NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one, Find the value of λ.

   Generalized Construction:
   We now that

$$\implies \mathbf{A}^\top = \frac{(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|} \tag{1}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \|\mathbf{B}+\mathbf{C}\| \tag{2}$$
$$\implies \mathbf{C} = \lambda\mathbf{e}_1 + \mathbf{D} \tag{3}$$

   were,

$$\implies \|\mathbf{B}+\mathbf{C}\| = \sqrt{(\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C})} \tag{4}$$

   From the Equation(2),We can do

$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{(\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C})} \tag{5}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{\|\mathbf{B}\|^2 + 2\left[\mathbf{B}^\top\mathbf{C}\right] + \|\mathbf{C}\|^2} \tag{6}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{\mathbf{B}^\top\mathbf{B} + 2\left[\mathbf{B}^\top\mathbf{C}\right] + \mathbf{C}^\top\mathbf{C}} \tag{7}$$

   Substitute the $\mathbf{C}$ Value in the Equation(7),We get

$$\implies \mathbf{A}^\top(\mathbf{B}+\lambda\mathbf{e}_1+\mathbf{D}) = \sqrt{\mathbf{B}^\top\mathbf{B} + 2\mathbf{B}^\top(\lambda\mathbf{e}_1+\mathbf{D}) + (\lambda\mathbf{e}_1+\mathbf{D})^\top(\lambda\mathbf{e}_1+\mathbf{D})} \tag{8}$$

   S.O.B.S,we get

$$\implies \left(\mathbf{A}^\top(\mathbf{B}+\lambda\mathbf{e}_1+\mathbf{D})\right)^2 = \mathbf{B}^\top\mathbf{B} + 2\mathbf{B}^\top(\lambda\mathbf{e}_1+\mathbf{D}) + \left((\lambda\mathbf{e}_1+\mathbf{D})^\top(\lambda\mathbf{e}_1+\mathbf{D})\right) \tag{9}$$
$$\implies \left(\mathbf{A}^\top\lambda\mathbf{e}_1\right)^2 + \left(\mathbf{A}^\top\mathbf{B}+\mathbf{D}\right)^2 + 2\left(\mathbf{A}^\top\lambda\mathbf{e}_1\right)\left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right) = \mathbf{B}^\top\mathbf{B} + 2\mathbf{B}^\top(\lambda\mathbf{e}_1+\mathbf{D}) + \lambda^2 + 2\lambda\mathbf{e}_1^\top\mathbf{D} + \mathbf{D}^\top\mathbf{D} \tag{10}$$
$$\implies (\lambda^2) + \left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right)^2 + 2\left(\mathbf{A}^\top\lambda\mathbf{e}_1\right)\left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right) = \mathbf{B}^\top\mathbf{B} + 2\lambda\left(\mathbf{B}^\top\mathbf{e}_1 + \mathbf{e}_1^\top\mathbf{D}\right) + \mathbf{D}^\top\mathbf{D} + \lambda^2 \tag{11}$$
$$\implies 2\lambda\left[\mathbf{A}^\top\mathbf{e}_1\mathbf{A}^\top(\mathbf{B}+\mathbf{D}) - \left(\mathbf{B}^\top\mathbf{e}_1 + \mathbf{e}_1^\top\mathbf{D}\right)\right] = \mathbf{B}^\top\mathbf{B} + 2\lambda\left(\mathbf{B}^\top\mathbf{e}_1 + \mathbf{e}_1^\top\mathbf{D}\right) + \mathbf{D}^\top\mathbf{D} - \left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right)^2 \tag{12}$$
$$\implies 2\lambda = \frac{\mathbf{B}^\top\mathbf{B} + 2\mathbf{B}^\top\mathbf{D} + \mathbf{D}^\top\mathbf{D} - \left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right)^2}{\left[\mathbf{A}^\top\mathbf{e}_1\mathbf{A}^\top(\mathbf{B}+\mathbf{D}) - \left(\mathbf{B}^\top\mathbf{e}_1 + \mathbf{e}_1^\top\mathbf{D}\right)\right]} \tag{13}$$
$$\implies \lambda = \frac{\mathbf{B}^\top\mathbf{B} + 2\mathbf{B}^\top\mathbf{D} + \mathbf{D}^\top\mathbf{D} - \left(\mathbf{A}^\top(\mathbf{B}+\mathbf{D})\right)^2}{2\left[\mathbf{A}^\top\mathbf{e}_1\mathbf{A}^\top(\mathbf{B}+\mathbf{D}) - \left(\mathbf{B}^\top\mathbf{e}_1 + \mathbf{e}_1^\top\mathbf{D}\right)\right]} \tag{14}$$

   Substitute the Given Data in Equation(14),

$$\mathbf{A} = \begin{pmatrix}1\\1\\1\end{pmatrix}; \mathbf{B} = \begin{pmatrix}2\\4\\-5\end{pmatrix}; \mathbf{C} = \begin{pmatrix}\lambda\\2\\3\end{pmatrix}$$

   we get,

$$\implies \lambda = \frac{45-14+13-36}{2(1(6)-2)} \tag{15}$$
$$\implies \lambda = \frac{44-36}{8} \tag{16}$$
$$\Longleftarrow \lambda = \frac{8}{8} \tag{17}$$
$$\implies \lambda = 1 \tag{18}$$