Problem 1 (0 points).

Assume that a simple regular language supports $*$ and ?, where ? matches any single character, and $*$ matches any multiple characters (can match nothing as well). Given a string S over alphabest Σ and a simple regular expression E (a string over $\Sigma \cup \{*\} \cup \{?\}$), describe an algorithm that decides whether S matches E.

Problem 2 (0 points).

Given a positive integer X with n digits, try to remove k ($k \leq n$) digits from X so that the number consisting of the remaining digits is as small as possible. You may assume that X has no leading zero and you are not allowed to change the relative order of the digits.

Problem 3 (0 points).

Recall from the class we talked about the sequence alignment problem where we use dynamic programming to calculate the edit distance between two strings. Given two strings A and B have the same length of n characters, and a threshold k, $0 < k < n$, design an algorithm to check whether the edit distance between A and B is less than k. Your algorithm should run in $O(nk)$ time.

Problem 4 (0 points).

You are given an undirected connected graph $G = (V, E)$ satisfying that $|E| = |V| + 5$. Each edge $e \in E$ has weight $w(e)$ and you may assume that all weights are distinct. Design an $O(|V|)$ time algorithm to find the minimum spanning tree of G, and prove the correctness of your algorithm.

Problem 5 (0 points).

Let $X = \{a_1, a_2, \cdots, a_n\}$, where $a_i \in \mathbb{R}^d$, $1 \leq i \leq n$. Let S be the set of all subsets of X that are linearly independent. Prove that (X, S) forms a matroid.