

# NCERT CLASS 12

## CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vectors  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one, Find the value of  $\lambda$ .

### Generalized Construction:

Let us assume that,

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \quad (1)$$

We now that

$$\Rightarrow \mathbf{C} = \lambda \mathbf{e}_1 + \mathbf{D} \quad (2)$$

and also we know that,

$$\Rightarrow \mathbf{A}^\top = \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} \quad (3)$$

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \quad (4)$$

Let us consider the L.H.S of Equation(4),and we get  $\mathbf{C}$  value from (2)

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) \quad (5)$$

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \lambda \mathbf{e}_1 + \mathbf{D}) \quad (6)$$

$$\Rightarrow (a_1 \ a_2 \ a_3) \left[ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \\ c_3 \end{pmatrix} \right] \quad (7)$$

$$\Rightarrow (a_1 \ a_2 \ a_3) \left[ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \right] \quad (8)$$

$$\Rightarrow a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) \quad (9)$$

Now let us consider R.H.S of Equation(4),we get,

$$\Rightarrow \sqrt{(\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C})} \quad (10)$$

$$\Rightarrow \sqrt{(b_1 + c_1 \ b_2 + c_2 \ b_3 + c_3) \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix}} \quad (11)$$

$$\Rightarrow \sqrt{(b_1 + c_1)^2 + (b_2 + c_2)^2 + (b_3 + c_3)^2} \quad (12)$$

We get Final Generalized Equation

$$\Rightarrow a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) = \sqrt{(b_1 + c_1)^2 + (b_2 + c_2)^2 + (b_3 + c_3)^2} \quad (13)$$

Substitute the Given Data in Equation(13),

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$$

we get,

$$\implies 1(2 + \lambda) + 1(4 + 2) + 1(-5 + 3) = \sqrt{(2 + \lambda)^2 + (4 + 2)^2 + (-5 + 3)^2} \quad (14)$$

$$\implies \lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)} \quad (15)$$

$$\implies (\lambda + 6)^2 = (\lambda)^2 + 4(\lambda) + 44 \quad (16)$$

$$\implies (\lambda)^2 + 12(\lambda) + 36 = (\lambda)^2 + 4(\lambda) + 44 \quad (17)$$

$$\implies 8(\lambda) = 8 \quad (18)$$

$$\implies \lambda = 1 \quad (19)$$