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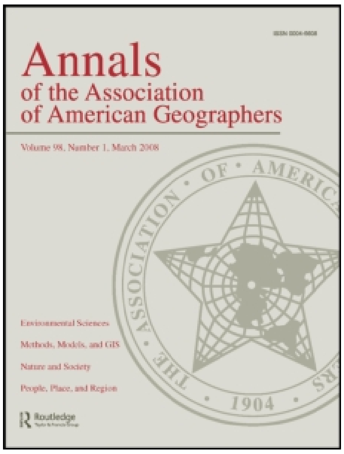

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Choropleth maps are commonly used to show statistical variation among map enumeration units. Mapmakers take into account numerous considerations and make many decisions to produce a product that will effectively communicate spatially complex information to the map user. One design consideration is the choice between classed or unclassed choropleth maps. Unclassed maps assign a unique color, shade, or pattern based on each unit's value. These maps are rich in information but might not be optimal for visual discrimination of regions or identifying values from a legend. Classed maps classify enumeration units based on unit values and in some cases consider geographic area per class or contiguity. These classed maps better delineate regions and interclass variation but are designed to eliminate visibility of intraclass variations. We present a method designed to use colors for choropleth classes and soft shadows to show intraclass variations associated with adjacent or nearby polygons. We conceptualize the choropleth data as a three-dimensional prism model under simulated illumination, with the height of each enumeration unit a function of its mapped value. Our user studies have demonstrated that participants were able to use soft shadows to better identify which of two adjacent units was of greater population density, regardless of whether units were in the same or different classes. Additionally, the resulting soft shadows rarely interfere with the map reader's ability to match color classes to a legend or to compare estimated differences in mean and variance of population density between two regions. *Key Words: cartography, choropleth maps, class intervals, illumination models, shading, shadowing.*

等值区域图通常用于显示地图中列举单位背景下的统计变化。制图者考虑到众多因素，作出诸多选择来生成一种产品，使之能有效地向地图用户传递复杂的空间信息。其中的一个设计考虑是选择分类图或者选择不分类的等值区域图。基于每一个单位的数值，不分类的地图为其分配一个独特的颜色、阴影、或模式。这种地图信息丰富，但对于视觉识别区域或者基于图例识别数值未必是最佳的选择。分类地图基于单位数值进行列举单位的划分，在某些情况下会考虑到每类或邻接的地理区域。这些分类地图可以更好地划分区域和标识类间的变化，但是在设计上，类内变化的可视性是不可得的。我们提出了一种新的方法，使用颜色来表现等值类别，使用软阴影来显示与相邻或附近的多边形相关联的类别内变化。我们将等值数据概念化为一个模拟照明下的三维棱镜模式，使每个列举单位的高度是其所对应数值的一个函数映射。我们的用户研究表明，参加者可以使用软阴影更好地确定哪两个相邻的单位具有更高的人口密度，无论这两个单位是否是在同一个分类类别。此外，该方法所产生的软阴影很少会影响到地图读者根据图例识别颜色类别的能力，或者是比较两个地区人口密度均值和方差估算差异的能力。*关键词：制图，等值区域图，类别间隔，光照模型，着色，阴影。*

Los mapas de coropletas se utilizan comúnmente para mostrar la variación estadística entre unidades cartográficas de enumeración. Quienes hacen los mapas toman en cuenta numerosas consideraciones y adoptan no pocas decisiones para lograr un producto que efectivamente comunique al usuario del mapa información espacialmente compleja. Una consideración de diseño es la escogencia entre mapas de coropletas clasificados o no clasificados. Los mapas no clasificados asignan un color, matiz o patrón único con base en el valor de cada unidad. Estos mapas son ricos en información pero podrían no ser lo mejor para la discriminación visual de regiones o para identificar valor a partir de una leyenda. Los mapas clasificados hacen la clasificación de las unidades de información con base en valores de la unidad y en algunos casos consideran el área geográfica por clase o por contigüidad. Estos mapas clasificados delinean mejor las regiones y la variación entre clases pero se diseñan para eliminar la visibilidad de las variaciones dentro de la clase. Nosotros proponemos un método diseñado para usar colores para clases de coropletas y sombreados suaves, con el fin de mostrar variaciones dentro de las clases asociadas con los polígonos adyacentes o cercanos. Conceptualizamos los datos coropléticos como un modelo de prisma tridimensional bajo iluminación simulada, en la que la altura de cada unidad de enumeración es una función de su valor mapeado. Nuestros estudios sobre usuarios de mapas han demostrado que los participantes utilizaron sin dificultad los sombreados suaves para identificar más fácilmente cuál entre dos unidades adyacentes tenía una mayor densidad de población, sin consideración a que las unidades fueran de la misma o diferente clase. Adicionalmente, los sombrados suaves resultantes raramente interfieren con la habilidad del lector del mapa para

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equiparar el color de las clases con una leyenda o de comparar las diferencias estimadas en media y varianza de la densidad población entre dos regiones. *Palabras clave: cartografía, mapas de coropletas, intervalos de clase, modelos de iluminación, matizado, sombreado.*

Choropleth maps show the variation in quantitative data among enumeration units such as countries, states, or counties (Robinson et al. 1995; Slocum et al. 2008). The variation throughout the mapped area is displayed using such visual variables as hue, spacing, or lightness (Slocum et al. 2008). In this sense, such maps are geographic graphs, or spatially arranged displays of statistical data (Robinson et al. 1995).

The utility of a choropleth map to the map user depends on a number of factors, both within and beyond the control of the cartographer. These include, but are not limited to, the geographic complexity of the phenomenon to be mapped, the decision to present data as either classed or unclassed, the method by which data will be symbolized, and the ability of the map user to interpret the resulting map. In our research, we focus on combining symbology from classed and unclassed data. We have devised a method to create illuminated choropleth maps in which soft shadows from an unclassed data model are used to add detail to classed choropleth maps symbolized by variations in hue–saturation combinations. We have conducted user studies demonstrating that this technique significantly improves the map reader's ability to identify local variations between adjacent enumeration units. Additionally, our illuminated choropleth maps do not generally interfere with the map user's ability to match map colors to a legend or to make regional comparisons of mean or variance between predefined geographic divisions.

Literature Review

The cartographic research of greatest relevance to our method can be categorized into two themes. The first focuses on the map user's ability to interpret map variations based on changes in choropleth symbols. This includes users outlining regions (e.g., Muller 1979; Mak and Coulson 1991); ranking map or region complexity (e.g., Olson 1975); answering questions related to maps, regions, or particular units (e.g., Olson 1975; Chang 1978; Brewer et al. 1997; Olson and Brewer 1997; MacEachren, Brewer, and Pickle 1998; Brewer and Pickle 2002); matching unit symbology to legends (e.g., Mak and Coulson 1991; Brewer and Pickle 2002); and discriminating among units (e.g., Mak and Coulson 1991).

The second theme, inextricably tied to the first, focuses on techniques designed to optimize the mapmaker's ability to display areas within regions as homogeneous and other areas as heterogeneous (e.g., Berry 1968; Dent 1999). This had led to an extensive discussion in the literature about how to best symbolize choropleth map units. Although statistical data were classified for map display since shortly after the first unclassed choropleth map appeared in 1826 (Robinson 1982), they only became a focus of study when Jenks and Caspall (1971) introduced methods to optimize the values of class breaks. Shortly thereafter, Tobler (1973) suggested assigning unique symbols to each enumeration unit to create unclassed choropleth maps.

Classed Choropleth Maps

Jenks and Caspall (1971) conceptualized the problem of finding optimal breaks as a series of three-dimensional (3D) models. A classed choropleth map would have a 3D model that assigns the height of each class to the average value within the class. In contrast, for an unclassed choropleth map the 3D model would assign height using each enumeration unit's value. Given this construct, the difference between the class mean and all enumeration unit values in the class can be measured and summed in various ways.

Jenks and Caspall summarized variations from the class mean using three different "error" metrics: tabular error, overview error, and boundary error. Minimizing tabular error is the method described in the current literature as the "optimal" method (Robinson et al. 1995; Slocum et al. 2008). It is based on the statistical research of Fisher (1958). Jenks and Caspall (1971) were the first to apply this research to defining class boundaries, an important basis on which much future research was built. Jenks (1977) detailed the method of iteratively finding class breaks so that within-class variations from the mean are minimized, looking only at the values and statistics associated with the tabular data and not using geographic data.

Jenks and Caspall's (1971) overview error takes into account geographic area by calculating a "volume" based on the 3D models. Each enumeration unit has an area and a "height" based on the difference between the class mean and its value. Variations of this volume can be minimized within each class in a manner similar to that described for tabular error. Although straightforward to implement using a geographic information system (GIS), most commercial algorithms do not account for overview error (Armstrong, Xiao, and Bennett 2003).

Other choropleth research focuses on issues related to optimally grouping polygons in the same class while still keeping tabular or overview error at a minimum. Monmonier (1972) was the first to tackle the issue of spatial clustering in creating clear and simple choropleth maps. Monmonier's concerns were justified later that decade by Chang (1978), who documented the preference of map readers for simpler, less fragmented map patterns. Monmonier (1972) used a taxonomic clustering algorithm to explicitly consider contiguity in selecting class intervals. He also included algorithms to balance statistical and geographical considerations. Olson (1975) used measures of spatial autocorrelation to analyze aspects of the overall look of classed maps. She found through user testing that concepts such as spatial complexity could be related to quantitative measures of autocorrelation.

Jenks and Caspall's (1971) third measure of error is boundary error. Returning to their 3D construct, ideal class boundaries would correspond to the enumeration unit borders with the largest changes in the mapped value. In other words, larger steps (or "cliffs") in the 3D enumeration unit model would ideally correspond to class boundaries. They stated the following: "The boundaries between shading on a choropleth map tend to dominate the visual impact of the representation, because sharp visual contrast occur along these lines. Map readers tend to assign significance to these boundaries and, as a result, often assume that they designate breaks in the configuration of the statistical surface" (229). Their strategy was to summarize and maximize variations in values at class breaks using various classifications. Varying class boundaries to address boundary error resulted in variations in their other two measures of error. They chose to give equal weights to all three error measures to define an optimal solution.

Their discussion of boundary error is important to our research for two reasons. First, Jenks and Caspall (1971) minimized boundary error to address a specific concern: the potential misinterpretation that large variations always occur at class boundaries and lesser or no variations always occur within classes. Second, the issue of minimizing boundary error is related to spatial contiguity but is not guaranteed to result in simpler, less fragmented map patterns as are the clustering methods discussed earlier. For example, a region might consist entirely of a single class as defined by tabular and overview error. If one enumeration unit is surrounded by units with lower value in that class, the central unit could be promoted to a new class on addition of boundary error into the classification process. Such results could yield a more complex, fragmented map pattern.

