(b) $\begin{bmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$$P(\lambda) = \det(\lambda I - A)$$

$$= \left| \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -7 \\ 0 & 0 & 1 & 2 \end{bmatrix} \right|$$

$$= \begin{vmatrix} \lambda-10 & 9 & 0 & 0 \\ -4 & \lambda+2 & 0 & 0 \\ 0 & 0 & \lambda+2 & 7 \\ 0 & 0 & -1 & \lambda-2 \end{vmatrix}$$

$$= \lambda-10 \begin{vmatrix} \lambda+2 & 0 & 0 \\ 0 & \lambda+2 & 7 \\ 0 & -1 & \lambda-2 \end{vmatrix} - 9 \begin{vmatrix} -4 & 0 & 0 \\ 0 & \lambda+2 & 7 \\ 0 & -1 & \lambda-2 \end{vmatrix}$$

$$+0-0$$

$$= \lambda-10\left[\lambda+2(\lambda+2)(\lambda-2)+7\right] - 9\left[-4(\lambda+2)(\lambda-2)+7\right]$$

$$= \lambda-10\left[\lambda^3-4\lambda+7\lambda+2\lambda^2-8+14\right] - 9\left[-4\lambda^2+16-28\right]$$

$$= \lambda^4+3\lambda^2+2\lambda^3+6\lambda-10\lambda^3-30\lambda-20\lambda^2-60+36\lambda^2$$
$$+108=0.$$

(15)

$$= 1\begin{vmatrix} 1 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} - 0 + 2\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{vmatrix} - 0$$

$$= 1[1(0) + 1(0) + 0] + 2[-1(0) - 1(0)]$$

$$= 1(0) + 2(0) = 0$$

If a matrix having all 0 in row or column so it's determinant is always equals to zero.

$$\lambda[\lambda(\lambda+2)(\lambda-1) - 0] + 1[-\lambda + 1 - 0] + 1$$

$$\lambda[\lambda(\lambda^2 - \lambda + 2\lambda - 2)] - \lambda + 1]$$

$$\lambda[\lambda^3 - \lambda^2 + 2\lambda^2 - 2\lambda - \lambda + 1$$

$$\lambda^4 - \lambda^3 + 2\lambda^3 - 2\lambda^2 - \lambda^2 + \lambda$$

$$\boxed{P(\lambda) = \lambda^4 - \lambda^3 + 2\lambda^3 - 3\lambda^2 + \lambda}$$

(7) Find characteristic equation of following matrix

(a) $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Sol:

As we know that

$P(\lambda) = (\lambda I - A) \det$

So,

$$\left| \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right|$$

$$= \begin{vmatrix} \lambda-1 & 0 & -2 & 0 \\ -1 & \lambda-1 & -1 & 0 \\ 0 & -1 & \lambda-2 & 0 \\ 0 & 0 & 0 & \lambda-1 \end{vmatrix}$$

(e) $\begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$

$\lambda = 2$

As we know that...

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$$

$$= \text{Null space: } \begin{bmatrix} -3 & 0 & -1 \\ -1 & 1 & 0 \\ 7 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$E_1: -3x_1 - x_3 = 0$

$E_2: -x_1 + x_2 = 0$

$E_3: 7x_1 - x_2 = 0$

$x_2 = 7x_1$

Eq(1) $\Rightarrow -x_1 + 7x_1 = 0$

$6x_1 = 0$

$x_1 = 0$

$x_2 = 0$

$$\text{Basis} = \begin{bmatrix} -1/3 \\ 1/3 \\ -1/3 \\ 1 \end{bmatrix}$$

(c) $\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$

$\lambda = -8$

$\lambda I - A$

$= \begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$

$= \begin{bmatrix} -6 & 0 & -1 \\ 6 & 6 & 0 \\ -19 & -5 & -4 \end{bmatrix}$

Null space $\begin{bmatrix} -6 & 0 & -1 \\ 6 & 6 & 0 \\ 19 & 5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Basis : $\begin{bmatrix} -1/6 \\ -1/6 \\ 1 \end{bmatrix}$ Ans

(11)

(iii) $\lambda_3 = 3$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

Null space $= \begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$E_1 : -x_1 - x_3 = 0$

$E_2 : 2x_1 + 2x_2 = 0$

$E_3 : 2x_1 + 2x_3 = 0$

Basis : $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

Basis $\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right)$ Ans.

6) Find bases for eigenspace of matrix.

