20/02/20

PRACTICAL - 9

SOLUTION OF THE WAVE EQUATION

Q1. Find the solution of the wave equation

$u_{tt} - u_{xx} = 0,\ 0 < x < 1,\ 0 \le t \le 4,$

$u(x, 0) = \text{Log}\left[1 + x^2\right],\ 0 \le x \le 1,\ u_t(x, 0) = 2,\ 0 \le x \le 1$

In[1]:=
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a = {Derivative[0, 2][u][x, t] - Derivative[2, 0][u][x, t] == 0,
    u[x, 0] == Log[1 + x^2], Derivative[0, 1][u][x, 0] == 2};
b = NDSolve[a, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. b, {x, 0, 1}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
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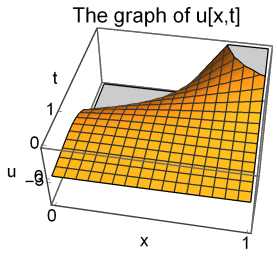
Out[3]=



Q2. Find the solution of the wave equation

$u_{tt} - u_{xx} = 0,\ -1 < x < 1,\ 0 \le t \le 4,$

$u(x, 0) = \text{Piecewise}[\{1 - x,\ 0 \le x \le 1\},\ \{1 + x,\ -1 \le x \le 0\}],$

$-1 \le x \le 1,\ u_t(x, 0) = 0,\ -1 \le x \le 1$

In[4]:=
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a = {Derivative[0, 2][u][x, t] - Derivative[2, 0][u][x, t] == 0,
    u[x, 0] == 1 + x, Derivative[0, 1][u][x, 0] == 0};
b = NDSolve[a, u[x, t], {x, -1, 1}, {t, 0, 4}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. b, {x, 0, 1}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
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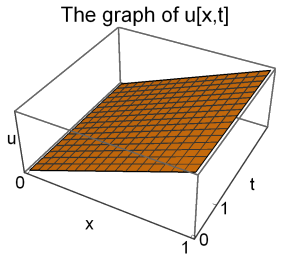
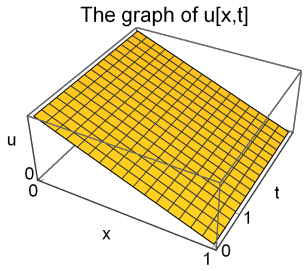
Out[6]=



In[7]:=
```
a = {Derivative[0, 2][u][x, t] - Derivative[2, 0][u][x, t] == 0,
    u[x, 0] == 1 - x, Derivative[0, 1][u][x, 0] == 0};
b = NDSolve[a, u[x, t], {x, -1, 1}, {t, 0, 4}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. b, {x, 0, 1}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
```

Out[9]=



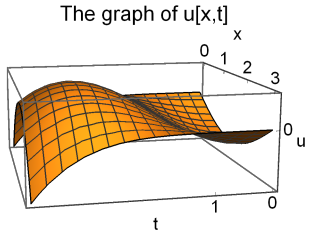
Q3. Find the solution of the wave equation

$u_{tt} - u_{xx} = 0,\ 0 < x < Pi,\ 0 \le t \le 4,$

$u(x, 0) = Sin[x],\ 0 \le x \le Pi,\ u_t(x, 0) = 0,\ 0 \le x \le Pi$

In[10]:=
```
a = {Derivative[0, 2][u][x, t] - Derivative[2, 0][u][x, t] == 0,
    u[x, 0] == Sin[x], Derivative[0, 1][u][x, 0] == 0};
b = NDSolve[a, u[x, t], {x, 0, Pi}, {t, 0, 4}, PrecisionGoal → 3] // Quiet;
Plot3D[u[x, t] /. b, {x, 0, Pi}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
```

Out[12]=



Q4. Find the solution of the wave equation

$u_{tt} - c^2 u_{xx} = F(x, t),\ 0 < x < 1,\ t > 0,\ u(x, 0) = f(x),\ u_t(x, 0) = g(x),\ c = 2;$

$F = 0,\ f = x^4,\ g = 0,\ u(0, t) = 0$

In[19]:=
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ClearAll[x, t, u]
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In[23]:=
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eqn1a =
  {∂t,t u[x, t] - 4 * ∂x,x u[x, t] == 0, u[x, 0] == x^4, Derivative[0, 1][u][x, 0] == 0, u[0, t] == 0};
sol1a = NDSolve[eqn1a, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 5] // Quiet;
Plot3D[u[x, t] /. sol1a, {x, 0, 1}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
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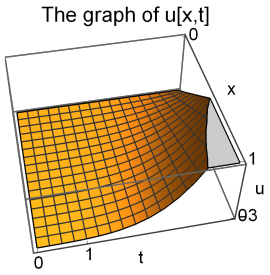
Out[25]=



Q4. Find the solution of the wave equation

$u_{tt} - c^2 u_{xx} = F(x, t),\ 0 < x < 10,\ t > 0,\ u(x, 0) = f(x),\ u_t(x, 0) = g(x),\ c = 3;$

$F = 0,\ f = 0,\ g = x^3,\ u_x(0, t) = 0$

In[32]:=
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eqn1a = {∂t,t u[x, t] - 9 * ∂x,x u[x, t] == 0, u[x, 0] == 0,
    Derivative[0, 1][u][x, 0] == x^3, Derivative[1, 0][u][0, t] == 0};
sol1a = NDSolve[eqn1a, u[x, t], {x, 0, 10}, {t, 0, 4}, PrecisionGoal → 5] // Quiet;
Plot3D[u[x, t] /. sol1a, {x, 0, 10}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 PlotLabel → "The graph of u[x,t]", Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
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Out[34]=