Solving systems of Ordinary Differential Equation

1. $\frac{dx}{dt} = 6x - y, \quad \frac{dy}{dt} = x + 4y$

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a = DSolve[{x'[t] == 6 x[t] - y[t], y'[t] == x[t] + 4 y[t]}, {x[t], y[t]}, t]
q = a /. {C[1] → 2, C[2] → 3}
Plot[Evaluate[{x[t], y[t]} /. q, {t, -1, 1}], PlotRange → {-100, 100},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of x[t] and y[t]"]
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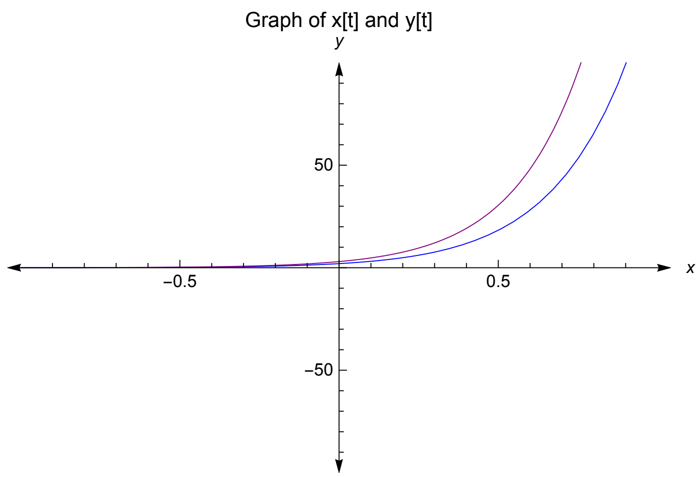
$$\{\{x[t] \to e^{5t}(1+t)\,C[1] - e^{5t}\,t\,C[2],\ y[t] \to e^{5t}\,t\,C[1] - e^{5t}(-1+t)\,C[2]\}\}$$

$$\{\{x[t] \to -3e^{5t}t + 2e^{5t}(1+t),\ y[t] \to -3e^{5t}(-1+t) + 2e^{5t}t\}\}$$



2. $\frac{dx}{dt} = 2x + 3y, \quad \frac{dy}{dt} = 4x - 2y$

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a = DSolve[{x'[t] == 2 x[t] + 3 y[t], y'[t] == 4 x[t] - 2 y[t]}, {x[t], y[t]}, t]
q = a /. {C[1] → 2, C[2] → 3}
Plot[Evaluate[{x[t], y[t]} /. q, {t, -1, 1}], PlotRange → {-100, 100},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of x[t] and y[t]"]
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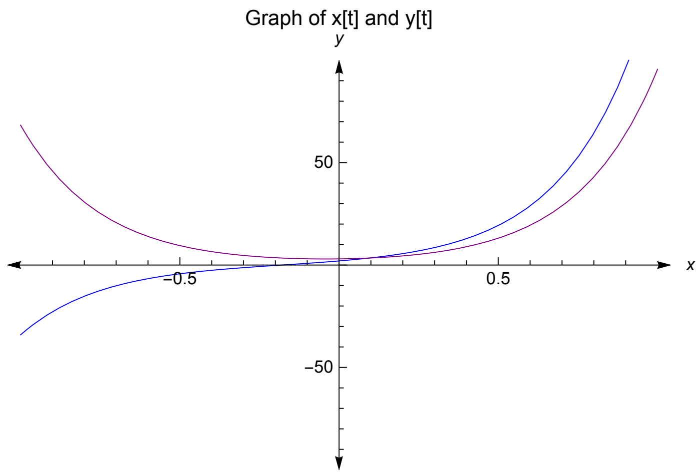
$$\left\{\left\{x[t] \to \frac{1}{4} e^{-4t} \left(1 + 3 e^{8t}\right) C[1] + \frac{3}{8} e^{-4t} \left(-1 + e^{8t}\right) C[2],\right.\right.$$
$$\left.\left. y[t] \to \frac{1}{2} e^{-4t} \left(-1 + e^{8t}\right) C[1] + \frac{1}{4} e^{-4t} \left(3 + e^{8t}\right) C[2]\right\}\right\}$$

$$\left\{\left\{x[t] \to \frac{9}{8} e^{-4t} \left(-1 + e^{8t}\right) + \frac{1}{2} e^{-4t} \left(1 + 3 e^{8t}\right), y[t] \to e^{-4t} \left(-1 + e^{8t}\right) + \frac{3}{4} e^{-4t} \left(3 + e^{8t}\right)\right\}\right\}$$



