Selected Problems Chapter 3
Introduction to Probability for Data Science
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Problem 3.2. Two dice are tossed. Let X be the absolute difference in the number of dots facing up.

    a. Find and plot the PMF of X.
    b. Find the probability $X \leq 2$.
    c. Find $\mathbb{E}[X]$ and $Var[X]$.

    Part a. The possible random variable states are $X(\Omega) = \{0, 1, 2, 3, 4, 5\}$. The probability mass function for this random variable is

$$p_X(0) = P(\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}) = \frac{1}{6}$$

$$p_X(1) = P(\{(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5),(5,4),(5,6),(6,5)\}) = \frac{5}{18}$$

$$p_X(2) = P(\{(1,3),(3,1),(2,4),(4,2),(3,5),(5,3),(4,6),(6,4)\}) = \frac{2}{9}$$

$$p_X(3) = P(\{(1,4),(4,1),(2,5),(5,2),(3,6),(6,3)) = \frac{1}{6}$$

$$p_X(4) = P(\{(1,5),(5,1),(2,6),(6,2)) = \frac{1}{9}$$

$$p_X(5) = P(\{(1,6),(6,1)) = \frac{1}{18}$$

Part b.

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= \frac{2}{3}$$

Part c.

$$\mathbb{E}[X] = 0 * \frac{1}{6} + 1 * \frac{5}{8} + 2 * \frac{2}{9} + 3 * \frac{1}{6} + 4 * \frac{1}{9} + 5 * \frac{1}{18}$$
$$= \frac{35}{18}$$

$$Var[X] = \mathbb{E}[X^2] - \mathbb{E}[X]$$
$$= \mathbb{E}[X^2] - \frac{35}{18}$$
$$= 0^2 * \frac{1}{6} + 1^2 * \frac{5}{8} + 2^2 * \frac{2}{9} + 3^2 * \frac{1}{6} + 4^2 * \frac{1}{9} + 5^2 * \frac{1}{18} - \frac{35}{18}$$
$$= \frac{665}{324}$$