CSE 565	**Martin Fürer**	**Algorithm Design and Analysis**

Assignment 10 Given: 11/27/18 Extended to: Wednesday 12/5/18

Exercises

1. Solve Exercise 1, Chapter 8 on page 505 of the Textbook.
(Independent Set versus Interval Scheduling)
2. Solve Exercise 5, Chapter 8 on page 506 of the Textbook.
(Hitting Set)
3. Solve Exercise 28, Chapter 8 on page 519 of the Textbook.
(Strongly Independent Set)

Problems

1. [15 points] Solve Problem 2, Chapter 8 on page 505 of the Textbook.
(Diverse customers)
2. [$5 \times 4 = 20$ points] Solve Problem 4, Chapter 8 on page 506 of the Textbook. (Four problems with resource allocations)
3. [20 points] Assume, we know that SAT is NP-complete. Prove that the following version of SAT is also NP-complete. In every clause either all literals are negated or all literals are unnegated. You can have clauses of both kinds.
4. [$5 + 5 + 20 = 30$ points] k-Coloring is the following decision problem.
Instance: $G = (V, E)$ (an undirected graph).
Question: Can the vertices V of G be colored with k colors such that no two adjacent vertices have the same color?
k-Coloring is actually NP-complete for all $k \geq 3$.
   (a) Give a short argument that 2-Coloring is in P.
   (b) Assume, you only know that 3-Coloring is NP-complete. Prove that 4-Coloring is NP-complete too.
   (c) Assume, you only know that 4-Coloring is NP-complete. Prove that 3-Coloring is NP-complete too.
   Hint: Construct a graph $f(G)$ that has (among other vertices) for every vertex v of G a pair of vertices (v', v''). Furthermore, $f(G)$ should have the property that whenever a 4-coloring colors vertex v_i of G with a color $c_i \in \{0, 1, 2, 3\}$, then there is 3-coloring of $f(G)$ coloring v'_i with color $c'_i \in \{0, 1\}$ and v''_i with color$c''_i \in \{0, 1\}$, where $c_i = 2c''_i + c'_i$.