NCERT CLASS 12

CHAPTER 10 : EXERCISE 5.13

1. The Scalar product of the Vector $(\hat{i}+\hat{j}+\hat{k})$ with a Unit Vector along the Sum of Vector $(2\hat{i}+4\hat{j}-5\hat{k})$ and $(\lambda\hat{i}+2\hat{j}+3\hat{k})$ is Equal to One.Find the Value of "λ".

CONSTRUCTION STEPS :

(a) Let us consider the three Different Vectors,

$$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \tag{1}$$
$$a_2\hat{i} + b_2\hat{j} + c_2\hat{k} \tag{2}$$
$$a_3\hat{i} + b_3\hat{j} + c_3\hat{k} \tag{3}$$

(b) Let us assume,The Sum of the two Vectors, Equation(2) and Equation(3),we get

$$(a_2 + a_3)\hat{i} + (b_2 + b_3)\hat{j} + (c_2 + c_3)\hat{k} \tag{4}$$

(c) Unit Vector of Equation(4),we get

$$\frac{(a_2 + a_3)\hat{i} + (b_2 + b_3)\hat{j} + (c_2 + c_3)\hat{k}}{\sqrt{(a_2 + a_3)^2 + (b_2 + b_3)^2 + (c_2 + c_3)^2}} \tag{5}$$

(d) As we know that,Scalar Product of Equation(1) and Equation(5) is Equals to 1,we get

$$(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) \times \frac{(a_2 + a_3)\hat{i} + (b_2 + b_3)\hat{j} + (c_2 + c_3)\hat{k}}{\sqrt{(a_2 + a_3)^2 + (b_2 + b_3)^2 + (c_2 + c_3)\hat{k}}} = 1 \tag{6}$$

(e) The final Equation is

$$\frac{a_1(a_2 + a_3) + b_1(b_2 + b_3) + c_1(c_2 + c_3)}{\sqrt{(a_2 + a_3)^2 + (b_2 + b_3)^2 + (c_2 + c_3)^2}} \tag{7}$$

INPUT MATRIX :

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & -5 \\ \lambda & 2 & 3 \end{pmatrix} \tag{8}$$

OUTPUT : when the Input Matrix is substituted in Equation(7),then we get

$$\frac{1(2 + \lambda) + 1(4 + 2) + 1(-5 + 3)}{\sqrt{(2 + \lambda)^2 + (4 + 2)^2 + (-5 + 3)^2}} = 1 \tag{9}$$

$$\frac{(2 + \lambda) + 6 - 2}{\sqrt{(2 + \lambda)^2 + 36 + 4}} = 1 \tag{10}$$

$$\frac{(2 + \lambda) + 4}{\sqrt{(2 + \lambda)^2 + 40}} = 1 \tag{11}$$

$$\frac{6 + \lambda}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1 \tag{12}$$

$$\sqrt{\lambda^2 + 4\lambda + 44} = 6 + \lambda \tag{13}$$

$$(6 + \lambda)^2 = \lambda^2 + 4\lambda + 44 \tag{14}$$

$$\lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44 \tag{15}$$

$$12\lambda - 4\lambda = 44 - 36 \tag{16}$$

$$8\lambda = 8 \tag{17}$$

$$\lambda = 1 \tag{18}$$