Recent research identifies additional criteria that can be used to define class breaks. Cromley (1996) used boundary error in a comparison of a number of classification methods. Armstrong, Xiao, and Bennett (2003) use a genetic algorithm that finds an optimal solution based on a number of criteria. They minimized measures of Jenks and Caspall's tabular error, aerial inequality among classes, and boundary error as defined by MacEachren's (1982) face complexity measure, as well as maximizing a reformulated Moran's I as a measure of spatial autocorrelation. They do not assign weights to these factors but rather find "Pareto optimal" solutions, ensuring that one criterion does not dominate another. The applications of evolutionary algorithms for comparing choropleth maps are discussed by Xiao and Armstrong (2005).

Unclassed Choropleth Maps

All class boundary research can be summarized as methods focused on finding the optimal (or set of optimal) class breaks based on one or more criteria. This research stems from the assumption that the values of a small number of class breaks is of vital importance. Other geographers suggest the possibility of diluting the importance of any particular class break by increasing their number, the logical limit being a different class for each value. This sort of map is referred to as an unclassed choropleth map. Tobler (1973) was the first to devise a method for creating such maps with a line plotter. Beginning with that article and comments of concern from Dobson (1973), the relative merits of classed versus unclassed choropleth maps have been the topic of much discussion.

Muller (1979) tested the ability of users to categorize areas of high, medium, and low density from unclassed choropleth maps of 1970 rural population density using the counties of Kentucky as enumeration units. Users closely replicated a choropleth map of the same data with three optimized classes. Muller (1979) argued that these results implied map users are able to identify regions from unclassed maps and that such maps offer the additional benefit of reproducing the data on which the map is based. Dobson (1980) argued that Muller's (1979) study focused on pattern delineation, without evaluation of more advanced map skills such as pattern memorization. Muller (1980) responded by underscoring the importance of recognizing map patterns and the suspect nature of class boundaries.

Results from more recent research underscore the complexity of issues involved in such comparisons. Gilmartin and Shelton (1989) found classed choropleth maps reduced map-processing time when compared to unclassed maps. Slocum et al. (2008) stated that classed maps are generally more effective than unclassed maps for the acquisition of specific information. They made the point that "The high accuracy of unclassed maps is, however, mathematical, not perceptual" (267). Nonetheless, this conclusion is based on maps with few classes (classed) versus maps with many classes (unclassed). Even then, results of such studies as Mersey (1990) and MacEachren (1982) are inconclusive about the effect of variation in the number of classes on some cognitive skills, such as the recall of specific information. In a non-user-based study, Cromley (2005) found more visual complexity of spatial patterns in classed versus unclassed maps by performing algebraic-to-graphic transformation lines to highlight the role of the maximum contrast principle.

Prism Maps

Our research focuses on adding more detailed information to classed choropleth maps using soft shadows from unclassed values. In these illuminated choropleth maps (Figure 1), the hue–saturation components of color of the units are based on the class. The shadows are based on an illumination model applied to a volumetric model of the enumeration units. In the volumetric model, each unit is extruded to a height based on the attribute being mapped. For a given illumination direction defined for the former model, the length of the shadows will be a function of the difference in values between adjacent units. These shadows act as a second unclassed visual variable, used with the intent of adding detail to classed choropleth maps in a manner that is perceptually intuitive.

Shadows are not defined as a visual variable, although they are often represented by changes in lightness. In the manner in which they vary, they are most similar to "perspective height," identified as a visual variable by Slocum et al. (2008). The perspective height variable extrudes area symbols into the third dimension based on value of some attribute. Such a "prism" display was used by Jenks and Caspall (1971) in Figure 10 of their seminal article (Figure 2A). In this example, shadows are not used to enhance the 3D effect; black areas represent sides of units. Also, Jenks and Caspall (1971) made no attempt to combine these extruded maps with classed choropleth maps such as their Figure 2 (in our Figure 2B) in a single display.

Such prism maps have the advantage of an excellent 3D effect. Interest in constructing such maps led to development of optical (Jenks and Brown 1966) and computer-automated (e.g., Franklin and Lewis 1978; Hilbert 1981) techniques. Prism maps continue to be popular today, especially in the mass media. Prism maps, however, also have disadvantages. Any map that is not planimetrically correct will have increased distortion in shape, size, distance, and direction. Additionally, some units might be hidden from view, and these are likely to change based on viewing direction. All of these factors might make typical choropleth map uses (such as identifying local or regional variations) more difficult.

Some of these issues were addressed by a method using stereoscopic vision to create two-dimensional choropleth maps with a true 3D appearance (Jensen 1978). Such maps overcome many of the issues of traditional prism maps. Users of such maps, however, will still be faced with the need for stereoscopic vision and the challenge of a limited field of view with such maps.

Our illuminated choropleth maps are planimetrically correct. Because hard shadows would obscure some units, we focus on creating *soft shadows* that vary the tone of class colors in a subtle manner. We use a clear-day illumination model so that maps match theoretical shading based on the distribution of light in the sky. In doing this, we are attempting to create a map display that is easily and intuitively visually interpreted. We realize that other methods, such as labeling population density values for each polygon, could provide even more information and can be an effective practice for maps with somewhat limited numbers of polygons, such as the states of the United States. We do not, however, feel that labeling would be a visually effective way to display maps with much more numerous polygons, such as our examples using the more than 3,000 counties of the conterminous United States. Finally, we conduct testing to ensure that users can correctly interpret shadows with respect to local variations, and that shadowing does not interfere with users' abilities to match unit colors to a legend or to make regional comparisons of mean and variance among large areas.

Our illuminated choropleth map, in its use of an attribute being displayed using multiple visual variables, shares similarities and has important differences when compared with traditional cartographic techniques. It

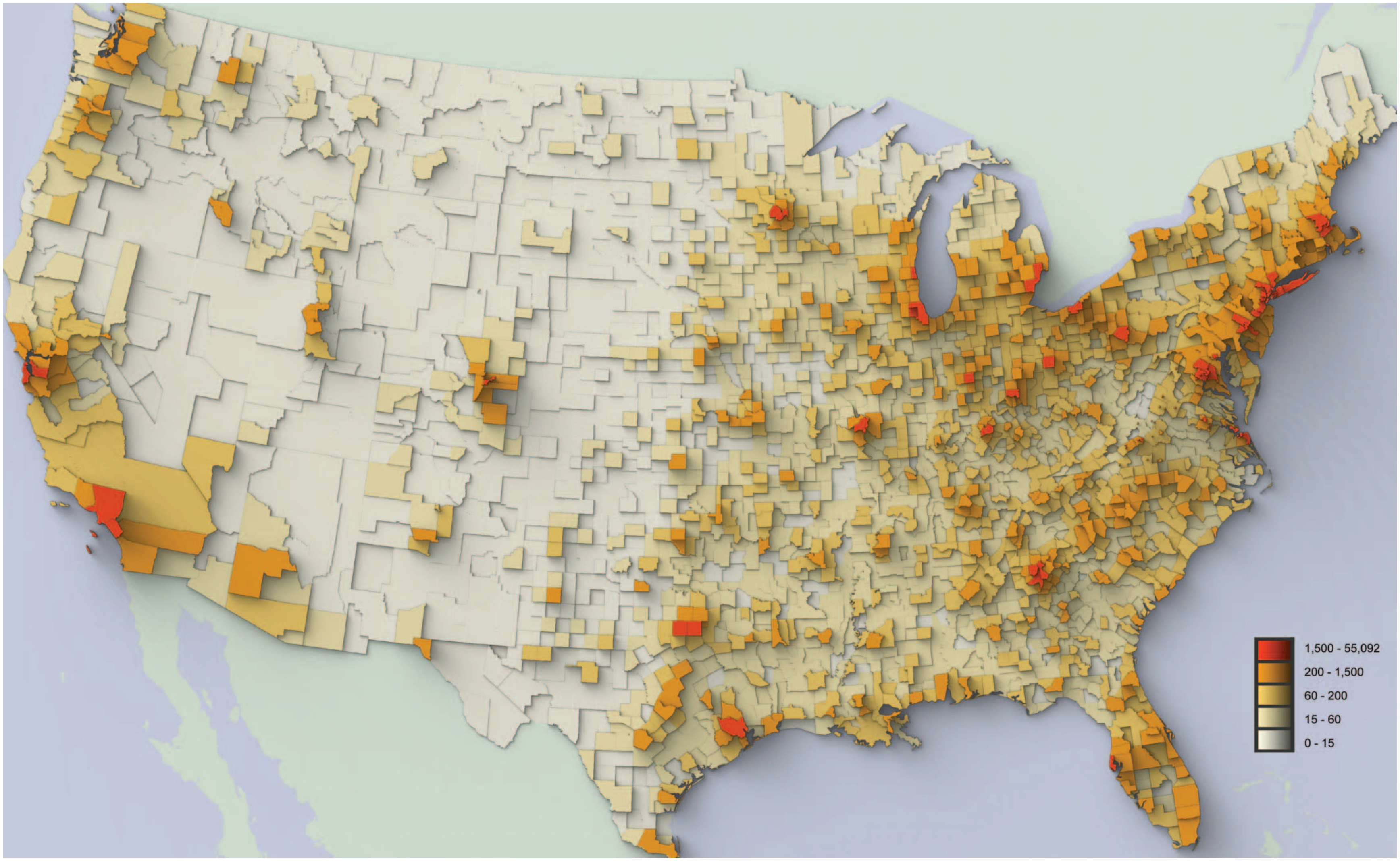

Figure 1. An illuminated choropleth map showing population density of counties in the conterminous United States.

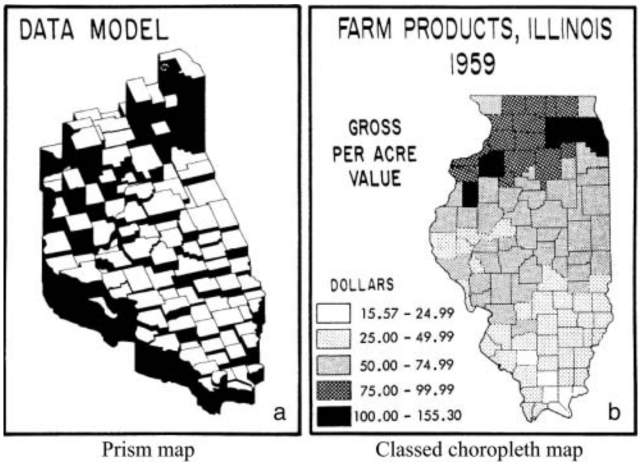

Figure 2. A comparison of a prism map with a classed choropleth map. *Source:* From Jenks and Caspall (1971). Used with permission from the Association of American Geographers.

is similar to a bivariate choropleth map, but our method maps only one attribute in two manners. It is also similar to maps of smoothly varying statistical surfaces such as topography that employ layer tinting with hill shading and shadowing.

