NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ.

Generalized Construction:
Let us assume that,

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b1 \\ b_2 \\ b_3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{1}$$

We now that

$$\implies \mathbf{C} = \lambda e_1 + \mathbf{D} \tag{2}$$

and also we know that,

$$\implies \mathbf{A}^\top = \frac{(\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} \tag{3}$$

$$\implies \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \tag{4}$$

Let us consider the L.H.S of Equation(4),and we get $\mathbf{C}$ value from (2)

$$\implies \mathbf{A}^\top (\mathbf{B} + \mathbf{C}) \tag{5}$$

$$\implies \mathbf{A}^\top (\mathbf{B} + \lambda e_1 + \mathbf{D}) \tag{6}$$

$$\implies \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \left[\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \\ c_3 \end{pmatrix} \right] \tag{7}$$

$$\implies \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \left[\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \right] \tag{8}$$

$$\implies a_1(b_1 + c_1) + a_2(b_2 + c_2) + a(b_3 + c_3) \tag{9}$$

Now let us consider R.H.S of Equation(4),we get,

$$\implies \sqrt{(\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C})} \tag{10}$$

$$\implies \sqrt{\begin{pmatrix} b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{pmatrix} \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \\ b_3 + c_3 \end{pmatrix}} \tag{11}$$

$$\implies \sqrt{(b_1 + c_1)^2 + (b_2 + c_2)^2 + (b_3 + c_3)^2} \tag{12}$$

We get Final Generalized Equation

$$\implies a_1(b_1 + c_1) + a_2(b_2 + c_2) + a(b_3 + c_3) = \sqrt{(b_1 + c_1)^2 + (b_2 + c_2)^2 + (b_3 + c_3)^2} \tag{13}$$

Substitute the Given Data in Equation(13),

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} ; \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$$

we get,

$$\implies 1\left(2+\lambda\right)+1\left(4+2\right)+1\left(-5+3\right)=\sqrt{\left(2+\lambda\right)^2+\left(4+2\right)^2+\left(-5+3\right)^2} \tag{14}$$

$$\implies \lambda+6=\sqrt{\left(\lambda^2+4\lambda+44\right)} \tag{15}$$

$$\implies \left(\lambda+6\right)^2=\left(\lambda\right)^2+4\left(\lambda\right)+44 \tag{16}$$

$$\implies \left(\lambda\right)^2+12\left(\lambda\right)+36=\left(\lambda\right)^2+4\left(\lambda\right)+44 \tag{17}$$

$$\implies 8\left(\lambda\right)=8 \tag{18}$$

$$\implies \lambda=1 \tag{19}$$