Practical -9

Draw the following sequences of function on the given interval ,and discuss the uniform conergence.

Ques. Show that sequence of the function $< x^n >$ is uniformly convergent on [0, 0.5]

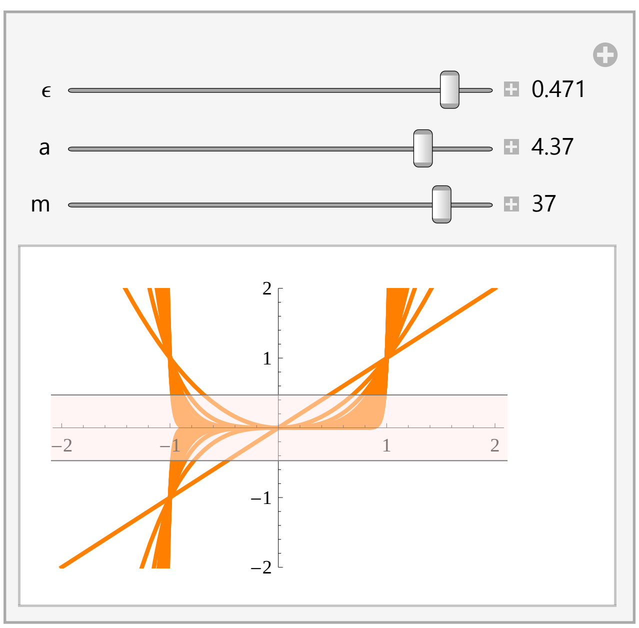
```
b[n_] := Maximize[{Abs[x^n - 0], 0 ≤ x ≤ 0.5}, x]
TableForm[Table[{n, NumberForm[N[b[n][[1]]], 12]}, {n, 1, 10}],
 TableHeadings → {{}, {"n", "|fn(x)-f(x)|"}}]
```

n	$\lvert f_n(x) - f(x) \rvert$
1	0.5
2	0.25
3	0.125
4	0.0625
5	0.03125
6	0.015625
7	0.0078125
8	0.00390625
9	0.001953125
10	0.0009765625

Graphically :

In[1]:=
```
f[n_, x_] := x^n
Manipulate[Plot[Table[f[n, x], {n, m}], {x, -2, 2},
  PlotRange → {-2, 2}, PlotStyle → {Orange, Thick}, Epilog →
   {Opacity[.5], LightPink, EdgeForm[GrayLevel[.5]], Rectangle[{-a, -ϵ}, {a, ϵ}]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 5, 0.01, Appearance → "Labeled"}, {m, 1, 40, 1, Appearance → "Labeled"}]
```

Out[2]=



Conclusion : The given sequence of functions converges uniformly to the function f (x) = 0 in any closed and bounded interval [-k, k], < 0 k < 1.

Ques. Show that sequence of the function $<\frac{x}{n}>$ is uniformly convergent on $[-1, 1]$

```
b[n_] := Maximize[{Abs[x/n - 0], -1 ≤ x ≤ 1}, x]
TableForm[Table[{n, NumberForm[N[b[n][[1]]], 12]}, {n, 1, 100}],
 TableHeadings → {{}, {"n", "|fn(x)-f(x)|"}}]
```