The Bivariate Map Analogy

Bivariate choropleth maps combine the display of two attributes on the same map. Initial bivariate maps focused on creating a matrix of easily identified color classes. Olson (1981) concluded that, although not without issues, students did gain information from such maps and found them interesting and appealing. Eyton (1984) devised a method in which the center of the legend-matrix is gray, with corners on the diagonals comprising two complementary colors and black and white. Brewer (1994) devised color schemes that take into account unipolar or bipolar attributes.

Other studies used an unclassed approach to bivariate choropleth maps. Techniques focused on the use of cross-hatched lines, with the spacing of horizontal and vertical lines varying with the two attributes of interest (Carstensen 1982, 1986a, 1986b; Lavin and Archer 1984). The resulting maps display tonal variations as well as variations in the size, dimension, and orientation of individual rectangles.

Illuminated choropleth maps are not bivariate maps but display one attribute using two different visual variables. Although hue–saturation components of color are assigned based solely on, in our example, population density values, shadows will change depending on conditions in the neighborhood. For example, an enumeration unit might cast a very short shadow if its neighbor in the direction opposite the illumination direction is nearly as densely populated. A unit in the most densely populated class might cast a long shadow if its neighbors have significantly lower population densities. Long shadows are not necessarily limited to the adjacent neighbor, depending again on local conditions.

Our tests indicate that users are not generally bothered by these shadows in matching class colors to a legend or making regional evaluations of mean and variance. Results are consistent with other perceptual studies that indicate that users are able to see continuous patterns through shaded regions, even if the patterns are represented as shades of gray (Adelson 1993, 2000). We suggest that shadows on illuminated choropleth maps do not offer a perceptual challenge to users because they are based on a 3D model illuminated in a predictable manner.

The Topographic Map Analogy

Early researchers pointed out the similarity of choropleth and topographic maps. Jenks and Caspall (1971, 218) stressed the impression a choropleth map will have on the map reader: "First, he may seek an overview of the statistical distribution from the choropleth map, much as he obtains the 'lay of the land' from a topographic map." Monmonier (1972) endorsed symbology that helps to display choropleth maps simply and clearly. He drew analogies to the varying contour intervals and classes of hypsometric tints used to create topographic maps that appear spatially organized. Tobler (1973) drew an analogy between selecting larger class intervals to generalizing a topographic surface by, for example, choosing a large contour interval.

We use contour mapping to discuss some of the similarities and differences between topographic and choropleth maps. Contour maps are often displayed with elevation values assigned to hypsometric or layer tints in a manner similar to which colors are assigned to classed choropleth maps. Contour maps represent the surface by a series of lines representing intersections of the terrain with planes evenly spaced in elevation; choropleth maps represent statistical variations by enumeration units that may vary in any manner. The former is appropriate for representing a surface of smooth variation, the latter for a surface of irregular steps between otherwise flat surfaces (i.e., the tops of the prisms). Given such a construct, a prism map would have contours on its vertical faces (e.g., Franklin and Lewis 1978).

Cartographic techniques include other methods for mapping terrain, such as hill shading and associated shadowing. Hill shading generally uses a simple directional illumination model to vary the shade of gray of individual map units (Horn 1982; Imhof 1982). The shade of gray is determined by the angular difference between the direction of illumination and the surface normal. Hill shading was first automated by calculating shading values for a small grid (Yoeli 1965), relating these shades to the density of black dots on a white background (Yoeli 1966), and using a computer-controlled electronic typewriter to print and overprint characters to match desired hill shades (Yoeli 1967). Subsequent efforts led to use of special characters on a line printer (Brassel, Little, and Peucker 1974) and finally continuous shades of gray on gray tone plotters (Peucker, Tichenor, and Rase 1974). This type of hill shading would be ineffectual for choropleth maps, as all enumeration units have the same (horizontal) orientation.

The same directional illumination model can also be used in terrain mapping to define areas in shadow. Although shadows provide important visual cues to local relief, they have a poor reputation in cartography because of their tendency to obscure local details (Imhof 1982). Applying a simple directional illumination model to the data in this study, we get a map with dark, sharp, hard shadows obscuring more areas (Figure 3B). More sophisticated clear-sky illumination models allow units beneath soft shadows to cast their own shadows (Figure 3A), as diffuse illumination from other sectors of the sky is not obscured. This is evident comparing shadowing resulting from the clear-day (Figure 3A) and directional or point source (Figure 3B) illumination models, as no units in cast directional shadows are creating their own shadows.

Additionally, the illumination model can serve to shade flat units. This shading results from diffuse light distributed throughout the sky being partially blocked at certain locales by high areas in the prism model that are not in the line of sun illumination. An example of an urban elevation model with equal illumination from all directions shows such shading patterns (Kennelly and Stewart 2006). Flat tops of buildings are rendered in many shades of gray, depending on how much of the virtual sky is obscured by other buildings. The implication for illuminated choropleth maps is that units more obscured will be shaded slightly darker than surrounding units in the same class, even if they are not covered by an obvious shadow. An example of this effect is the relatively darker shade of some class colors in northeastern counties, which are not apparently beneath soft shadow.

In summary, illuminated choropleth maps and topographic maps with layer tinting and hill shading represent very different types of geographic phenomena, but similar shading methods can be used to represent them. In terrain maps, elevation layer tinting applies the same colors over continuous regions within specified ranges of values. These colors are modulated by a derivative map based on a directional illumination model that defines shading based on local relief. Our illuminated choropleth method applies the same colors over potentially less continuous, stepped surfaces within specified classes. These colors are modulated by a different derivative map based on a more sophisticated illumination model that defines soft shadowing based on attribute variations among adjacent or proximal polygons.

User Study

Methods

We began with U.S. Census Bureau county data for the conterminous United States in a GIS-based vector format. Data were taken from the last decennial report of 2000 and were projected into an Albers equal area map projection. In addition to counties, independent cities were included as county equivalents. We calculated population density in people per square mile for the 3,184 polygons representing 3,109 counties or county equivalents and converted data into a one-kilometer resolution raster format. Using this sample size, all polygons were represented by at least one grid cell. We used this grid as input for a custom application written in the C++ programming language to "illuminate" the choropleth map, as described later.

Population density values ranged from 0 to 55,092 individuals per square mile. We classified the data into five categories. One of the goals of our study was to evaluate the user's ability to discern randomly selected units of various class colors with soft shadows. The Jenks optimal method, however, did not lend itself to this sort of analysis. Using the Jenks method, for example, only four clustered county equivalents of relatively small size fall into the class of highest population density of more than 21,170 (the New York City boroughs of Manhattan, Brooklyn, the Bronx, and Queens; Figure 4).

We chose to define our own class breaks so that a greater number of counties or county equivalents of different classes would be distributed across the map.

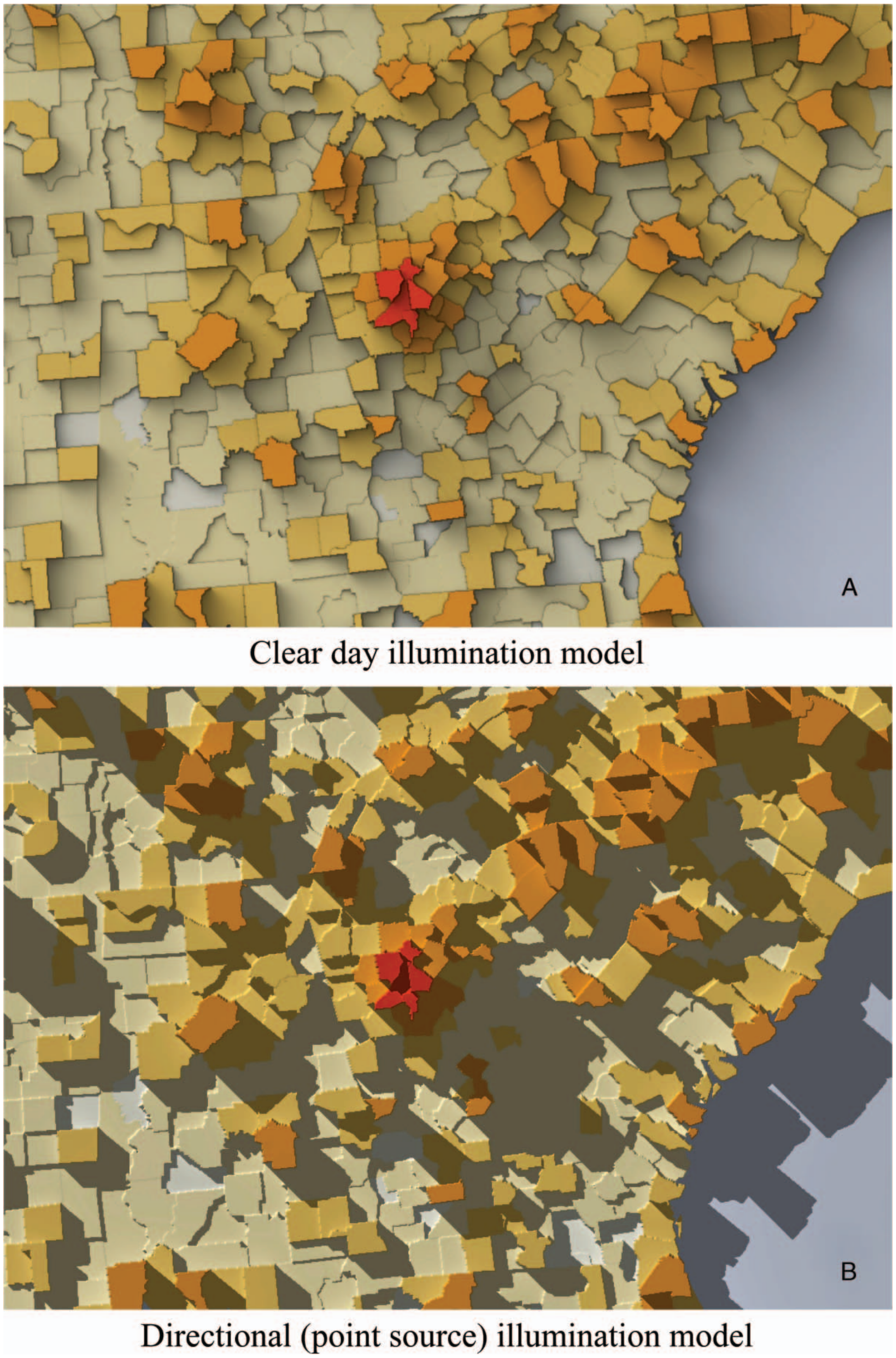

Figure 3. A comparison of illuminated choropleth maps shaded with (A) a clear day illumination model and (B) a directional (point source) illumination model.

