Practical 9

To perform Laurent series expansion of the given function f (z) around a given point z

Definition : If f (z) is analytic in an annulus $R_1 < |z - z_0| < R_2$, the series

$$\sum_{n=-\infty}^{\infty} c_n (z - z_o)^n$$

is called the laurent series expension of f.

Question 1) $f(z) = \frac{(\text{Sinz} - 1)}{z^4}$ around $z = 0$

```
In[1]:= f[z_] = (Sin[z] - 1)/z^4;
zo = 0;
L[z_] = Normal[Series[f[z], {z, zo, 10}]];
Print["Laurent series expantion of ", f[z], " around z=", zo, " is : \n", L[z], "+....."]
```

Laurent series expantion of $\frac{-1 + \text{Sin}[z]}{z^4}$ around z=0 is :

$$-\frac{1}{z^4} + \frac{1}{z^3} - \frac{1}{6z} + \frac{z}{120} - \frac{z^3}{5040} + \frac{z^5}{362880} - \frac{z^7}{39916800} + \frac{z^9}{6227020800} + ...$$

Question 2) $f[z] = z^2 \text{Exp}\left[\frac{1}{z}\right]$; $z0 = \infty$

```
In[ ]:= f[z_] = z^2 e^(1/z);
zo = ∞;
L[z_] = Normal[Series[f[z], {z, zo, 10}]];
Print["Laurent series expantion of ", f[z], " around z=", zo, " is : \n", L[z], "+....."]
```

Laurent series expantion of $e^{\frac{1}{z}} z^2$ around $z=\infty$ is :

$$\frac{1}{2} + \frac{1}{479001600 z^{10}} + \frac{1}{39916800 z^9} + \frac{1}{3628800 z^8} + \frac{1}{362880 z^7} + \frac{1}{40320 z^6} + \frac{1}{5040 z^5} + \frac{1}{720 z^4} + \frac{1}{120 z^3} + \frac{1}{24 z^2} + \frac{1}{6z} + z + z^2 + ...$$

Question 3) $f(z) = \frac{\text{Cot}[z]}{z^4}$ around $z = 0$

In[∘]:=
```
f[z_] = Cot[z]/z^4;
zo = 0;
L[z_] = Normal[Series[f[z], {z, zo, 10}]];
Print["Laurent series expantion of ", f[z], " around z=", zo, " is : \n", L[z], "+....."]
```

Laurent series expantion of $\frac{\text{Cot}[z]}{z^4}$ around z=0 is :

$$\frac{1}{z^5} - \frac{1}{3 z^3} - \frac{1}{45 z} - \frac{2 z}{945} - \frac{z^3}{4725} - \frac{2 z^5}{93555} - \frac{1382 z^7}{638512875} - \frac{4 z^9}{18243225} + ...$$

Ques 4. f[z] = 1/(z-2) around z = 0

In[5]:=
```
f[z_] = 1 / (z - 2);
z0 = 0;
L[z_] = Normal[Series[f[z], {z, z0, 12}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{1}{-2+z}$ around z = 0 is :

$$-\frac{1}{2} - \frac{z}{4} - \frac{z^2}{8} - \frac{z^3}{16} - \frac{z^4}{32} - \frac{z^5}{64} - \frac{z^6}{128} - \frac{z^7}{256} - \frac{z^8}{512} - \frac{z^9}{1024} - \frac{z^{10}}{2048} - \frac{z^{11}}{4096} - \frac{z^{12}}{8192}$$

Ques 5. f[z] = 1/(z-2) around z = ∞

In[9]:=
```
f[z_] = 1 / (z - 2);
z0 = ∞;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{1}{-2+z}$ around z = ∞ is :

$$\frac{512}{z^{10}} + \frac{256}{z^9} + \frac{128}{z^8} + \frac{64}{z^7} + \frac{32}{z^6} + \frac{16}{z^5} + \frac{8}{z^4} + \frac{4}{z^3} + \frac{2}{z^2} + \frac{1}{z}$$

