Selected Problems Chapter 1
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Problem 8 (a).

Proof. We will prove this by contradiction. Suppose the n^{th} root of k is rational. Choose $p, q \in \mathbb{Z}$, where $q \neq 0$, such that the n^{th} root $r = \frac{p}{q}$. Then $k = r^n = \frac{p^n}{q^n}$. Since k is an integer, q must divide p. This r is an integer, and therefore k is a perfect n^{th} root, a contradiction. $\square$

Problem 8 (b).

A natural number is either a perfect n^{th} root or it is not. If it is not a perfect n^{th} root, By (a), we know the n^{th} root must be irrational. If it is a perfect n^{th} root, by definition the n^{th} root must be a an integer.