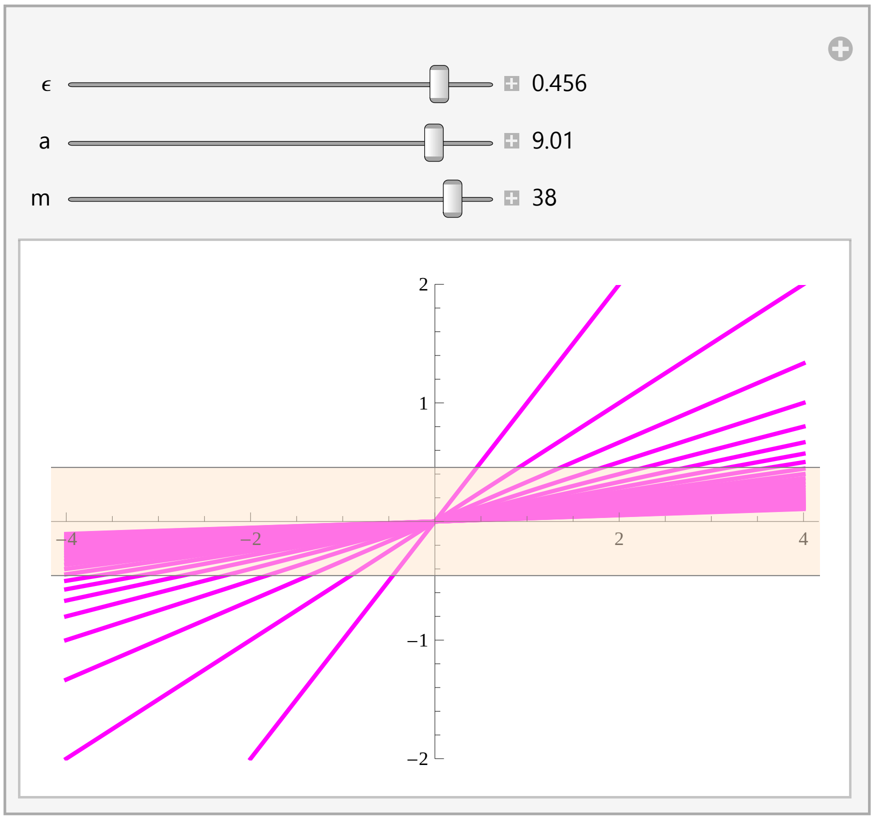
n	$\lvert f_n(x)-f(x)\rvert$
1	1.
2	0.5
3	0.333333333333
4	0.25
5	0.2
6	0.166666666667
7	0.142857142857
8	0.125
9	0.111111111111
10	0.1
11	0.0909090909091
12	0.0833333333333
13	0.0769230769231
14	0.0714285714286
15	0.0666666666667
16	0.0625
17	0.0588235294118
18	0.0555555555556
19	0.0526315789474
20	0.05
21	0.047619047619
22	0.0454545454545
23	0.0434782608696
24	0.0416666666667
25	0.04
26	0.0384615384615
27	0.037037037037
28	0.0357142857143
29	0.0344827586207
30	0.0333333333333
31	0.0322580645161
32	0.03125
33	0.030303030303
34	0.0294117647059
35	0.0285714285714
36	0.0277777777778
37	0.027027027027
38	0.0263157894737
39	0.025641025641
40	0.025
41	0.0243902439024
42	0.0238095238095
43	0.0232558139535
44	0.0227272727273
45	0.0222222222222

46	0.0217391304348
47	0.0212765957447
48	0.0208333333333
49	0.0204081632653
50	0.02
51	0.0196078431373
52	0.0192307692308
53	0.0188679245283
54	0.0185185185185
55	0.0181818181818
56	0.0178571428571
57	0.0175438596491
58	0.0172413793103
59	0.0169491525424
60	0.0166666666667
61	0.016393442623
62	0.0161290322581
63	0.015873015873
64	0.015625
65	0.0153846153846
66	0.0151515151515
67	0.0149253731343
68	0.0147058823529
69	0.0144927536232
70	0.0142857142857
71	0.0140845070423
72	0.0138888888889
73	0.013698630137
74	0.0135135135135
75	0.0133333333333
76	0.0131578947368
77	0.012987012987
78	0.0128205128205
79	0.0126582278481
80	0.0125
81	0.0123456790123
82	0.0121951219512
83	0.0120481927711
84	0.0119047619048
85	0.0117647058824
86	0.0116279069767
87	0.0114942528736
88	0.0113636363636
89	0.0112359550562
90	0.0111111111111
91	0.010989010989
92	0.0108695652174
93	0.010752688172
94	0.0106382978723
95	0.0105263157895
96	0.0104166666667
97	0.0103092783505
98	0.0102040816327
99	0.010101010101
100	0.01

Graphically :

In[3]:=
```
g[n_, x_] := x/n
Manipulate[Plot[Table[g[n, x], {n, m}], {x, -4, 4},
  PlotRange → {-2, 2}, PlotStyle → {Magenta, Thick}, Epilog → {Opacity[.5],
    LightOrange, EdgeForm[GrayLevel[.5]], Rectangle[{-a, -ϵ}, {a, ϵ}]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 10, 0.01, Appearance → "Labeled"},
 {m, 1, 40, 1, Appearance → "Labeled"}]
```

Out[4]=



Conclusion : The given sequence of functions converges uniformly to the function f (x) = 0 in any closed and bounded interval [a, b], b > a.

