Practical 8

Draw and discuss the pointwise convergence of the following sequences

Ques1. Discuss the pointwise convergence of the sequence $f(x, n) = x^n$.

```
f[x_, n_] := x^n
L1 = Limit[f[x, n], n → ∞, Assumptions → x == -1];
L2 = Limit[f[x, n], n → ∞, Assumptions → x == 1];
L3 = Limit[f[x, n], n → ∞, Assumptions → -1 < x < 1];
L4 = Limit[f[x, n], n → ∞, Assumptions → x > 1];
Print["lim_{n→∞} f(x) = ", L1, " for x=-1"]
Print["lim_{n→∞} f(x) = ", L2, " for -x=1"]
Print["lim_{n→∞} f(x) = ", L3, " for -1<x<1"]
Print["lim_{n→∞} f(x) = ", L4, " for x>1"]
```

$\lim_{n\to\infty} f(x) = e^{2 i \,\text{Interval}[\{0,\pi\}]}$ for $x=-1$

$\lim_{n\to\infty} f(x) = 1$ for $-x=1$

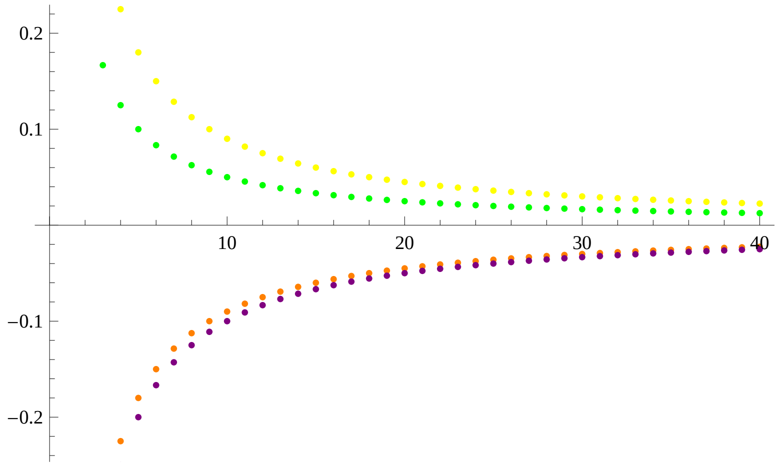
$\lim_{n\to\infty} f(x) = 0$ for $-1<x<1$

$\lim_{n\to\infty} f(x) = \infty$ for $x>1$

Conclusion : The given sequence is convergent in $x \in (-1, 1]$ and divergent in $x > 1$

Checking Pointwise convergence graphically :

```
a = ListPlot[Table[f[0.9], {n, 1, 40}], PlotStyle → Yellow];
b = ListPlot[Table[f[0.5], {n, 1, 40}], PlotStyle → Green];
c = ListPlot[Table[f[-0.9], {n, 1, 40}], PlotStyle → Orange];
d = ListPlot[Table[f[-1], {n, 1, 40}], PlotStyle → Purple];
Show[a, b, c, d]
```



Conclusion : The given function is convergent in the interval (-1, 1]

Ques 2. $f(x, n) = \frac{x}{n}$

```
f[x_, n_] := x/n
L5 = Limit[f[x, n], n → ∞, Assumptions → x > 0];
L6 = Limit[f[x, n], n → ∞, Assumptions → x < 0];
Print["lim_(n→∞) f(x) = ", L5, " for x>0"]
Print["lim_(n→∞) f(x) = ", L6, " for x<0"]
```

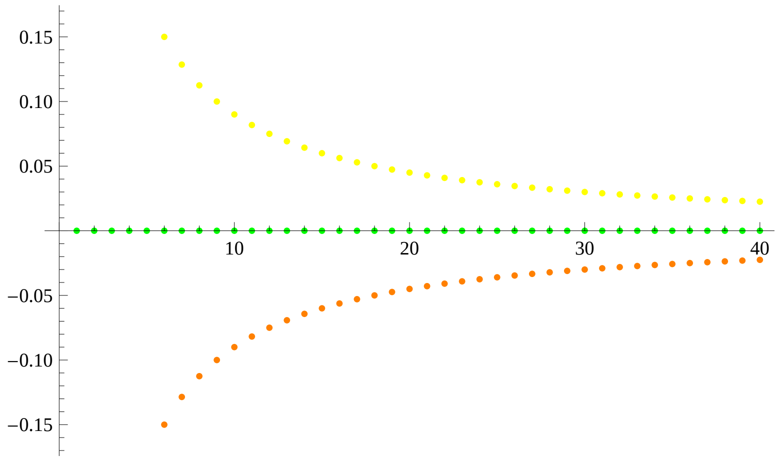
$\lim_{n\to\infty} f(x) = 0$ for $x>0$

$\lim_{n\to\infty} f(x) = 0$ for $x<0$

Conclusion : The given sequence is convergent ∀ x ϵ ℝ.

