z) , z = 2 , n = 30

In[37]:=

```
f[z_] = 1 / (1 - z);
z0 = 2;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 30}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

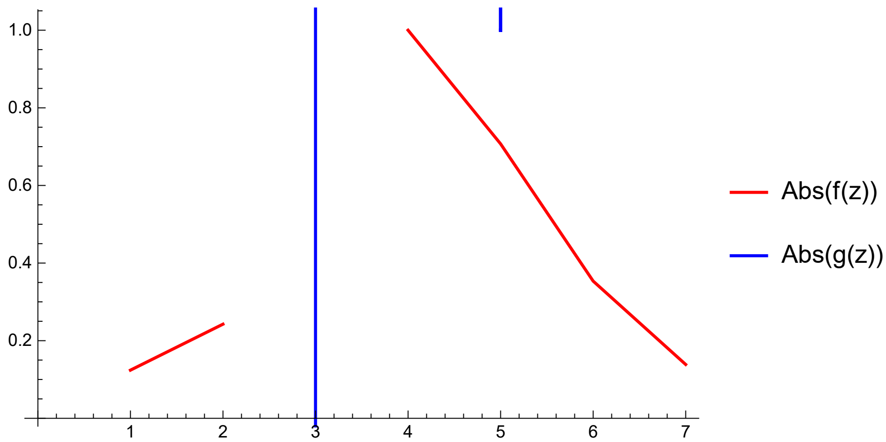
The given function $f(z) = \frac{1}{1-z}$

The Taylor series Expansion of f(z) around z=2is

$-3-(-2+z)^2+(-2+z)^3-(-2+z)^4+(-2+z)^5-(-2+z)^6+(-2+z)^7-(-2+z)^8+(-2+z)^9-(-2+z)^{10}+(-2+z)^{11}-(-2+z)^{12}+(-2+z)^{13}-(-2+z)^{14}+(-2+z)^{15}-(-2+z)^{16}+(-2+z)^{17}-(-2+z)^{18}+(-2+z)^{19}-(-2+z)^{20}+(-2+z)^{21}-(-2+z)^{22}+(-2+z)^{23}-(-2+z)^{24}+(-2+z)^{25}-(-2+z)^{26}+(-2+z)^{27}-(-2+z)^{28}+(-2+z)^{29}-(-2+z)^{30}+z$

Power: Infinite expression $\frac{1}{0}$ encountered.

Out[45]=



Ques 6. f [z] = cos [z] , z = 1 , n = 15

In[46]:=

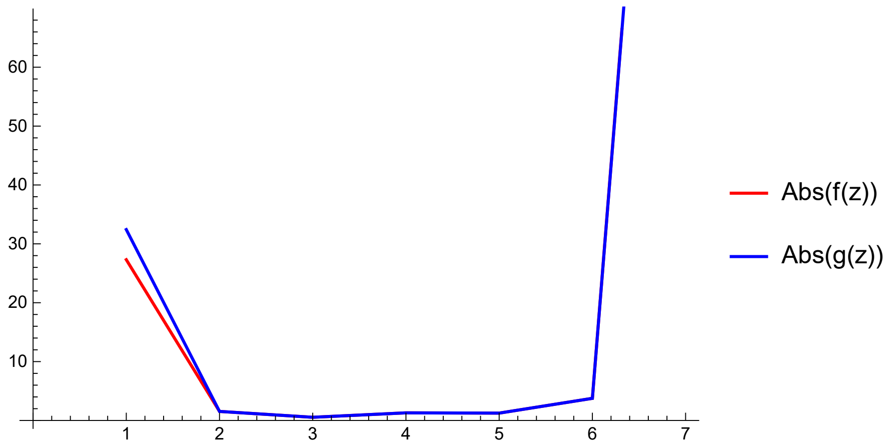
```
f[z_] = Cos[z];
z0 = 1;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 15}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z)=Cos[z]