Ques. Show that sequence of the function $< \frac{x^2}{n} >$ is uniformly convergent on [0, 8]

```
w[n_] := Maximize[{Abs[x^2/n - 0], 0 ≤ x ≤ 8}, x]
TableForm[Table[{n, NumberForm[N[w[n][[1]]], 12]}, {n, 1, 10}],
 TableHeadings → {{}, {"n", "|fn(x)-f(x)|"}}]
```

n	$\lvert f_n(x)-f(x)\rvert$
1	64.
2	32.
3	21.3333333333
4	16.
5	12.8
6	10.6666666667
7	9.14285714286
8	8.
9	7.11111111111
10	6.4

Ques. Show that sequence of the function $< f_n(x) = \frac{x}{x+n} >$ is uniformly convergent on $[0, 8]$

```
w[n_] := Maximize[{Abs[x/(x + n) - 0], 0 ≤ x ≤ 10}, x]
TableForm[Table[{n, NumberForm[N[w[n][[1]]], 12]}, {n, 1, 10}],
 TableHeadings → {{}, {"n", "|fn(x)-f(x)|"}}]
```

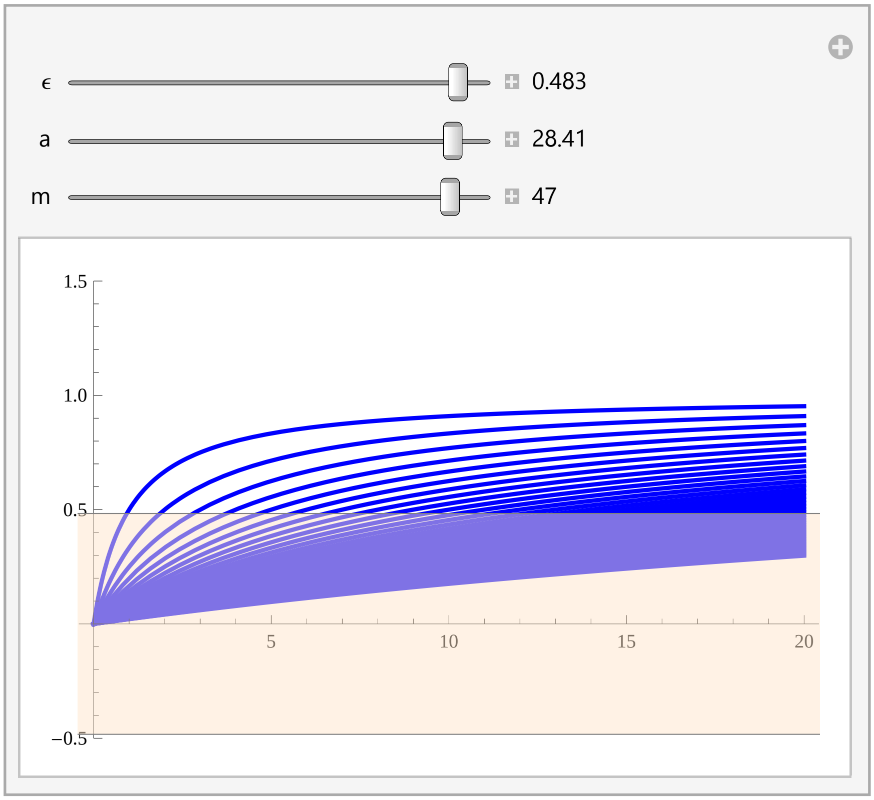
n	$\lvert f_n(x)-f(x)\rvert$
1	0.909090909091
2	0.833333333333
3	0.769230769231
4	0.714285714286
5	0.666666666667
6	0.625
7	0.588235294118
8	0.555555555556
9	0.526315789474
10	0.5

Graphically :

In[13]:=

```
l[n_, x_] := x/(x + n)
Manipulate[Plot[Table[l[n, x], {n, m}], {x, 0, 20},
  PlotRange → {-0.5, 1.5}, PlotStyle → {Blue, Thick}, Epilog → {Opacity[.5],
    LightOrange, EdgeForm[GrayLevel[.5]], Rectangle[{-a, -ϵ}, {a, ϵ}]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 30, 0.01, Appearance → "Labeled"},
 {m, 1, 50, 1, Appearance → "Labeled"}]
```

