Please clearly write your name and your PSU ID (i.e., xyz1234) on top of your paper.

Problem 1 (4 points). For each pairs of functions below, indicate one of the three: $f = O(g)$, $f = \Omega(g)$, or $f = \Theta(g)$.

1. $f(n) = 2^n$, $g(n) = (\log n)^{\log n}$
2. $f(n) = n^2 \cdot (\log n)^3$, $g(n) = 2^{3 \cdot \log_2 n}$

Problem 2 (3 points). Solve this recursion: $T(n) = 4 \cdot T(n/\sqrt{2}) + n^4$.

Problem 3 (3 points). Design an instance of the convex hull problem such that the run of the Graham-Scan algorithm on your instance will execute the pop operations *exactly three times and these three pop operations are continuous* (that is, if you list all the push and pop operations along the run of the algorithm, the three pop operations should be adjacent to each other). Your instance should contain 6 points on 2D plane and any three of them are not on the same line.

Problem 1

Please not that $\log f(n) = \Theta(\log g(n))$ does not imply $f(n) = \Theta(g(n))$. E.g: $f(n) = n^2$, $g(n) = n^3$. If $\log f(n)$ has definitely higher order (not same order) than $\log g(n)$, "take log on both" work

1. $\log f(n) = n$ $\log g(n) = \log n \log\log n$

   $f(n) = \Omega(g(n))$

2. $g(n) = 2^{3 \cdot \log_2 n} = n^3$

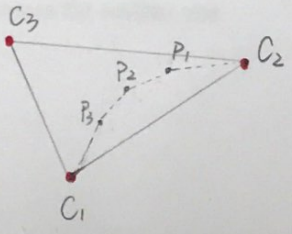
   $f(n) = O(g(n))$

Problem 2

$\log_{\sqrt{2}} 4 = 4$

$T(n) = \Theta(n^4 \log n)$ or $O(n^4 \log n)$ either is correct

Problem 3



key point: $C_1 \to C_2 \to P_1 \to P_2 \to P_3$ should be convex

When checking C_3, P_1, P_2, P_3 all pop out of stack