Selected Problems Chapter 1
Linear Algebra Done Right, Sheldon Axler, 3rd Edition

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Problem 1.A.2. Show that $\frac{-1+\sqrt{3}i}{2}$ is a cube root of 1 (meaning that its cube equals 1.)

Proof. We can use the definition of complex multiplication :

$$\begin{aligned}\left(\frac{-1+\sqrt{3}i}{2}\right)^3 &= \left(\frac{-1+\sqrt{3}i}{2}\right)^2\left(\frac{-1+\sqrt{3}i}{2}\right)\\ &= \left(\frac{-1-\sqrt{3}i}{2}\right)\left(\frac{-1+\sqrt{3}i}{2}\right)\\ &= \frac{1}{4}+\frac{-\sqrt{3}i}{2}+\frac{\sqrt{3}i}{2}+\frac{3}{4}\\ &= 1\end{aligned}$$

□

Problem 1.A.3. Find two distinct roots of i.

Let $z = (a+bi)$ be some root of i. We have :

$$z^2 = (a+bi)^2 = a^2 - b^2 + 2abi = i$$

Since i has no real component, this means that $a^2 - b^2 = 0$. Also, since the coefficient of i is 1, $2ab = 1$, which also means that a, b must have the same sign. Thus, $a = b$, and

$$\begin{aligned}2ab = 2a^2 &= 1\\ a^2 &= \frac{1}{2}\\ a = b &= \pm\frac{1}{\sqrt{2}}\end{aligned}$$

,

so the two solutions are $z = (\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})$ and $z = (-\frac{1}{\sqrt{2}} + \frac{-i}{\sqrt{2}})$.