Out[14]=

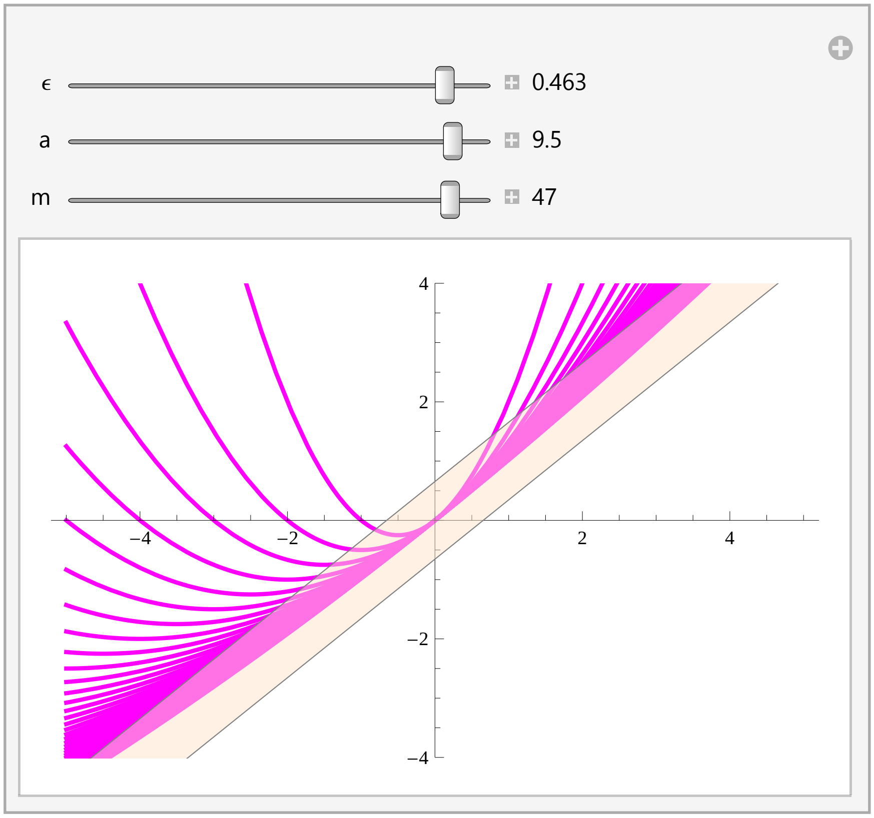


Conclusion : The given sequence of functions converges uniformly to the function $f(x) = 0$ in any closed and bounded interval $[0, k]$, $k > 0$.

Ques. Show that sequence of the function $\left\langle \frac{(x)^2 + n x}{n} \right\rangle$ is uniformly convergent on $[-5, 5]$

In[9]:=
```
h[n_, x_] := ((x)^2 + n x)/n
Manipulate[Plot[Table[h[n, x], {n, m}], {x, -5, 5}, PlotRange → {-4, 4},
  PlotStyle → {Magenta, Thick}, Epilog → {Opacity[.5], LightOrange,
    EdgeForm[GrayLevel[.5]], Rotate[Rectangle[{-a, -ϵ}, {a, ϵ}], π/4]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 10, 0.01, Appearance → "Labeled"},
 {m, 1, 50, 1, Appearance → "Labeled"}]
```