The Taylor series Expansion of f(z) around z=1is

$$\mathrm{Cos}[1]-\frac{1}{2}(-1+z)^2\,\mathrm{Cos}[1]+\frac{1}{24}(-1+z)^4\,\mathrm{Cos}[1]-\frac{1}{720}(-1+z)^6\,\mathrm{Cos}[1]+\frac{(-1+z)^8\,\mathrm{Cos}[1]}{40320}-\frac{(-1+z)^{10}\,\mathrm{Cos}[1]}{3628800}+\frac{(-1+z)^{12}\,\mathrm{Cos}[1]}{479001600}-\frac{(-1+z)^{14}\,\mathrm{Cos}[1]}{87178291200}-(-1+z)\,\mathrm{Sin}[1]+\frac{1}{6}(-1+z)^3\,\mathrm{Sin}[1]-\frac{1}{120}(-1+z)^5\,\mathrm{Sin}[1]+\frac{(-1+z)^7\,\mathrm{Sin}[1]}{5040}-\frac{(-1+z)^9\,\mathrm{Sin}[1]}{362880}+\frac{(-1+z)^{11}\,\mathrm{Sin}[1]}{39916800}-\frac{(-1+z)^{13}\,\mathrm{Sin}[1]}{6227020800}+\frac{(-1+z)^{15}\,\mathrm{Sin}[1]}{1307674368000}$$

