NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one, Find the value of λ.

Generalized Construction:
We now that

$$\implies \mathbf{C} = \lambda e_1 + \mathbf{D} \tag{1}$$

and also we know that,

$$\implies \mathbf{A}^\top = \frac{(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|} \tag{2}$$

$$\implies \mathbf{A}^\top (\mathbf{B}+\mathbf{C}) = \|\mathbf{B}+\mathbf{C}\| \tag{3}$$

Let us consider the L.H.S of Equation(3),and we get $\mathbf{C}$ value from (1)

$$\implies \mathbf{A}^\top (\mathbf{B}+\mathbf{C}) \tag{4}$$

$$\implies \mathbf{A}^\top (\mathbf{B}+\lambda e_1+\mathbf{D}) \tag{5}$$

Now let us consider R.H.S of Equation(3),we get,

$$\implies \sqrt{(\mathbf{B}+\mathbf{C})^\top (\mathbf{B}+\mathbf{C})} \tag{6}$$

We get Final Generalized Equation

$$\implies \mathbf{A}^\top (\mathbf{B}+\lambda e_1+\mathbf{D}) = \sqrt{(\mathbf{B}+\mathbf{C})^\top (\mathbf{B}+\mathbf{C})} \tag{7}$$

Substitute the Given Data in Equation(7),

$$\mathbf{A} = \begin{pmatrix}1\\1\\1\end{pmatrix}; \mathbf{B} = \begin{pmatrix}2\\4\\-5\end{pmatrix}; \mathbf{C} = \begin{pmatrix}\lambda\\2\\3\end{pmatrix}$$

we get,

$$\implies 1(2+\lambda)+1(4+2)+1(-5+3) = \sqrt{(2+\lambda)^2+(4+2)^2+(-5+3)^2} \tag{8}$$

$$\implies \lambda+6 = \sqrt{(\lambda^2+4\lambda+44)} \tag{9}$$

$$\implies (\lambda+6)^2 = (\lambda)^2+4(\lambda)+44 \tag{10}$$

$$\implies (\lambda)^2+12(\lambda)+36 = (\lambda)^2+4(\lambda)+44 \tag{11}$$

$$\implies 8(\lambda) = 8 \tag{12}$$

$$\implies \lambda = 1 \tag{13}$$