3. $x_1' = x_2$, $x_2' = -5 x_1 + 4 x_2$

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a = DSolve[{x₁'[t] == x₂[t], x₂'[t] == -5 x₁[t] + 4 x₂[t]}, {x₁[t], x₂[t]}, t]
q = a /. {C[1] → 5, C[2] → 1}
Plot[Evaluate[{x₁[t], x₂[t]} /. q, {t, -1, 1}], PlotRange → {-50, 50},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of x₁[t] and x₂[t]"]
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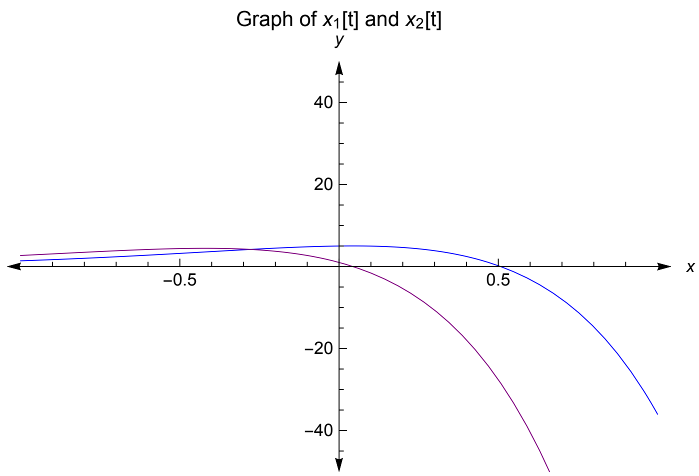
$$\left\{\left\{x_1[t] \to e^{2t} C[1] \left(\mathrm{Cos}[t] - 2\,\mathrm{Sin}[t]\right) + e^{2t} C[2]\,\mathrm{Sin}[t],\; x_2[t] \to -5\, e^{2t} C[1]\,\mathrm{Sin}[t] + e^{2t} C[2] \left(\mathrm{Cos}[t] + 2\,\mathrm{Sin}[t]\right)\right\}\right\}$$

$$\left\{\left\{x_1[t] \to 5\, e^{2t} \left(\mathrm{Cos}[t] - 2\,\mathrm{Sin}[t]\right) + e^{2t}\,\mathrm{Sin}[t],\; x_2[t] \to -25\, e^{2t}\,\mathrm{Sin}[t] + e^{2t} \left(\mathrm{Cos}[t] + 2\,\mathrm{Sin}[t]\right)\right\}\right\}$$