Out[54]=



Ques7. f [z] = sin[z] , z = 1 , n = 20

In[55]:=

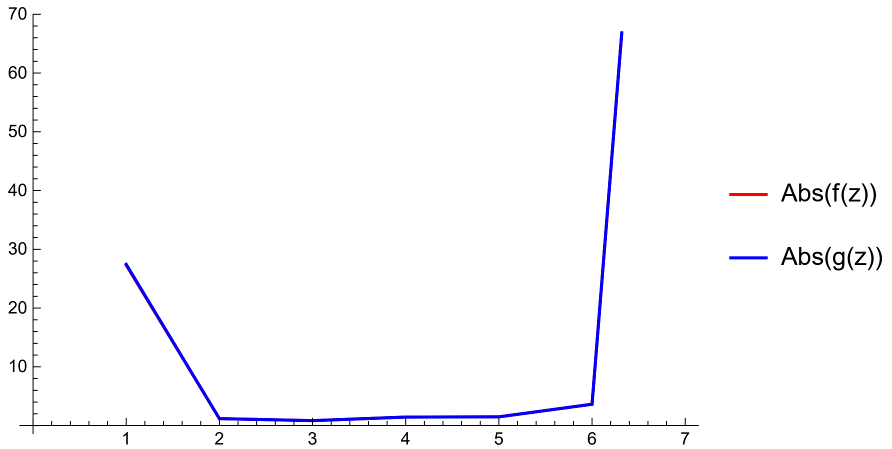
```
f[z_] = Sin[z];
z0 = 1;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 20}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z)=Sin[z]

The Taylor series Expansion of f(z) around z=1is

$$(-1+z)\,\mathrm{Cos}[1]-\frac{1}{6}(-1+z)^3\,\mathrm{Cos}[1]+\frac{1}{120}(-1+z)^5\,\mathrm{Cos}[1]-\frac{(-1+z)^7\,\mathrm{Cos}[1]}{5040}+\frac{(-1+z)^9\,\mathrm{Cos}[1]}{362\,880}-\frac{(-1+z)^{11}\,\mathrm{Cos}[1]}{39\,916\,800}+\frac{(-1+z)^{13}\,\mathrm{Cos}[1]}{6\,227\,020\,800}-\frac{(-1+z)^{15}\,\mathrm{Cos}[1]}{1\,307\,674\,368\,000}+\frac{(-1+z)^{17}\,\mathrm{Cos}[1]}{355\,687\,428\,096\,000}-\frac{(-1+z)^{19}\,\mathrm{Cos}[1]}{121\,645\,100\,408\,832\,000}+\mathrm{Sin}[1]-\frac{1}{2}(-1+z)^2\,\mathrm{Sin}[1]+\frac{1}{24}(-1+z)^4\,\mathrm{Sin}[1]-\frac{1}{720}(-1+z)^6\,\mathrm{Sin}[1]+\frac{(-1+z)^8\,\mathrm{Sin}[1]}{40\,320}-\frac{(-1+z)^{10}\,\mathrm{Sin}[1]}{3\,628\,800}+\frac{(-1+z)^{12}\,\mathrm{Sin}[1]}{479\,001\,600}-\frac{(-1+z)^{14}\,\mathrm{Sin}[1]}{87\,178\,291\,200}+\frac{(-1+z)^{16}\,\mathrm{Sin}[1]}{20\,922\,789\,888\,000}-\frac{(-1+z)^{18}\,\mathrm{Sin}[1]}{6\,402\,373\,705\,728\,000}+\frac{(-1+z)^{20}\,\mathrm{Sin}[1]}{2\,432\,902\,008\,176\,640\,000}$$

Out[63]=



Ques 8. f [z] = sin[z^2] , z = 1 , n = 15

In[64]:=

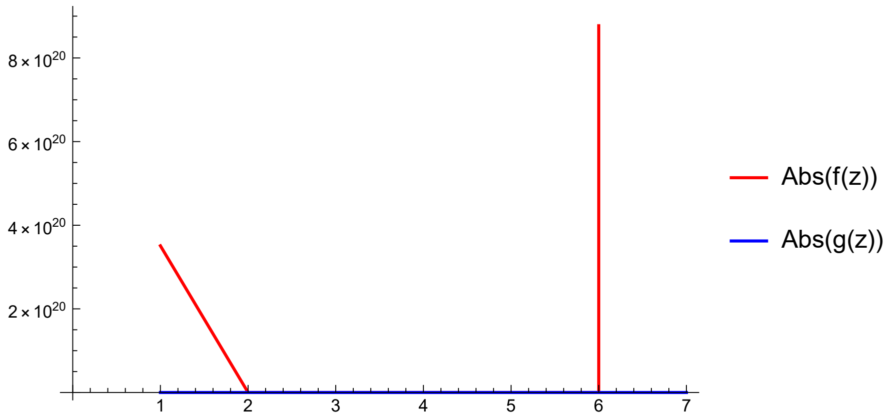
```
f[z_] = Sin[z^2];
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 15}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z)=Sin[z^2]

