

NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ .

Solution:

From the given data, We get

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \quad (1)$$

We now that

$$\mathbf{A}^\top \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} = 1 \quad (2)$$

$$\Rightarrow \mathbf{A}^\top = (1 \ 1 \ 1) \quad (3)$$

From that,

$$\Rightarrow (\mathbf{B} + \mathbf{C}) = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (4)$$

$$\Rightarrow \|\mathbf{B} + \mathbf{C}\| = \sqrt{(\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C})} = \sqrt{\lambda^2 + 4\lambda + 44} \quad (5)$$

From the Equation (2) we get,

$$\mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \quad (6)$$

$$(1 \ 1 \ 1) \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} = \sqrt{(\lambda^2 + 4\lambda + 44)} \quad (7)$$

$$\lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)} \quad (8)$$

$$(\lambda + 6)^2 = (\lambda)^2 + 4(\lambda) + 44 \quad (9)$$

$$(\lambda)^2 + 12(\lambda) + 36 = (\lambda)^2 + 4(\lambda) + 44 \quad (10)$$

$$8(\lambda) = 8 \quad (11)$$

$$\Rightarrow \lambda = 1 \quad (12)$$