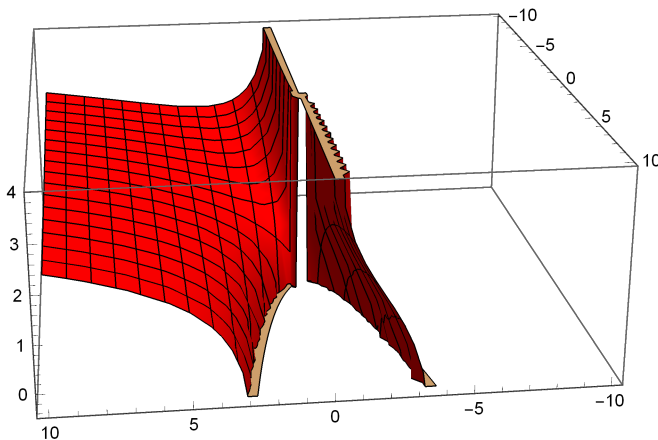


# Plotting the integral surfaces of a given first order Partial Differential Equation

1.  $xu_x + yu_y = x e^{-u} ; u(x, x^2) = 0$

```
a = DSolve[{x * D[u[x, y], x] + y * D[u[x, y], y] == x * e^(-u[x, y]), u[x, x^2] == 0},
  u[x, y], {x, y}] // Quiet;
Print["The general solution of the given PDE is ", a]
Plot3D[u[x, y] /. a, {x, -10, 10}, {y, -10, 10}, PlotStyle -> {Red}]
```

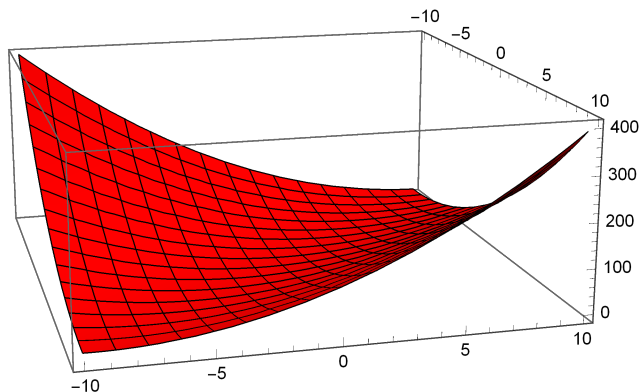
The general solution of the given PDE is  $\left\{ \left\{ u[x, y] \rightarrow \text{Log} \left[ 1 + x - \frac{y}{x} \right] \right\} \right\}$



2.  $u_x - u_y = 1 ; u(x, 0) = x^2$

```
a = DSolve[{D[u[x, y], x] - D[u[x, y], y] == 1, u[x, 0] == x^2}, u[x, y], {x, y}] // Quiet;
Print["The general solution of the given PDE is ", a]
Plot3D[u[x, y] /. a, {x, -10, 10}, {y, -10, 10}, PlotStyle -> {Red}]
```

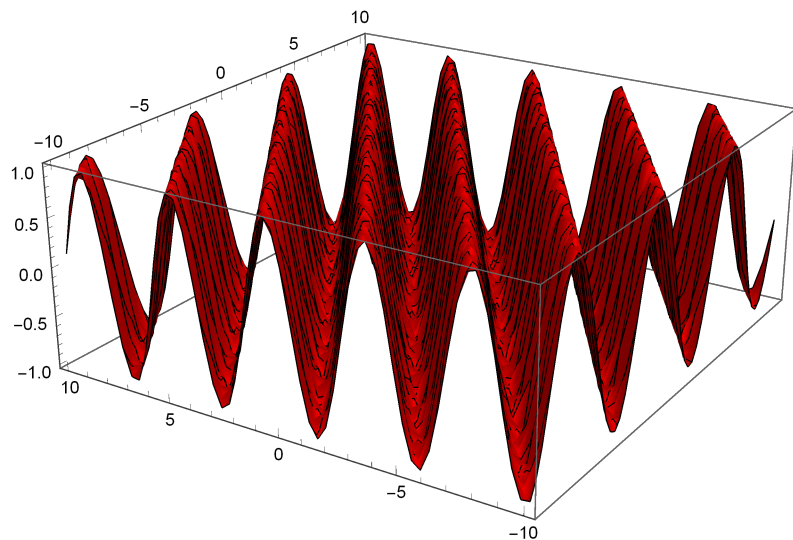
The general solution of the given PDE is  $\left\{ \left\{ u[x, y] \rightarrow x^2 - y + 2xy + y^2 \right\} \right\}$



3.  $\times 3 u_x + 2 u_y = 0$  ;  $u[x, 0] = \text{Sin}[x]$

```
a = DSolve[
  {3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0, u[x, 0] == Sin[x]}, u[x, y], {x, y}] // Quiet;
Print["The general solution of the given PDE is ", a]
Plot3D[u[x, y] /. a, {x, -10, 10}, {y, -10, 10}, PlotStyle -> {Red}]
```

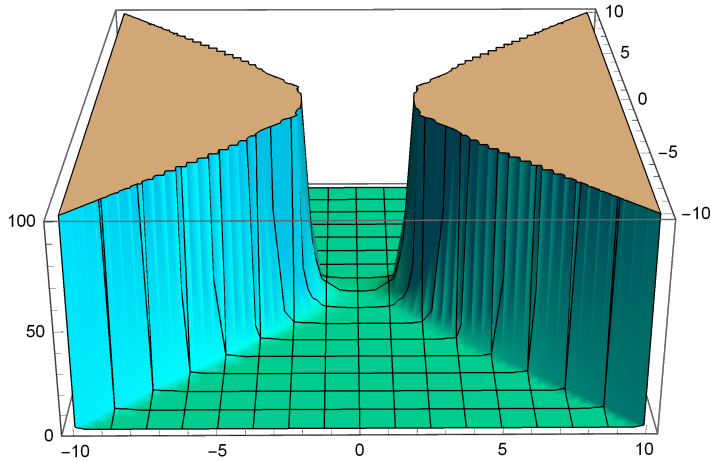
The general solution of the given PDE is  $\left\{ \left\{ u[x, y] \rightarrow \text{Sin}\left[\frac{1}{2} (2x - 3y)\right] \right\} \right\}$



$$4. yu_x + xu_y = 0; u[0, y] = e^{-y^2}$$

a =

```
DSolve[{y * D[u[x, y], x] + x * D[u[x, y], y] == 0, u[0, y] == e^-y^2}, u[x, y], {x, y}] // Quiet;
Print["The general solution of the given PDE is ", a]
Plot3D[u[x, y] /. a, {x, -10, 10}, {y, -10, 10}, PlotStyle -> {Cyan}, PlotRange -> {0, 100}]
The general solution of the given PDE is {{u[x, y] -> e^{x^2-y^2}}}
```



$$5. xu_x + xyu_y = 2xy; u[x, x^2] = 2$$

```
a = DSolve[{x * D[u[x, y], x] + x * y * D[u[x, y], y] == 2 * x * y, u[x, x^2] == 2},
u[x, y], {x, y}] // Quiet;
Print["The general solution of the given PDE is ", a]
Plot3D[u[x, y] /. a, {x, -10, 10}, {y, -10, 10}, PlotStyle -> {Red}]
```

The general solution of the given PDE is  $\left\{ \left\{ u[x, y] \rightarrow 2 \left( 1 + y - 4 \operatorname{ProductLog} \left[ -\frac{1}{2} \sqrt{e^{-x} y} \right]^2 \right) \right\} \right\}$

