

NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ .

Generalized Construction:

We now that

$$\Rightarrow \mathbf{A}^\top = \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} \quad (1)$$

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \quad (2)$$

were,

$$\Rightarrow \mathbf{C} = \lambda e_1 + \mathbf{D} \quad (3)$$

Let us consider the L.H.S of Equation(2),and we get \mathbf{C} value from (3)

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) \quad (4)$$

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \lambda e_1 + \mathbf{D}) \quad (5)$$

Now let us consider R.H.S of Equation(2),we get \mathbf{C} value from (3),

$$\Rightarrow \|\mathbf{B} + \mathbf{C}\| \quad (6)$$

$$\Rightarrow \sqrt{(\mathbf{B} + \lambda e_1 + \mathbf{D})^\top (\mathbf{B} + \lambda e_1 + \mathbf{D})} \quad (7)$$

We get Final Generalized Equation

$$\Rightarrow \mathbf{A}^\top (\mathbf{B} + \lambda e_1 + \mathbf{D}) = \sqrt{(\mathbf{B} + \lambda e_1 + \mathbf{D})^\top (\mathbf{B} + \lambda e_1 + \mathbf{D})} \quad (8)$$

Substitute the Given Data in Equation(8),

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$$

we get,

$$\Rightarrow 1(2 + \lambda) + 1(4 + 2) + 1(-5 + 3) = \sqrt{(2 + \lambda)^2 + (4 + 2)^2 + (-5 + 3)^2} \quad (9)$$

$$\Rightarrow \lambda + 6 = \sqrt{(\lambda^2 + 4\lambda + 44)} \quad (10)$$

$$\Rightarrow (\lambda + 6)^2 = (\lambda)^2 + 4(\lambda) + 44 \quad (11)$$

$$\Rightarrow (\lambda)^2 + 12(\lambda) + 36 = (\lambda)^2 + 4(\lambda) + 44 \quad (12)$$

$$\Rightarrow 8(\lambda) = 8 \quad (13)$$

$$\Rightarrow \lambda = 1 \quad (14)$$