NCERT CLASS 12

CHAPTER 10: EXERCISE 5.13

1. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda \hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ .

Generalized Construction:

Let us assume that,

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$
 (1)

We now that

$$\implies \mathbf{C} = \lambda \mathbf{e}_1 + \mathbf{D} \tag{2}$$

and also we know that,

$$\implies \mathbf{A}^{\top} = \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} \tag{3}$$

$$\implies \mathbf{A}^{\top} (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \tag{4}$$

Let us consider the L.H.S of Equation (4), and we get C value from (2)

$$\implies \mathbf{A}^{\top} (\mathbf{B} + \mathbf{C}) \tag{5}$$

$$\implies \mathbf{A}^{\top} (\mathbf{B} + \lambda \mathbf{e}_1 + \mathbf{D}) \tag{6}$$

$$\implies \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \begin{bmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \\ c_3 \end{pmatrix} \end{bmatrix} \tag{7}$$

$$\implies a_1(b_1+c_1) + a_2(b_2+c_2) + a_3(b_3+c_3) \tag{8}$$

Now let us consider R.H.S of Equation(4), we get,

$$\implies \sqrt{\left(\mathbf{B} + \mathbf{C}\right)^{\top} \left(\mathbf{B} + \mathbf{C}\right)} \tag{9}$$

$$\Rightarrow \sqrt{(b_1 + c_1 \quad b_2 + c_2 \quad b_3 + c_3) \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix}}$$
 (10)

$$\implies \sqrt{(b_1 + c_1)^2 + (b_2 + c_2)^2 + (b_3 + c_3)^2} \tag{11}$$

We get Final Generalized Equation

$$\implies a_1(b_1+c_1) + a_2(b_2+c_2) + a(b_3+c_3) = \sqrt{(b_1+c_1)^2 + (b_2+c_2)^2 + (b_3+c_3)^2}$$
 (12)

Substitute the Given Data in Equation (12),

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$$

we get,

$$\implies 1(2+\lambda) + 1(4+2) + 1(-5+3) = \sqrt{(2+\lambda)^2 + (4+2)^2 + (-5+3)^2}$$
(13)

$$\implies \lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)} \tag{14}$$

$$\implies (\lambda + 6)^2 = (\lambda)^2 + 4(\lambda) + 44 \tag{15}$$

$$\implies (\lambda)^2 + 12(\lambda) + 36 = (\lambda)^2 + 4(\lambda) + 44 \tag{16}$$

$$\implies 8(\lambda) = 8 \tag{17}$$

$$\implies \lambda = 1$$
 (18)