Out[10]=

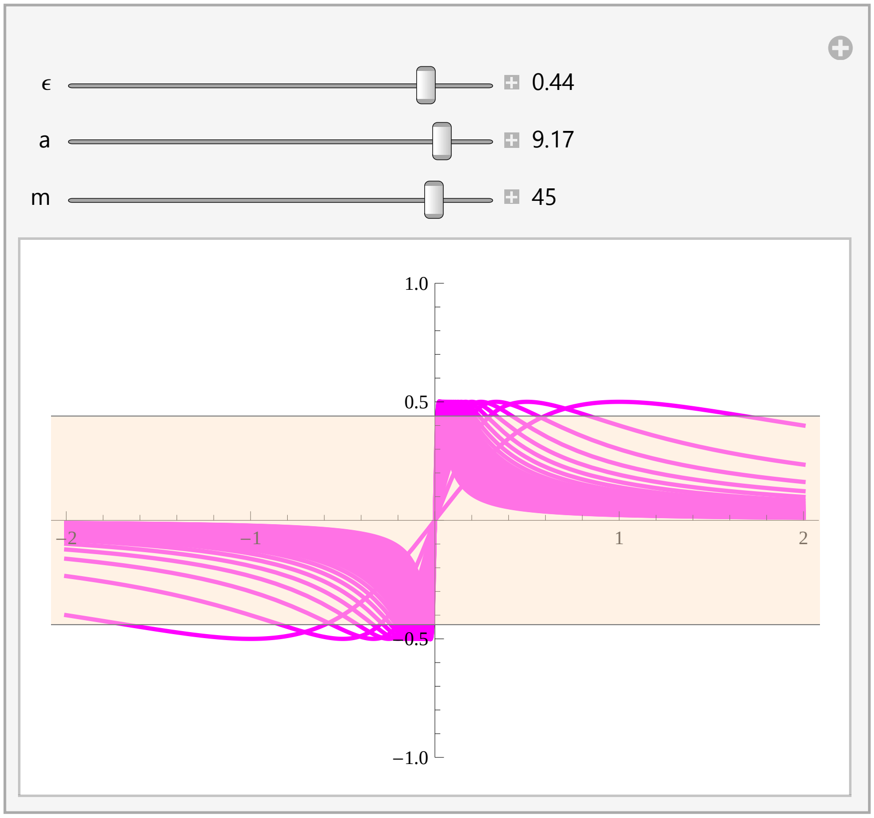


Conclusion : The given sequence of functions converges uniformly to the function f (x) = x in any closed and bounded interval [-k, k], k > 0.

Ques. Show that sequence of the function $< \frac{n x}{1 + n^2 x^2} >$ is uniformly convergent on [-2, 2]

In[7]:=
```
b[n_, x_] := n x / (1 + n^2 x^2)
Manipulate[Plot[Table[b[n, x], {n, m}], {x, -2, 2},
  PlotRange → {-1, 1}, PlotStyle → {Magenta, Thick}, Epilog → {Opacity[.5],
    LightOrange, EdgeForm[GrayLevel[.5]], Rectangle[{-a, -ϵ}, {a, ϵ}]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 10, 0.01, Appearance → "Labeled"},
 {m, 1, 50, 1, Appearance → "Labeled"}]
```

Out[8]=

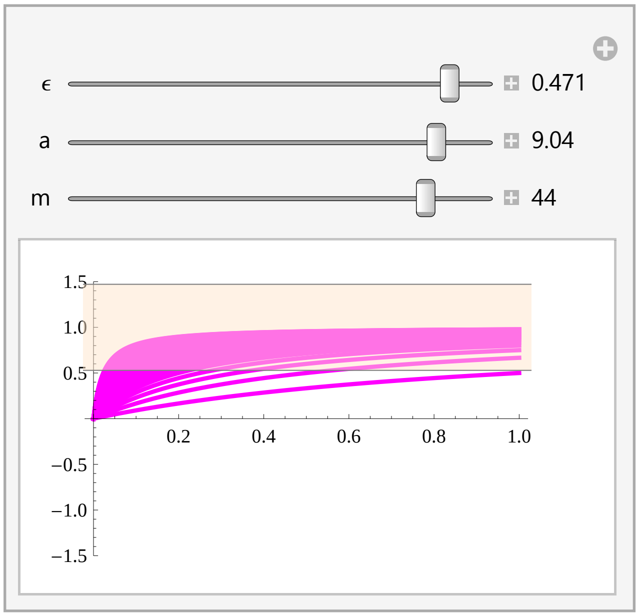


Conclusion : The given sequence of functions converges uniformly to the function f (x) = 0 in any interval [1, k], k > 1.

Ques. Show that sequence of the function < $\frac{n x}{1 + n x}$ > is uniformly convergent on [0, 1]

In[5]:=
```
z[n_, x_] := (n x)/(1 + n x)
Manipulate[Plot[Table[z[n, x], {n, m}], {x, 0, 1}, PlotRange → {-1.5, 1.5},
  PlotStyle → {Magenta, Thick}, Epilog → {Opacity[.5], LightOrange,
    EdgeForm[GrayLevel[.5]], Rectangle[{-a, 1 - ϵ}, {a, 1 + ϵ}]}],
 {ϵ, 0.01, 0.5, 0.001, Appearance → "Labeled"},
 {a, 0, 10, 0.01, Appearance → "Labeled"},
 {m, 1, 50, 1, Appearance → "Labeled"}]
```

Out[6]=



Conclusion : The given sequence of functions converges uniformly to the function f (x) = 1 in any closed and bounded interval [a, b], b > a > 0.