

5/03/20

PRACTICAL - 10

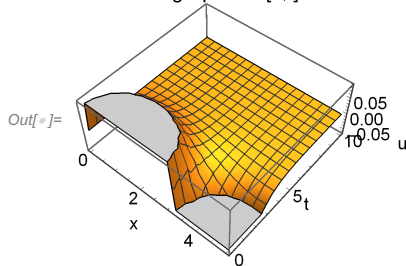
### SOLUTION OF ONE DIMENSIONAL HEAT EQUATION

Q1. Find the solution of the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < 5, \quad 0 \leq t, \quad u(x, 0) = \sin[x], \\ 0 \leq x \leq 5, \quad u(0, t) = 0 \quad t \geq 0, \quad u(5, t) = 0 \quad t \geq 0$$

```
In[ ]:= eqn1a = {D[u[x, t], t] - D[D[u[x, t], x], x] == 0, u[x, 0] == Sin[x], u[0, t] == 0, u[5, t] == 0};  
sol1a = NDSolve[eqn1a, u[x, t], {x, 0, 5}, {t, 0, 10}, PrecisionGoal -> 3] // Quiet;  
Plot3D[u[x, t] /. sol1a, {x, 0, 5}, {t, 0, 10},  
  AxesLabel -> {"x", "t", "u"}, PlotLabel -> "The graph of u[x,t]"]
```

The graph of u[x,t]

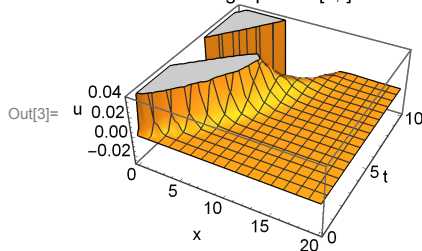


Q2. Find the solution of the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < 20, \quad 0 < t, \quad u(x, 0) = 0, \quad 0 \leq x \leq 20, \\ u(0, t) = t^2 \sin[t] \quad t \geq 0, \quad u(20, t) = 0 \quad t \geq 0$$

```
In[1]:= eqn1 = {D[u[x, t], t] - D[D[u[x, t], x], x] == 0, u[x, 0] == 0, u[0, t] == t^2 Sin[t], u[20, t] == 0};  
sol1 = NDSolve[eqn1, u[x, t], {x, 0, 20}, {t, 0, 10}, PrecisionGoal -> 3] // Quiet;  
Plot3D[u[x, t] /. sol1, {x, 0, 20}, {t, 0, 10},  
  AxesLabel -> {"x", "t", "u"}, PlotLabel -> "The graph of u[x,t]"]
```

The graph of u[x,t]



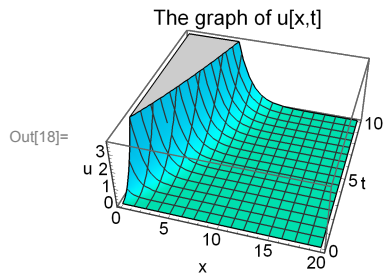
Q3. Find the solution of the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < 20, \quad 0 < t, \quad u(x, 0) = 0, \\ 0 \leq x \leq 20, \quad u(0, t) = t^2 \quad t \geq 0, \quad u(20, t) = 0 \quad t \geq 0$$

```

In[16]:= eqn1b = {∂tu[x, t] - ∂x,xu[x, t] == 0, u[x, 0] == 0, u[0, t] == t2, u[20, t] == 0};
sol1b = NDSolve[eqn1b, u[x, t], {x, 0, 20}, {t, 0, 10}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. sol1b, {x, 0, 20}, {t, 0, 10}, AxesLabel → {"x", "t", "u"},
PlotLabel → "The graph of u[x,t]", PlotStyle → {Cyan}]

```



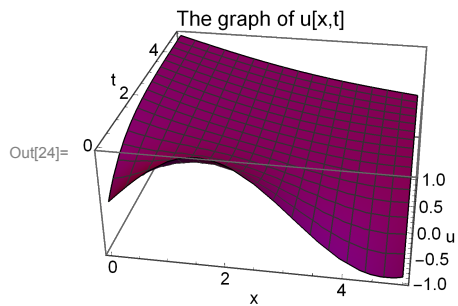
Q3. Find the solution of the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < 5, \quad 0 < t, \quad u(x, 0) = \sin[x], \\ 0 \leq x \leq 5, \quad u(0, t) = 1 \quad t \geq 0, \quad u(5, t) = 0 \quad t \geq 0$$

```

In[22]:= eqn1k = {∂tu[x, t] - ∂x,xu[x, t] == 0, u[x, 0] == Sin[x], u[0, t] == 1, u[5, t] == 0};
sol1k = NDSolve[eqn1k, u[x, t], {x, 0, 5}, {t, 0, 10}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. sol1k, {x, 0, 5}, {t, 0, 5}, AxesLabel → {"x", "t", "u"},
PlotLabel → "The graph of u[x,t]", PlotStyle → {Purple}]

```



Q5. Find the solution of the heat equation

$$u_t - u_{xx} = 0, \quad 0 < x < 10, \quad 0 < t, \quad u(x, 0) = \tanh[x], \\ 0 \leq x \leq 10, \quad u(0, t) = t \quad t \geq 0, \quad u(10, t) = 0 \quad t \geq 0$$

```

In[25]:= eqn1p = { $\partial_t u[x, t] - \partial_{x,x} u[x, t] == 0$ ,  $u[x, 0] == \text{Tanh}[x]$ ,  $u[0, t] == t$ ,  $u[10, t] == 0$ };
sol1p = NDSolve[eqn1p, u[x, t], {x, 0, 10}, {t, 0, 4}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. sol1p, {x, 0, 10}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
PlotLabel → "The graph of u[x,t]", PlotStyle → {Pink}]

```