Although similar to concerns of overview error discussed previously, we did not optimize reduction of this error, as our goal was not to balance volumes within each class but to make available more locales of classes that might be randomly sampled. Beginning with initial Jenks optimal breaks at 805, 3,166, 8,379, and 20,705, we adjusted class breaks to 15, 60, 200, and 1,500 (Figure 5).

We applied a sequential color scheme based primarily on changes in hue–saturation combinations from yellow through orange to red to these classes (Brewer, Hatchard, and Harrower 2003; Harrower and Brewer

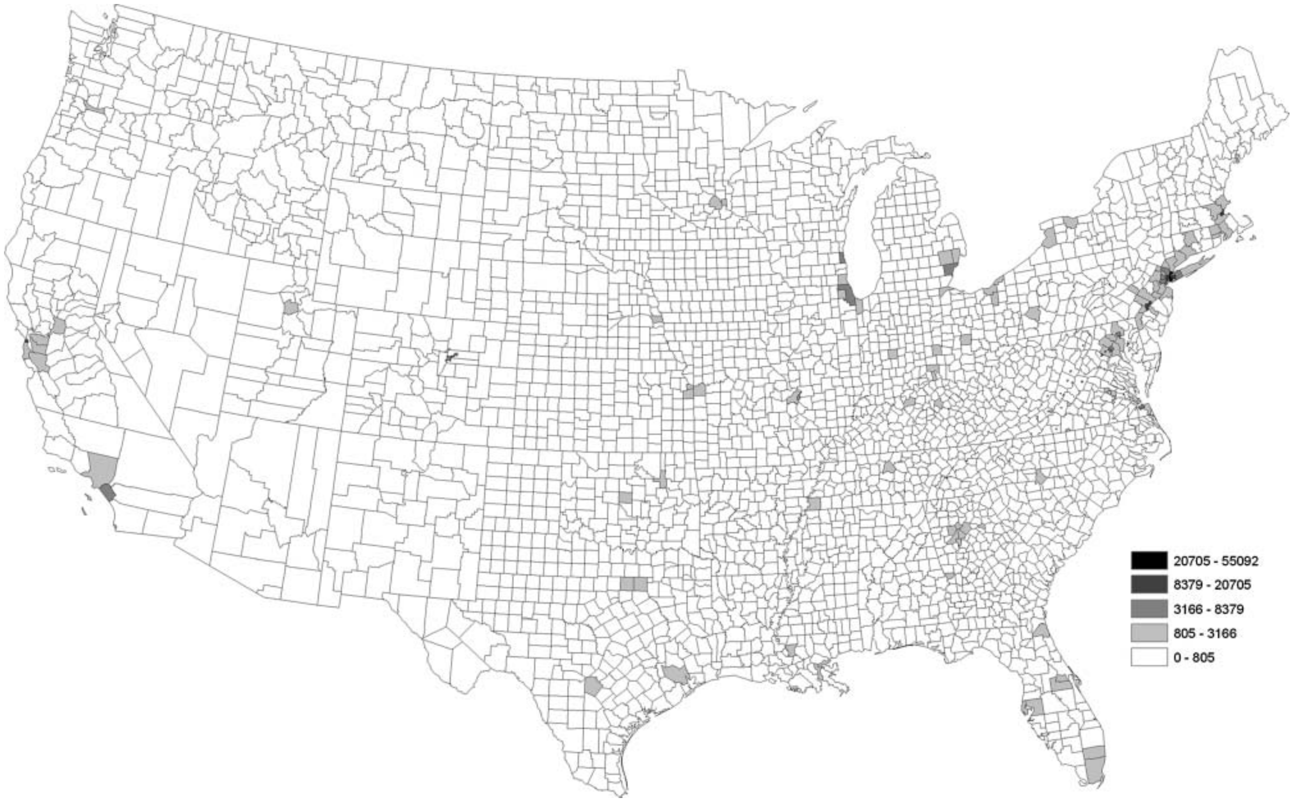

Figure 4. A classed population density map of counties in the conterminous United States using Jenks's optimal method.

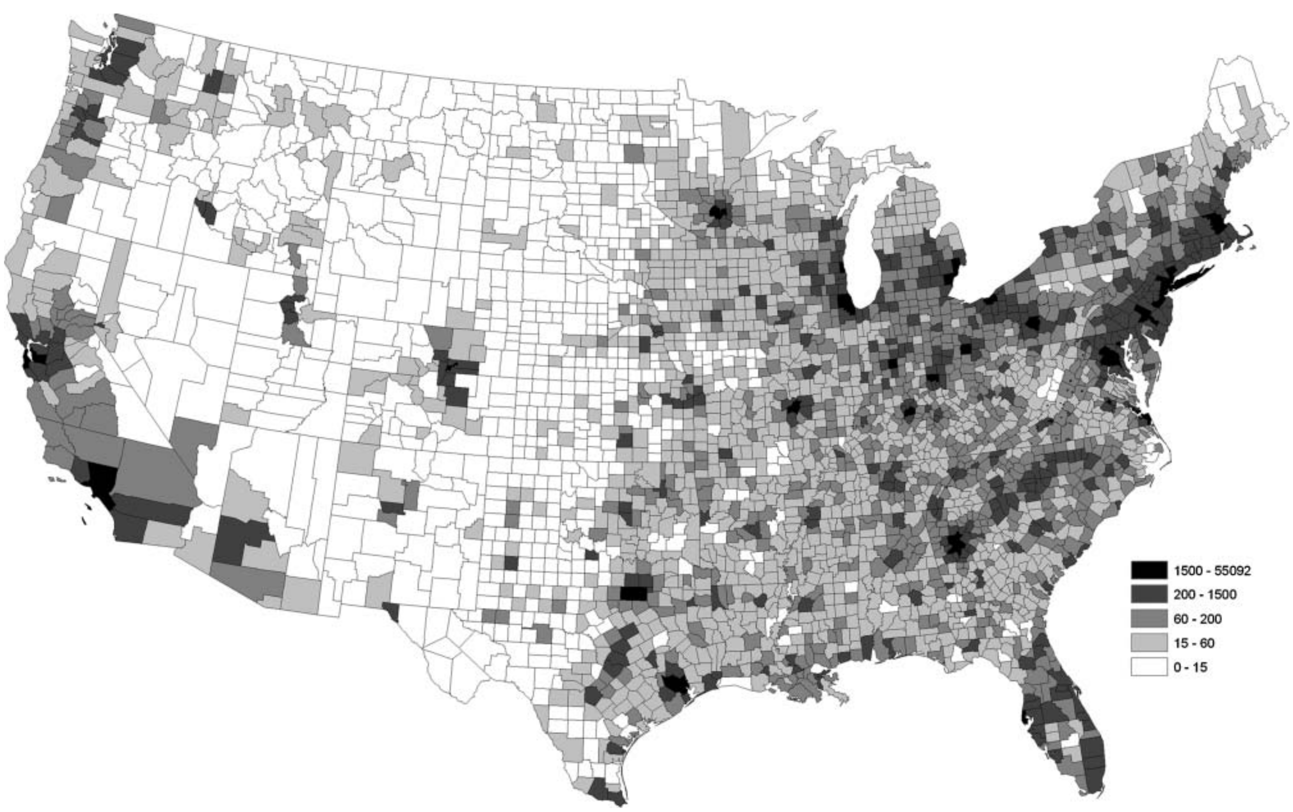

Figure 5. A classed population density map of counties in the conterminous United States using class breaks that provide a wider distribution of more densely populated classes.

2003; Brewer 2005). We opted to keep the lightness of all colors relatively high, with the value of the hue–saturation value specifications remaining above 97 percent. This would allow nearly the entire gamut of value variations as soft shadows to overprint the colors in the final color version of the map. It can be noted, however, that colors ranging around the color circle from yellow to red have very different luminosities (Brewer 1994; Kennelly and Kimerling 2004; Slocum et al. 2008). Whereas changing value of our light yellow from 0 percent to 100 percent varies luminosity by about 98 percent, doing the same to red only varies its luminosity by about 55 percent, as red is a less luminous color. Assigning red to our areas of highest population density minimizes issues associated with its decreased dynamic range of luminosity, as these units are more likely to cast shadows than be partially obscured by them.

Illumination Model

Many realistic sky models have been developed. The classic "overcast sky" of Moon and Spencer (1942) provides the sky radiance in a particular direction as

$$0.33\ L_Z(1 + 2\sin\theta) \tag{1}$$

where L_Z is the radiance at the zenith and θ is the angle between the particular direction and the horizontal. This model provides three times as much illumination at the zenith as at the horizon. The CIE Standard General Sky (Commission Internationale de l'Eclairage 2001) provides a much more elaborate formula in which five parameters can be set to model various skies, from clear to partly cloudy to overcast (Darula and Kittler 2002). The model we used for Figure 6 is a clear-day illumination model.

In our illuminated choropleth maps, the gridded data were treated as a surface, with the surface height at each point being a function of the population density at that point. The surface was illuminated with a clear-day sky in which most light arrived from all directions equally, with twenty-one times as much light arriving from the direction of the sun, which was placed at a 45-degree elevation above the horizon in the northwest.

Figure 6. An unclassed map showing a prism model of county population density, with heights normalized by an exponential function in Figure 7. The prism model is shaded according to a clear-day illumination model.

The radiance of the sky in a particular direction was

$$0.05 + \cos^{500}\theta \quad (2)$$

where θ is the angle between the particular sky direction and the sun. The cosine term, with its high exponent, ensured that the brightest light came from the direction of the sun and that the sun's contribution tapered to zero at about 7 degrees away from the sun's direction. This resulted in soft shadows at the base of "cliffs" on the surface (caused by the decrease in the amount of visible sky, and hence, total shadowing, at the cliff base) and a somewhat diffuse shadow cast from the sun by high parts of the surface.

The computation of the surface illumination was done using the method of Kennelly and Stewart (2006). Their method computes an approximation of the horizon at each grid point. Using the horizon, their method computes the illumination at a grid point by integrating the sky illumination over the sky area that is above the horizon and reflecting that light in proportion to the surface albedo (0.6 in our case). We scaled the computed surface illumination to the range 0 to 255 so that it could be represented compactly as a gridded map display.

Due to the discrete nature of the horizon approximation, long shadows can appear somewhat fanned out, especially in the vicinity of localized spikes in the surface. That effect can be reduced with greater computation time in the horizon algorithm (to produce a more accurate horizon), but the participants in our study did not report this to be distracting.

