Approximating solution to Initial Value Problems using the Runge Kutta Method

```
rkmethod[a0_, b0_, b_, m0_, f] := Module[{a = a0, y0 = b0, j, m = m0}, h = (b - a)/m;
  x = Table[a + (j - 1) * h, {j, 1, m + 1}];
  y = Table[y0, {j, 1, m + 1}];
  i = Table[j, {j, 0, m}];
  For[j = 1, j ≤ m, j++, k1 = h f[x[[j]], y[[j]]];
   k2 = h f[x[[j]] + 1/2 h, y[[j]] + 1/2 k1];
   k3 = h f[x[[j]] + 1/2 h, y[[j]] + 1/2 k2];
   k4 = h f[x[[j]] + h, y[[j]] + k3];
   k = (k1 + 2 k2 + 2 k3 + k4)/6;
   y[[j + 1]] = y[[j]] + k;];
  Return[
   TableForm[
    Table[Transpose[{N[i], N[x], N[y]}]],
    TableHeadings → {{}, {"i", "x", "y"}}]];]
```

1. $\frac{dy}{dx} = 1 + \frac{y}{x}$, $1 \le x \le 6$, $y[1] = 1$. Find $y[6]$.

```
f[x_, y_] := 1 + y/x
rkmethod[1, 1, 6, 20, f]
```

i	x	y
0.	1.	1.
1.	1.25	1.52891
2.	1.5	2.10816
3.	1.75	2.72928
4.	2.	3.38624
5.	2.25	4.07453
6.	2.5	4.79066
7.	2.75	5.53183
8.	3.	6.29575
9.	3.25	7.08054
10.	3.5	7.88457
11.	3.75	8.70648
12.	4.	9.54507
13.	4.25	10.3993
14.	4.5	11.2682
15.	4.75	12.1511
16.	5.	13.0471
17.	5.25	13.9556
18.	5.5	14.876
19.	5.75	15.8077
20.	6.	16.7504

2. $\frac{dy}{dx} = \sqrt{y}\, x,\ 2 \le x \le 3,\ y[2] = 4.$ **Find** $y[3]$.

```
f[x_, y_] := √y x
rkmethod[2, 4, 3, 5, f]
```

i	x	y
0.	2.	4.
1.	2.2	4.8841
2.	2.4	5.95359
3.	2.6	7.23609
4.	2.8	8.76158
5.	3.	10.5625