The Taylor series Expansion of f(z) around z=0is

$$z^2 - \frac{z^6}{6} + \frac{z^{10}}{120} - \frac{z^{14}}{5040}$$

Out[72]=



Ques 9. f[z] = Exp[z^2], z = 1, n = 7

In[73]:=

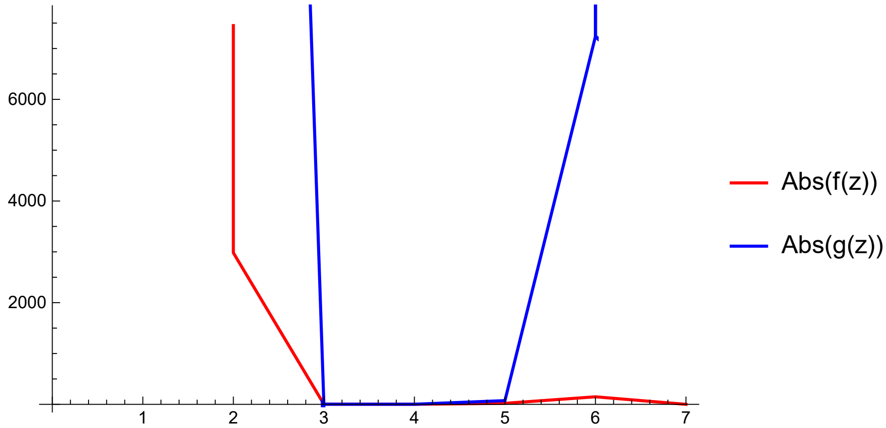
```
f[z_] = Exp[z^2];
z0 = 1;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 7}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function $f(z) = e^{z^2}$

The Taylor series Expansion of $f(z)$ around z=1is

$$e + 2 e (-1 + z) + 3 e (-1 + z)^2 + \frac{10}{3} e (-1 + z)^3 + \frac{19}{6} e (-1 + z)^4 + \frac{13}{5} e (-1 + z)^5 + \frac{173}{90} e (-1 + z)^6 + \frac{407}{315} e (-1 + z)^7$$

Out[81]=



Ques 10. f[z] = Sinh[z], z = 0, n = 4

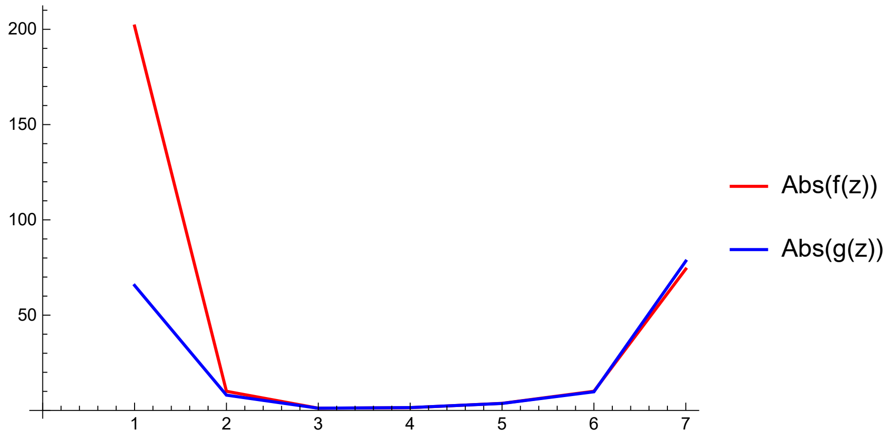
In[82]:=
```
f[z_] = Sinh[z];
z0 = 0;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 4}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z)=Sinh[z]

The Taylor series Expansion of f(z) around z=0is

$$z + \frac{z^3}{6}$$

Out[90]=



Ques 11. f[z] = tan[z], z = 1, n = 4

In[91]:=

```
f[z_] = Tan[z];
z0 = 1;
Z = {-6 + 4 I, -3 - I, 1, 1 + I, 2 + I, 3 + 2 I, 5 + 6 I};
g[z_] = Normal[Series[f[z], {z, z0, 4}]];
Print["The given function f(z)=", f[z]];
Print["The Taylor series Expansion of f(z) around z=", z0, "is \n", g[z]]
k =
  ListLinePlot[Table[Abs[f[z]], {z, Z}], PlotLegends → {"Abs(f(z))"}, PlotStyle → {Red}];
h = ListLinePlot[Table[Abs[g[z]], {z, Z}],
   PlotLegends → {"Abs(g(z))"}, PlotStyle → {Blue}];
Show[
 k,
 h]
```

The given function f(z)=Tan[z]

The Taylor series Expansion of f(z) around z=1is

$$\text{Tan}[1] + (-1+z)\left(1+\text{Tan}[1]^2\right) + (-1+z)^2\left(\text{Tan}[1]+\text{Tan}[1]^3\right) + \frac{1}{3}(-1+z)^3\left(1+4\,\text{Tan}[1]^2+3\,\text{Tan}[1]^4\right) + \frac{1}{3}(-1+z)^4\left(2\,\text{Tan}[1]+5\,\text{Tan}[1]^3+3\,\text{Tan}[1]^5\right)$$

Out[99]=