Our first attempts at illuminating the surface produced poor results. Most of the height differences between adjacent counties occurred in lower, less dense parts of the surface (particularly in the Midwestern states), and the scale of those differences was insignificant compared to the scale of the largest heights. Those very small differences were insufficient to cast shadows.

To enhance small height differences and, hence, to cast shadows in these areas, we transformed the heights with an exponential function that increased small heights more than it did high heights and resulted in the highest polygons having shadows of reasonable length on our map. We used the formula

$$H = 0.0025(D/D_{max})^{0.455}\ D_{max} \quad (3)$$

where H is the new "height" of the unit being scaled (measured in people per square mile), D_{max} is the maximum population density of all units, and D is the original population density of the particular unit being scaled. This transformation increases the subtle differences between the many units of low population density. At the same time, it suppresses the units of highest population density, those most likely to cast long shadows prone to artifacts of the method. These new heights were used only for the illumination model and the resulting shading. Classes were assigned based on the original, unadjusted values of population density. The height transformation function is shown as Figure 7.

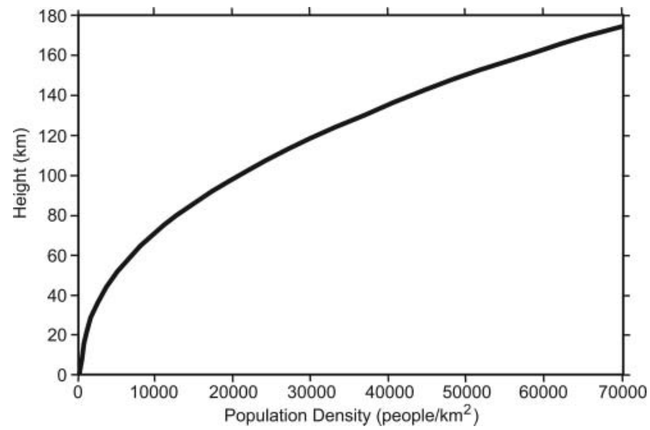

Figure 7. A graph of prism height versus population density, illustrating the exponential function used to create a more even distribution of population density values for illumination modeling.

User Testing

The illuminated choropleth method provides a fine-grained view of the data (Figure 6). Combining these soft shadows with classes symbolized by color reveals relative values within the same class, especially between adjacent polygons (Figure 1). The illuminated method also lets us see the underlying enumeration units within a class when those samples are sufficiently different to cause shadowing. These assumed benefits are balanced by potential disadvantages. First, the shadowing might obscure the class coloring. Second, the shadowing might confound the perception of aggregated characteristics, such as the mean or variance of a region.

We performed a formal user study to determine whether these advantages and potential disadvantages were statistically significant. We made the following hypotheses:

H1: The illuminated method improves a person's ability to determine which of two adjacent counties has a higher population density.
H2: The illuminated method does not affect a person's ability to determine the class of a county, given a map legend.
H3: The illuminated method does not affect a person's ability to determine which of two aggregate regions has a higher mean population density or higher variance in population density.

Test Cases

A computer program was written in C++ to draw on a computer screen a choropleth map of population densities in the conterminous United States, both with and without illumination. The program randomly selected sites (counties, pairs of adjacent counties, or U.S. Census Bureau divisions) zoomed to the area of interest, highlighted the sites of interest, recorded mouse clicks, and kept track of response time. The program operated in four modes, corresponding to four different tasks. For each task, a part of the map was shown shaded or unshaded and the subject was asked to click. The four tasks were as follows.

1. Pair selection task. Two adjacent counties were centered on the screen and highlighted with blue circles. The participant was asked to click on the county of higher population density. In some cases, the selected, adjacent units were in different classes (Figure 8). In other cases, the units were in the same class (Figure 9).
2. Legend matching task. A county was centered on the screen and highlighted with a blue circle. A legend of class colors was shown on the upper right corner of the screen. The participant was asked to click in the menu on the color that matched that of the highlighted county (Figure 10).
3. Region mean task. The conterminous United States was shown divided into the eleven U.S. Census Bureau divisions and spatially separated for visual clarity. Two randomly selected regions were highlighted, with all other regions assigned one shade of gray. Participants were asked to click on the region of greater mean population density (Figure 11).
4. Region variance task. The same method was used to display two divisions. Participants were asked to click on the region of greater variance. The presentation was identical to Figure 11.

Experimental Setup

We tested forty-one participants, who were faculty, graduate students, and staff in geography and computer science departments. Participants were divided into experienced and nonexperienced groups: We considered as experienced those individuals with some graduate experience in computer science graphic rendering with shading and those individuals with some graduate experience in geography with cartographic hill shading. Of the forty-one participants, sixteen were experienced and twenty-four were nonexperienced.

Each participant performed four groups of tasks, in this order:

1. Legend matching tasks for eighteen counties. Each county was shown once shaded and once unshaded for a total of thirty-six tasks.
2. Pair selection tasks for twelve pairs of adjacent counties from the same class and for twelve pairs of adjacent counties from different classes. Each pair was shown once shaded and once unshaded, for a total of forty-eight tasks.
3. Region mean tasks for fifteen pairs of regions, with each pair shown once shaded and once unshaded.
4. Region variance tasks for fifteen pairs of regions, with each pair shown once shaded and once unshaded.

Within each of these four groups of tasks, the same tasks (i.e., the same counties, county pairs, or region pairs) were used, but the order of the tasks and illuminated versus nonilluminated displays were randomized to reduce learning bias.

Before a participant's trial, the class colors were explained, as was the "higher is denser" representation used in the illumination method. Each participant was trained on four matching tasks, six pair selection tasks, four region mean tasks, and four region variance tasks. Variance was explained as being the degree to which densities varied from the mean. During the trial, the participant's responses were recorded for later analysis. Each trial took approximately twenty minutes. After the trial, the participants was asked for subjective evaluations of the illumination.

User Study Results

The two conditions under which each task was performed were "with illumination" and "without illumination." For each task, a participant's performance was defined as the fraction of correct responses made by the

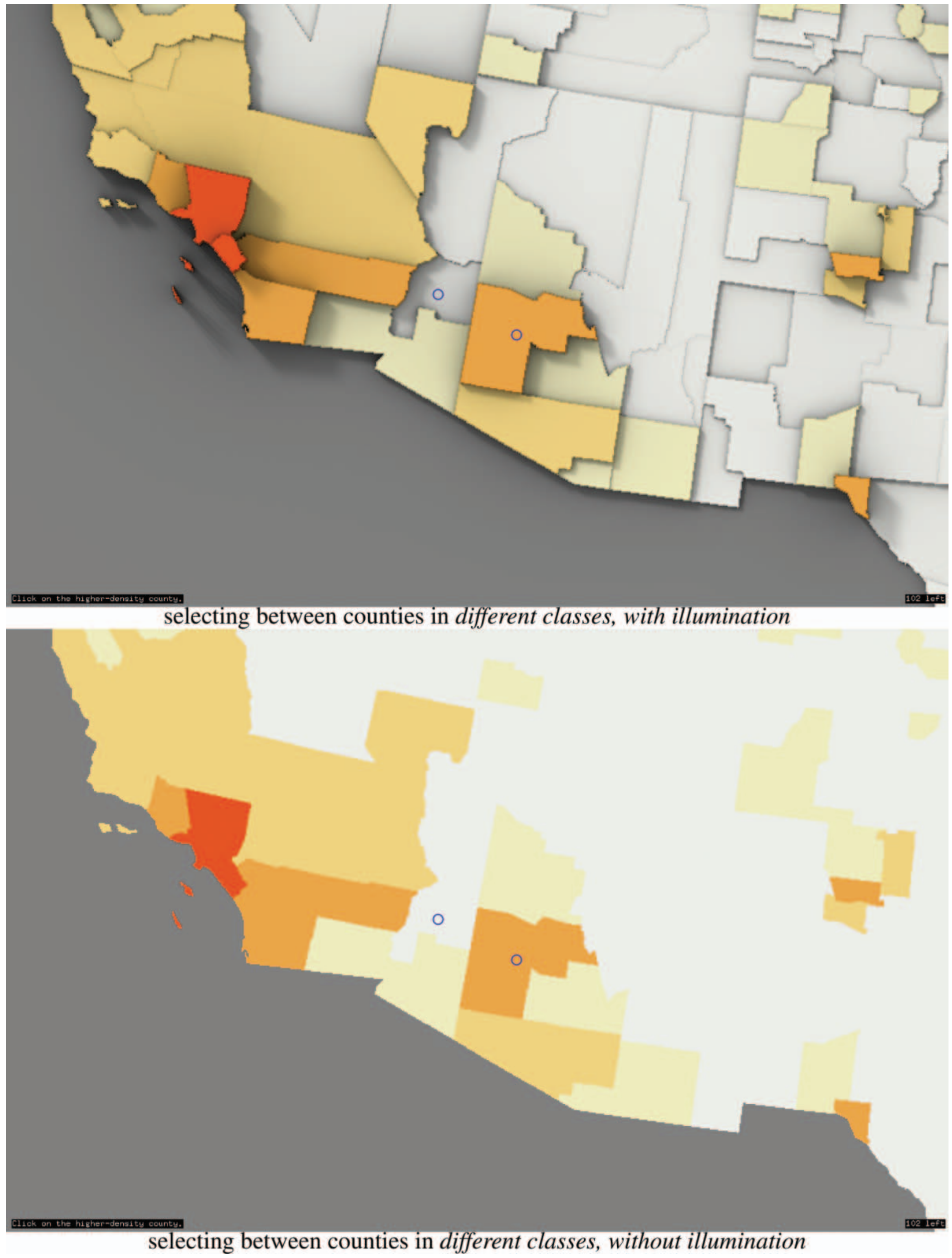

Figure 8. Selection mode (different classes): The participant was asked to click on the denser of the two counties in different classes indicated with the blue circles near the center of the screen.

participant. A participant's completion time was defined as the time between the presentation of the task on the screen and his or her subsequent mouse click.

Student's t test for paired data was used to determine whether the participants performed differently under the two conditions. Two one-sided t tests (TOSTs) were made to determine whether participants performed equivalently. We considered a difference in performance of up to 10 percent to be equivalent. In the following results, we show the 95 percent confidence interval of the mean of each measure as mean $\pm$ 1.96 standard error. An alpha level of 0.05 was used in all tests. All results are summarized in Table 1.

Pair Selection Task

Participants performed significantly better ($p < 0.02$) at selecting the denser of two adjacent counties of the same class when using illumination (accuracy 0.75 ± 0.09) than when not using illumination (accuracy 0.65 ± 0.04). The performance increase was even more substantial among experienced participants (0.88 versus 0.67, $p < 0.001$). No conclusion could be drawn about the relative performance of nonexperienced participants under the two conditions.

