NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The Scalar product of the Vector $(\hat{i}+\hat{j}+\hat{k})$ with a Unit Vector along the Sum of Vector $(2\hat{i}+4\hat{j}-5\hat{k})$ and $(\lambda\hat{i}+2\hat{j}+3\hat{k})$ is Equal to One.Find the Value of "λ".

CONSTRUCTION STEPS :

(a) Let us consider the given Data,and find λ Value

$$\mathbf{A}=\begin{pmatrix}1\\2\\\lambda\end{pmatrix};\mathbf{B}=\begin{pmatrix}1\\4\\2\end{pmatrix};\mathbf{C}=\begin{pmatrix}1\\-5\\3\end{pmatrix} \tag{1}$$

(b) We know that ,

$$\mathbf{A}^{\top}\left(\frac{(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|}\right)=1 \tag{2}$$

(c) since from given data,we get

$$\implies \mathbf{A}^{\top}=\begin{pmatrix}1&2&\lambda\end{pmatrix} \tag{3}$$

$$\implies (\mathbf{B}+\mathbf{C})=\begin{pmatrix}2\\-1\\5\end{pmatrix} \tag{4}$$

$$\implies (\mathbf{B}+\mathbf{C})^{\top}=\begin{pmatrix}2&-1&5\end{pmatrix} \tag{5}$$

(d) For $\|\mathbf{B}+\mathbf{C}\|$,

$$\|\mathbf{B}+\mathbf{C}\|=\sqrt{(\mathbf{B}+\mathbf{C})^{\top}\cdot(\mathbf{B}+\mathbf{C})} \tag{6}$$

$$\|\mathbf{B}+\mathbf{C}\|=\sqrt{\begin{pmatrix}2&-1&5\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\5\end{pmatrix}} \tag{7}$$

$$\implies \|\mathbf{B}+\mathbf{C}\|=\sqrt{30} \tag{8}$$

(e) Substitute the Values in Equation(2),then we get

$$\begin{pmatrix}1&2&\lambda\end{pmatrix}\begin{pmatrix}\frac{2}{\sqrt{30}}\\\frac{-1}{\sqrt{30}}\\\frac{5}{\sqrt{30}}\end{pmatrix}=1 \tag{9}$$

$$\frac{5\lambda}{\sqrt{30}}=1 \tag{10}$$

$$\implies \lambda=1.093151 \tag{11}$$

$$\implies \lambda\approx 1 \tag{12}$$

$$\tag{13}$$

Hence $\therefore \lambda$=1;