Fall 2018, CMPSC 465: Exam 3.
Closed book and closed notes, no 'cheat sheet', no calculators allowed.
Please don't use cell phones during the exam.
Answer questions in the space provided.
The exam is for 40 points.

NAME: _____   SECTION:_____

1. Complete the pseudocode for the InsertAfter(p, e) operation in a doubly-linked list, where a
   new node with element e is inserted between node p and its successor. What is the worst-case
   asymptotic running time?
   (**5 points**)

   Algorithm insertAfter(p, e):

   Create a new node v

   v.element $\leftarrow e$

   _____   /* link v to its predecessor */

   _____   /* link v to its successor */

   _____   /* link p's old successor to v */

   _____   /* link p to its new successor, v */

   return v

   Running time: _____

2. Describe how to implement a queue using two stacks. What is the worst-case running time of
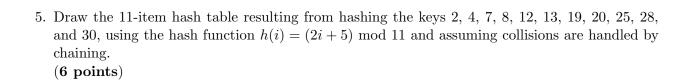   the `enqueue`() and `dequeue`() methods in this case?
   (**6 points**)

3. A *matched string* is a sequence of {, }, (,), [, and] characters that are properly matched. For example, "{{()[]}}" is a matched string, but this "{{()]}" is not, since the second { is matched with a]. Show how to use a stack so that, given a string of length n, you can determine if it is a matched string in O(n) time.
(**6 points**)

4. Suppose we perform a DeleteMax operation on the max heap $H = [11, 8, 7, 3, 5, 6, 2, 1]$ (the heap is stored here implicitly in the form of an array). Show the steps performed after deletion to restore the heap order of elements.
(**5 points**)

5. Draw the 11-item hash table resulting from hashing the keys 2, 4, 7, 8, 12, 13, 19, 20, 25, 28, and 30, using the hash function $h(i) = (2i + 5) \bmod 11$ and assuming collisions are handled by chaining.
(**6 points**)

6. Let T be an ordered tree with more than one node. Is it possible that the preorder traversal of T visits the nodes in the same order as the postorder traversal of T? If so, give an example; otherwise, argue why this cannot occur. Likewise, is it possible that the preorder traversal of T visits the nodes in the reverse order of the postorder traversal of T? If so, give an example; otherwise, argue why this cannot occur.
(**6 points**)

7. Suppose T is a binary search tree of height 4 (including the external nodes) that is storing all the integers in the range from 1 to 15, inclusive. Suppose further that you do a search for the number 11. Explain why it is impossible for the sequence of numbers you encounter in this search to be (9, 12, 10, 11).

Answer.

A binary tree whose height is 4 including the external nodes has only 4 levels of *internal* nodes. A binary tree with 4 levels of internal nodes can hold at most

$$2^4 - 1 = 15$$

internal nodes. Since T must store all 15 of the integers $1,\dots,15$, the tree must contain the maximum possible number of internal nodes for its height. This is only possible if T is a *perfectly balanced* (complete/perfect) binary tree: every internal node in the top three levels has two children, and the bottom internal level is completely filled.

For a binary search tree, there is exactly **one** such perfectly balanced arrangement of $1,\dots,15$: the element in each position is forced by the in-order (sorted) ordering. In particular, the root must be the median value,

$$8,$$

its children must be 4 and 12, the next level must be $2,6,10,14$, and the leaves must be $1,3,5,7,9,11,13,15$.

Therefore **any** search in T must begin at the root, whose value is 8. Searching for 11 then proceeds:

$$8 \;(\,11>8,\ \text{go right}\,) \;\to\; 12 \;(\,11<12,\ \text{go left}\,) \;\to\; 10 \;(\,11>10,\ \text{go right}\,) \;\to\; 11,$$

so the sequence of numbers encountered is $(8, 12, 10, 11)$.

The proposed sequence $(9, 12, 10, 11)$ starts with 9, but the first number encountered must be the root, which is forced to be 8. Hence it is impossible for the search sequence to be $(9, 12, 10, 11)$.