COMP232 SECTION (PP)

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Presented to Professor Hassan Hajjdiab

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I certify that this submission is my original work and meets the Faculty's Expectations of Originality.

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Date: October 6

1.

Due 6/10/2023 Mustafa Alawadi 40217764

Assignment 2

1. a) $p \to q$
$\neg p$
$\therefore \neg q$

If p then q
Not p thus not q

Modus Tollens

p	q	$\neg p \land (p \to q) \to \neg p$
F	F	T
F	T	F
T	F	T
T	T	T

It's invalid ~~Tautology~~ contengency: $(\neg p \land (p \to q)) \to \neg q$

B) $p \to q$
$\neg q$
$\therefore \neg p$

If p then q
not q thus not p

Modus Tollens.

p	q	$(\neg q \land (p \to q)) \to \neg p$
F	F	T
F	T	T
T	F	T
T	T	T

It's valid Tautology: $(\neg q \land (p \to q)) \to \neg p$

C) $p \land \neg p$

If p and not p (always false)
Then q

$\therefore q$

p	q	$(p \land \neg p) \to q$
F	F	T
F	T	T
T	F	T
T	T	T

Valid Tautology: $(p \land \neg p) \to q$

D) $p \to (q \to r)$
$q \to (p \to r)$
$\therefore (p \lor q) \to r$

p	q	$p \to \dots$
T	T	T
F	F	T
T	T	T
T	F	F
T	T	T
T	T	F
T	T	T
T	T	T

$((p \to (q \to r)) \land (q \to (p \to r))) \to (p \lor q) \to r$

invalid contingency

e) $(p \to q) \land (r \to s)$

$\underline{p \lor r}$

$\therefore q \lor s$

$(((p \to q) \land (r \to s)) \land (p \lor r)) \to q \lor s$ valid Tautology

p	q	r	s	$(((p \to q) \land (r \to s)) \land (p \lor r)) \to q \lor s$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

2.

A) p: Democrats win r: Republican wins
q: Economy is up s: unemployement is up

$p \to q \land (r \to s)$
$p \lor r$

$\therefore s \lor q$

$(((p \to q) \land (r \to s)) \land (p \lor r)) \to s \lor q$

p	q	r	s	$((p \to q) \land (r \to s)) \land (p \lor r)) \to s \lor q$
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

Valid Tautology

b) p: Democrats win   r: Republicans win
q: Economy is up   s: Unemployement is up

$(p \vee r) \to (s \vee q)$ If p or r, then s or q
$\underline{p \wedge \neg s}$ P and ¬s, thus q
$\therefore q$

valid Tautology: $(((p \vee r) \to (s \vee q)) \wedge (p \wedge \neg s)) \to q$

p	q	r	s	$((((p \vee r) \to (s \vee q)) \wedge (p \wedge \neg s)) \to q$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	T
F	F	F	F	T
F	T	F	T	T
F	T	T	F	T
F	T	T	T	T
T	T	F	F	T
T	F	F	T	T
T	F	T	F	T
T	F	T	T	T
T	F	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	T

c) p: Democrats win r: Republicans win
q: The economy is up s: unemployement is up

$p \rightarrow s$
$r \rightarrow q$
$p \oplus r$
$\neg p$

$\therefore q$

unvalid, contingency: $\big((p \rightarrow s) \wedge (r \rightarrow q) \wedge (p \oplus r) \wedge (\neg p)\big) \rightarrow q$

p	q	r	s	$((p \rightarrow s) \wedge (r \rightarrow q) \wedge (p \oplus r) \wedge (\neg p)) \rightarrow q$
F	F	F	F	F
F	F	F	T	F
F	F	T	F	F
F	F	T	T	T
F	T	F	F	F
F	T	F	T	F
F	T	T	F	F
F	T	T	T	T
T	F	F	F	F
T	F	F	T	F
T	F	T	F	F
T	F	T	T	F
T	T	F	F	F
T	T	F	T	F
T	T	T	F	F
T	T	T	T	F

3.

s_1
s_2
$\vdots$
s_n
$\therefore c$

3.

p: If it rains — r: Sailing race will be held

q: If it is foggy — s: The lifesaving demonstration will go on

T: Trophy will be awarded

Hypotheses:

$(\neg p \vee \neg q) \rightarrow (r \wedge s)$, $r \rightarrow T$, $\neg T$

conclusion: p

Step	Reason
~~Premise~~ 1. $(\neg p \vee \neg q) \rightarrow (r \wedge s)$	Premise
2. $r \wedge s$	Simplification ~~Modus Ponens~~
3. $r \rightarrow T$	Premise
4. T	Modus Ponens
5. $\neg T$	Premise

4.

