Fall 2018, CMPSC 465: Exam 4.
Closed book and closed notes, no 'cheat sheet', no calculators allowed.
Please don't use cell phones during the exam.
Answer questions in the space provided.
The exam is for 40 points.

Name: _____   Section:_____

1. A bipartite graph is a graph whose vertices can be partitioned into two sets such that there are
   no edges between vertices in the same set. Prove that an undirected graph is bipartite if and
   only if it contains no cycles of odd length.
   (**10 points**)

2. Your job is to prepare a lineup of n awardees at an award ceremony. You are given a list of m constraints of the form "i wants to receive an award before j." If you violate a constraint, it might upset the affected award recipient (i) and then i may leave the ceremony. Give an algorithm that prepares such a lineup, (or says that it is not possible) in $O(m + n)$ time. (**8 points**)

3. Run the strongly connected components algorithm on the following directed graph. Whenever there is a choice of vertices, pick the one that is alphabetically first.
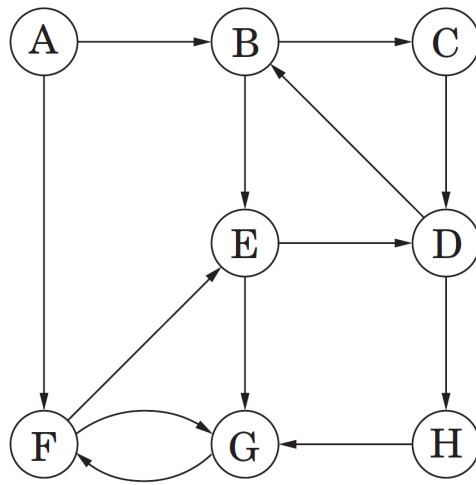
(**8 points**)

4. Prove that the following statement is true, or give a counterexample to disprove it: when we perform depth-first search on a directed graph G, for any nodes u and v, the two intervals $[\text{pre}(u), \text{post}(u)]$ and $[\text{pre}(v), \text{post}(v)]$ are either disjoint or one is contained within the other.
(**7 points**)

5. Design a linear-time algorithm which, given an undirected graph G and a particular edge e in it, determines whether G has a cycle containing e.
(**7 points**)