(a) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

Soln:- (1) $\lambda_1 = 1$

$\because \lambda I - A$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

Null space $\begin{bmatrix} -4 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

$E_1 : -4x_1 - x_2$
$E_2 = 2x_2 = 0$
$x_2 = 0$
$E_1 : -4x_1 - 0$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(2) $\lambda_2 = 2$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & -1 \\ -2 & 1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Null space $\begin{bmatrix} -2 & 0 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

E_1: $-2x_1 + x_3 = 0$
E_2: $2x_1 + x_2 = 0$
E_3: $2x_1 + 3x_3 = 0$
Add eq$_1$ & eq$_2$:

$$\begin{aligned} -\cancel{2x_1} + x_3 &= \\ \cancel{2x_1} + 3x_3 &= \\ \hline -2x_3 &= 0 \\ x_3 &= 0 \end{aligned}$$

basis: $\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$

$$-(\lambda^3+6\lambda^2+12\lambda+8)=0$$

$$\lambda^3+6\lambda^2+12\lambda+8=0$$

$$\lambda(\lambda^2+6\lambda+12)=8 \qquad \pm1, \pm2, \pm4, \pm8$$

$$\begin{array}{r|l} & \lambda^2-4\lambda+4 \\ \hline \lambda-2 & \cancel{\lambda^3}-6\lambda^2+12\lambda-8 \\ & -\lambda^3 \mp 2\lambda^2 \\ \hline & -4\cancel{\lambda^2}+12\lambda-8 \\ & -\cancel{4}\lambda^2 \pm 8\lambda \\ \hline & 4\lambda-\cancel{8} \\ & 4\lambda-8 \\ \hline & 0 \end{array}$$

$$(\lambda-2)(\lambda^2-4\lambda+4)=0$$

$$(\lambda-2)[(\lambda-2)(\lambda-2)]=0$$

$$[\lambda_1=2,\ \lambda_2=2,\ \lambda_3=2].$$

(6).

(b) $\begin{bmatrix} 3 & 0 & -5 \\ 1/5 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$

$$-(\lambda^3 + 6\lambda^2 - 6\lambda - 4) = 0$$

$$\lambda(\lambda^2 + 6\lambda - 6 - 4) = 0$$

$$\lambda(\lambda^2 + 6\lambda - 10) = 0$$

~~x~~ Applying quadratic equaction

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-6 \pm \sqrt{6^2 - 4(1)(-10)}}{2(6)}$$

$$\lambda = \frac{-6 + \sqrt{36 + 40}}{12} \Rightarrow \frac{-6 + \sqrt{76}}{2}$$

$$\lambda = \frac{-6 + 2\sqrt{19}}{2} \Rightarrow \frac{\cancel{2}(-3 + \sqrt{19})}{\cancel{2}}$$

$$[\lambda_1 = -3 + \sqrt{19},\ \lambda_2 = -3 - \sqrt{19},\ \lambda_3 = 0]$$

Ans

(7)

(c) $\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 48 & 5 & -4 \end{bmatrix}$

$-(\lambda^3+8\lambda^2-8\lambda+20)=0$

$\lambda^3+8\lambda^2-8\lambda+20=0$

$\lambda(\lambda^2+8\lambda-8)=-20$

$\lambda(\lambda^2+8\lambda-8)=-20$

$\lambda_3=-20$

$\lambda=\dfrac{-8\pm\sqrt{8^2-4(1)(-8)}}{2(1)}$

$\lambda=\dfrac{-8\pm\sqrt{64+32}}{2} \Rightarrow \dfrac{-8\pm\sqrt{96}}{2}$

$\lambda=\dfrac{-8\pm4\sqrt{6}}{2} \Rightarrow \dfrac{\cancel{2}(-4\pm2\sqrt{6})}{\cancel{2}}$

$\left[\lambda_{1,2}=\dfrac{-4\pm2\sqrt{6}}{2}, \lambda_3=-20\right]$

Ans

5. Find the eigenvalues of the matrices.

a) $$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

From Q#4 we get

$$\cancel{P(\lambda)} \quad 0 = -(\lambda^3 - 6\lambda^2 + 11\lambda - 6)$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\lambda(\lambda^2 - 6\lambda + 11 - 6) = 0$$

$$\lambda(\lambda^2 - 6\lambda + 5) = 0$$

$$\lambda(\lambda - 5)(\lambda - 1) = 0$$

$\lambda_1 = 5$
$\lambda_2 = 1$
$\lambda_3 = 0$

Ans

(5)

