Selected Problems Chapter 2
Probability Theory, Grinstead/Snell, Second Edition

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Problem 4 Statement. Describe in words the events specified by the following subsets of

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(a) E = {HHH, HHT, HTH ,HTT}
(b) E = {HHH, TTT}
(c) E = {HHT, HTH, THH}
(d) E = {HHT, HTH ,HTT, THH ,THT ,TTH, TTT}

Problem 4 .

(a) The first flip is H.
(b) All flips are of the same type.
(c) Exactly 2 Heads.
(d) At least 1 T.

Problem 5 Statement. What are the probabilities of the events described in Exercise 4?

Problem 5 .

(a) 1/2
(b) 1/4
(c) 3/8
(d) 7/8

Problem 6 Statement. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on the face. (For example, a six is three times as probable as a two.) What is the probability of getting an even number in one throw?

Problem 6 . Let $s = P(6)$. Since the probabilities are proportional to the number of dots on the face, we have :

$$P(\Omega) = 1 = s + \frac{5}{6}s + \frac{4}{6}s + \frac{3}{6}s + \frac{2}{6}s + \frac{1}{6}s,$$

thus, $s = \frac{2}{7}$, and

$$\begin{aligned}
P(6) &= \frac{2}{7} \\
P(5) &= \frac{10}{42} \\
P(4) &= \frac{8}{42} \\
P(3) &= \frac{6}{42} \\
P(2) &= \frac{4}{42} \\
P(1) &= \frac{2}{42}
\end{aligned}$$

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Problem 10 Statement. For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to these two bodies, 60 percent pass the House, 80 percent pass the Senate, and 90 percent pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.

Problem 10 .

The sample space would be

$$\Omega = \{TT, FF, FT, TF\},$$

where T represents passing and F represents not passing. We know that

$$P(A \cap B) = P(A) + P(B) - P(A \cup B).$$

Let A be the event that a bill passes in the House and B be the event that a bill passes in the senate. Then,

$$\begin{aligned}
P(A \cap B) =& P(A) + P(B) - P(A \cup B) \\
=& \frac{6}{10} + \frac{8}{10} - \frac{9}{10} \\
=& \frac{5}{10}
\end{aligned}$$

Problem 13 Statement. In a horse race, the odds that Romance will win are listed as 2 : 3 and that Downhill will win are 1 : 2. What odds should be given for the event that either Romance or Downhill wins?

Problem 13.

11 : 4

Problem 16 Statement. In a fierce battle not less than 70 percent of soldiers lost one eye, not less than 75 percent lost one ear, not less than 80 percent lost one hand, and not less than 85 percent lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand and one leg?

Problem 16.

Let A be the event of losing one eye, B be the event of losing one ear, C be the event of losing one hand, and D be the event of losing one leg. Then the probability we are looking for is

$$\begin{aligned}
P(A \cap B \cap C \cap D) =& 1 - P(\overline{A \cap B \cap C \cap D}) \\
=& 1 - P(\overline{A} \cup \overline{B} \cup \overline{C} \cup \overline{D}) \\
=& 1 - P(\overline{A}) + P(\overline{B}) + P(\overline{C}) + P(\overline{D}) \\
\geq& 1 - \frac{3}{10} - \frac{1}{4} - \frac{2}{10} - \frac{3}{20} \\
=& \frac{1}{10}
\end{aligned}$$

Problem 18 Statement. (a) for events $A_1, \ldots, A_n$, prove that

$$P(A_1 \cup \cdots \cup A_n) \leq P(A_1) + \cdots + P(A_n)$$

(b) For events A and B, prove that

$$P(A \cap B) \geq P(A) + P(B) - 1$$

Problem 18 (a).

Proof. Let $S = \{j \in \mathbb{N} \mid \forall A_1, \ldots, A_j \text{ events}, P(A_1 \cup \cdots \cup A_j) \leq P(A_1) + \cdots + P(A_j)\}$. We want that $S = \mathbb{N} \setminus \{1\}$.

Base Case: Given events A_1 and A_2 in the sample space, we want the theorem to hold true. Since $P(A_1 \cap A_2) \geq 0$, we have

$$\begin{aligned}P(A_1 \cup A_2) =& P(A_1) + P(A_2) - P(A_1 \cap A_2)\\ \leq& P(A_1) = P(A_2),\end{aligned}$$

it follow that $2 \in S$.

Inductive Step : By the principle of mathematical induction, it suffices to show that $\forall j \in S, j+1 \in S$. Given $j \in S$ and given $A_1, \ldots, A_{j+1}$ events, we want $j+1 \in S$. We have

$$\begin{aligned}P(A_1 \cup \cdots \cup A_{j+1}) &= P((\bigcup_{i=1}^{j} A_i) \cup A_{j+1})\\ &\leq P(\bigcup_{i=1}^{j} A_i) + P(A_{j+1})\\ &\leq P(A_1) + \cdots + P(A_{j+1}),\end{aligned}$$

as desired.

$\square$
