NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ.

Generalized Construction:
We now that

$$\implies \mathbf{A}^\top = \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} \tag{1}$$

$$\implies \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \tag{2}$$

were,

$$\implies \mathbf{C} = \lambda e_1 + \mathbf{D} \tag{3}$$

Let us consider the L.H.S of Equation(2),and we get $\mathbf{C}$ value from (3)

$$\implies \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) \tag{4}$$

$$\implies \mathbf{A}^\top (\mathbf{B} + \lambda e_1 + \mathbf{D}) \tag{5}$$

Now let us consider R.H.S of Equation(2),we get,

$$\implies \sqrt{(\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C})} \tag{6}$$

We get Final Generalized Equation

$$\implies \mathbf{A}^\top (\mathbf{B} + \lambda e_1 + \mathbf{D}) = \sqrt{(\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C})} \tag{7}$$

Substitute the Given Data in Equation(7),

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} ; \mathbf{C} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$$

we get,

$$\implies 1(2 + \lambda) + 1(4 + 2) + 1(-5 + 3) = \sqrt{(2 + \lambda)^2 + (4 + 2)^2 + (-5 + 3)^2} \tag{8}$$

$$\implies \lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)} \tag{9}$$

$$\implies (\lambda + 6)^2 = (\lambda)^2 + 4(\lambda) + 44 \tag{10}$$

$$\implies (\lambda)^2 + 12(\lambda) + 36 = (\lambda)^2 + 4(\lambda) + 44 \tag{11}$$

$$\implies 8(\lambda) = 8 \tag{12}$$

$$\implies \lambda = 1 \tag{13}$$