(4)

(c) $\begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$

$P(\lambda) = \det(A - \lambda I)$

$= \begin{vmatrix} 5-\lambda & 0 & 1 \\ 1 & 1-\lambda & 0 \\ -7 & 1 & -\lambda \end{vmatrix}$

$= 5-\lambda\left[(-\lambda+\lambda^2)\right] - 0 + 1\left[1+7(1-\lambda)\right]$

$= 5-\lambda[\lambda^2-\lambda] + 7 - 7\lambda + 1$

$= 5\lambda^2 - 5\lambda - \lambda^3 + \lambda^2 + 7 - 7\lambda + 1$

$= -\lambda^3 + 6\lambda^2 - 12\lambda + 8$

$= \boxed{P(\lambda) = -(\lambda^3 + 6\lambda^2 - 12\lambda + 8)}$

(b) $\begin{bmatrix} 3 & 0 & -5 \\ 1/5 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$

$$\therefore P(\lambda) = \det(A - \lambda I)$$

$$= \left| \begin{bmatrix} 3 & 0 & -5 \\ 1/5 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 3-\lambda & 0 & -5 \\ 1/5 & -1-\lambda & 0 \\ 1 & 1 & -2-\lambda \end{vmatrix}$$

$$= 3-\lambda\left[(-1-\lambda)(-2-\lambda)\right] - 0 + (-5)\left[\frac{1}{5} - (-1-\lambda)\right]$$

$$= 3-\lambda\left[2+\lambda+2\lambda+\lambda^2\right] - 5\left[\frac{1+5+5\lambda}{5}\right]$$

$$= \left[6 - 2\lambda + 9\lambda - 9\lambda^2 + 3\lambda^2 - \lambda^3\right] - \left[\frac{5+5+5\lambda}{5}\right]$$

$$= \frac{5\left[-\lambda^3 - 6\lambda^2 + 7\lambda + 6\right] - (10+5\lambda)}{5}$$

$$= \frac{\cancel{5}\left[\left[-\lambda^3 - 6\lambda^2 + 7\lambda + 6\right] - \cancel{5}(2+\lambda)\right]}{\cancel{5}}$$

$$= -\lambda^3 - 6\lambda^2 + 7\lambda + 6 - 2 - \lambda$$

$$\therefore \boxed{P(\lambda) = -\lambda^3 - 6\lambda^2 + 6\lambda + 4}$$

(3)

(c) $\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$

$= \det(A - \lambda I)$

$= \left| \begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$

$= \begin{vmatrix} -2-\lambda & 0 & 1 \\ -6 & -2-\lambda & 0 \\ 19 & 5 & -4-\lambda \end{vmatrix}$

$= -2-\lambda\left[(-2-\lambda)(-4-\lambda)\right] - 0 + 1\left[-30 - 19(-2-\lambda)\right]$

$= -2-\lambda\left[8+2\lambda+4\lambda+\lambda^2\right] + \left[-30+38+19\lambda\right]$

$= -2-\lambda\left[\lambda^2+6\lambda+8\right] + \left[19\lambda+8\right]$

$= -2\lambda^2 - \lambda^3 - 12 - 6\lambda^2 - 16 - 8\lambda + 19\lambda + 8$

$= -\lambda^3 - 8\lambda^2 + 8\lambda - 20$

$= \boxed{P(\lambda) = -(\lambda^3 + 8\lambda^2 - 8\lambda + 20)}$

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EXERCISE # 7.1:-

4) Find the characteristic equation

a $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$$P(\lambda) = \det(A - \lambda I)$$

$$= \left| \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{vmatrix}$$

$$= 4-\lambda\left((1-\lambda)*(1-\lambda)\right) - 0 + 1(0 + 2 - 2\lambda)$$

$$= 4-\lambda(1-\lambda-\lambda+\lambda^2) + 2 - 2\lambda$$

$$= 4-\lambda(\lambda^2 - 2\lambda + 1) + 2 - 2\lambda$$

$$= 4\lambda^2 - \lambda^3 - 8\lambda + 2\lambda^2 + 4 - \lambda + 2 - 2\lambda$$

$$= -\lambda^3 + 6\lambda^2 - 10\lambda - \lambda + 6$$

$$= \boxed{-(\lambda^3 - 6\lambda^2 + 11\lambda - 6).}$$ Ans