Fall 2018, CMPSC 465: Exam 5.
Closed book and closed notes, no ‘cheat sheet’, no calculators allowed.
Please don’t use cell phones during the exam.
Answer questions in the space provided.
The exam is for 40 points.

NAME: ______________________________ SECTION:____________

1. Arbitrage is the use of discrepancies in currency exchange rates to make a profit. For example, there may be a small window of time during which 1 U.S. dollar buys 0.75 British pounds, 1 British pound buys 2 Australian dollars, and 1 Australian dollar buys 0.70 U.S. dollars. At such a time, a smart trader can trade one U.S. dollar and end up with $0.75 \times 2 \times 0.7 = 1.05$ U.S. dollars, which is a profit of 5%. Suppose that there are n currencies $c_1, \ldots, c_n$ and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j. Devise and analyze an algorithm to determine the maximum value of
   $R[c_1, c_{i_1}] \cdot R[c_{i_1}, c_{i_2}] \cdots R[c_{i_{k-1}}, c_{i_k}] \cdot R[c_{i_k}, c_1]$
   (10 points)

2. Design and analyze an algorithm that takes as input an undirected graph $G = (V, E)$ and determines whether G contains a simple cycle of length four. Its running time should be at most $O(|V|^3)$. You may assume that the input graph is represented either as an adjacency matrix or with adjacency lists, whichever makes your algorithm simpler.
(**8 points**)

3. You are given a strongly connected directed graph $G = (V, E)$ with positive edge weights along with a particular node $v_0 \in V$. Give an efficient algorithm for finding shortest paths between all pairs of nodes, with the one restriction that these paths must all pass through v_0.
(8 points)

4. Can we modify Dijkstra's algorithm to solve the single-source longest path problem by changing minimum to maximum? If so, then prove that the algorithm is correct. If not, provide a counterexample.
(7 points)

5. Argue that in a breadth-first search, the distance label assigned to a vertex v is independent of the order in which the vertices appear in each adjacency list. Also, show using an example that the breadth-first tree computed by BFS can depend on the ordering of vertices in the adjacency list representation of the graph.
(7 points)