BSM307
İşaretler ve Sistemler

Dr. Seçkin Arı

Giriş

Tanıtım

- Seçkin Arı
- Ofis #1161
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- Kaynak
  - Ders Notları
  - A. V. Oppenheim, A.S. Wilsky ve S.H. Nawab, *Signals and Systems,* Prentice Hall (Pearson)
  - J.G. Proakis ve D.G. Manolakis, *Digital Signal Processing,* Pearson

- Değerlendirme
  - 3 Kısa sınav
  - 1 Ara sınav
  - 1 Dönem sonu sınavı

Ders İçeriği

- Ayrık Zaman İşaret ve Sistemler
  - Birim Darbe Cevabı
  - Fark Denklemleri
  - Durum Denklemleri
  - z-Dönüşümü
  - Ters z-Dönüşümü
- Sürekli Zaman İşaret ve Sistemler
  - Fourier Seri Açılımı
  - Fourier Dönüşümü
  - Örnekleme

İçerik

- İşaret (Signal)
- Sistem
- Ayrık (Kesikli) Zaman İşaret ve Sistemler
- Sürekli Zaman İşaret ve Sistemler
- Bağımsız Değişken Dönüşümleri
- Birim Darbe ve Birim Basamak Fonksiyonları

İşaret

İşaret

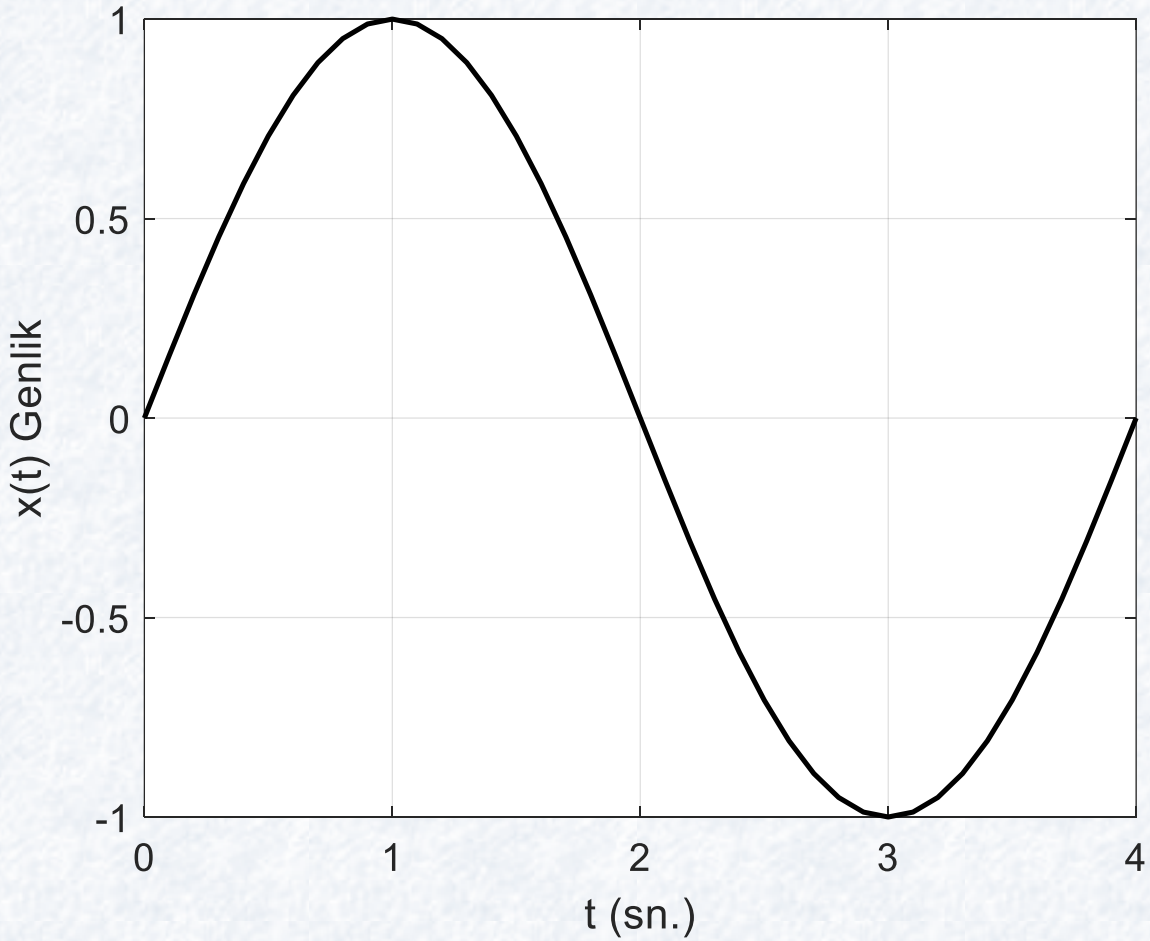
- Matematiksel bir fonksiyon
  - ♦ Fiziksel değişimler
  - ♦ Bilgi taşıyan
  - ♦ Bağımsız değişken: zaman
  - ♦ Bağımlı değişken: voltaj, akım, basınç, sıcaklık, akış hızı, vs....

Örnek İşaretler

- Akıllı telefonlar arasında paylaşılan bilgiler
- Elektromanyetik dalgalar
- Ses
- Görüntü
- Audio
- Video
- Banka faiz oranları
- Borsa indisi
- Döviz oranları
- Tıbbi görüntüler
- EKG
- EEG
- Seçim sonuçları
- Sınav sonuçları

