

$$1) \quad T(n) = T\left(\frac{n}{2}\right) + 1$$

$T(n)$  is an increasing function

$$a = 1 \quad b = 2 \quad d = 0$$

$$b^d = 1$$

$$a = b^d \Rightarrow T(n) \in \Theta(n^d \log n)$$

$$T(n) \in \Theta(\log n)$$

1) b) While our last algorithm solves the problem with  $\Theta(n)$  complexity, first algorithm solves it in  $\Theta(n^3)$ . On the other hand, last algorithm uses  $\Theta(n)$  space, while the first one uses  $\Theta(1)$ . Dynamic programming type solution is better.

2)  $T(n) = 2T(\frac{n}{2}) + 1$   $T(n)$  is an increasing function.

$a=2$   $b=2$   $d=0$

$b^d = 1 < a = 2$   $b^d < a \rightarrow T(n) \in \Theta(n^{\log_b a})$   
 $T(n) \in \Theta(n)$

3)  $T(n) = T(n-1) + T(n-2) + \dots + T(n-k)$   $\nwarrow$  number of recursive call

$T(n) = F(n) + F(n-1) + \dots + F(n-k+1)$

$F(n) = \sum_{i=0}^k 1 = n \in \Theta(n)$

$T(n) = \Theta(kn)$

4)  $T(n) = 2T(\frac{n}{2}) + n$   $T(n)$  is an increasing function

$a=2$   
 $b=2$   
 $d=1$

$a = b^d \Rightarrow T(n) \in \Theta(n^d \log n)$

$T(n) \in \Theta(n \log n)$

5) Brute Force:  $n = \text{power}$

$$T(n) = \sum_{i=0}^{n-1} 1 = n \in \Theta(n)$$

Divide and conquer:

$$T(n) = T\left(\frac{n}{2}\right) + 1$$

$$a = 1$$

$$b = 2$$

$$d = 0$$

$a = b^d$

$$\Rightarrow \Theta(n^d \log n) = \Theta(\log n)$$

$$T(n) \in \Theta(\log n)$$