We also include histograms to highlight the differences we detected with the preceding summary statistics

Figure 9. Selection mode (same class): The participant was asked to click on the denser of the two counties in the same class indicated with the blue circles near the center of the screen.

(Figure 12). Without shading, participants have a fairly equal distribution of correct choices, with scores varying between 40 percent and 90 percent. With shading, more than half (twenty-one) of the participants scored in the 90 percent to 100 percent range (with thirteen perfect scores). Another point to note is that no participants scored terribly low without shading. With shading, two participants scored less than 10 percent. These individuals were not experienced with shading techniques and might have mentally inverted the 3D model.

Also worth noting is that participants scored much better (accuracy 0.65) than random chance (accuracy 0.50) when not using illumination, even though there was no difference in the appearance of the two counties. Two participants reported that they used the strategy of picking as denser the county that was closer to a major population center.

For adjacent counties of the same class, participants took more time when using illumination (3.0 ± 0.7 seconds) than when not using illumination (2.5 ± 0.5 seconds). The difference was significant ($p = 0.010$).

Participants performed about the same at selecting the denser of two adjacent counties of different classes

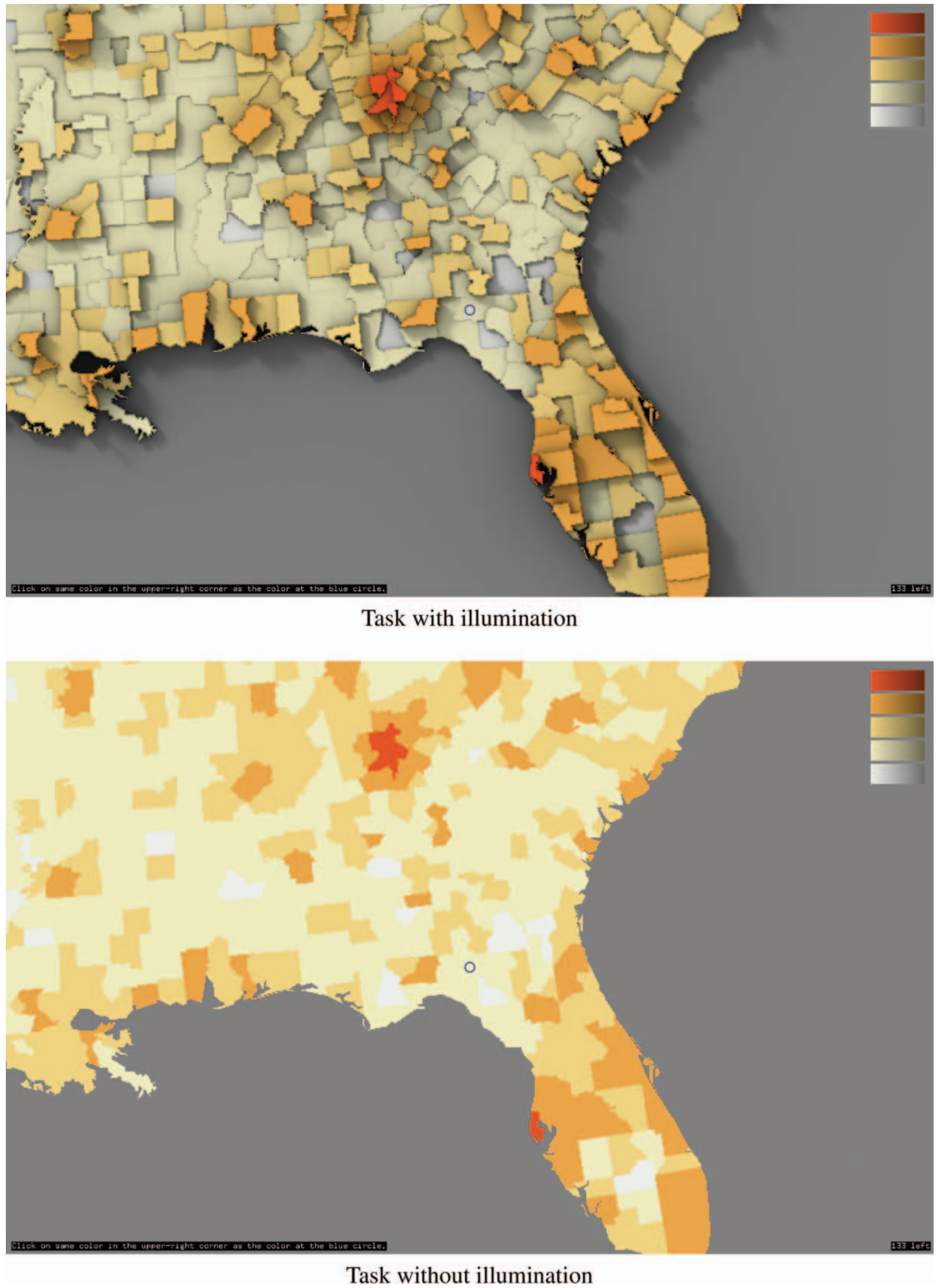

Figure 10. Legend matching task: The participant was asked to click on the legend color that matched that of the county with the blue circle near the center of the screen.

when using illumination (accuracy 0.92 ± 0.03) than when not using illumination (accuracy 0.94 ± 0.02). The two conditions were equivalent ($p < 0.001$) for an effect size of 10 percent, regardless of whether the participant was experienced or nonexperienced. In other words, our conclusion that the two conditions are equivalent (i.e., that there is at most a 10 percent difference in accuracy) has a 0.1 percent likelihood ($p < 0.001$) of being incorrect.

For adjacent counties of different classes, participants took more time when using illumination (2.2 ± 0.3 seconds) than when not using illumination (1.9 ± 0.2 seconds). The difference was significant ($p < 0.001$).

Legend Matching Task

Participants performed equivalently at classifying counties when using illumination (accuracy 0.89 ± 0.04) and when not using illumination (accuracy 0.91 ± 0.03). The two conditions were statistically equivalent for both experienced ($p < 0.002$) and nonexperienced ($p < 0.003$) participants. Participants took equivalent

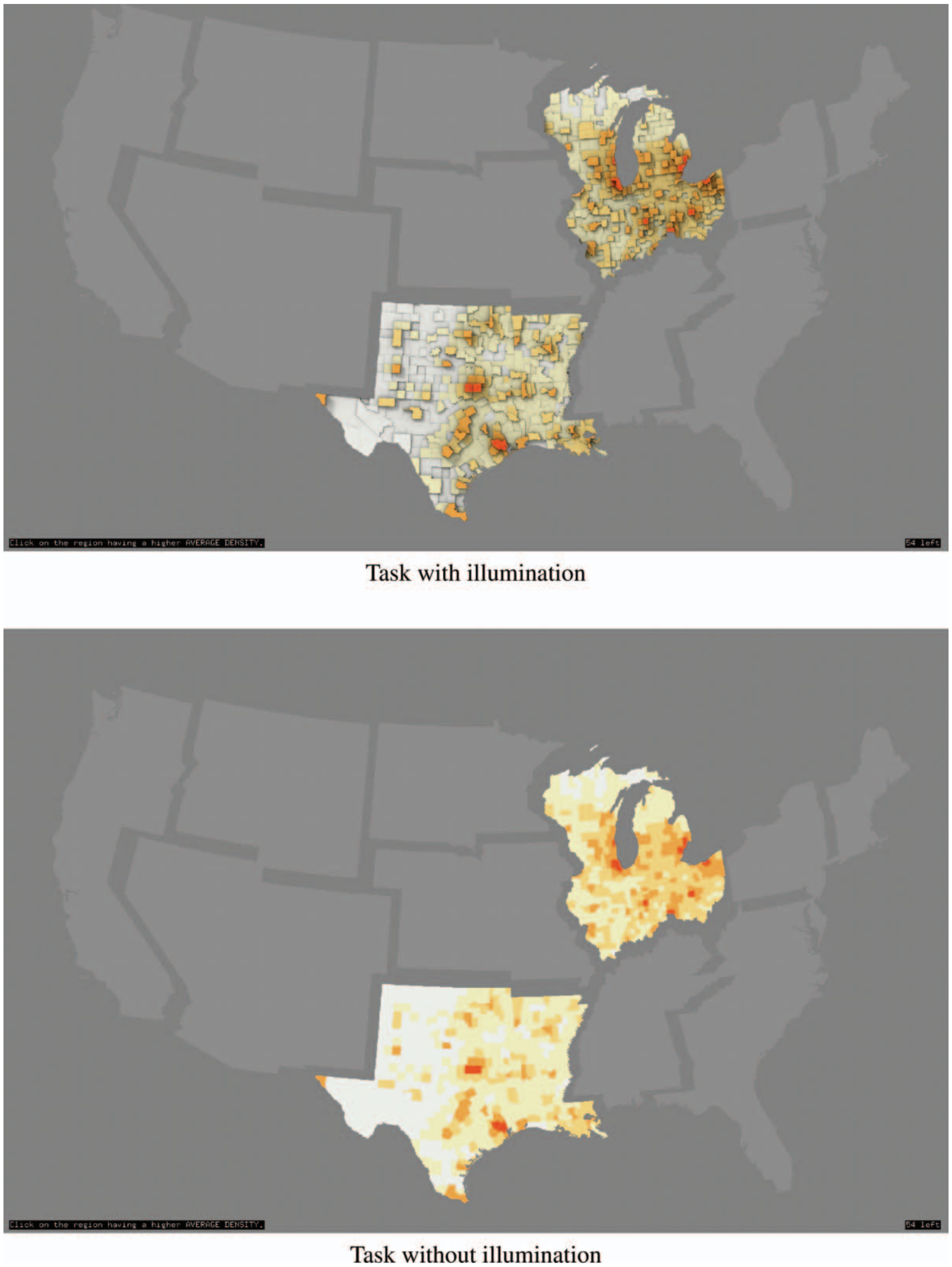

Figure 11. Region mean and variance modes: The participant was asked to click on the region that had greater mean population density (in one test) or greater variance in population density (in another test).

time to classify when using illumination (3.41 ± 0.42 seconds) and when not using illumination (3.38 ± 0.35 seconds, $p < 0.029$) considering an effect size of 10 percent (i.e., 0.34 seconds).

We did, however, notice that participants performed poorly when classifying a few particular counties: those that fell in the deep shadow of a much denser adjacent county. With the sun placed in the northwest, deep shadows were cast on counties to the southeast of counties with much greater population densities. An example of one such site is shown in Figure 13. In these cases, the participants had a mean accuracy of 0.40 ± 0.15 with illumination and 0.95 ± 0.07 without illumination. The difference was significant ($p < 0.001$).