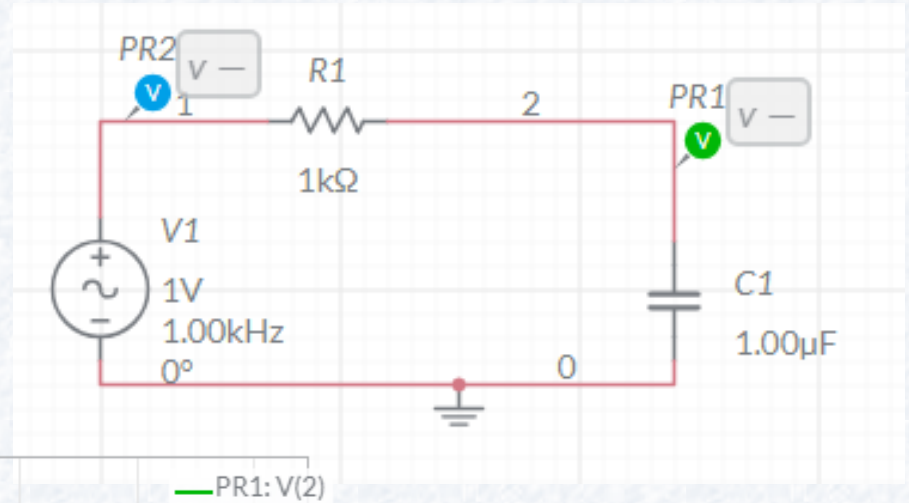
İşaret Türleri

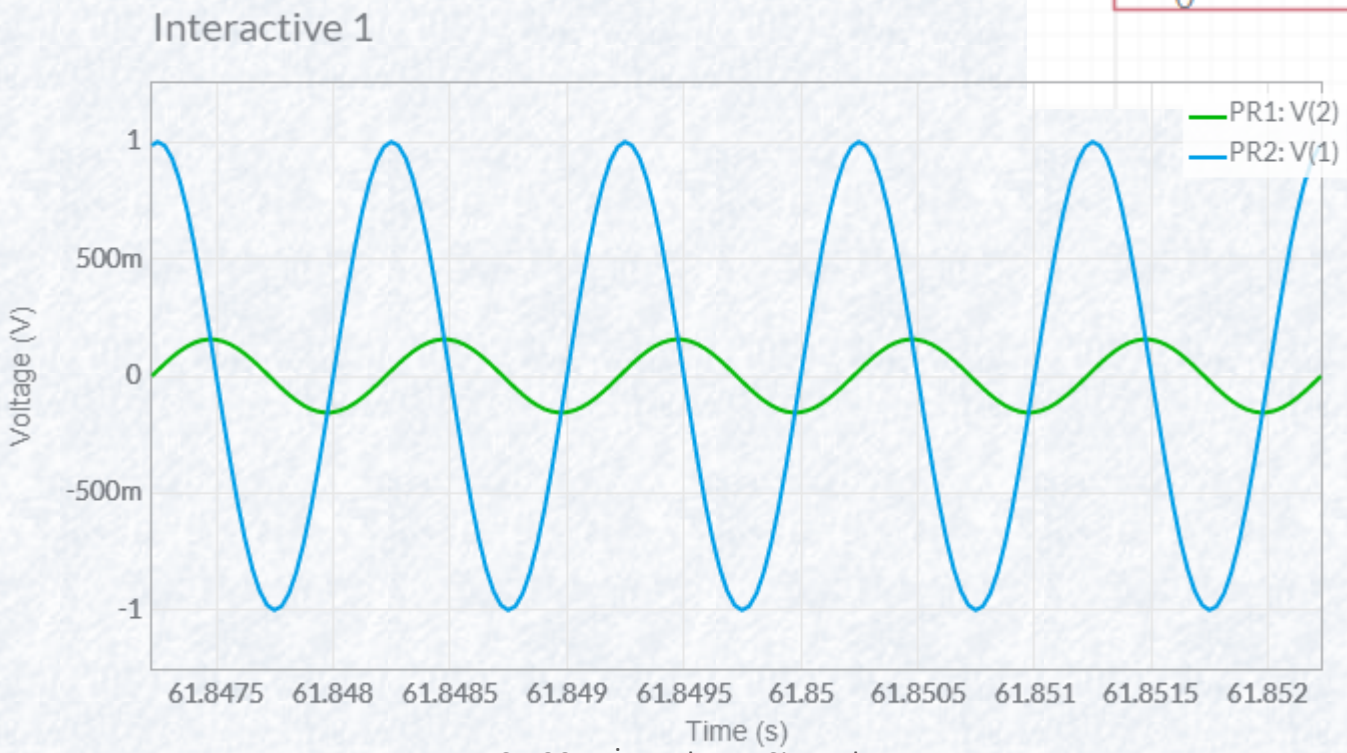
- Sürekli Zaman İşaret
- $x(t)$



İşaret Türleri

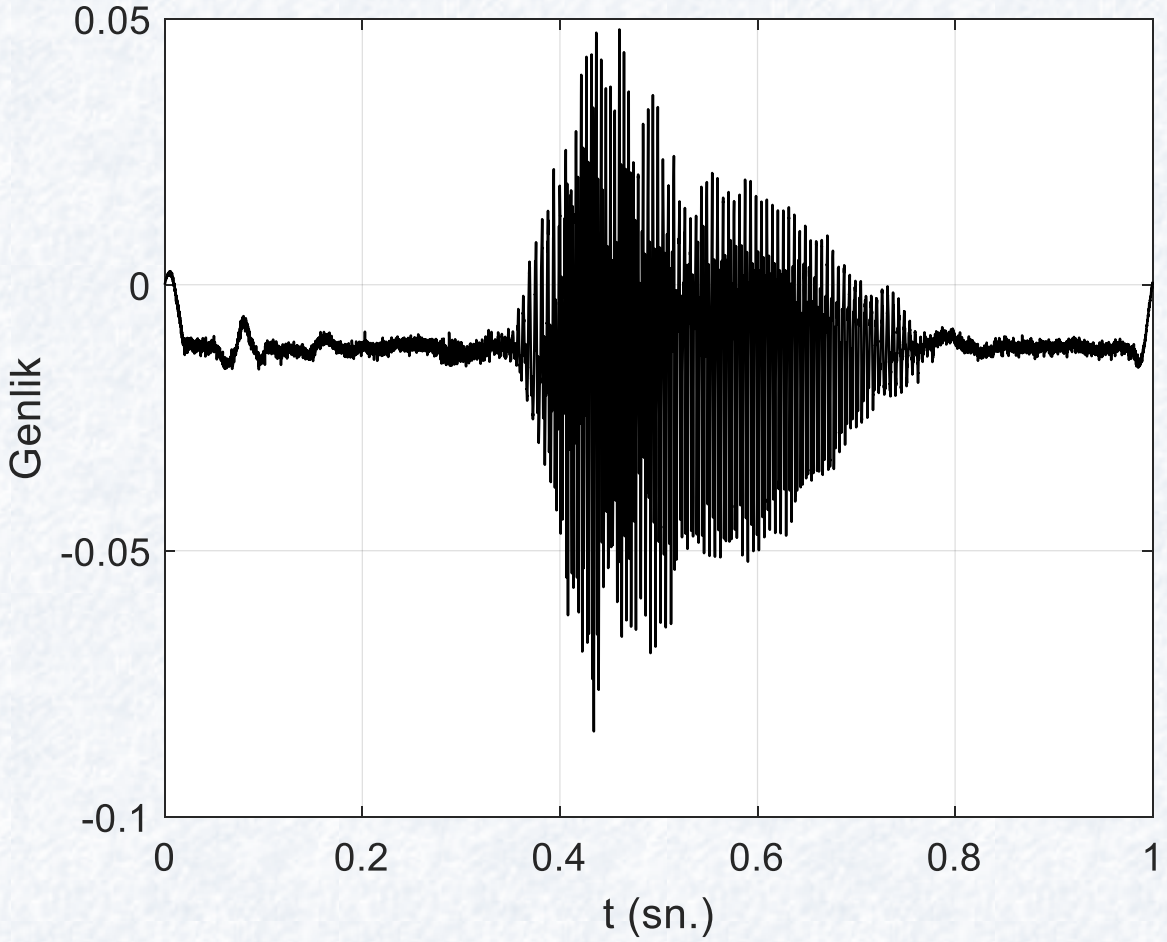
- Sürekli Zaman İşaret - $x(t)$
  - RC devre çıkışı