4. $C(x)$: x is in class
$F(x)$: x has been to France
$L(x)$: x has visited Louvre

$\exists x (C(x) \wedge F(x))$ — Hypothesis
$C(x) \wedge F(x)$
$C(x)$: simplification
$F(x)$: simplification
$\exists x (F(x) \rightarrow L(x))$ — Hypothesis
$\exists F(x) \rightarrow L(x)$
$L(x)$
$C(x) \wedge L(x)$ conjuction
$\exists x (C(x) \wedge L(x))$ — Exisitential generalization

5.

5. A) If x is irrational, then x^2 is also irrational.
Proving by contradiction, if x is rational then x^2 is rational
let $x = \frac{a}{b}$, where $b \neq 0$

$$x^2 = \frac{a^2}{b^2}, \text{ for } a, b\ 2$$

Because the relation is true, we conclude that for x as real number, if x^2 is irrational then x is also irrational.

B) If $3n-5$ is even, then n is odd
Proof by contraposition, if n is even, then $3n-5$ is odd,
let $n = 2k$

$$3(2k) - 5 = 6k - 5$$

$6k$ similar to $2k$ will always be even, when -5, will always be a number that's odd, thus by proof of contraposition if $3n-5$ is even, then n is odd

c) Since the value of x is the absolute value, we can check for two cases, one for non-negative x and one for a negative x

Case 1: if $x = 0$, and $|x| = 0$
Case 2: if $x < 0$, and $|-(-x)|$
$x < 0$ is smaller than $|-x|$

d) x and y are odd integers, $x = 2k+1$, $y = 2t+1$
for k, m be integers

$$x + y; (2k+1) + (2t+1) = 2k + 2t + 2$$
$$= 2(k+t+1)$$

replace $(k+t+1) = m$
so $2m$ is even, thus proven by direct proof

e) If $x = (-2, 2)$, then leg $x = -2$ and $y = (-4, 4)$, then leg $y = -4$ then $-2 - (-4) = 2$, therefore if: $(-2) - (-4) = 2$

6.

6\. 1. $\forall x\, (P(x) \vee Q(x))$ — Premise

2\. $P(a) \vee Q(x)$ — Universal instantiation

3\. $\forall x((\neg P(x) \wedge Q(x)) \to R(x))$ — Premise

4\. $\neg P(x) \wedge Q(x) \to R(x)$ — universal instantiation

5\. $\neg(\neg P(x) \wedge Q(x)) \vee R(x)$ — Law of Implication

6\. $P(x) \vee \neg Q(x) \vee R(x)$ — De Morgan's Law

7\. $P(x) \vee R(x)$ — Resolution from 2 and 6

8\. $R(x) \vee P(x)$ — commutative law

9\. $\neg(\neg R(x)) \vee P(x)$ — Reventing

10\. $\neg R(x) \to P(x)$ — Law of implilation

11\. $\forall x(\neg R(x) \to P(x))$ — universal generalization

Thus, $\forall x(\neg R(x) \to P(x))$ is true.

7.

7\. a) If m, n be negative integers there is a solution.

Assume for all m, n in positive integers, $m^3 + 2n^2 = 36$. has at least one solution.

$m^3 + 2n^2 = 36$

let $m = 2x$ and $n = 2y$, where x and y are positive integers

Substitute into the equation: $(2x)^3 + 2(2y)^2 = 36$

$8x^3 + 8y^2 = 36$

$2x^3 + 2y^2 = 9$

Since LHS have even equality and on RHS it's odd equality, we conclude that it isn't possible for $m^3 + 2n^2 = 36$ to have any solutions in positive integers.

b) $n = 2k$

$n^3 + n = (2k)^3 + 2k$

$2^3 k^3 + 2k$

$8k^3 + 2k$

Let's factor 2 from $8k^3 + 2k$

$2(4k^3 + k)$

$4k^3 + k$ by m

$(2m)$, Thus, by direct proof for every n, $n^3 + n$ is even

c) n is odd, for $n+2$ is odd.

Using contradiction to prove

n is even, $n = 2k$

$2k + 2$ is even

$2(k+1)$

replace $(k+1)$ by m

$2m$ is even, thus n is odd, $n+2$ is odd

d) $n \times (n+1)$

n is even, $n = 2k$

$(2k) \times (2k+1)$

$4k^2 + 2k$

$2(2k^2 + k)$

$2^2k + k$ replace by m

$2(m)$ proved using direct proofs