Region Mean and Variance Tasks

The two region tasks required the participants to roughly estimate the aggregate measures of mean and variance for regions consisting of many counties. Given a pair of regions, participants were asked to click on the region of greater mean or variance.

For the region *mean* task, there was no significant difference in performance with illumination (accuracy

Table 1. All performance and time results from the user study

Task	Measure	Participants	With illumination	Without illumination	Conclusions
Pair selection					
Same class	Performance	All	0.75 ± 0.04	0.65 ± 0.09	Different ($p < 0.02$)
		Experienced	0.88 ± 0.08	0.67 ± 0.07	Different ($p < 0.001$)
		Nonexperienced	0.66 ± 0.13	0.64 ± 0.05	No conclusion
	Time	All	3.0 ± 0.7 sec	2.5 ± 0.5 sec	Different ($p = 0.010$)
Different class	Performance	All	0.92 ± 0.03	0.94 ± 0.02	Equivalent ($p < 0.001$)
		Experienced	0.94 ± 0.03	0.96 ± 0.02	Equivalent ($p < 0.001$)
		Nonexperienced	0.91 ± 0.03	0.93 ± 0.03	Equivalent ($p < 0.001$)
	Time	All	2.2 ± 0.3 sec	1.9 ± 0.2 sec	Different ($p < 0.001$)
Legend matching	Performance	All	0.89 ± 0.04	0.91 ± 0.03	Equivalent ($p < 0.001$)
		Experienced	0.92 ± 0.03	0.94 ± 0.04	Equivalent ($p < 0.002$)
		Nonexperienced	0.88 ± 0.06	0.90 ± 0.05	Equivalent ($p < 0.003$)
	Time	All	3.41 ± 0.42 sec	3.38 ± 0.35 sec	Equivalent ($p < 0.029$)
Regional					
Mean	Performance	All	0.97 ± 0.01	0.97 ± 0.02	Equivalent ($p < 0.001$)
		Experienced	0.98 ± 0.02	0.97 ± 0.03	Equivalent ($p < 0.001$)
		Nonexperienced	0.96 ± 0.02	0.97 ± 0.02	Equivalent ($p < 0.001$)
	Time	All	3.7 ± 0.6 sec	3.3 ± 0.4 sec	No conclusion
Variance	Performance	All	0.61 ± 0.06	0.63 ± 0.06	Equivalent ($p < 0.001$)
		Experienced	0.66 ± 0.08	0.66 ± 0.09	Equivalent ($p < 0.001$)
		Nonexperienced	0.57 ± 0.08	0.60 ± 0.07	Equivalent ($p < 0.01$)
	Time	All	5.1 ± 0.6 sec	4.5 ± 0.7 sec	Different ($p = 0.042$)

0.97 ± 0.01) or without illumination (accuracy 0.97 ± 0.02). The two conditions were statistically equivalent ($p < 0.001$). For the region mean, participants took more time when using illumination (3.7 ± 0.6 seconds)

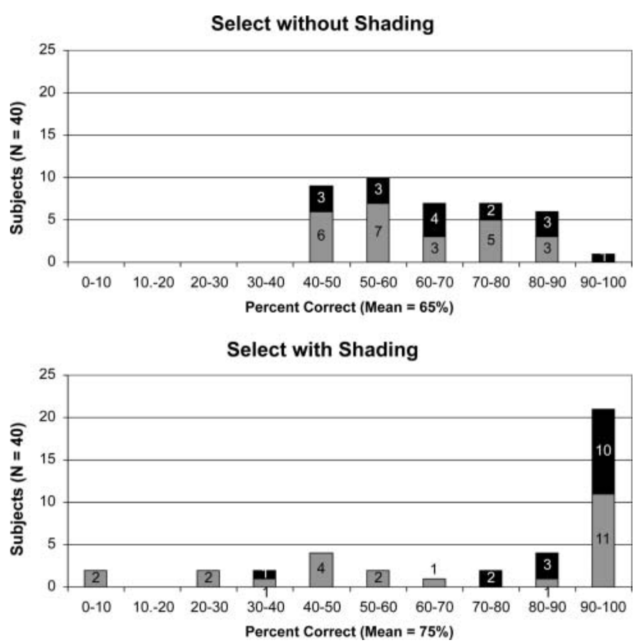

Figure 12. Histograms showing performance of participants in selecting more densely populated of adjacent counties in the same class, without and with shading. Black sections of the bars represent users experienced with shading; gray sections represent those without experience.

than when not using illumination (3.3 ± 0.4 seconds), although no statistical conclusion could be reached.

For the region *variance* task, participants had much more difficulty, although again the performance was equivalent ($p < 0.001$) with illumination (accuracy 0.61 ± 0.06) and without illumination (accuracy 0.63 ± 0.06). For the region variance, participants took more time when using illumination (5.1 ± 0.6 seconds) than when not using illumination (4.5 ± 0.7 seconds). The difference was significant ($p = 0.042$).

All of the participants reported that they found the region variance task to be much more difficult than the other tasks. Five participants reported that they tried to envision histograms of the class frequency for each region and to estimate the variance from the histogram, a daunting mental task.

Posttrial Survey Responses

After the trial, each participant was asked to evaluate three statements on a five-point Likert scale with response choices of strongly agree (SA), agree (A), neutral (N), disagree (D), and strongly disagree (SD). Responses from all forty participants were gathered. Participants generally agreed that illumination helped them to understand variations within regions and did not agree that illumination made it more difficult to determine class or see boundaries. The statements are

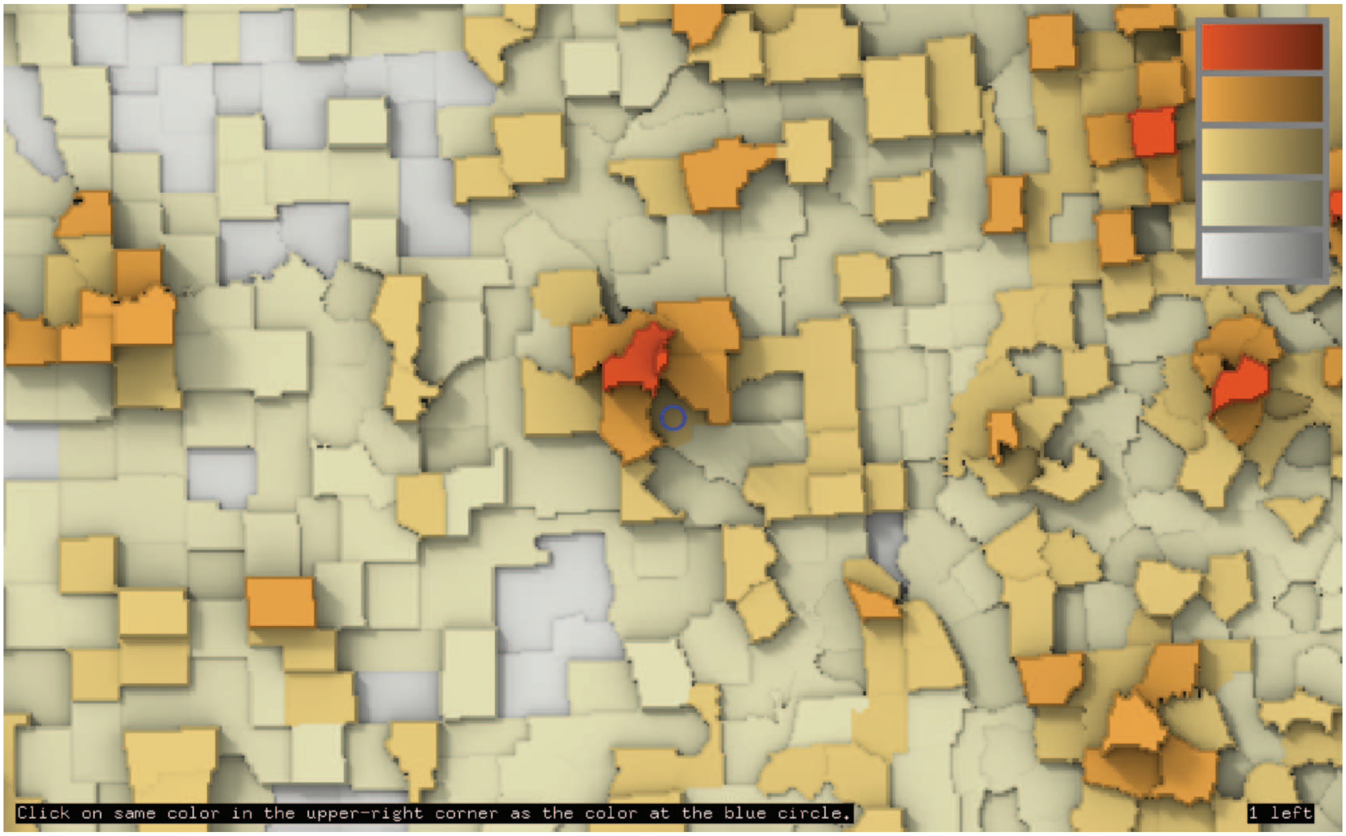

Figure 13. An example in which legend matching is difficult because the county being classified (indicated earlier with the blue circles) is southeast of a county with much greater population density, which casts a shadow on it.

presented here, and user responses are summarized in Figure 14 for these statements.

A. The illuminated choropleth map gave me a better understanding of variations within a region (consisting of many counties) than did the unshaded choropleth map.
B. It was more difficult to determine the class of a county in the illuminated choropleth map than in the unshaded map.
C. It was easier to see the boundaries of counties on the unshaded choropleth map, compared to the illuminated choropleth map.

Participants were also asked if they had any further comments about the illuminated choropleth map. Several said that the shaded map "was visually nice" or "looked more accurate" or "was easier to interpret." One comment came in various phrasings from four participants, who said that the color cue, where deeper colors represent regions of denser population, competed with the illumination cue, where lighter shades correspond to higher, denser regions that are not in shadow. One participant said that the orange class colors were "close" and hard to distinguish in shadow.

Discussion

The real strength revealed by this study is the users' abilities to use soft shadows to identify local variations within classes. It is readily apparent that an illuminated choropleth map is more detailed than its counterpart without shadows. Our study shows that this detail adds information to the map in a manner that many users, especially experienced individuals, are able to understand. We see the statistically significant improvement in performance of nonexperienced participants as an indication of the intuitive nature of shadows and the even greater improvement in performance of experienced participants as an indication of a capacity to learn to interpret such shadows.