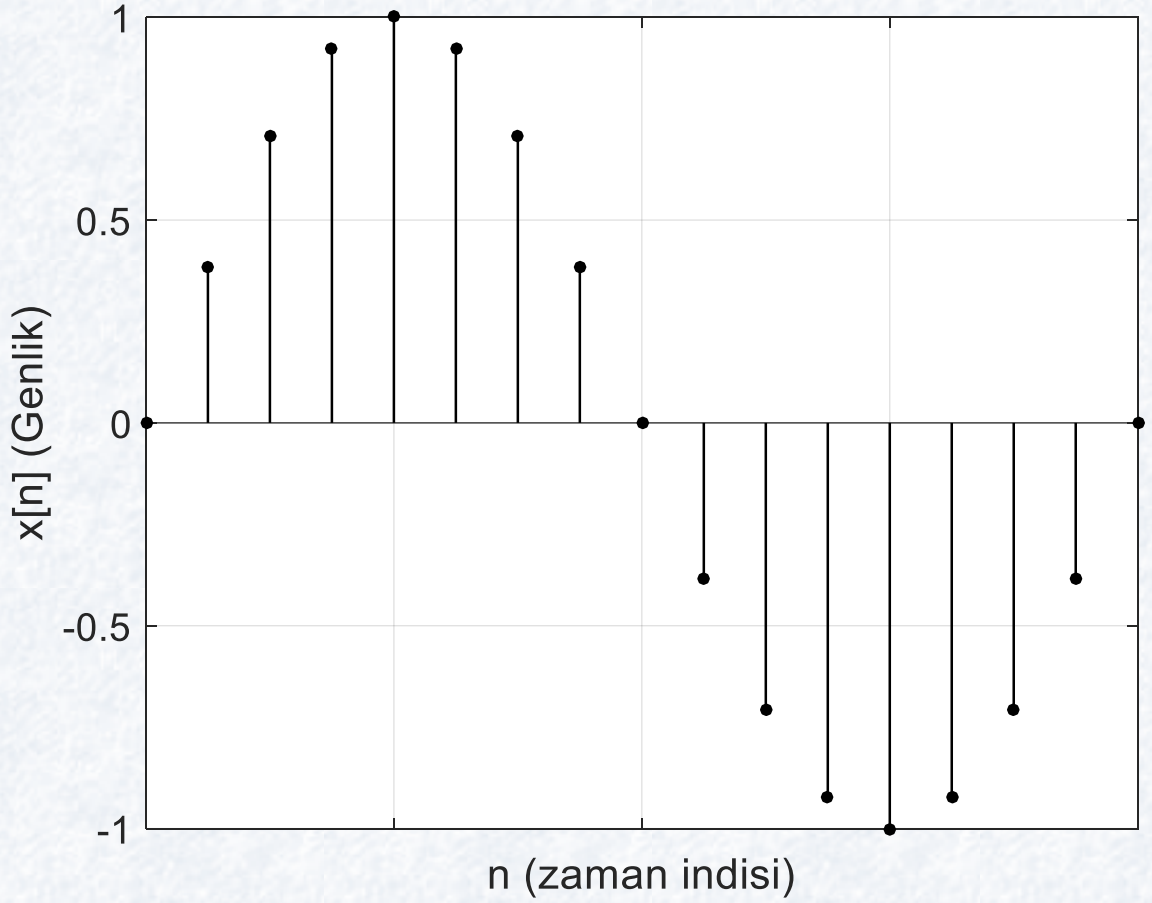
İşaret Türleri

- Sürekli Zaman İşaret - $x(t)$
  - Ses



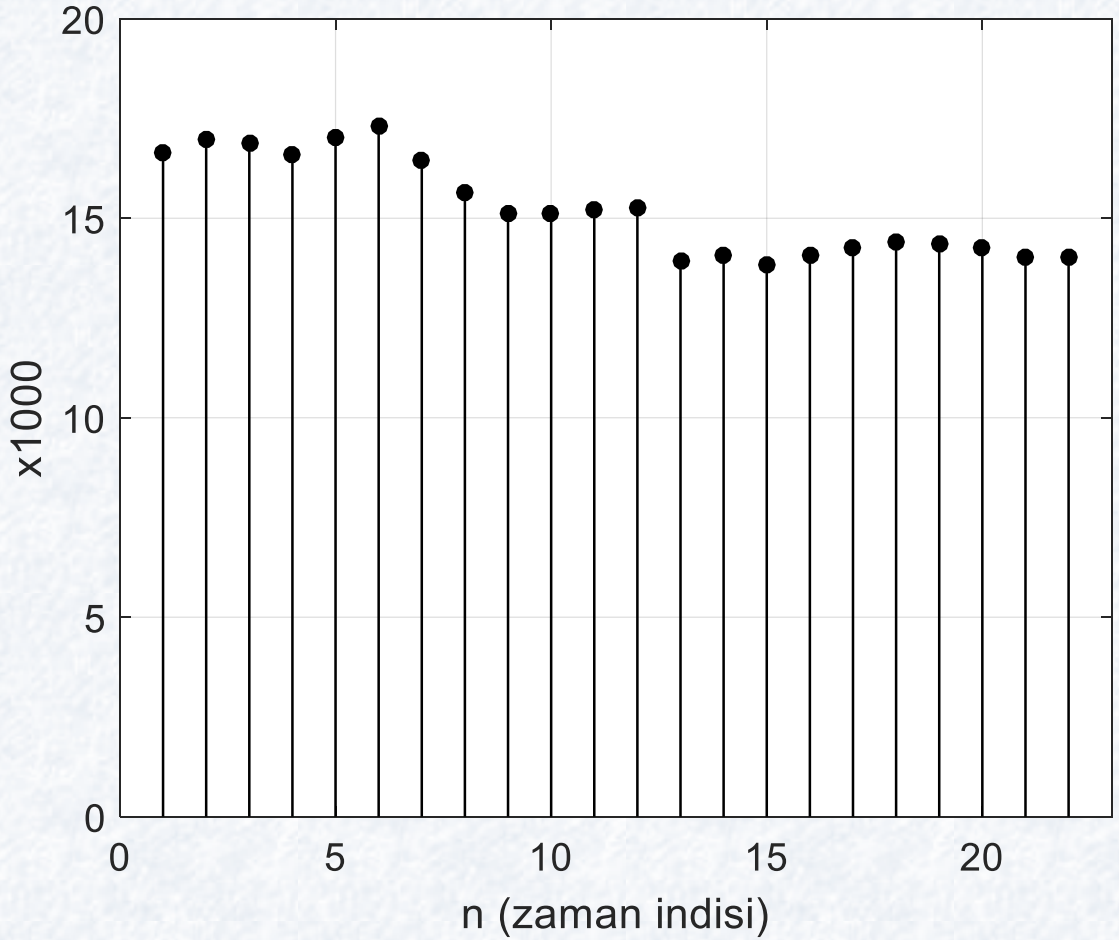
İşaret Türleri

- Ayrık Zaman İşaret
- $x[n]$



İşaret Türleri

- Ayrık Zaman İşaret - $x[n]$
  - BIST



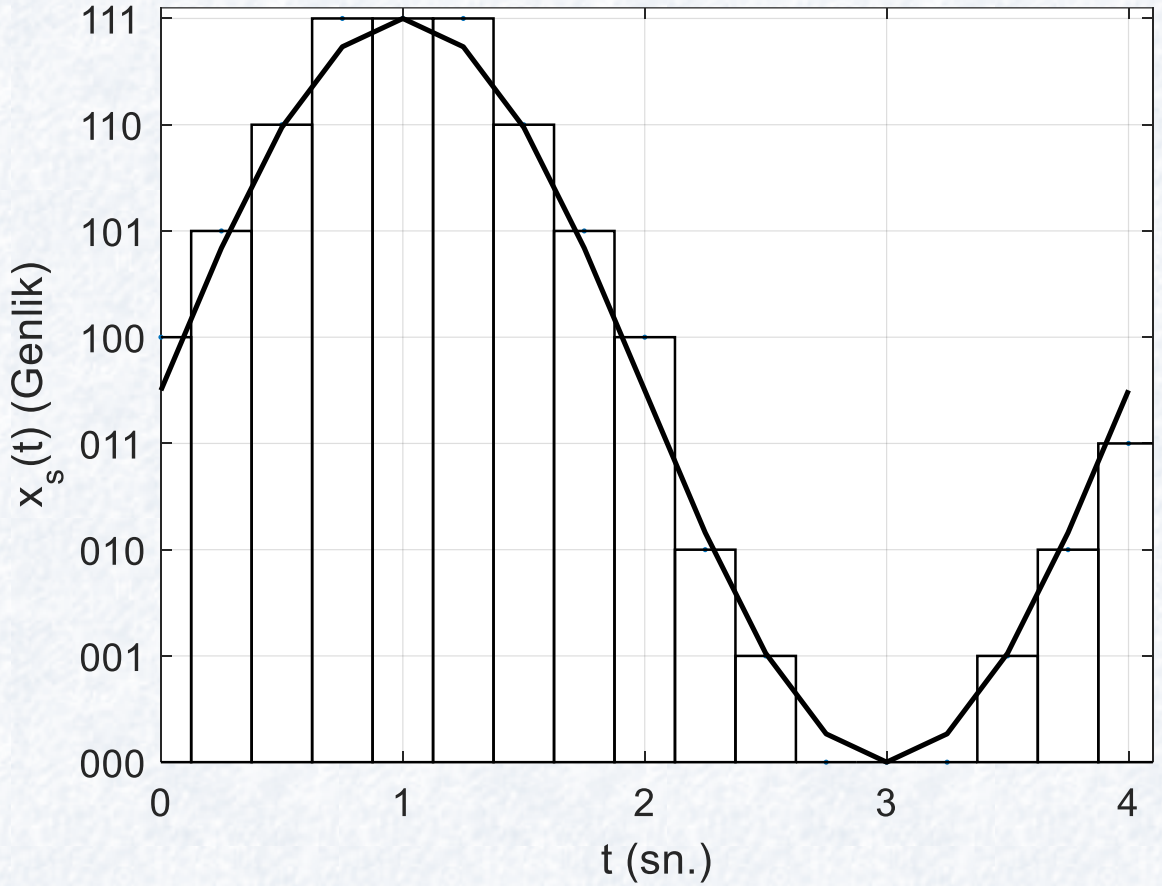
İşaret Türleri

- Ayrık Zaman İşaret - $x[n]$
  - ♦ Görüntü

29	29	29	29	29	29	29
29	29	29	29	29	29	29
173	173	173	173	29	29	29
7	7	7	173	173	173	173
7	7	173	173	7	173	173
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7
173	173	173	173	173	7	7
173	7	173	7	173	7	7

İşaret Türleri

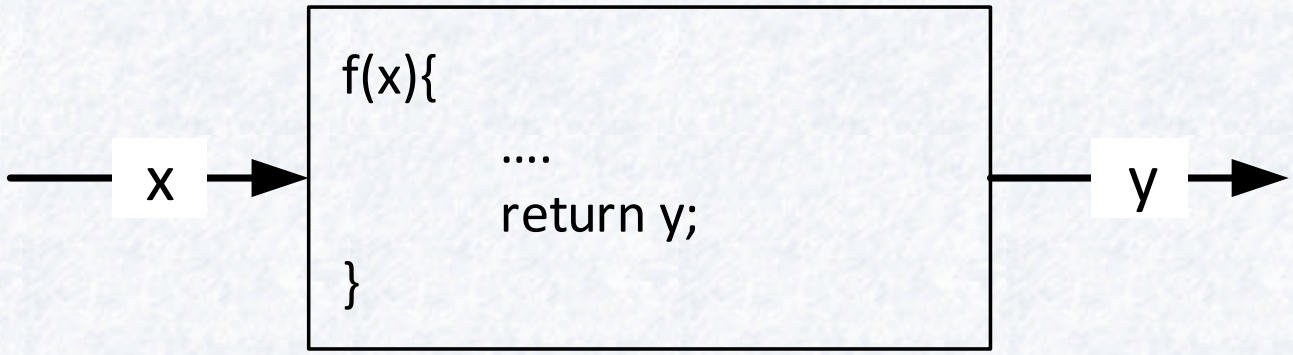
- Sayısal İşaret, kuantalanmış işaret
- $x_s(t)$



Sistem

Sistem

- Giriş işaretini işleyip çıkış işareti oluşturma
  - ♦ Fiziksel
  - ♦ Matematiksel
  - ♦ Bilişimsel

