COMP335

Assignment 3

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Assignment 3

1. a)

$$n \neq m-1 \longleftrightarrow (n \geq m) \vee (n < m-1)$$

Therefore, the context-free grammar is:

$S \to A \mid B \mid \lambda$
$A \to aAb \mid aA \mid ab$
$B \to aBb \mid Bb \mid bb$

b)

$S \to AB \mid C$
$A \to aA \mid \varepsilon$
$B \to bBc \mid \varepsilon$
$C \to bC \mid \varepsilon$

c) $\overline{L} = \{w \in \{a,b\}^* : w = a^n b^m, n \neq m\} \cup \{wbav : w, v \in \{a,b\}^*\}$

Thus, the context-free grammar is:

$S \to A \mid B \mid C$
$A \to aAb \mid aA \mid a$
$B \to aBb \mid Bb \mid b$
$C \to DbaD$
$D \to aD \mid bD \mid \lambda$

2. a) 00101

Leftmost	Rightmost
$S \to A1B$	$S \to A1B$
$\to 0A1B$	$\to A10B$
$\to 00A1B$	$\to A101B$
$\to 001B$	$\to A101$
$\to 0010B$	$\to 0A101$
$\to 00101B$	$\to 00A101$
$\to 00101$	$\to 00101$

B) 1001

Leftmost	Rightmost
$S \to A1B$	$S \to A1B$
$\to 1B$	$\to A10B$
$\to 10B$	$\to A100B$
$\to 100B$	$\to A1001B$
$\to 1001B$	$\to A1001$
$\to 1001$	$\to 1001$

c) 000111

Leftmost	Rightmost
$S \to A1B$	$S \to A1B$
$\to 0A1B$	$\to A11B$
$\to 00A1B$	$\to A111B$
$\to 000A1B$	$\to A111$
$\to 0001B$	$\to 0A111$
$\to 00011B$	$\to 00A111$
$\to 000111B$	$\to 000A111$
$\to 000111$	$\to 000111$

3. To prove that $L(G) = L$, we need to show that every string generated by CFG G is also in L, and vice-versa.

Proving by induction we need first a Base case:

Base for CFG G is in L: we have S, which is the start symbol and it derives ε, thus $S \to \varepsilon$.

Inductive hypothesis: Assume that any string w derived from S, wwR is in L.

Derivations: In the case $S \to 0S0$ and $S \to 1S1$
Since S derives w, S can also derive $0w0$ or $1w1$
If we assume $w = x(wR)$ in which x is a string,
we get $0x(wR)0$ or $1x(wR)1$ by applying $0S0$ or $1S1$
Concatinating both sides gives $0x(wR)0x(wR)0$ or $1x(wR)1x(wR)1$
and that's in L, so all strings by CFG G is also in L.

Proving by induction for every string in L by CFG G

Base case: for ε, then $S \to \varepsilon$

Inductive hypothesis: we can assume that for any string w in L, the string can be derived from S.

Derivations: If we allow w to be any string in L, we'll have $w = (wR)$ for an x

Thus we can form $0x(wR)0$ or $1x(wR)1$ by using $S \to 0S0$ and $S \to 1S1$.

4. a) The grammar is indeed ambiguous since the string aaba can be formed by 2 different leftmost derivations.

$$S \Rightarrow SS \Rightarrow SSS \Rightarrow aSS \Rightarrow aabS \Rightarrow aaba$$
$$S \Rightarrow SS \Rightarrow aS \Rightarrow aSS \Rightarrow aabS \Rightarrow aaba$$

An unambiguous example is: $S \rightarrow Sa \mid Sab \mid a \mid ab$

b) The grammar is indeed ambiguas as the string a has 2 leftmost derivations:

$$S \Rightarrow ABA \Rightarrow aABA \Rightarrow a\varepsilon BA \Rightarrow a\varepsilon\varepsilon A \Rightarrow a\varepsilon\varepsilon\varepsilon = a$$
$$S \Rightarrow ABA \Rightarrow \varepsilon BA \Rightarrow \varepsilon\varepsilon A \Rightarrow \varepsilon\varepsilon a = a$$

An unambiguas version is:

$$S \rightarrow ABA \mid AB \mid BA \mid A \mid B \mid \lambda$$
$$A \rightarrow aA \mid a$$
$$B \rightarrow bB \mid b$$

c) The grammar is ambiguous since the string aaabb can be formed by two leftmost derivations.

$$S \Rightarrow aSb \Rightarrow aaaSbb \Rightarrow aaa\varepsilon bb = aaabb$$
$$S \Rightarrow aaSb \Rightarrow aaaSbb \Rightarrow aaa\varepsilon bb = aaabb$$

An unambiguous version can be:

$$S \rightarrow A \mid \varepsilon$$
$$A \rightarrow aAb \mid B \mid ab$$
$$B \rightarrow aaBb \mid aab$$

