NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one, Find the value of λ.

Generalized Construction:
Let us assume that,

$$\mathbf{A}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix},\mathbf{B}=\begin{pmatrix}b1\\b_2\\b_3\end{pmatrix},\mathbf{C}=\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} \tag{1}$$

We now that

$$\implies \mathbf{C}=\lambda\mathbf{e}_1+\mathbf{D} \tag{2}$$

and also we know that,

$$\implies \mathbf{A}^\top=\frac{(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|} \tag{3}$$

$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C})=\|\mathbf{B}+\mathbf{C}\| \tag{4}$$

Let us consider the L.H.S of Equation(4),and we get $\mathbf{C}$ value from (2)

$$\implies \mathbf{A}^\top(\mathbf{B}+\mathbf{C}) \tag{5}$$

$$\implies \mathbf{A}^\top(\mathbf{B}+\lambda\mathbf{e}_1+\mathbf{D}) \tag{6}$$

$$\implies \begin{pmatrix}a_1 & a_2 & a_3\end{pmatrix}\left[\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}+\begin{pmatrix}c_1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\c_2\\c_3\end{pmatrix}\right] \tag{7}$$

$$\implies a_1(b_1+c_1)+a_2(b_2+c_2)+a(b_3+c_3) \tag{8}$$

Now let us consider R.H.S of Equation(4),we get,

$$\implies \sqrt{(\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C})} \tag{9}$$

$$\implies \sqrt{\begin{pmatrix}b_1+c_1 & b_2+c_2 & b_3+c_3\end{pmatrix}\begin{pmatrix}b_1+c_1\\b_2+c_2\\b_3+c_3\end{pmatrix}} \tag{10}$$

$$\implies \sqrt{(b_1+c_1)^2+(b_2+c_2)^2+(b_3+c_3)^2} \tag{11}$$

We get Final Generalized Equation

$$\implies a_1(b_1+c_1)+a_2(b_2+c_2)+a(b_3+c_3)=\sqrt{(b_1+c_1)^2+(b_2+c_2)^2+(b_3+c_3)^2} \tag{12}$$

Substitute the Given Data in Equation(12),

$$\mathbf{A}=\begin{pmatrix}1\\1\\1\end{pmatrix};\mathbf{B}=\begin{pmatrix}2\\4\\-5\end{pmatrix};\mathbf{C}=\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$$

we get,

$$\implies 1(2+\lambda)+1(4+2)+1(-5+3)=\sqrt{(2+\lambda)^2+(4+2)^2+(-5+3)^2} \tag{13}$$

$$\implies \lambda+6=\sqrt{(\lambda^2+4\lambda+44)} \tag{14}$$

$$\implies (\lambda+6)^2=(\lambda)^2+4(\lambda)+44 \tag{15}$$

$$\implies (\lambda)^2+12(\lambda)+36=(\lambda)^2+4(\lambda)+44 \tag{16}$$

$$\implies 8(\lambda)=8 \tag{17}$$

$$\implies \lambda=1 \tag{18}$$