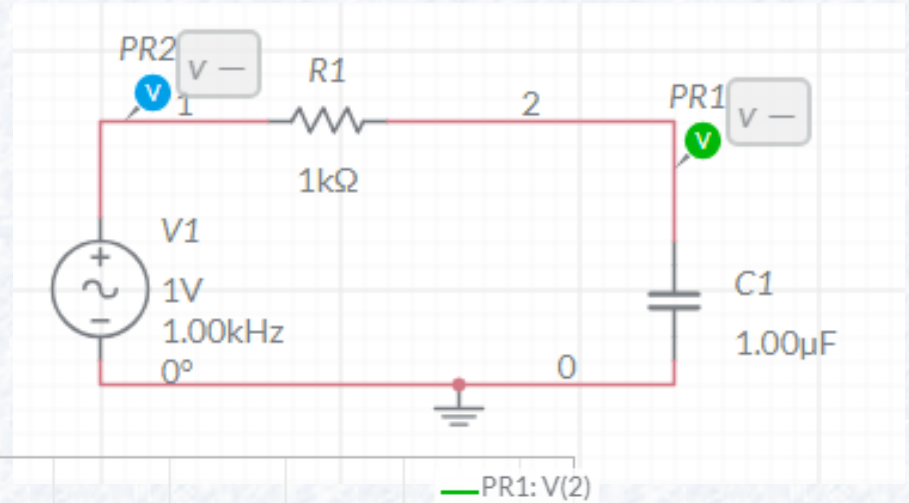


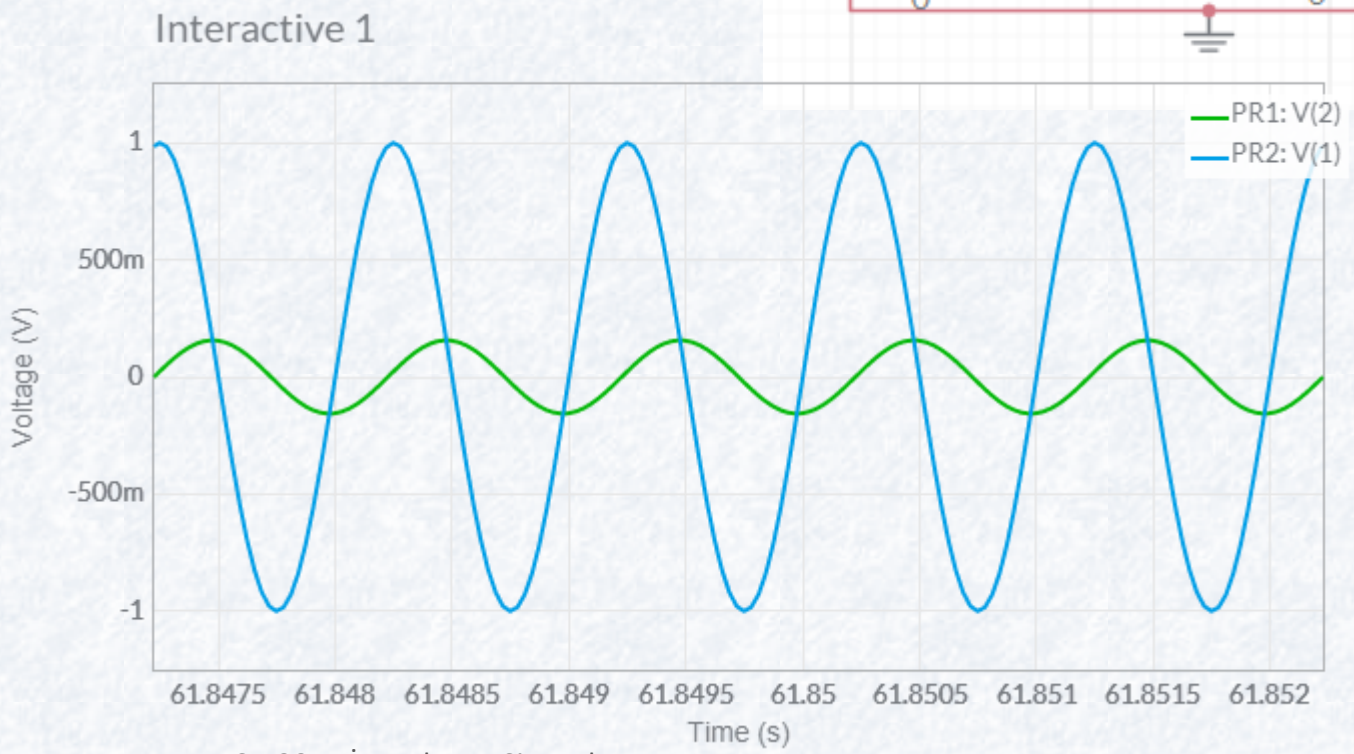
Sistem Türleri

- Sürekli Zaman Sistemler
  - RC devreleri (Voltaj, akım…)
  - Mekanik sistemler
    - Yay
    - Havuz
    - Taşıt

Sistem

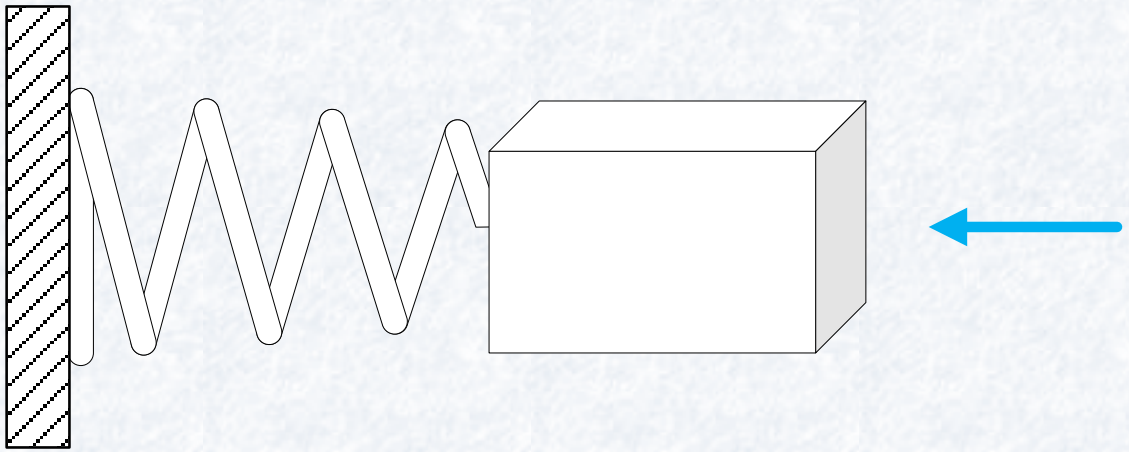
- RC devreleri (Voltaj, akım…)
  - ♦ Giriş: AC voltaj kaynağı
  - ♦ Çıkış: Kapasitör gerilimi

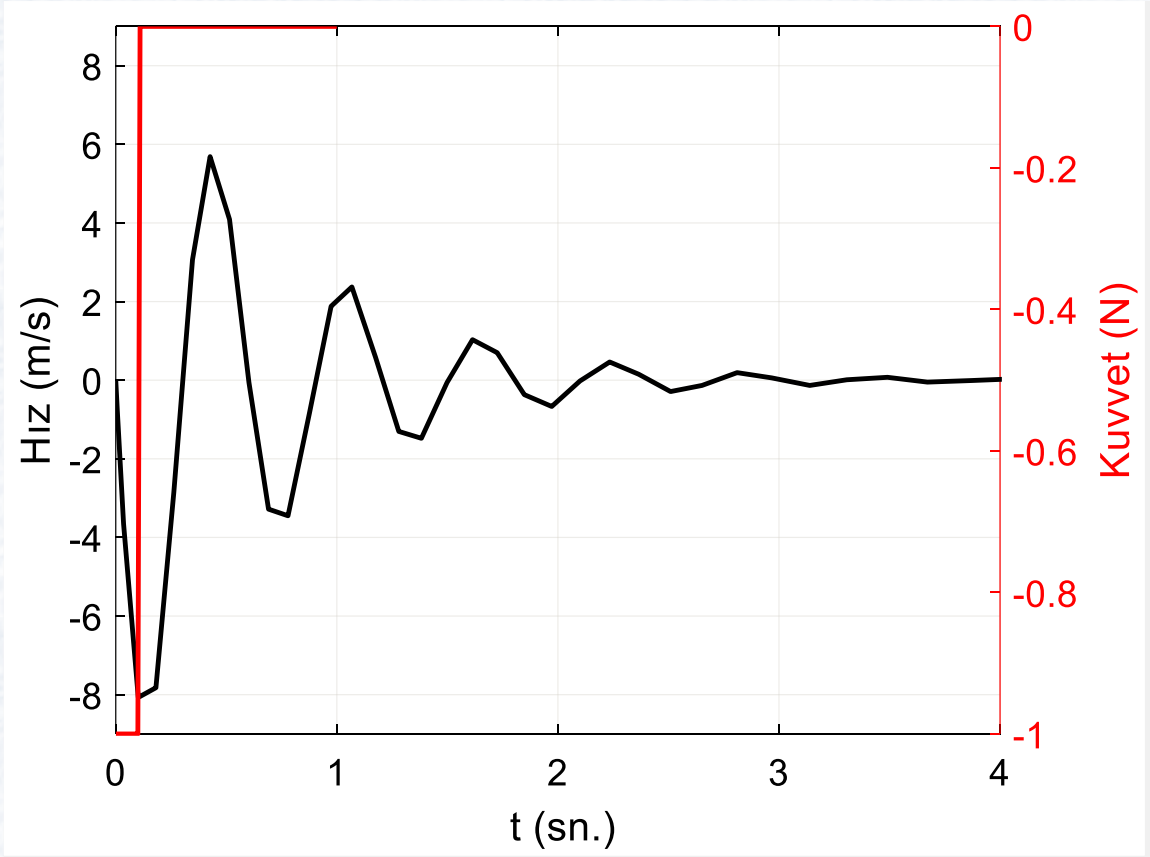




Sistem

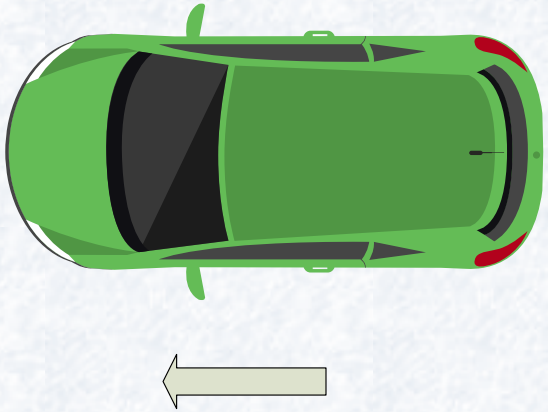
- Kütle yay sistemleri
  - ♦ Giriş: Uygulanan kuvvet
  - ♦ Çıkış: Hız (Yer değiştirme)





