1.1 $A=\{a,b,c\}$

- $b\in A$ V
- $\emptyset\subset A$ V
- $\{\emptyset\}\subset A$ F
- $\{c,d\}\not\subset A$ V
- $\{a,\{c\}\}\subset A$ F

1.2 $A=\{x\in\mathbb{R} \mid x^2>25\}\cap\{x\in\mathbb{R} \mid 2x\in\mathbb{Z}\}$

- $5\in A$ F
- $-\frac{11}{2}\in A$ V
- $\frac{100}{3}\in A$ F
- $\mathbb{N}\subset A$ F
- $\{\frac{7}{6}, 6\}\subset A$ F

1.3 $A=\{a,e,i,o,u\}$

- $\emptyset\in P(A)$ V
- $a\in P(A)$ F
- $\{i,u\}\subset P(A)$ F
- $\{e,o\}\in P(A)$ V
- $\{\{e\},\{o\}\}\subset P(A)$ V

1.4

$A=\{x\in\mathbb{R} \mid P(x)=0\}$
$B=\{x\in\mathbb{R} \mid Q(x)=0\}$

1) $P(x)\cdot Q(x)=0 \longrightarrow A\cup B$

2) $\begin{cases} P(x)=0 \\ Q(x)=0 \end{cases} \longrightarrow A\cap B$

1.5

Dim. $A\cap B=\emptyset \iff P(A)\cap P(B)=\{\emptyset\}$

Suppongo che $A=\{a_1,a_2,\_\_ a_n\}\neq B=\{b_1,b_2 \_\_ b_m\}$ $m,n\in\mathbb{N}$

$A\cap B=\emptyset \iff a_i\neq b_j \, , \; i,j\in\mathbb{N}$

Essendo $P(A)=\{\emptyset,\{a_1\},\{a_2\},\_\_,A\}$ e
$P(B)=\{\emptyset,\{b_1\},\{b_2\} \_\_ B\}$

$$P(A) \cap P(B) = \{\emptyset\} \quad \text{c.v.d.}$$

$\boxed{1.6}$

- $P(A \cap B) = P(A) \cap P(B)$? $\iff \begin{cases} P(A \cap B) \subseteq P(A) \cap P(B) \\ P(A) \cap P(B) \subseteq P(A \cap B) \end{cases}$

$A \cap B = \{x \mid x \in A \text{ e } x \in B\}$

$$\begin{aligned} X \in P(A \cap B) &\iff X \subseteq (A \cap B) \iff X \subseteq A \text{ e } X \subseteq B \\ &\iff X \in P(A) \text{ e } X \in P(B) \\ &\iff X \in P(A) \cap P(B) \\ &\implies P(A \cap B) \subseteq P(A) \cap P(B) \end{aligned}$$

$$\begin{aligned} X \in P(A) \cap P(B) &\iff X \in P(A) \text{ e } X \subseteq P(B) \\ &\iff X \subseteq A \text{ e } X \subseteq B \\ &\iff X \subseteq (A \cap B) \\ &\iff X \in P(A \cap B) \\ &\implies P(A) \cap P(B) \subseteq P(A \cap B) \end{aligned}$$

$\implies P(A \cap B) = P(A) \cap P(B)$ VERO

- $P(A \cup B) = P(A) \cup P(B)$? $\iff \begin{cases} P(A \cup B) \subseteq P(A) \cup P(B) \\ P(A) \cup P(B) \subseteq P(A \cup B) \end{cases}$

$A \cup B = \{x \mid x \in A \text{ oppure } x \in B\}$

$$X \in P(A \cup B) \iff X \subseteq (A \cup B) \;\cancel{\implies}\; X \subseteq A \text{ oppure } X \subseteq B$$

$\implies P(A \cup B) = P(A) \cup P(B)$ FALSO

$\boxed{1.7}$ Dimostrare $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$$\Longleftrightarrow \begin{cases} (A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C) \\ (A \cup C) \cap (B \cup C) \subseteq (A \cap B) \cup C \end{cases}$$

$x \in (A \cap B) \cup C \Longleftrightarrow x \in (A \cap B)$ oppure $x \in C$

$\Longleftrightarrow x \in A$ oppure C e $x \in B$ oppure C

$\Longleftrightarrow x \in (A \cup C)$ e $x \in (B \cup C)$

$x \in (A \cup C) \cap (B \cup C)$

$x \in (A \cup C) \cap (B \cup C) \Longleftrightarrow x \in (A \cup C)$ e $x \in (B \cup C)$

$\Longleftrightarrow x \in A$ oppure $x \in C$ e

$x \in B$ oppure $x \in C$

$\Longleftrightarrow [x \in A$ e $x \in B]$ oppure $x \in C$

$\Longleftrightarrow x \in (A \cap B)$ oppure $x \in C$

$\Longleftrightarrow x \in (A \cap B) \cup C$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ c.v.d.

