NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one, Find the value of λ.

Generalized Construction:
We now that

$$\implies \mathbf{A}^\top = \frac{(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|} \tag{1}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \|\mathbf{B}+\mathbf{C}\| \tag{2}$$
$$\implies \mathbf{C} = \lambda\mathbf{e}_1 + \mathbf{D} \tag{3}$$

were,

$$\implies \|\mathbf{B}+\mathbf{C}\| = \sqrt{(\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C})} \tag{4}$$

From the Equation(2),We can do

$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{(\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C})} \tag{5}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{\|\mathbf{B}\|^2 + 2\left[\mathbf{B}^\top\mathbf{C}\right] + \|\mathbf{C}\|^2} \tag{6}$$
$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \sqrt{\mathbf{B}\mathbf{B}^\top + 2\left[\mathbf{B}^\top\mathbf{C}\right] + \mathbf{C}^\top\mathbf{C}} \tag{7}$$

Substitute the $\mathbf{C}$ Value in the Equation(7),We get

$$\implies \mathbf{A}^\top(\mathbf{B}+\lambda\mathbf{e}_1+\mathbf{D}) = \sqrt{\mathbf{B}\mathbf{B}^\top + 2\mathbf{B}^\top(\lambda\mathbf{e}_1+\mathbf{D}) + (\lambda\mathbf{e}_1+\mathbf{D})^\top(\lambda\mathbf{e}_1+\mathbf{D})} \tag{8}$$
$$\implies \lambda = \frac{\sqrt{\mathbf{B}\mathbf{B}^\top + 2\left(\mathbf{B}^\top\lambda\mathbf{e}_1 + \mathbf{B}^\top\mathbf{D}\right) + \left(\lambda\mathbf{e}_1+\mathbf{D}^\top\right)(\lambda\mathbf{e}_1+\mathbf{D})} - \mathbf{A}^\top(\mathbf{B}+\mathbf{D})}{\mathbf{A}^\top\mathbf{e}_1} \tag{9}$$

Substitute the Given Data in Equation(9),

$$\mathbf{A} = \begin{pmatrix}1\\1\\1\end{pmatrix}; \mathbf{B} = \begin{pmatrix}2\\4\\-5\end{pmatrix}; \mathbf{C} = \begin{pmatrix}\lambda\\2\\3\end{pmatrix}$$

we get,

$$\implies \lambda = \frac{\sqrt{45 + 2(2\lambda-7) + \lambda^2 + 13} - 6}{1} \tag{10}$$
$$\implies \lambda = \sqrt{44 + 4\lambda + \lambda^2} - 6 \tag{11}$$
$$\implies \lambda + 6 = \sqrt{44 + 4\lambda + \lambda^2} \tag{12}$$
$$\implies (\lambda+6)^2 = 44 + 4\lambda + \lambda^2 \tag{13}$$
$$\implies \lambda^2 + 36 + 12\lambda = 44 + 4\lambda + \lambda^2 \tag{14}$$
$$\implies 12\lambda - 4\lambda = 44 - 36 \tag{15}$$
$$\impliedby 8\lambda = 8 \tag{16}$$
$$\implies \lambda = 1 \tag{17}$$