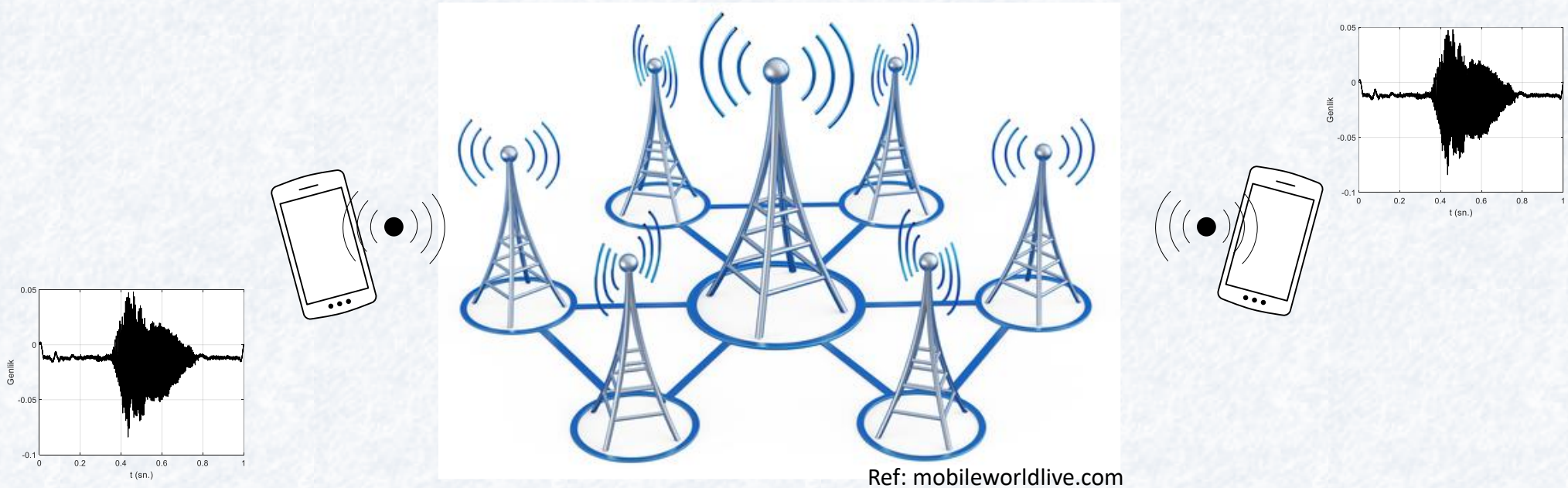
Sistem

- Taşıt
  - ♦ Giriş: Gaz pedal açısı
  - ♦ Çıkış: Hız



Sistem

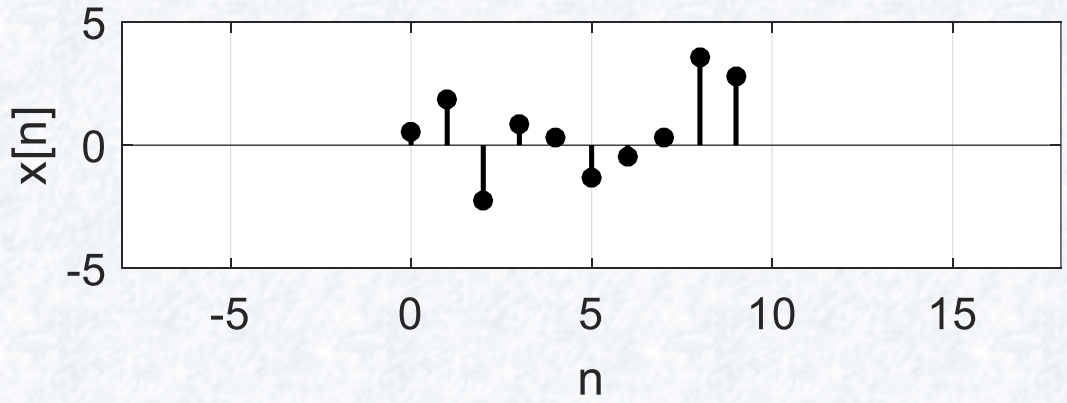
- Cep Telefonu
  - Giriş: Ses
  - Çıkış: Ses



Ref: mobileworldlive.com

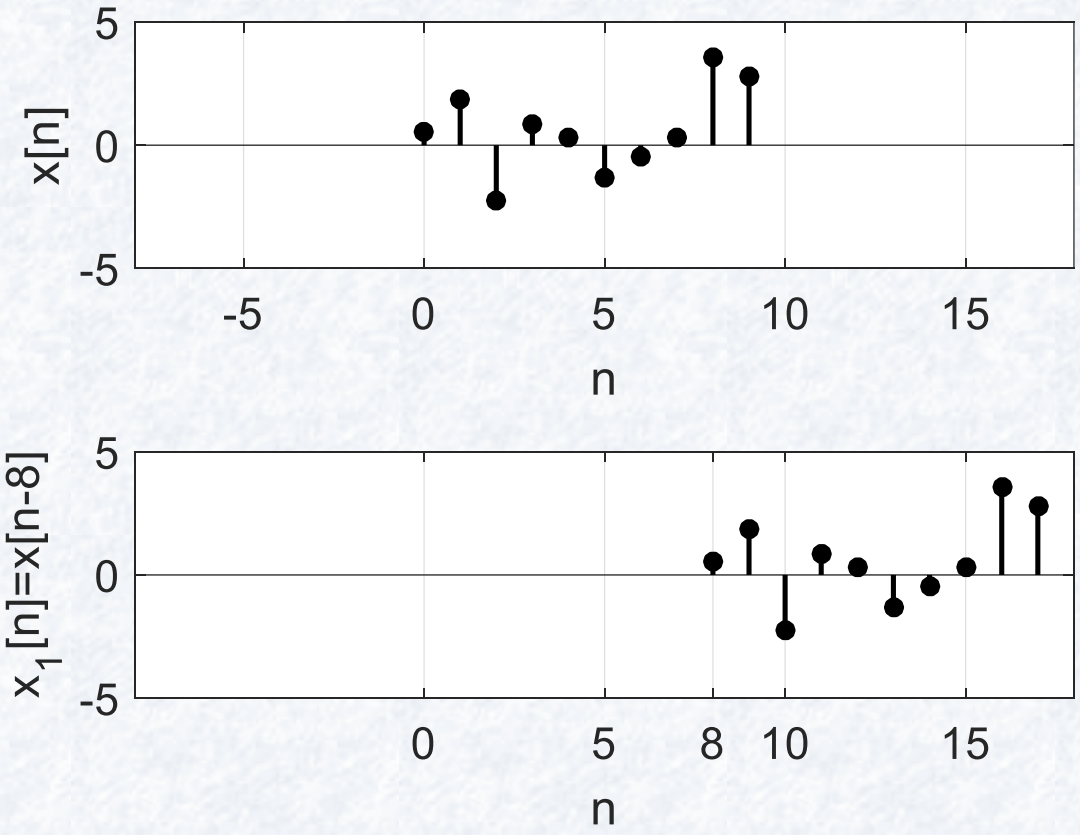
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_1[n] = x[n-8]$



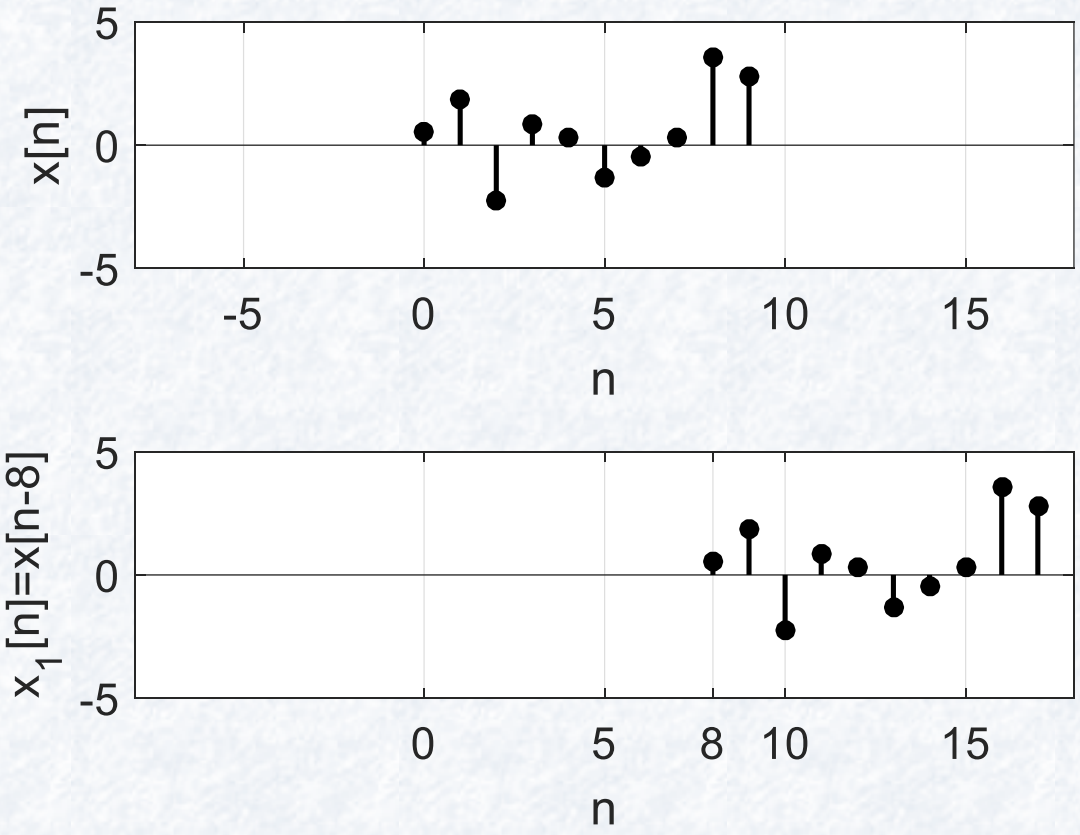
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_1[n] = x[n-8]$



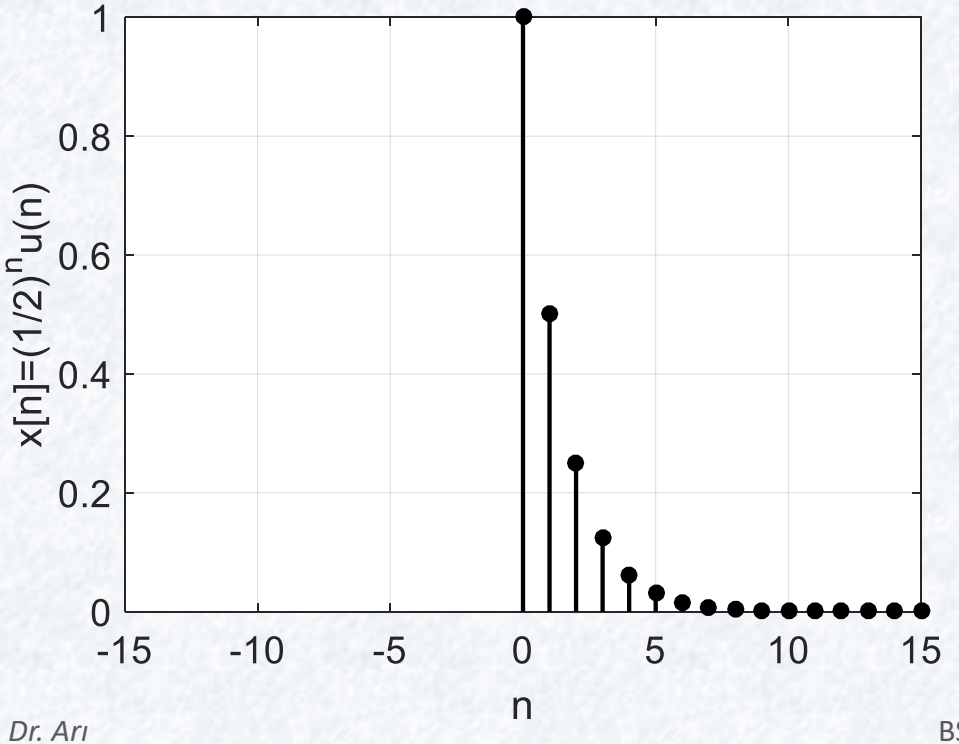
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_1[n] = x[n-8]$
    - Geçmiş