$\boxed{1.9}$ Dimostrare $C_X(A \cup B) = C_X(A) \cap C_X(B)$

$$L \Longleftrightarrow \begin{cases} C_X(A \cup B) \subseteq C_X(A) \cap C_X(B) \\ C_X(A) \cap C_X(B) \subseteq C_X(A \cup B) \end{cases}$$

$x \in C_X(A \cup B) \Longleftrightarrow x \in X \setminus (A \cup B) \Longleftrightarrow x \in X$ e $x \notin A$ e $x \notin B$

$\Longleftrightarrow x \in X \setminus A$ e $x \in X \setminus B$

$\Longleftrightarrow x \in C_X(A) \cap C_X(B)$

$x \in C_X(A) \cap C_X(B) \Longleftrightarrow x \in X \setminus A$ e $x \in X \setminus B$

$\Longleftrightarrow x \in X$ e $x \notin A$ e $x \notin B$

$\Longleftrightarrow x \in X$ e $x \notin (A \cup B)$

$\Longleftrightarrow x \in X \setminus (A \cup B) \Longleftrightarrow x \in C_X(A \cup B)$

$\boxed{1.11}$

Dimostrare che :

- $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$

$$x \in X \setminus (A \cap B) \Rightarrow x \in X \text{ e } x \notin (A \cap B)$$
$$\Rightarrow [x \in X \text{ e } x \notin A] \text{ oppure } [x \in X \text{ e } x \notin B]$$
$$\Rightarrow x \in X \setminus A \text{ oppure } x \in X \setminus B$$
$$\Rightarrow x \in (X \setminus A) \cup (X \setminus B)$$
$$x \in (X \setminus A) \cup (X \setminus B) \Rightarrow x \in X \text{ e } (x \notin A \text{ e } x \notin B)$$
$$\Rightarrow x \in X \text{ e } x \notin A \cap B$$
$$\Rightarrow x \in X \setminus (A \cap B)$$

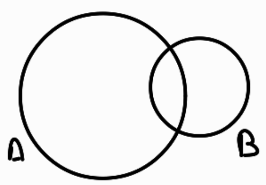
- $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$

$$x \in X \setminus (A \cup B) \Rightarrow x \in X \text{ e } x \notin (A \cup B)$$
$$\Rightarrow x \in X \text{ ma } x \notin A \text{ e } x \in X \text{ ma } x \notin B$$
$$\Rightarrow x \in X \setminus A \text{ e } x \in X \setminus B$$
$$\Rightarrow x \in (X \setminus A) \cap (X \setminus B)$$
$$x \in (X \setminus A) \cap (X \setminus B) \Rightarrow x \in X \text{ ma } x \notin A \text{ e } x \in X \text{ ma } x \notin B$$
$$\Rightarrow x \in X \text{ ma } x \notin A \text{ oppure } x \notin B$$
$$\Rightarrow x \in X \text{ ma } x \notin A \cup B$$
$$\Rightarrow x \in X \setminus (A \cup B)$$

$\boxed{1.12}$

Dimostrare che $A \cup B \setminus A = C_B(A \cap B)$

$$x \in (A \cup B \setminus A) \Rightarrow x \in (A \cup B) \text{ ma } x \notin A \Rightarrow$$
$$x \in B \text{ ma } x \notin A \cap B \Rightarrow x \in B \setminus (A \cap B)$$
$$\Rightarrow x \in C_B \setminus (A \cap B)$$



$$x \in C_B(A\cap B) \Rightarrow x\in B\setminus(A\cap B) \Rightarrow x\in B \text{ ma } x\notin (A\cap B)$$
$$\Rightarrow x\in B \text{ e } x\notin A \text{ e } x\notin A\cap B$$
$$\Rightarrow x\in B \text{ o } x\in A \text{ ma } x\notin A$$
$$\Rightarrow x\in (A\cup B\setminus A)$$

$\boxed{1.13}$

- $A\cap B\subseteq C$ V
- $\{d,e\}\in P(C\cap D)$ V
- $(A\times B)\cap(C\times D)=\emptyset$ F
- $(v,v)\in A\times B\setminus B\times C$ V
- $(e,p)\in A\times D$ F
- $\{b,l,u\}\subseteq P(B\cup D)$ V

$\boxed{1.14}$ Partizioni di $A=\{a,b,c,d\}$

$$P(A)=\{\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{c,d\},\{b,d\},\{a,b,c\},$$
$$\{a,b,d\},\{a,c,d\},\{b,c,d\},A\}$$

$Q_1=\{\{a\},\{b\},\{c\},\{d\}\}$

$Q_2=\{\{a\},\{b\},\{c,d\}\}$

$Q_3=\{\{a\},\{c\},\{b,d\}\}$

$Q_4=\{\{a\},\{d\},\{b,c\}\}$

$Q_5=\{\{b\}\{c\},\{a,d\}\}$

$Q_6=\{\{b\},\{d\},\{a,c\}\}$

$Q_7=\{\{c\},\{d\},\{a,b\}\}$

$Q_8=\{\{a\},\{b,c,d\}\}$

$Q_9=\{\{b\},\{a,c,d\}\}$

$Q_{10}=\{\{c\},\{a,b,d\}\}$

$Q_{11}=\{\{d\},\{a,b,c\}\}$

$Q_{12}=\{\{a,b,c,d\}\}$

$\boxed{1.15}$ R e' ricoprimento? sí perche $\bigcup_{y=mx} = X\times Y$

R e' partizione? No perche tutte le rette hanno l'origine in comune.

$\boxed{1.16}$ $|X|=n$ $P_k=\{A\subset X \mid |A|=k\}$ $k=0,1,\ldots,n$

$P=\{P_k\}$ e' partizione di $P(X)$? $|P|=?$

se $|A|=0 \to P_0=\{\emptyset\}$ e $|P_0|=1$

se $|A|=1 \to P_1=\{A_1,A_2,\ldots A_n\}$ e $|P_1|=n$

se $|A|=n \to P_n=\{X\}$ e $|P_n|=1$

se $|A|=2$, $P_2 = \{\{a,b\} \subset X \mid a \neq b\}$ e così via per tutti i k

i sottoinsiemi di X della stessa dimensione differiscono tra loro per almeno un elemento, perciò gli elementi del suo insieme delle parti sono tutti disgiunti. $\Rightarrow$ P è partizione di $P(X)$

$|P| = n+1$