

Fall 2018, CMPSC 465: Exam 1.

Closed book and closed notes, no ‘cheat sheet’, no calculators allowed.

Please don’t use cell phones during the exam.

Answer questions in the space provided.

The exam is for 40 points (4 problems, 10 points each).

NAME:

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1. Using the Big-O and Big-Omega definitions, show that the function  $f(n) = n^3 + n$  is  $O(n^3)$  and  $\Omega(n)$ .

**Solution:**

2. Given the terms in the Fibonacci sequence, find a constant  $c < 1$  such that  $F_n \leq 2^{cn}$  for all  $n \geq 0$ . Show using induction that your answer is correct.

**Solution:**

3. Solve the recurrence  $T(n) = 3T(n/3) + n$  using the recursion tree method. Express your answer in the  $\Theta$ -notation. In the recursion tree method, you
- draw a recursion tree,
  - determine its height,
  - estimate the work associated with nodes at each level, and
  - simplify the summation to come up with a closed-form expression for the running time.

Assume that  $T(1) = \Theta(1)$  and that  $n$  is a power of 3.

**Solution:**

4. Solve the recurrence  $T(n) = T(n - 1) + \log n$  using the iterative substitution method.

**Solution:**