Fall 2018, CMPSC 465: Exam 4 (practice).
Closed book and closed notes, no 'cheat sheet', no calculators allowed.
Please don't use cell phones during the exam.
Answer questions in the space provided.
The exam is for 40 points.

NAME: _____ SECTION: _____

1. There are two types of professional wrestlers: "babyfaces" ("good guys") and "heels" ("bad guys"). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.
   (**8 points**)

2. The police department in the city of Computopia has made all streets one-way. The mayor contends that there is still a way to drive legally from any intersection in the city to any other intersection, but the opposition is not convinced. A computer program is needed to determine whether the mayor is right. However, the city elections are coming up soon, and there is just enough time to run a linear-time algorithm.

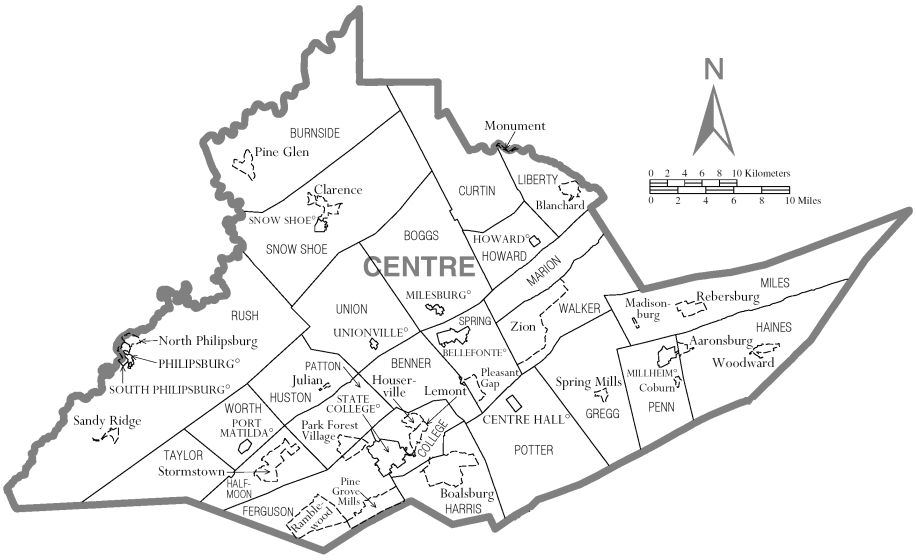
   - Formulate this problem graph-theoretically, and explain why it can indeed be solved in linear time.
   - Suppose it now turns out that the mayor's original claim is false. She next claims something weaker: if you start driving from town hall, navigating one-way streets, then no matter where you reach, there is always a way to drive legally back to the town hall. Formulate this weaker property as a graph-theoretic problem, and carefully show how it too can be checked in linear time.

   (**10 points**)

3. Your job is to arrange n ill-behaved children in a straight line, facing front. You are given a list of m statements of the form "i hates j." If i hates j, then you do not want to put i somewhere behind j, because then i is capable of throwing something at j. Give an algorithm that orders the line, (or says that it is not possible) in $O(m+n)$ time.
(**8 points**)

4. Professor Madduri claims that the algorithm for strongly connected components would be simpler if it used the reversed graph in the second depth-first search as well, and scanned the vertices in the order of *increasing* post identifiers. Does this simpler algorithm always produce correct results?
(**7 points**)

5. The map below shows the townships and boroughs in Centre County, PA. Suppose we want to color this map, so that neighboring townships/boroughs have different colors. We want to determine the minimum number of colors required to do such a coloring. Formulate this "map coloring" problem as a graph-theoretic problem. What is a lower bound on the number of colors required, and a possible upper bound? Briefly explain your answers. Image source:



https://commons.wikimedia.org/wiki/File:Map_of_Centre_County,_Pennsylvania.png

(**7 points**)