# Mid-term 1, Part 1

Started: Oct 5 at 6:05pm

# Quiz Instructions

## Question 1

3 pts

Let $A$, $B$, $C$, and $D$ be 4 strings over alphabet $\Sigma$. Let $d(\cdot,\cdot)$ be the edit distance defined in class. We use $AB$ to denote the concatenation of $A$ and $B$. Which one of the following is true?

- ○ All other 3 choices are not true
- ○ $d(A,C) + d(B,C) \le d(AB,C)$
- ○ $d(A,B) + d(B,C) \le d(A,C)$
- ○ $d(A,B) + d(C,D) \le d(AC,BD)$

## Question 2

3 pts

Let $G$ be a directed graph possibly with negative cycles. Assume that $G$ has 10 vertices, and $s$ and $v$ are two of them. Define $dist(k,v)$ as the length of the shortest path from $s$ to $v$ using at most $k$ edges. Which one of the following is true?

- ○ If $dist(9,v) = dist(i,v)$ for all $i = 10, 11, \cdots$, then $G$ must not contain negative cycle reachable from $s$.
- ○ If $G$ contains negative cycles reachable from $s$, then $dist(9,v) > dist(10,v)$.
- ○ $dist(10,v) = dist(11,v)$, no matter $G$ contains negative cycle or not.
- ○ If $dist(9,v) > dist(11,v)$, then $G$ must contain negative cycle reachable from $s$.

## Question 3

3 pts

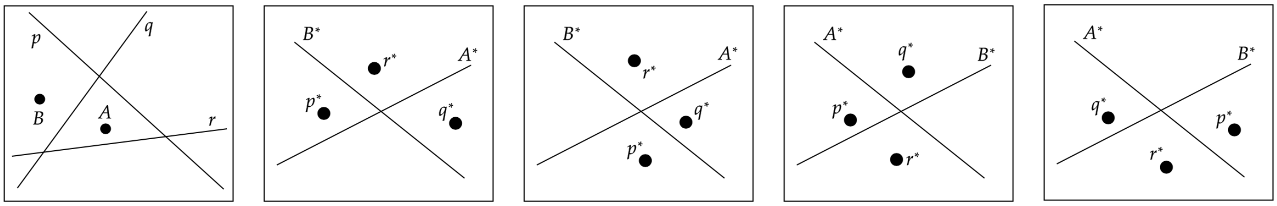
Which of the following is the asymptotic solution of recurrence $T(n) = 2 \cdot T(n/2) + \log n$ ?

- ○ $\Theta(n)$
- ○ $\Theta(\log n)$
- ○ $\Theta(n \cdot \log\log n)$
- ○ $\Theta(n \cdot \log n)$

## Question 4

**3 pts**

For the two points $A$ and $B$ and three lines $p$, $q$ and $r$ on the primal plane, which one gives their correct dual points and dual lines?




primal dual 1 dual 2 dual 3 dual 4

- ○ dual 2
- ○ dual 4
- ○ dual 3
- ○ dual 1

## Question 5

**3 pts**

Let $S$ be an array with $n$ distinct integers. Similar to the selection algorithm introduced in class, we partition $S$ into $n/17$ sub-arrays, each of which contains 17 numbers. Let $x$ be the median of the medians of the $n/17$ sub-arrays. How many numbers in $S$ are guaranteed larger than $x$?

- ○ $6n/17$
- ○ $5n/17$
- ○ $9n/34$
- ○ $11n/34$

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**Problem 1 (3 points).**

- Correct. (Note: A more accurate statement would be "all other 3 choices are not always true".)
- False. Counter-example: $A = G$, $B = G$, and $C = GG$.
- False. Counter-example: $A = G$, $B = T$, and $C = G$.
- False. Counter-example: $A = TT$, $B = T$, $C = G$, and $D = TG$.

**Problem 2 (3 points).**

- False. $dist(9, v) = dist(i, v)$ for all $i \geq 10$ implies that there does not exist a path from $s$ to $v$ that go through a negative cycle. But it is still possible that $G$ contains negative cycle reachable from $s$—it's just that such negative cycle cannot reach $v$. In fact, to show that $G$ does not contain negative cycle reachable from $s$, we need this statement to hold for *every* vertex $v$.
- False.
- False.
- True. (Proved in class.)

**Problem 3 (3 points).**

Think about a lower bound and a upper bound of $\log n$: asymptotically $n^0 \leq \log n \leq n^{0.1}$:
$T(n) = 2T(n/2) + n^0$ gives $T(n) = \Theta(n)$, using master's theorem;
$T(n) = 2T(n/2) + n^{0.1}$ gives $T(n) = \Theta(n)$, using master's theorem.
Therefore, $T(n) = 2T(n/2) + \log n$ must also give $T(n) = \Theta(n)$.

In fact, a more general form of master's theorem solves the recurrence: $T(n) = aT(n/b) + \Theta(n^d \cdot \log^s n)$.

$$T(n) = \begin{cases} \Theta(n^{\log_b a}) & \log_b a > d \\ \Theta(n^d \cdot \log^{s+1} n) & \log_b a = d \\ \Theta(n^d \cdot \log^s n) & \log_b a < d \end{cases}$$

**Problem 4 (3 points).**

- Dual 1: False. Point $A$ is below line $p$ in the primal plane, so in the dual plane $p^*$ should be below line $A^*$.
- Dual 2: False. The slope of $r$ is larger than the slope of $p$ in the primal plane, so in the dual plane, the $x$-coordinate of $r^*$ should be larger than that of $p^*$. This option will become true if $p^*$ can be moved slightly to the left so that its $x$-coordinate can be less than that of $r^*$.
- Dual 3: False.
- Dual 4: False.

Note: This problem were regraded so that full points were granted to any answer.

**Problem 5 (3 points).**

Among all $n/17$ medians, $n/34$ of them are guaranteed larger than $x$. There are $n/34$ subarrays, each of which contains 8 numbers that are guaranteed larger than $x$. In total, there are $n/34 + 8n/34 = 9n/34$ numbers guaranteed larger than $x$.