

Fall 2018, CMPSC 465: Homework 5 and Exam 5 (practice).

No collaboration permitted. Please upload your solutions to Canvas by 9 pm on November 13th. Solutions will be available soon after the deadline.

1. We are given a directed graph $G = (V, E)$ on which each edge $\langle u, v \rangle$ has an associated value $r(u, v)$, which is a real number in the range $(0, 1)$ that represents the reliability of a communication channel from vertex u to vertex v . We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.
(10 points)

2. You are given a directed graph $G(V, E)$ with (possibly negative) weighted edges, along with a specific node $s \in V$ and a tree $T = (V, E'), E' \subseteq E$. Give an algorithm that checks whether T is a shortest-path tree for G with starting point s . Your algorithm should run in linear time.
(7 points)

3. Draw a simple directed weighted graph G with 8 vertices and 16 edges, such that G contains a minimum-weight cycle with at least 4 edges. Show that the Bellman-Ford algorithm will find this cycle.
(8 points)

4. Design an efficient algorithm for finding the longest shortest path connecting any two vertices in an acyclic weighted directed graph. Also, analyze the time complexity of your algorithm.
(8 points)

5. Explain why there are no nontree forward edges with respect to a BFS tree constructed for a directed graph.
(**7 points**)