Pointwise Convergence

Discuss the pointwise convergence of the sequence of functions
$f(x, n) = x^n$

```
f[x_, n_] := x^n
L1 = Limit[f[x, n], n → ∞, Assumptions → x < -1];
L2 = Limit[f[x, n], n → ∞, Assumptions → x == -1];
L3 = Limit[f[x, n], n → ∞, Assumptions → -1 < x < 1];
L4 = Limit[f[x, n], n → ∞, Assumptions → x == 1];
L5 = Limit[f[x, n], n → ∞, Assumptions → x > 1];
Print[lim_{n→∞} f(x)= , L1, " for x<-1"]
Print[lim_{n→∞} f(x)= , L2, " for x=-1"]
Print[lim_{n→∞} f(x)= , L3, " for -1<x<1"]
Print[lim_{n→∞} f(x)= , L4, " for x=1"]
Print[lim_{n→∞} f(x)= , L5, " for x>1"]
```

$\lim_{n\to\infty} f(x) =$ Interval[{−∞, ∞}] for x<-1

$\lim_{n\to\infty} f(x) = e^{2\,i\,\text{Interval}[\{0,\pi\}]}$ for x=-1

$\lim_{n\to\infty} f(x) = 0$ for -1<x<1

$\lim_{n\to\infty} f(x) = 1$ for x=1

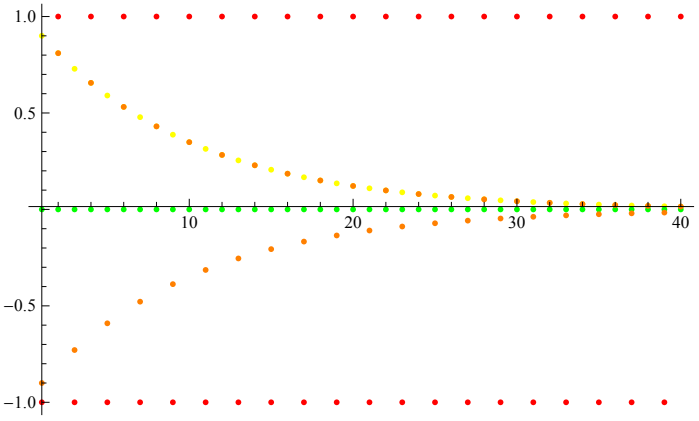
$\lim_{n\to\infty} f(x) = \infty$ for x>1

Conclusion : The given sequence is convergent $\forall\ x \in (-1,1]$ and divergent otherwise

Checking Pointwise convergence Graphically :

```
a = ListPlot[Table[f[0.9, n], {n, 1, 40}], PlotStyle → Yellow, PlotRange → All];
b = ListPlot[Table[f[0, n], {n, 1, 40}], PlotStyle → Green, PlotRange → All];
c = ListPlot[Table[f[-1, n], {n, 1, 40}], PlotStyle → Red, PlotRange → All];
d = ListPlot[Table[f[-0.9, n], {n, 1, 40}], PlotStyle → Orange, PlotRange → All];
```

```
Show[a, b, c, d]
```



Conclusion : The given function is pointwise convergent.