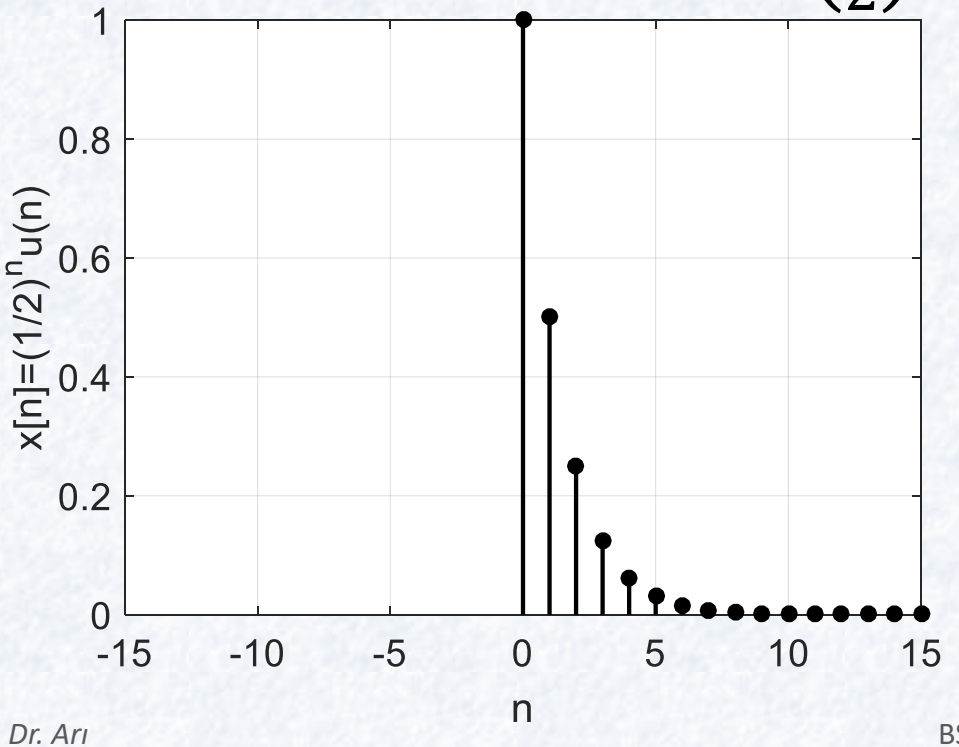
Örnek 1

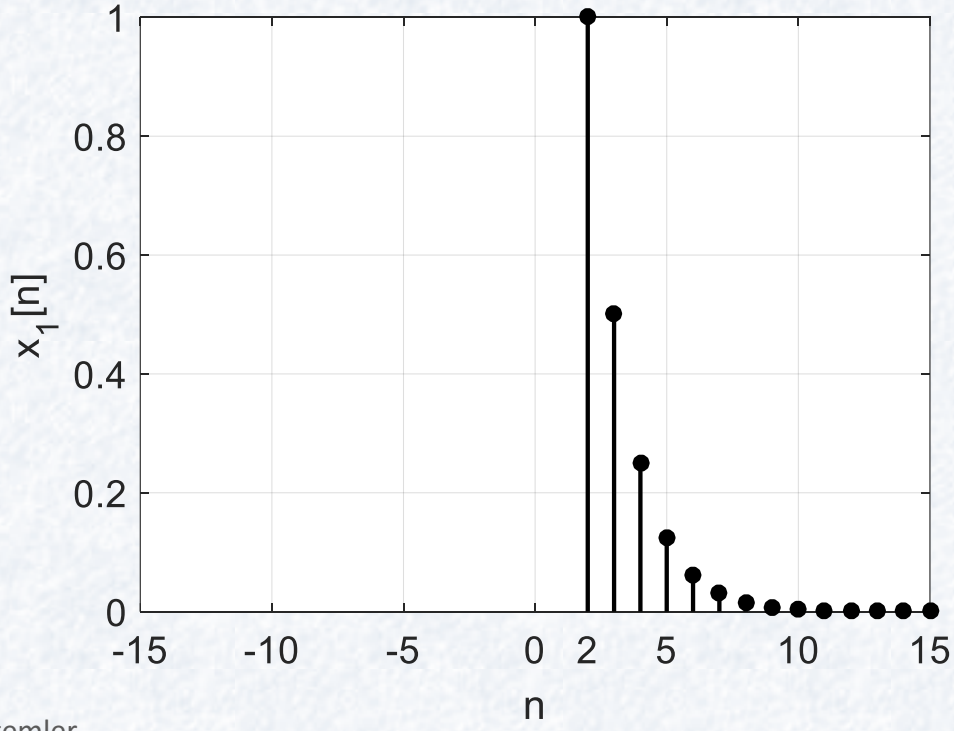
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_1[n] = x[n-2] = ?$



Örnek

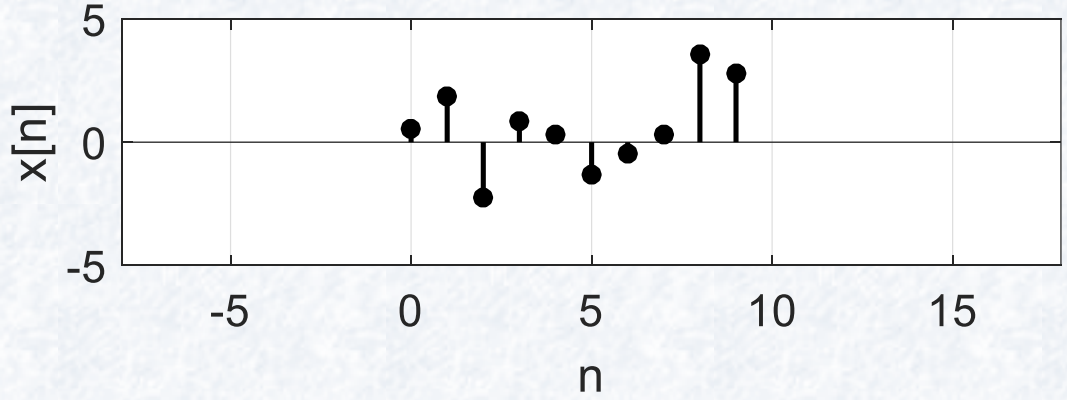
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_1[n] = x[n-2] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$





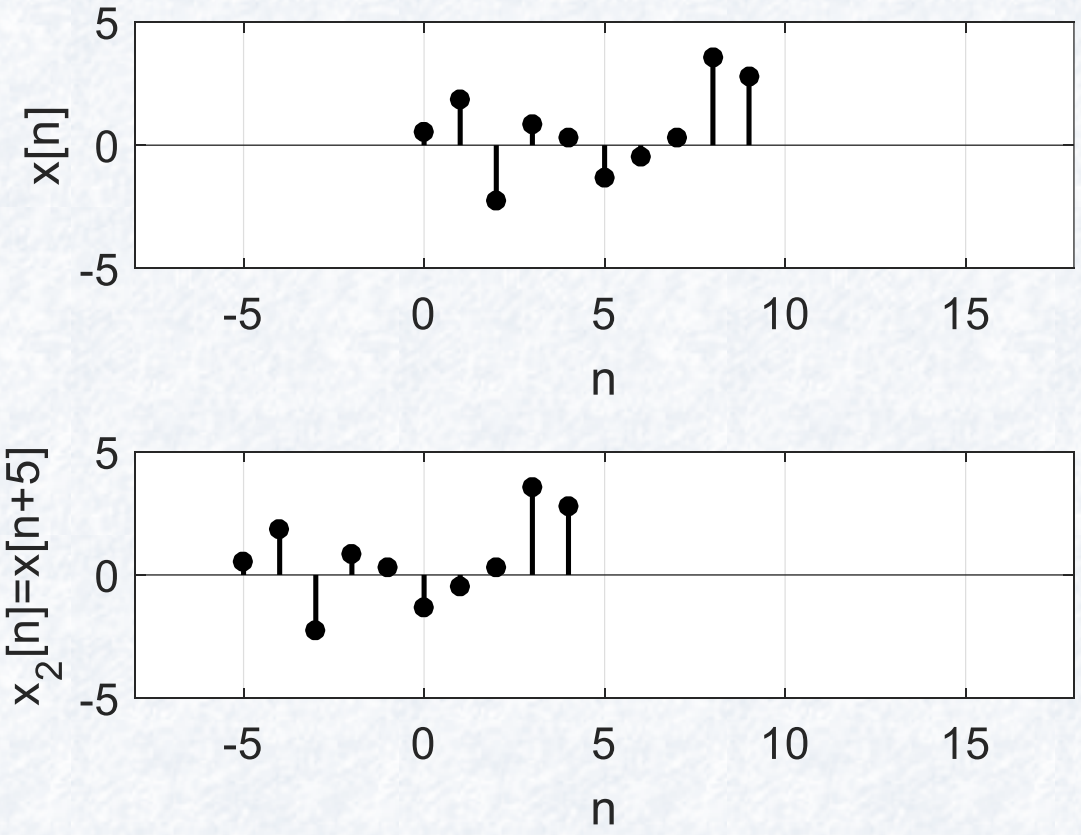
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_2[n] = x[n+5]$