4. $\frac{du}{dt} = 3u + 4v$, $\frac{dv}{dt} = -4u + 3v$

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a = DSolve[{u'[t] == 3 u[t] + 4 v[t], v'[t] == -4 u[t] + 3 v[t]}, {u[t], v[t]}, t]
q = a /. {C[1] → 5, C[2] → 1}
Plot[Evaluate[{u[t], v[t]} /. q, {t, -1, 1}], PlotRange → {-50, 50},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of u[t] and v[t]"]
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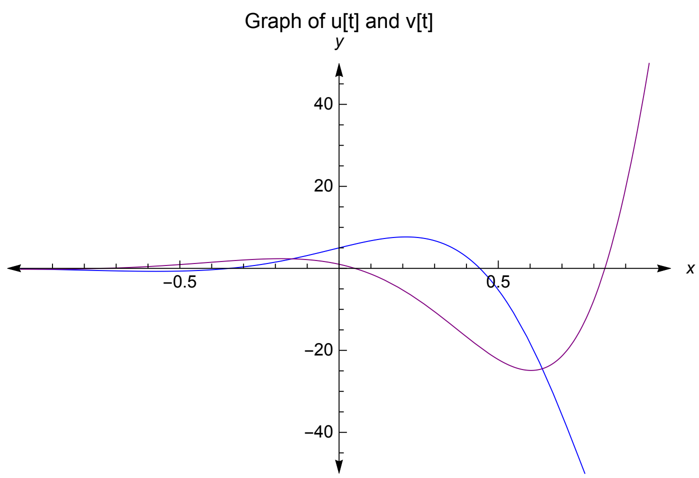
$\{\{u[t] \to e^{3t} C[1] \text{Cos}[4t] + e^{3t} C[2] \text{Sin}[4t], v[t] \to e^{3t} C[2] \text{Cos}[4t] - e^{3t} C[1] \text{Sin}[4t]\}\}$

$\{\{u[t] \to 5 e^{3t} \text{Cos}[4t] + e^{3t} \text{Sin}[4t], v[t] \to e^{3t} \text{Cos}[4t] - 5 e^{3t} \text{Sin}[4t]\}\}$



5. $\frac{dx}{dt} = x + 2y + 1$, $\frac{dy}{dt} = -x + y + t$

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a = DSolve[{x'[t] == x[t] + 2 y[t] + 1, y'[t] == -x[t] + y[t] + t}, {x[t], y[t]}, t]
q = a /. {C[1] → 25, C[2] → 20}
Plot[Evaluate[{x[t], y[t]} /. q, {t, -1, 1}], PlotRange → {-50, 50},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of x[t] and y[t]"]
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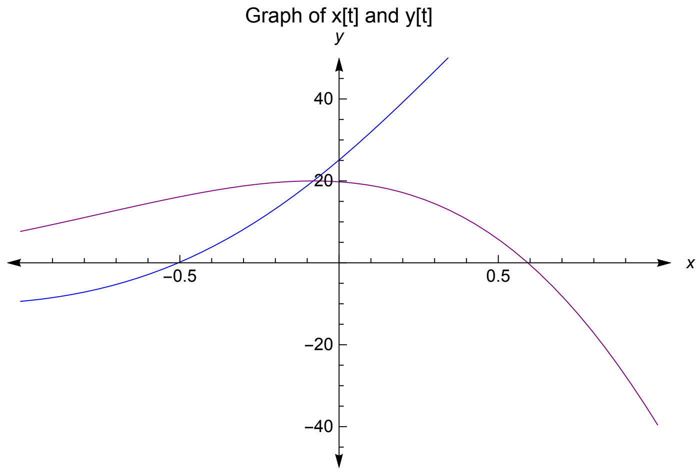
$$\Big\{\Big\{x[t] \to e^t C[1] \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, e^t C[2] \text{Sin}[\sqrt{2}\, t] + \frac{1}{9} \text{Cos}[\sqrt{2}\, t] \left((1+6t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (2+3t) \text{Sin}[\sqrt{2}\, t]\right) + \frac{\text{Sin}[\sqrt{2}\, t] \left(-2 (2+3t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (1+6t) \text{Sin}[\sqrt{2}\, t]\right)}{9\sqrt{2}},$$

$$y[t] \to e^t C[2] \text{Cos}[\sqrt{2}\, t] - \frac{e^t C[1] \text{Sin}[\sqrt{2}\, t]}{\sqrt{2}} - \frac{\text{Sin}[\sqrt{2}\, t] \left((1+6t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (2+3t) \text{Sin}[\sqrt{2}\, t]\right)}{9\sqrt{2}} + \frac{1}{18} \text{Cos}[\sqrt{2}\, t] \left(-2 (2+3t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (1+6t) \text{Sin}[\sqrt{2}\, t]\right)\Big\}\Big\}$$

