Selected Problems Chapter 2
Introduction to Probability for Data Science
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March 30, 2022

Problem 2.1. A space S and three of its subsets are given by $S = \{1, 3, 5, 7, 9, 11\}$, $A = \{1, 3, 5\}$, $B = \{7, 9, 11\}$, and $C = \{1, 3, 9, 11\}$. Find $A \cap B \cap C$, $A^c \cap B$, $A - C$, and $(A - B) \cup B$.

1. $A \cap B \cap C = \{\}$
2. $A^c \cap B = B$
3. $A - C = \{5\}$
4. $(A - B) \cup B = S$

Problem 2.3. Simplify the following sets.
(a). $[1, 4] \cap ([0, 2] \cup [3, 5])$
(b). $([0, 1] \cup [2, 3])^c$
(c). $\bigcap_{i=1}^{\infty} (\frac{-1}{n}, \frac{1}{n})$
(d). $\bigcup_{i=1}^{\infty} [5, 8 - \frac{1}{2^n}]$

(a). $[1, 2] \cup [3, 4]$
(b). $(-\infty, 0) \cup (1, 2) \cup (3, \infty)$
(c). $\{0\}$
(d). $[5, 8)$

Problem Theorem 2.5 Part 2. Prove that $(A \cup B)^c = A^c \cap B^c$.

Proof. We'll prove this with a series of equivalences:

$$\begin{aligned} x \in (A \cup B)^c &\iff x \notin A \cup B \\ &\iff x \notin A \text{ and } x \notin B \\ &\iff x \in A^c \text{ and } x \in B^c \\ &\iff x \in A^c \cap B^c \\ &\iff x \in A^c \cap B^c \end{aligned}$$

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