Approximating solution to Initial Value Problems using the Euler' s Method

```
mEuler[a0_, b0_, y0_, m0_, f] := Module[{a = a0, b = b0, j, m = m0}, h = (b - a) / m;
  x = Table[a + (j - 1) * h, {j, 1, m + 1}];
  y = Table[y0, {j, 1, m + 1}];
  For[j = 1, j ≤ m, j++, y[[j + 1]] =
     y[[j]] + h/2 (f[x[[j]], y[[j]]] + f[x[[j]] + h, y[[j]] + h * f[x[[j]], y[[j]]]]);];
  Return[
   TableForm[
    Table[Transpose[{N[x], N[y]}]],
    TableHeadings → {{}, {"x", "y"}}]];]
```

1. $\frac{dy}{dx} = 1 + \frac{y}{x}$, $1 \le x \le 6$, $y[1] = 1$. Find $y[6]$.

```
f[x_, y_] := 1 + y/x
mEuler[1, 6, 1, 20, f]
```

x	y
1.	1.
1.25	1.525
1.5	2.10083
1.75	2.71883
2.	3.37286
2.25	4.05835
2.5	4.77178
2.75	5.51033
3.	6.27168
3.25	7.05394
3.5	7.85547
3.75	8.67491
4.	9.51105
4.25	10.3628
4.5	11.2294
4.75	12.1098
5.	13.0034
5.25	13.9095
5.5	14.8276
5.75	15.757
6.	16.6973

```
s = DSolve[{y'[x] == 1 + y[x]/x, y[1] == 1}, y[x], x]
TableForm[Table[Transpose[{x, y[x]} /. s], {x, 1, 6, 0.25}],
 TableHeadings → {{}, {"x", "y[x]"}}]
```

{{y[x] → x + x Log[x]}}

x	y[x]
1.	1.
1.25	1.52893
1.5	2.1082
1.75	2.72933
2.	3.38629
2.25	4.07459
2.5	4.79073
2.75	5.5319
3.	6.29584
3.25	7.08063
3.5	7.88467
3.75	8.70658
4.	9.54518
4.25	10.3994
4.5	11.2683
4.75	12.1512
5.	13.0472
5.25	13.9557
5.5	14.8761
5.75	15.8079
6.	16.7506

2. $\frac{dy}{dx} = \sqrt{y}\, x$, $2 \le x \le 3$, y[2] = 4. Find y[3].

```
f[x_, y_] := √y x
mEuler[2, 3, 4, 10, f]
```

x	y
2.	4.
2.1	4.42025
2.2	4.88355
2.3	5.39312
2.4	5.95234
2.5	6.56472
2.6	7.23395
2.7	7.96384
2.8	8.75836
2.9	9.62165
3.	10.558