Selected Problems Chapter 6
Linear Algebra Done Right, Sheldon Axler, 3rd Edition

Mustaf Ahmed

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Problem Inner Product Bilinearity. Let V be a vector space equipped with an inner product $\langle .,. \rangle : V \times V \to F$. Show that the inner product is bilnear.

Proof. There are two cases: $F = \mathbb{R}$ or $F = \mathbb{C}$. Assume that $F = \mathbb{R}$. We'll first show additivity in the second slot. We have

$$\begin{aligned}
\langle u, v + w \rangle &= \overline{\langle v + w, u \rangle} \\
&= \overline{\langle v, u \rangle + \langle w, u \rangle} \\
&= \langle v, u \rangle + \langle w, u \rangle \\
&= \overline{\langle u, v \rangle} + \overline{\langle u, w \rangle} \\
&= \langle u, v \rangle + \langle u, w \rangle
\end{aligned}$$

For homogenity in the second slot, we have

$$\begin{aligned}
\langle u, \lambda v \rangle &= \overline{\langle \lambda v, u \rangle} \\
&= \overline{\lambda \langle v, u \rangle} \\
&= \lambda \langle v, u \rangle \\
&= \lambda \overline{\langle u, v \rangle} \\
&= \lambda \langle u, v \rangle
\end{aligned}$$

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