Fall 2018, CMPSC 465: Take-home Exam 2.
No collaboration, no discussion with others permitted. You may refer to your notes, material on the Canvas class site, and the three class textbooks, but you are not permitted to access any other resource. Please read the statement on academic integrity in the syllabus.
Solutions must be typed and electronically prepared (use LaTeX or Word). Scans and photographs won't be graded. Please submit a single PDF file on Canvas.
The submission deadline is **9 pm on November 30th**. It will be strictly enforced.
The exam is for 100 points, and each problem is for 20 points.

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1. Devise an $O(n \log n)$ algorithm for the following problem: Given n points in the plane, find the pair with the shortest distance between them.
2. A mother vertex in a directed graph $G = (V, E)$ is a vertex v such that all other vertices in G can be reached by a directed path from v.
   (a) Give an $O(n + m)$ algorithm to test whether a given vertex v is a mother of G.
   (b) Give an $O(n + m)$ algorithm to test whether graph G contains a mother vertex.
3. In a **side-scrolling video game**, a character moves through an environment from, say, left-to-right, while encountering obstacles, attackers, and prizes. The goal is to avoid or destroy the obstacles, defeat or avoid the attackers, and collect as many prizes as possible while moving from a starting position to an ending position. We can model such a game with a graph, G, where each vertex is a game position, given as an (x, y) point in the plane, and two such vertices, v and w, are connected by an edge, given as a straight line segment, if there is a single movement that connects v and w. Furthermore, we can define the cost, $c(e)$, of an edge to be a combination of the time, health points, prizes, etc., that it costs our character to move along the edge e (where earning a prize on this edge would be modeled as a negative term in this cost). A path, P, in G is **monotone** if traversing P involves a continuous sequence of left-to-right movements, with no right-to-left moves. Thus, we can model an optimal solution to such a side-scrolling computer game in terms of finding a minimum-cost monotone path in the graph, G, that represents this game. Describe and analyze an efficient algorithm for finding a minimum-cost monotone path in such a graph, G.
4. Exercise 3.20 from the textbook.
5. Exercise 4.22 from the textbook.