

Selected Problems Chapter 3

Introduction to Probability for Data Science

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Problem 3.2. Two dice are tossed. Let X be the absolute difference in the number of dots facing up.

- a. Find and plot the PMF of X .
- b. Find the probability $X \leq 2$.
- c. Find $\mathbb{E}[X]$ and $Var[X]$.

Part a. The possible random variable states are $X(\Omega) = \{0, 1, 2, 3, 4, 5\}$. The probability mass function for this random variable is

$$p_X(0) = P(\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}) = \frac{1}{6}$$

$$p_X(1) = P(\{(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)\}) = \frac{5}{18}$$

$$p_X(2) = P(\{(1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4)\}) = \frac{2}{9}$$

$$p_X(3) = P(\{(1, 4), (4, 1), (2, 5), (5, 2), (3, 6), (6, 3)\}) = \frac{1}{6}$$

$$p_X(4) = P(\{(1, 5), (5, 1), (2, 6), (6, 2)\}) = \frac{1}{9}$$

$$p_X(5) = P(\{(1, 6), (6, 1)\}) = \frac{1}{18}$$

Part b.

$$\begin{aligned}P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\&= \frac{2}{3}\end{aligned}$$

Part c.

$$\begin{aligned}\mathbb{E}[X] &= 0 * \frac{1}{6} + 1 * \frac{5}{8} + 2 * \frac{2}{9} + 3 * \frac{1}{6} + 4 * \frac{1}{9} + 5 * \frac{1}{18} \\&= \frac{35}{18}\end{aligned}$$

$$\begin{aligned}\text{Var}[X] &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\&= \mathbb{E}[X^2] - \left(\frac{35}{18}\right)^2 \\&= 0^2 * \frac{1}{6} + 1^2 * \frac{5}{8} + 2^2 * \frac{2}{9} + 3^2 * \frac{1}{6} + 4^2 * \frac{1}{9} + 5^2 * \frac{1}{18} - \frac{35^2}{18^2} \\&= \frac{665}{324}\end{aligned}$$