Approximating solution to Initial Value Problems using the Euler' s Method

$\frac{dy}{dx} = f[x, y],\ y[a0] = y0,\ a0 \le x \le b0,$ m0 is step size. Find $y[b0]$.

```
Euler[a0_, b0_, y0_, m0_, f] := Module[{a = a0, b = b0, j, m = m0}, h = (b - a) / m;
  x = Table[a + (j - 1) * h, {j, 1, m + 1}];
  y = Table[y0, {j, 1, m + 1}];
  i = Table[j, {j, 0, m}];
  For[j = 1, j ≤ m, j++, y[[j + 1]] = y[[j]] + h * f[x[[j]], y[[j]]];];
  Return[
   TableForm[
    Table[Transpose[{N[i], N[x], N[y]}]],
    TableHeadings → {{}, {"i", "x", "y"}}]];]
```

1. $\frac{dy}{dx} = 1 + \frac{y}{x},\ 1 \le x \le 6,\ y[1] = 1$

```
f[x_, y_] := 1 + y/x
Euler[1, 6, 1, 20, f]
```

i	x	y
0.	1.	1.
1.	1.25	1.5
2.	1.5	2.05
3.	1.75	2.64167
4.	2.	3.26905
5.	2.25	3.92768
6.	2.5	4.61409
7.	2.75	5.3255
8.	3.	6.05963
9.	3.25	6.8146
10.	3.5	7.5888
11.	3.75	8.38086
12.	4.	9.18958
13.	4.25	10.0139
14.	4.5	10.853
15.	4.75	11.7059
16.	5.	12.572
17.	5.25	13.4506
18.	5.5	14.3411
19.	5.75	15.243
20.	6.	16.1557

```
s = DSolve[{y'[x] == 1 + y[x]/x, y[1] == 1}, y[x], x]
TableForm[Table[Transpose[{x, y[x]} /. s], {x, 1, 6, 0.25}],
 TableHeadings → {{}, {"x", "y[x]"}}]
```

```
{{y[x] → x + x Log[x]}}
```

x	y[x]
1.	1.
1.25	1.52893
1.5	2.1082
1.75	2.72933
2.	3.38629
2.25	4.07459
2.5	4.79073
2.75	5.5319
3.	6.29584
3.25	7.08063
3.5	7.88467
3.75	8.70658
4.	9.54518
4.25	10.3994
4.5	11.2683
4.75	12.1512
5.	13.0472
5.25	13.9557
5.5	14.8761
5.75	15.8079
6.	16.7506

2. $\frac{dy}{dx} = \sqrt{y}\, x,\ 2 \le x \le 3,\ y[2] = 4$

```
f[x_, y_] := √y x
Euler[2, 3, 4, 20, f]
```

	i	x	y
	0.	2.	4.
	1.	2.05	4.2
	2.	2.1	4.41006
	3.	2.15	4.63056
	4.	2.2	4.86189
	5.	2.25	5.10444
	6.	2.3	5.35861
	7.	2.35	5.62482
	8.	2.4	5.90349
	9.	2.45	6.19505
	10.	2.5	6.49996
	11.	2.55	6.81864
	12.	2.6	7.15158
	13.	2.65	7.49923
	14.	2.7	7.86208
	15.	2.75	8.24061
	16.	2.8	8.63532
	17.	2.85	9.04673
	18.	2.9	9.47533
	19.	2.95	9.92167
	20.	3.	10.3863

```
s = DSolve[{y'[x] == Sqrt[y[x]] x, y[2] == 4}, y[x], x]
TableForm[Table[Transpose[{x, y[x]} /. s], {x, 2, 3, 0.05}],
 TableHeadings → {{}, {"x", "y[x]"}}]
```

$$\left\{\left\{y[x] \to \frac{1}{16}\left(144 - 24\,x^2 + x^4\right)\right\}, \left\{y[x] \to \frac{1}{16}\left(16 + 8\,x^2 + x^4\right)\right\}\right\}$$

	x	y[x]
	2.	4.
	2.	4.
	2.05	3.80006
	2.05	4.20506
	2.1	3.60051
	2.1	4.42051
	2.15	3.40172
	2.15	4.64672
	2.2	3.2041
	2.2	4.8841
	2.25	3.00806
	2.25	5.13306
	2.3	2.81401
	2.3	5.39401
	2.35	2.62238
	2.35	5.66738
	2.4	2.4336
	2.4	5.9536
	2.45	2.24813
	2.45	6.25313
	2.5	2.06641
	2.5	6.56641
	2.55	1.88891
	2.55	6.89391
	2.6	1.7161
	2.6	7.2361
	2.65	1.54847
	2.65	7.59347
	2.7	1.38651
	2.7	7.96651
	2.75	1.23071
	2.75	8.35571
	2.8	1.0816
	2.8	8.7616
	2.85	0.939688
	2.85	9.18469
	2.9	0.805506
	2.9	9.62551
	2.95	0.679594
	2.95	10.0846
	3.	0.5625
	3.	10.5625