The amount of time users spent in selecting the unit of higher population density also merits discussion. In the case of units in the same class, users spent significantly more time making a decision. This could

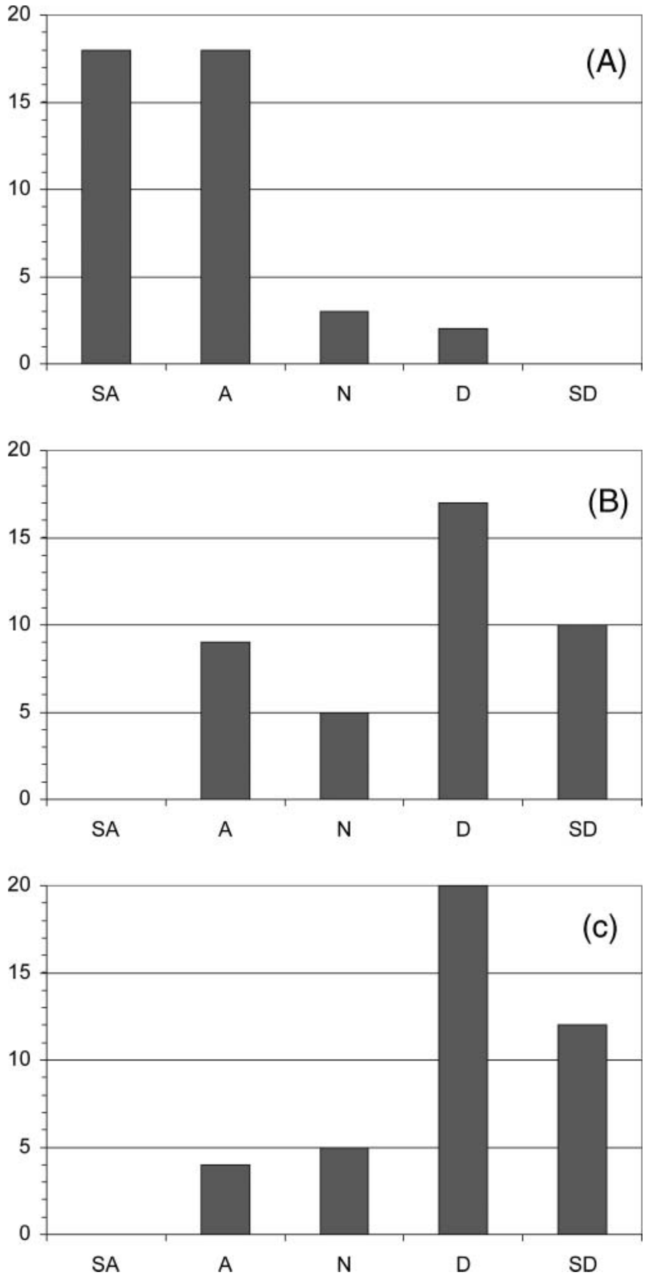

Figure 14. Participant responses to the posttrial survey. See text for statements A, B, and C.

be thought of as time used wisely. Instead of guessing which of two units of identical orange color is higher, users were busy incorporating information from shadows into their decision. This effort shows in their improved performance.

The significant increase in the amount of time users took to select the unit of higher population density when comparing between different classes with illumination is a more unexpected finding. We would have predicted that users would have had faster response times, as both of the visual variables, class color and shadows, resulting from changes in perspective height, are designed to lead them to the same conclusion. We speculate that users might have been using this time to integrate these visual cues.

The user study shows that the illuminated choropleth method does not interfere with map users' abilities to utilize choropleth maps for the other tasks tested in the study. The shading of class colors does not prevent users from matching colors to a color legend and does not require significantly more time. The only exceptions we found were a few places where small areas were entirely overlain and obscured by the darkest parts of these shadows. These shadows could be toned down using other illumination models that increase the diffuse brightness at the expense of the directional, but such displays would have a less noticeable shadowing effect. We would caution against using illuminated choropleth maps with clear-day illumination if important areas in the map design process are identified as at risk of falling under such umbrage.

Another important functionality of choropleth maps to which our method appears to do no harm is the user's ability to compare the mean or variance of the attribute values between two regions. Our study indicates that users tend to be able to estimate and compare mean values, although this takes more time with soft shadows. Our results imply that all of the additional detail provided by soft shadows is not interfering with the user's ability to synthesize a large amount of data over a region.

Our study also indicates that users tend not to be able to estimate and compare variance values. In one respect, finding similar results in selecting the region with greater variance is a good thing. It again implies that our method is not decreasing the users' performance. In another respect, however, we had hoped that the information provided by soft shadows of varying lengths would improve the users' abilities to compare variance among regions. We suggest that the real problem lies in the difficulty of applying the concept of variance, a decidedly more complex statistic than the mean, to a choropleth map.

Conclusions

Our illuminated choropleth method uses unclassed heights of enumeration units to cast shadows that add detail and information to classed choropleth displays. Our goal is to provide the map user with the ability to determine local relative changes between adjacent or close-by enumeration units in the same class, without compromising the ability to compare unit colors to a legend or to compare aggregate measures between larger regions.

We note that our results test only a few tasks that a user might choose to perform with an illuminated choropleth map. We note also that the most important difference is the user's ability to differentiate between the relative population density of two *adjacent* enumeration units. Further assessment of this method could explore the ability of users to compare the relative density of *nonadjacent* polygons in a similar manner. Although much more complex, we can imagine scenarios in which such displays could prove useful. For example, if three square counties form an east–west-oriented rectangle, and the western county casts a shadow on the central county, which in turn casts a shadow on the eastern county, the user could conclude that the western county is higher than the eastern. Complexities of shape and more complex variations in relative height, combined with greater numbers of polygons, would certainly complicate this task in a manner that is difficult to predict.

Several different levels of complexity enter into making and interpreting a choropleth map. This complexity often is a reflection of considerations that might be to some degree in potential conflict. For example, cartographers hope to reveal map patterns, but distributions of geographic data are spatially complex and might be difficult to summarize well on any map. Monmonier (1972, 208) pointed out that, even if class boundaries account for all numerical values and spatial arrangements, "it must be recognized that the statistical and the geographical distributions are not always cooperative."

Another example of this complexity and potential conflict is reflected in the broad spectrum of studies focused on the number of choropleth classes, including ones endorsing fewer classes, more classes, and no classes. From a mapmaker's perspective, opting to create an unclassed map allows display of all potential values, as well as eliminating the assimilating duties of the mapmaker. As Muller (1980, 107) commented, "The embedded classification implies an interpretation which is always questionable." If trying to moderate complexity, however, MacEachren's (1982, 31) finding that "the number of class intervals has a greater effect on complexity than does the pattern of the distribution mapped" highlights the benefit of such questionable interpretation.

A third example of this complexity and potential conflict is the way in which the map is used. A map user might not be concerned whether a map is classed or unclassed, as long as it provides useful information. Gilmartin and Shelton (1989, 43) pointed to the challenges of meeting potentially disparate needs: "Since it is not possible to predict or control whether map readers will use a choropleth map to look for regional trends or to obtain tabular data for specific units on the map (or both), the cartographer must, ideally, try to design the map so that it will fulfill both potential applications."

We see our illuminated choropleth map as one attempt to move toward harmony. In our map, enumeration units of varying values will be represented by changes in color, shadows, or both, but this will not occur in all geographic locations. Areas of relatively high heterogeneity within classes will be highlighted by stronger shadows, and boundaries of relatively high homogeneity between classes will be demarcated by weaker shadows. Our method also incorporates visual variables from classed and unclassed maps in a manner designed to share the visual harmony of objects viewed under natural lighting. Our testing indicates that users are able to effectively use this display in a local and regional sense.

Returning to the debate over the relative merits of classed versus unclassed choropleth maps, we see our method useful for addressing some challenges, although not designed to take on others. In the case of classed choropleth maps, we demonstrate that illuminated choropleth maps reveal local intraclass detail not captured in traditional classed maps. This only has been tested, however, for the relative height of adjacent polygons. No additional information regarding more detailed absolute values within the units' classes can be interpreted from illuminated choropleth maps. Additionally, no additional relative or absolute information can be interpreted from illuminated choropleth maps versus classed choropleth maps when comparing widely separated units.

Optimized class boundaries have been an important focus of classed choropleth maps. Even while minimizing tabular, overview, and boundary error, however, such a method might not optimally represent the terrain of a prism map everywhere. Take, for example, a map of two classes, each made up of numerous units, with the region of higher values centered within that of the lower. All units in the lower class have the same value. Units in the central region increase in value in a consistent manner from west to east. Although the boundaries of the region of higher values have three very different characteristic (cliff, ramp, and incremental steps), all sides would appear as the same sort of boundary on a classed choropleth map. The distinct stepwise increase from west to east and the large cliff at the eastern edge, however, would be apparent on an illuminated choropleth map.

Regarding comparisons to unclassed choropleth maps, illuminated choropleth maps reveal very detailed local patterns of relative variation in a manner that our study indicates users are able to interpret. Additionally, these changes are revealed using shading that generally occurs only at the edge of most sized units, allowing significant area for display of the class color. Also, although not addressed in our user study, we are able in some instances to see shading resulting from variations between adjacent units as small as one person per square mile, or a 1 percent change in population density. We suggest this might be simpler to detect for adjacent polygons than color or pattern variations representing similar sized changes on unclassed choropleth maps.

We realize that our method is not without issues and might not be useful for every potential application of choropleth mapping. For example, our testing indicated that users are not able to match unit colors inside strong shadows to a legend. Also, we tested the user's ability to pick the higher of two adjacent polygons but not two polygons in the same class separated by some distance. As shadows generally are cast on adjacent polygons, we would assume this task to be more challenging. Also, users were tasked with identifying relative changes in population density; we would not expect our method to assist in more closely identifying absolute density values within a class.

We also realize that this illumination approach is not the only approach that could exploit the prism model to enhance choropleth maps. For example, a perspective view with hidden surfaces revealed via semitransparency could also help to visualize the same information. An area obscured from sight by one prism might be more visible using partial transparency than another area obscured by two. We suggest this technique would face different but similarly complex challenges when compared to our method. For example, as our method allows users to identify colors and other shadows beneath soft shadows, a perspective view method might require users to identify color and changes in height on overlapping semitransparent surfaces. Alternatively, a dynamic prism map would allow the user to interact with the display, looking at surfaces from multiple perspectives. It should be noted, however, that realistic shading and shadowing are often used to enhance such computer graphics displays. We opt to focus our methods on creating a static, planimetrically correct map.

Jenks and Caspall (1971) concluded that their research is focused on the definition, measurement, and reduction of error for classed choropleth data but that "We have not, on the other hand, provided the cartographer with a measure of the carrying capacity of a map" (243). Although they did not define carrying capacity or how it can be measured, they referred to "visual static" associated with an increase in the number of classes. We consider our method an attempt to limit this visual static, at the same time enhancing the visual signal.

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