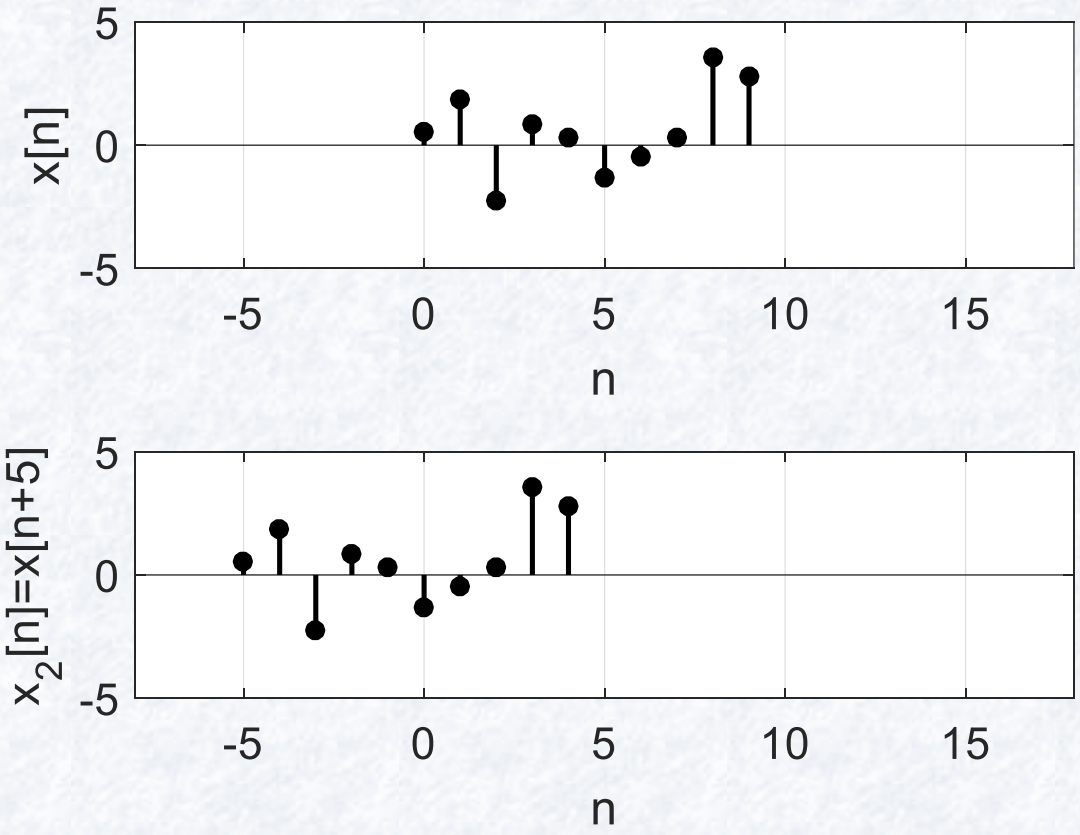
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_2[n] = x[n+5]$



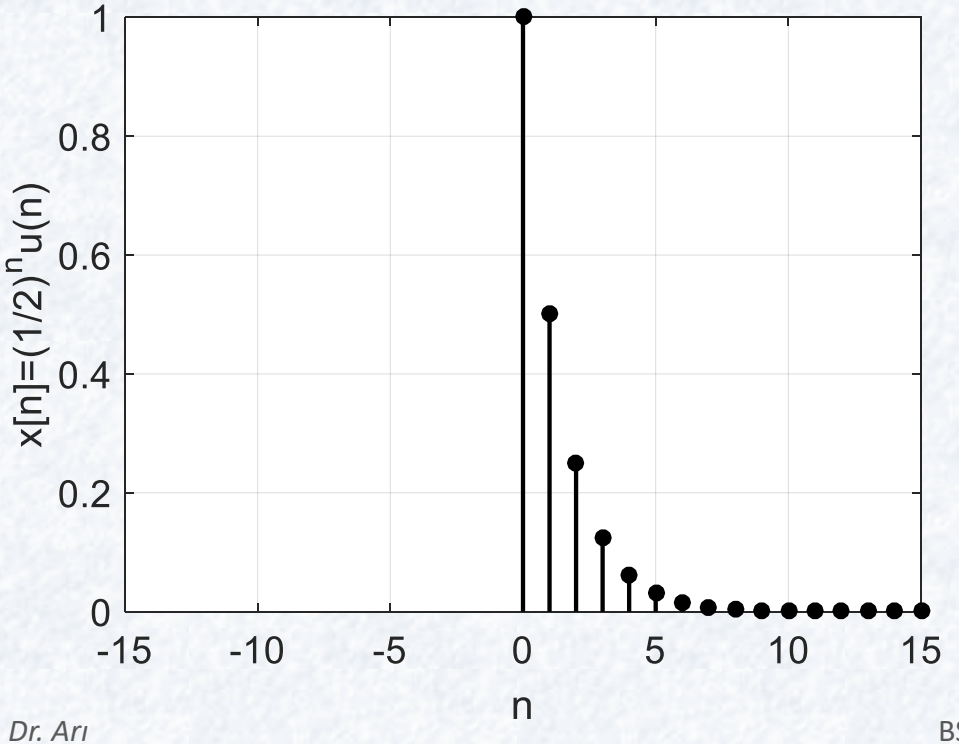
Bağımsız Değişken Dönüşümü

- Zamanda Öteleme
  - $x_2[n] = x[n+5]$
    - Gelecek



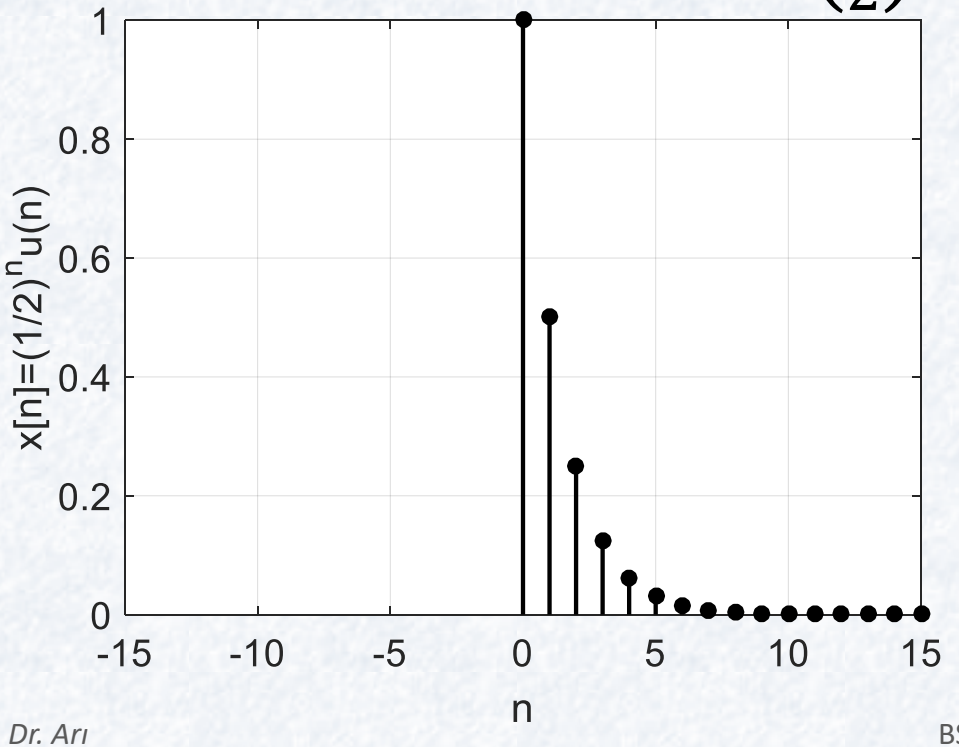
Örnek 2

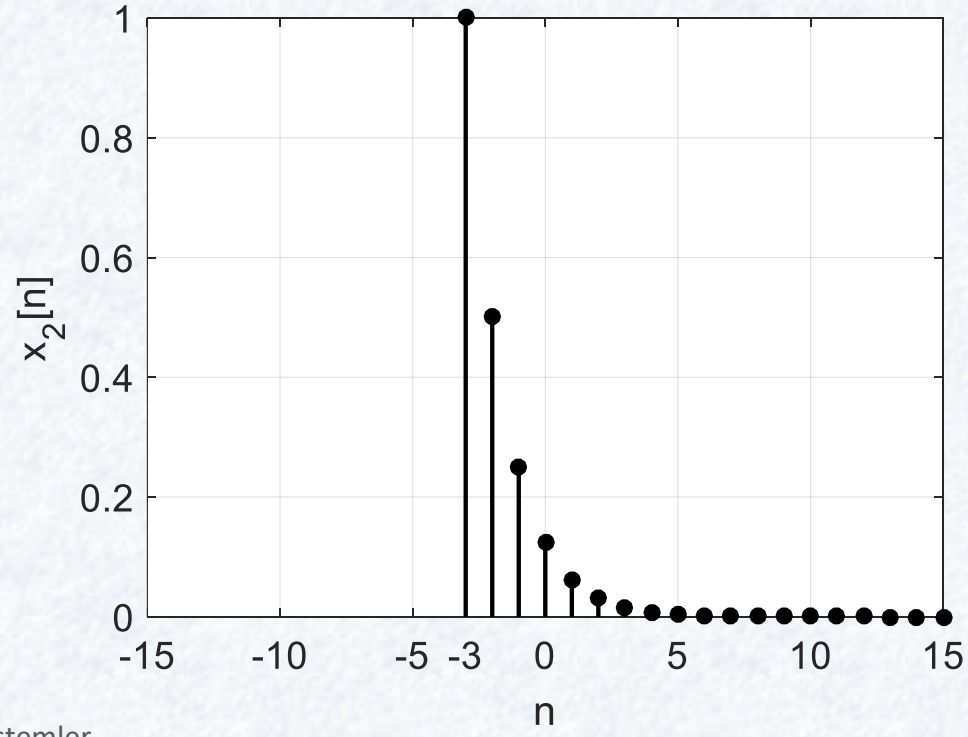
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_2[n] = x[n+3] = ?$