$$\Big\{\Big\{x[t] \to 25\, e^t \text{Cos}[\sqrt{2}\, t] + 20\sqrt{2}\, e^t \text{Sin}[\sqrt{2}\, t] + \frac{1}{9} \text{Cos}[\sqrt{2}\, t] \left((1+6t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (2+3t) \text{Sin}[\sqrt{2}\, t]\right) + \frac{\text{Sin}[\sqrt{2}\, t] \left(-2 (2+3t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (1+6t) \text{Sin}[\sqrt{2}\, t]\right)}{9\sqrt{2}}, y[t] \to 20\, e^t \text{Cos}[\sqrt{2}\, t] -$$

$$\frac{25\, e^t \text{Sin}[\sqrt{2}\, t]}{\sqrt{2}} - \frac{\text{Sin}[\sqrt{2}\, t] \left((1+6t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (2+3t) \text{Sin}[\sqrt{2}\, t]\right)}{9\sqrt{2}} + \frac{1}{18} \text{Cos}[\sqrt{2}\, t] \left(-2 (2+3t) \text{Cos}[\sqrt{2}\, t] + \sqrt{2}\, (1+6t) \text{Sin}[\sqrt{2}\, t]\right)\Big\}\Big\}$$


Graph of x[t] and y[t]
y
40
20
−20
−40
−0.5
0.5
x

6. $\frac{dx}{dt} = y + 1 - t$, $\frac{dy}{dt} = x - t$

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a = DSolve[{x'[t] == y[t] + 1 - t, y'[t] == x[t] - t}, {x[t], y[t]}, t]
q = a /. {C[1] → 25, C[2] → 20}
Plot[Evaluate[{x[t], y[t]} /. q, {t, -1, 1}], PlotRange → {-50, 50},
 PlotStyle → {Blue, Purple}, AxesStyle → Arrowheads[{-0.02, 0.02}],
 AxesLabel → {x, y}, PlotLabel → "Graph of x[t] and y[t]"]
```

$$\Big\{\Big\{x[t] \to \frac{1}{4} e^{-t}\left(-1+e^{2t}\right)\left(-e^{t}-e^{-t}\left(-1-2t\right)\right) + \frac{1}{4} e^{-t}\left(1+e^{2t}\right)\left(e^{t}+e^{-t}\left(1+2t\right)\right) + \frac{1}{2} e^{-t}\left(1+e^{2t}\right) C[1] + \frac{1}{2} e^{-t}\left(-1+e^{2t}\right) C[2],$$
$$y[t] \to \frac{1}{4} e^{-t}\left(1+e^{2t}\right)\left(-e^{t}-e^{-t}\left(-1-2t\right)\right) + \frac{1}{4} e^{-t}\left(-1+e^{2t}\right)\left(e^{t}+e^{-t}\left(1+2t\right)\right) + \frac{1}{2} e^{-t}\left(-1+e^{2t}\right) C[1] + \frac{1}{2} e^{-t}\left(1+e^{2t}\right) C[2]\Big\}\Big\}$$

$$\Big\{\Big\{x[t] \to 10\, e^{-t}\left(-1+e^{2t}\right) + \frac{25}{2} e^{-t}\left(1+e^{2t}\right) + \frac{1}{4} e^{-t}\left(-1+e^{2t}\right)\left(-e^{t}-e^{-t}\left(-1-2t\right)\right) + \frac{1}{4} e^{-t}\left(1+e^{2t}\right)\left(e^{t}+e^{-t}\left(1+2t\right)\right), y[t] \to \frac{25}{2} e^{-t}\left(-1+e^{2t}\right) + 10\, e^{-t}\left(1+e^{2t}\right) +$$
$$\frac{1}{4} e^{-t}\left(1+e^{2t}\right)\left(-e^{t}-e^{-t}\left(-1-2t\right)\right) + \frac{1}{4} e^{-t}\left(-1+e^{2t}\right)\left(e^{t}+e^{-t}\left(1+2t\right)\right)\Big\}\Big\}$$