Checking Pointwise convergence Graphically :

```
a = ListPlot[Table[f[0.9], {n, 1, 40}], PlotStyle → Yellow];
b = ListPlot[Table[f[0], {n, 1, 40}], PlotStyle → Green];
c = ListPlot[Table[f[-0.9], {n, 1, 40}], PlotStyle → Orange];
Show[a, b, c]
```



Conclusion : The given function is pointwise convergent.

Ques 3. $f(x) = \frac{x}{x+n}$

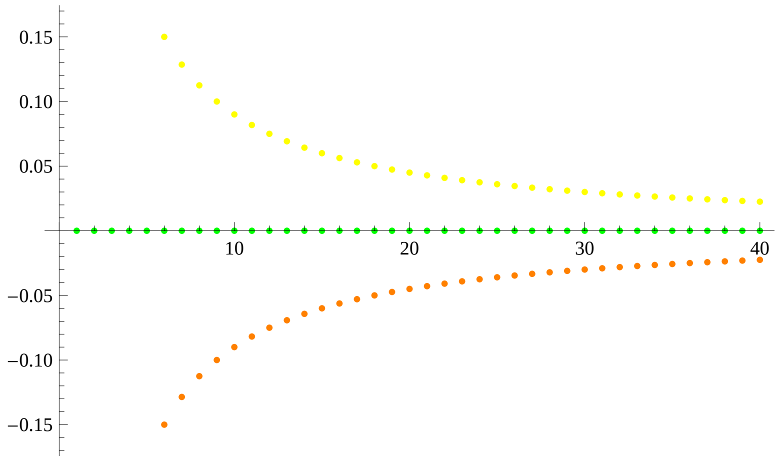
```
f[x_, n_] := x/(x + n)
L7 = Limit[f[x, n], n → ∞, Assumptions → -∞ < x < ∞];
Print["lim_{n→∞} f(x) = ", L7, " for -∞<x<∞"]
```

$\lim_{n\to\infty} f(x) = 0$ for $-\infty<x<\infty$

Conclusion : The given sequence is convergent $\forall\ x \in \mathbb{R}$.

Checking Pointwise convergence Graphically :

```
a = ListPlot[Table[f[0.9], {n, 1, 40}], PlotStyle → Yellow];
b = ListPlot[Table[f[0], {n, 1, 40}], PlotStyle → Green];
c = ListPlot[Table[f[-0.9], {n, 1, 40}], PlotStyle → Orange];
Show[a, b, c]
```



Conclusion : The given function is pointwise convergent.

Ques 4. $f(x) = \dfrac{nx}{1+(nx)^2}$

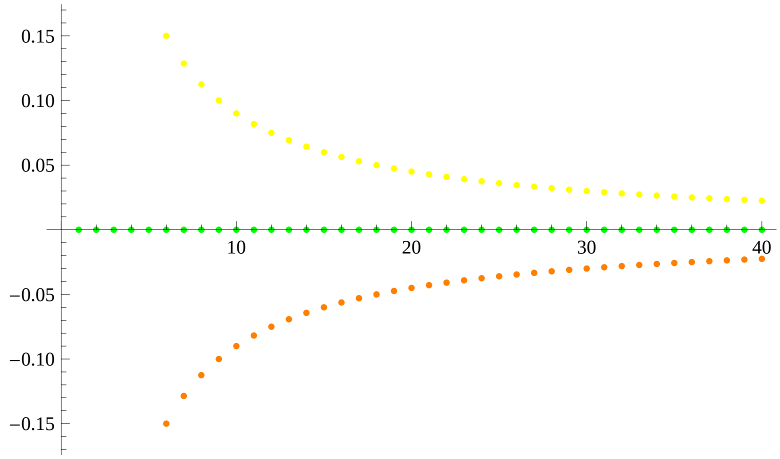
```
f[x_, n_] := n x / (1 + (n x)^2)
L8 = Limit[f[x, n], n → ∞, Assumptions → -∞ < x < ∞];
Print["lim_{n→∞} f(x) = ", L8, " for -∞<x<∞"]
```

$\lim_{n\to\infty} f(x) = 0$ for $-\infty<x<\infty$

Conclusion : The given sequence is convergent $\forall\ x \in \mathbb{R}$.

Checking Pointwise convergence Graphically :

```
a = ListPlot[Table[f[0.9], {n, 1, 40}], PlotStyle → Yellow];
b = ListPlot[Table[f[0], {n, 1, 40}], PlotStyle → Green];
c = ListPlot[Table[f[-0.9], {n, 1, 40}], PlotStyle → Orange];
Show[a, b, c]
```



Conclusion : The given function is pointwise convergent.