Ques 6. f[z] = z/((z-2)(z-3)) around z = 0

In[13]:=
```
f[z_] = z / ((z - 2) (z - 3));
z0 = 0;
L[z_] = Normal[Series[f[z], {z, z0, 12}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{z}{(-3+z)\,(-2+z)}$ around z = 0 is :

$$\frac{z}{6} + \frac{5 z^2}{36} + \frac{19 z^3}{216} + \frac{65 z^4}{1296} + \frac{211 z^5}{7776} + \frac{665 z^6}{46656} + \frac{2059 z^7}{279936} + \frac{6305 z^8}{1679616} + \frac{19171 z^9}{10077696} + \frac{58025 z^{10}}{60466176} + \frac{175099 z^{11}}{362797056} + \frac{527345 z^{12}}{2176782336}$$

Ques 7. f[z] = Sin[z]/z^3 around z = 0

In[17]:=
```
f[z_] = Sin[z] / (z^3);
z0 = 0;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{\text{Sin}[z]}{z^3}$ around z = 0 is :

$$-\frac{1}{6}+\frac{1}{z^2}+\frac{z^2}{120}-\frac{z^4}{5040}+\frac{z^6}{362\,880}-\frac{z^8}{39\,916\,800}+\frac{z^{10}}{6\,227\,020\,800}$$

Ques 8. f[z] = z^3/(z-1)^3 around z = 2

In[21]:=
```
f[z_] = z^3 / (z - 1) ^3;
z0 = 2;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{z^3}{(-1+z)^3}$ around z = 2 is :

$$8-12\,(-2+z)+18\,(-2+z)^2-25\,(-2+z)^3+33\,(-2+z)^4-42\,(-2+z)^5+52\,(-2+z)^6-63\,(-2+z)^7+75\,(-2+z)^8-88\,(-2+z)^9+102\,(-2+z)^{10}$$

Ques 9. f[z] = 1/z around z = 0

In[25]:=
```
f[z_] = 1 / z;
z0 = 0;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{1}{z}$ around z = 0 is :

$$\frac{1}{z}$$

Ques 10.a) f[z] = z^4/((z-1)(z-4)) around z = 1

In[29]:=
```
f[z_] = z^4 / ((z - 1) (z - 4));
z0 = 1;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{z^4}{(-4+z)\,(-1+z)}$ around z = 1 is :

$$-\frac{13}{9}-\frac{1}{3\,(-1+z)}-\frac{67}{27}(-1+z)-\frac{175}{81}(-1+z)^2-\frac{256}{243}(-1+z)^3-\frac{256}{729}(-1+z)^4-\frac{256\,(-1+z)^5}{2187}-\frac{256\,(-1+z)^6}{6561}-\frac{256\,(-1+z)^7}{19\,683}-\frac{256\,(-1+z)^8}{59\,049}-\frac{256\,(-1+z)^9}{177\,147}-\frac{256\,(-1+z)^{10}}{531\,441}$$

Ques 10.b) f[z] = z^4/((z-1)(z-4)) around z=4

In[33]:=
```
f[z_] = z^4 / ((z - 1) (z - 4));
z0 = 4;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{z^4}{(-4+z)\ (-1+z)}$ around z = 4 is :

$$\frac{512}{9} + \frac{256}{3\ (-4+z)} + \frac{352}{27}\ (-4+z) + \frac{80}{81}\ (-4+z)^2 + \frac{1}{243}\ (-4+z)^3 - \frac{1}{729}\ (-4+z)^4 + \frac{(-4+z)^5}{2187} - \frac{(-4+z)^6}{6561} + \frac{(-4+z)^7}{19\,683} - \frac{(-4+z)^8}{59\,049} + \frac{(-4+z)^9}{177\,147} - \frac{(-4+z)^{10}}{531\,441}$$

Ques 11. f[z] = z^2/(z-2) around z = 2

In[37]:=
```
f[z_] = z^2 / (z - 2);
z0 = 2;
L[z_] = Normal[Series[f[z], {z, z0, 10}]];
Print["Laurent Series Expansion of ", f[z], " around z = ", z0 , " is :\n ", L[z]]
```

Laurent Series Expansion of $\frac{z^2}{-2+z}$ around z = 2 is :

$$2 + \frac{4}{-2+z} + z$$