5. a) States: q_0, q_1, q_2, q_3

starting state: q_0

final state: q_3

Alphabet: $\{a, b\}$

Stack: $\{Z_0\}$

$(q_0, \varepsilon, Z_0) \to (q_1, Z_0)$

$(q_1, a, Z_0) \to (q_1, aZ_0)$

$(q_1, a, a) \to (q_1, aa)$

$(q_1, b, a) \to (q_2, \varepsilon)$

$(q_2, b, a) \to (q_2, \varepsilon)$

$(q_2, \varepsilon, Z_0) \to (q_3, Z_0)$

b) States: q_0, q_1, q_2

Starting states: q_0

Alphabet: $\{a, b, c\}$

stack: $\{Z_0\}$

$(q_0, \varepsilon, Z_0) \to (q_1, Z_0)$

$(q_1, a, Z_0) \to (q_1, aZ_0)$

$(q_1, a, a) \to (q_1, aa)$

$(q_1, b, a) \to (q_1, \varepsilon)$

$(q_1, b, a) \to (q_2, \varepsilon)$

$(q_2, c, a) \to (q_2, \varepsilon)$

$(q_2, \varepsilon, Z_0) \to (q_2, \varepsilon)$

$(q_2, \varepsilon, Z_0) \to (q_0, Z_0)$

c) States : q_0, q_1, q_2, q_3

Starting state : q_0

Final State : q_3

Alphabet : $\{a, b\}$

Stack : $\{Z_0\}$

$(q_0, \varepsilon, Z_0) \rightarrow (q_1, Z_0)$

$(q_1, a, Z_0) \rightarrow (q_1, aZ_0)$

$(q_1, b, Z_0) \rightarrow (q_1, bZ_0)$

$(q_1, a, a) \rightarrow (q_1, aa)$

$(q_1, b, b) \rightarrow (q_1, bb)$

$(q_1, a, b) \rightarrow (q_2, \varepsilon)$

$(q_1, b, a) \rightarrow (q_2, \varepsilon)$

$(q_2, \varepsilon, Z_0) \rightarrow (q_3, Z_0)$

6. a) $S \to AB \mid aB$
$A \to abb \mid \varepsilon$
$B \to bbA$
$C \to BC$
$D \to a$

Removing ε Productions

$S \to AB \mid aB \mid B$
$A \to abb$
$B \to bbA \mid bb$

Eliminating productions of single terminals

$S \to AB \mid X_a B \mid B$
$A \to X_a X_b X_b$
$B \to X_b X_b A \mid X_b X_b$

Simplifying for $Y \to X_b X_b$

$S \to AB \mid X_a B \mid B$
$A \to X_a Y$
$B \to YA \mid Y$
$C \to BC$
$Y \to X_b X_b$
$X_a \to a$
$X_b \to b$

b) $S \rightarrow A \mid Ba$
$A \rightarrow B$
$B \rightarrow S \mid a$

Introducing $X_a \rightarrow a$

$S \rightarrow A \mid BX_a$
$A \rightarrow B$
$B \rightarrow S \mid X_a$

Replacing

$S \rightarrow A \mid BX_a$
$A \rightarrow S \mid X_a$

Finally

$S \rightarrow BX_a \mid X_a$
$B \rightarrow BX_a \mid X_a$
$X_a \rightarrow a$

7. $S \to aAA$

$A \to aS \mid bS \mid a$

States: $Q = \{q_0, q_1\}$

Input: $\Sigma = \{a, b\}$

start state: symbol is S

Transition function: δ

$\delta(q_0, \varepsilon, \varepsilon) = (q_1, S)$

$\delta(q_1, \varepsilon, S) = (q_1, AAa)$

$\delta(q_1, \varepsilon, A) = (q_1, SbSa)$

$\delta(q_1, a, a) = (q_1, \varepsilon)$

$\delta(q_1, b, b) = (q_1, \varepsilon)$

8. a) States: $\{q\}$

Input: $\{a, b\}$

Stack: $\{A, B, Z_0\}$

Transition function: δ

Production rules:

$S \rightarrow qZ_0$

$qZ_0 \rightarrow aX$

$qZ_0 \rightarrow \varepsilon$

$qA \rightarrow aY$

$qA \rightarrow \varepsilon$

$qB \rightarrow bY$

$qB \rightarrow \varepsilon$

S, X, Y are non-terminal states

b) 1) we need to start with a start symbol S

$S \rightarrow qZ_0$

$qZ_0 \rightarrow aX$

$qA \rightarrow aY$

$qAA \rightarrow aY$

$qAAA \rightarrow bY$

$qAB \rightarrow bY$

$qBB \rightarrow aY$

$qB \rightarrow aY$

$q \rightarrow \varepsilon$