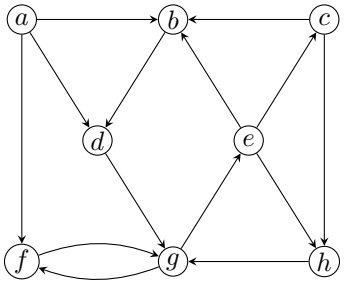
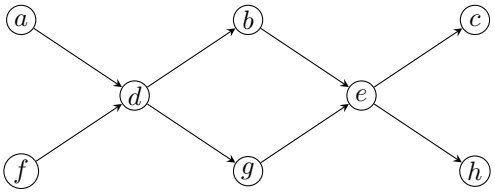
Problem 1 (10 points). What is the maximum number of edges a directed acyclic graph with n vertices can have? Prove your answer.

Problem 2 (10 points). Draw the meta-graph for the following directed graph.



Problem 3 (10 points). How many different linearizations for the following graph?



Problem 4 (20 points). Let $G = (V, E)$ be a DAG. Design an $O(|E|)$ time algorithm to decide whether there is only one possible linearization for G.

Problem 5 (20 points). Your job is to prepare a lineup of n awardees at an award ceremony. You are given a list of m constraints of the form: awardee i wants to receive his award before awardee j. Design an algorithm to either give such a lineup that satisfies all constraints, or return that it is not possible. Your algorithm should run in $O(m+n)$ time.

Problem 6 (30 points). All streets in city X are one-way. Residents want to know whether there is a way to drive legally from any intersection in the city to any other intersection.

1. Formulate this problem as a graph problem and design a linear time algorithm for it.
2. Suppose that the answer for the above question is false. Residents further want to know whether the following weaker statement is true: if you start driving from city town hall (assume that it locates at one intersection), then no matter where you reach, there is always a way to drive legally back to the city town hall. Formulate this problem as a graph problem and design a linear time algorithm for it.