Plotting the characteristics of the first order PDE

1. $xyu_x + \left(x^2 + y^2\right) u_y = 0$

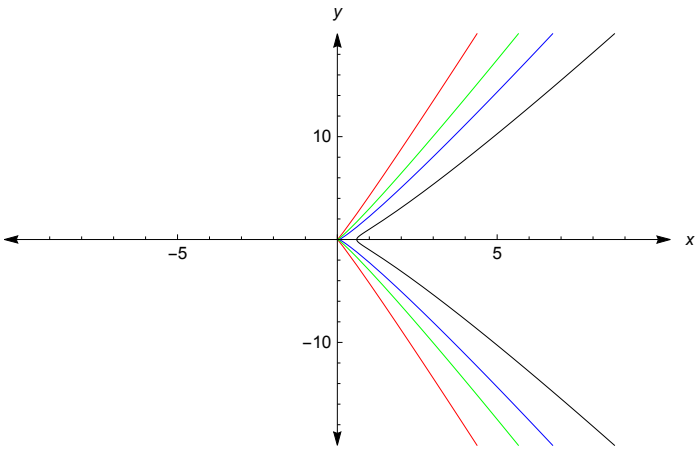
The characteristics are determined by

$$\frac{dx}{xy} = \frac{dy}{x^2 + y^2} = \frac{du}{0} \qquad \ldots (1)$$

```
a = DSolve[y'[x] == (x^2 + (y[x])^2)/(x * y[x]), y, x];
Print["The characteristic curves are y =", a[[1]],
 "and y = ", a[[2]]]
Plot[{y[x] /. {a[[1]], a[[2]]} /. C[1] → 1, y[x] /. {a[[1]], a[[2]]} /. C[1] → 18,
  y[x] /. {a[[1]], a[[2]]} /. C[1] → 5, y[x] /. {a[[1]], a[[2]]} /. C[1] → 9},
 {x, -10, 10}, PlotRange -> {-20, 20}, AxesLabel → {x, y},
 AxesStyle → Arrowheads[{-0.02, 0.02}], PlotStyle → {Black, Red, Blue, Green}]
```

The characteristic curves are $y = \left\{y \to \text{Function}\left[\{x\}, -x\sqrt{C[1] + 2\,\text{Log}[x]}\right]\right\}$

and $y = \left\{y \to \text{Function}\left[\{x\}, x\sqrt{C[1] + 2\,\text{Log}[x]}\right]\right\}$



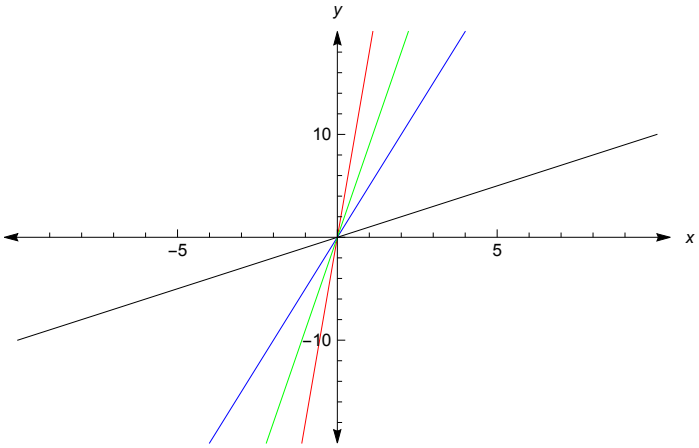
2. $xu_x + yu_y = 0$

The characteristics are determined by

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{0} \qquad .. (1)$$

```
a = DSolve[y'[x] == y[x]/x, y, x];
Print["The characteristic curves are y =", a[[1]]]
Plot[{y[x] /. {a[[1]]} /. C[1] → 1, y[x] /. {a[[1]]} /. C[1] → 18,
  y[x] /. {a[[1]]} /. C[1] → 5, y[x] /. {a[[1]]} /. C[1] → 9},
 {x, -10, 10}, PlotRange -> {-20, 20}, AxesLabel → {x, y},
 AxesStyle → Arrowheads[{-0.02, 0.02}], PlotStyle → {Black, Red, Blue, Green}]
```

```
The characteristic curves are y = {y → Function[{x}, x C[1]]}
```



3. $u_x - u_y = 1$

The characteristics are determined by

$$\frac{dx}{1} = \frac{dy}{-1} = \frac{du}{1} \qquad \ldots (1)$$

```
a = DSolve[y'[x] == -1, y, x];
Print["The characteristic curves are y =", a[[1]]]
Plot[{y[x] /. {a[[1]]} /. C[1] → 1, y[x] /. {a[[1]]} /. C[1] → 12,
  y[x] /. {a[[1]]} /. C[1] → 5, y[x] /. {a[[1]]} /. C[1] → 9},
 {x, -10, 10}, PlotRange -> {-20, 20}, AxesLabel → {x, y},
 AxesStyle → Arrowheads[{-0.02, 0.02}], PlotStyle → {Black, Red, Blue, Green}]
```

```
The characteristic curves are y = {y → Function[{x}, -x + C[1]]}
```