

Family of characteristic curves

$$1. uu_x + u_y = 1; \quad x = 2s^2, \quad y = 2s, \quad u = 0, \quad s > 0$$

Family of characteristic curves is

$$2x - y^2 - 8s^2 + 4sy = 0$$

and its Envelope is

$$y^2 = 4x$$

```
a = DSolve[
  {x'[t] == u[t], y'[t] == 1, u'[t] == 1, x[s] == 2 s^2, y[s] == 2 s, u[s] == 0}, {x, y, u}, t]
{b} = x[t] /. a;
{c} = y[t] /. a;
Print["Family of characteristic curves is given by ", Eliminate[{x == b, y == c}, t]]
{{u -> Function[{t}, -s + t], x -> Function[{t},  $\frac{1}{2} (5 s^2 - 2 s t + t^2)$ ], y -> Function[{t}, s + t]}}
```

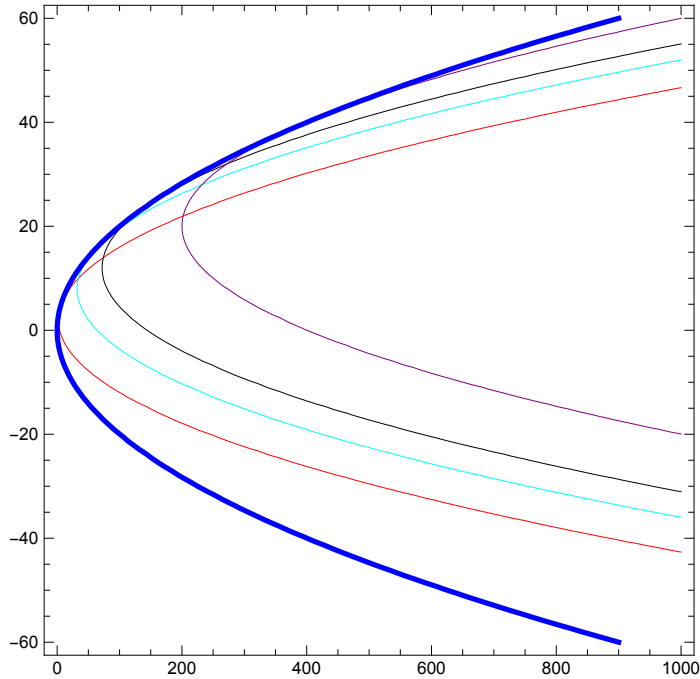
Family of characteristic curves is given by $8s^2 - 4sy + y^2 = 2x$

```

F[s_] := 2 x - y^2 - 8 s^2 + 4 s * y
Print["Envelope of the family of characteristic curves is given by ",
  Eliminate[{F[s] == 0, D[F[s], s] == 0}, s]]
ContourPlot[{F[1] == 0, F[6] == 0, F[4] == 0, F[10] == 0, y^2 == 4 x}, {x, 0, 1000},
  {y, -60, 60}, ContourStyle -> {Red, Black, Cyan, Purple, {Blue, Thickness[0.008]}}]

```

Envelope of the family of characteristic curves is given by $y^2 = 4x$



$$2. u_t + uu_x = 0, \quad x(s) = \frac{s^2}{2}, \quad t(s) = s, \quad u(s) = s$$

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a = DSolve[{t'[z] == 1, x'[z] == u[z], u'[z] == 0, x[s] == s^2/2, t[s] == s, u[s] == s}, {x, t, u}, z]

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{b} = t[z] /. a;

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{c} = x[z] /. a;

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Print["Family of characteristic curves is given by ", Eliminate[{t == b, x == c}, z]]

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{{t -> Function[{z}, z], u -> Function[{z}, s], x -> Function[{z}, 1/2 (-s^2 + 2 s z)]}}

```

Family of characteristic curves is given by $s^2 + 2x = 2st$

```

F[s_] := s^2 + 2 x - 2 s t
Print["Envelope of the family of characteristic curves is given by ",
  Eliminate[{F[s] == 0, D[F[s], s] == 0}, s]]
ContourPlot[{F[1] == 0, F[6] == 0, F[4] == 0, F[10] == 0, t^2 == 2 x}, {t, 0, 10},
  {x, -60, 60}, ContourStyle -> {Red, Black, Cyan, Purple, {Blue, Thickness[0.008]}}]

```

Envelope of the family of characteristic curves is given by $2x = t^2$