Örnek

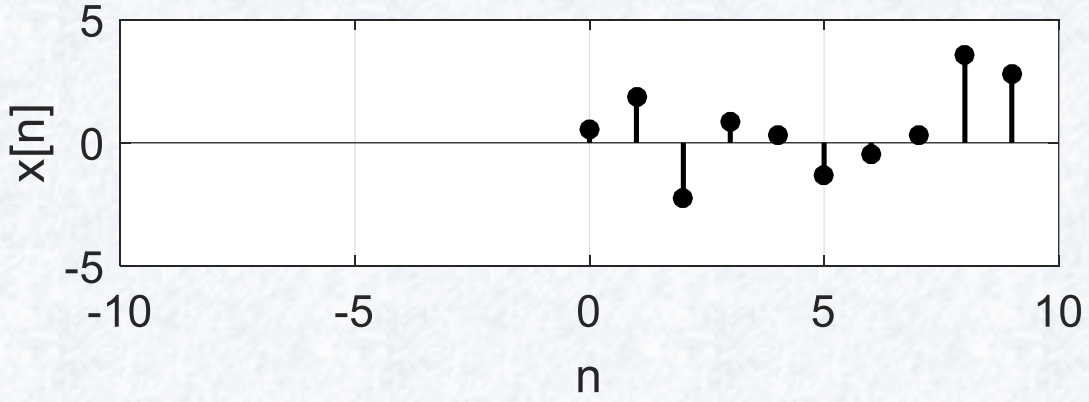
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_2[n] = x[n+3] = \left(\frac{1}{2}\right)^{n+3} u(n+3)$





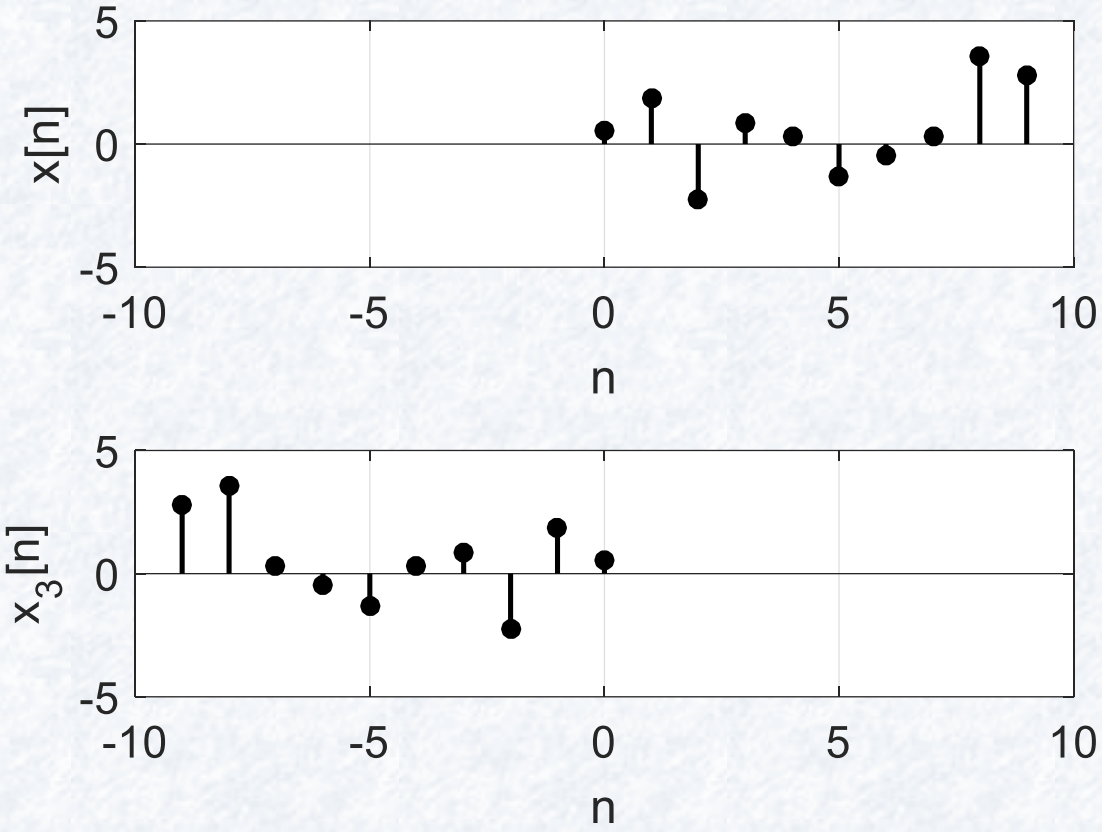
Bağımsız Değişken Dönüşümü

- Zamanda Ters Çevirme
  - $x_3[n] = x[-n]$



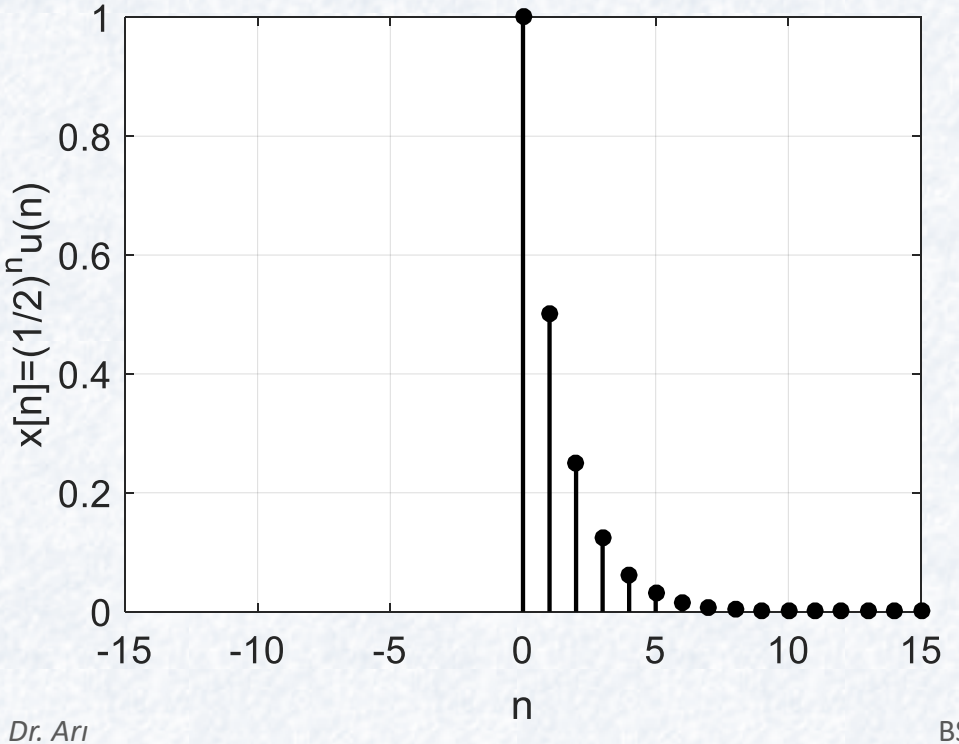
Bağımsız Değişken Dönüşümü

- Zamanda Ters Çevirme
  - $x_3[n] = x[-n]$



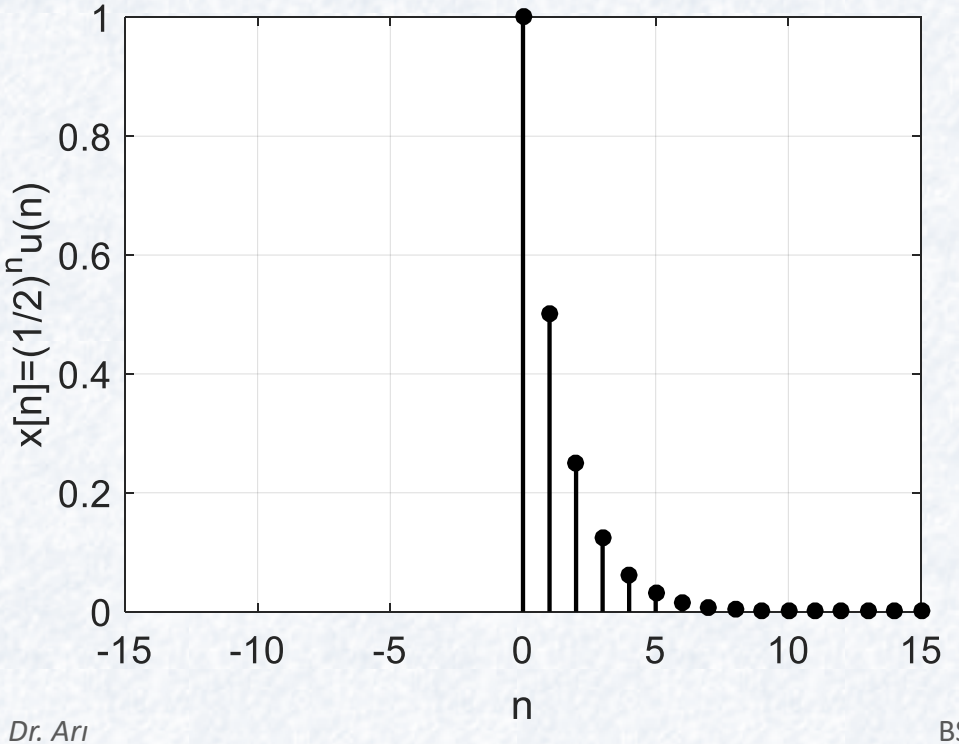
Örnek 3

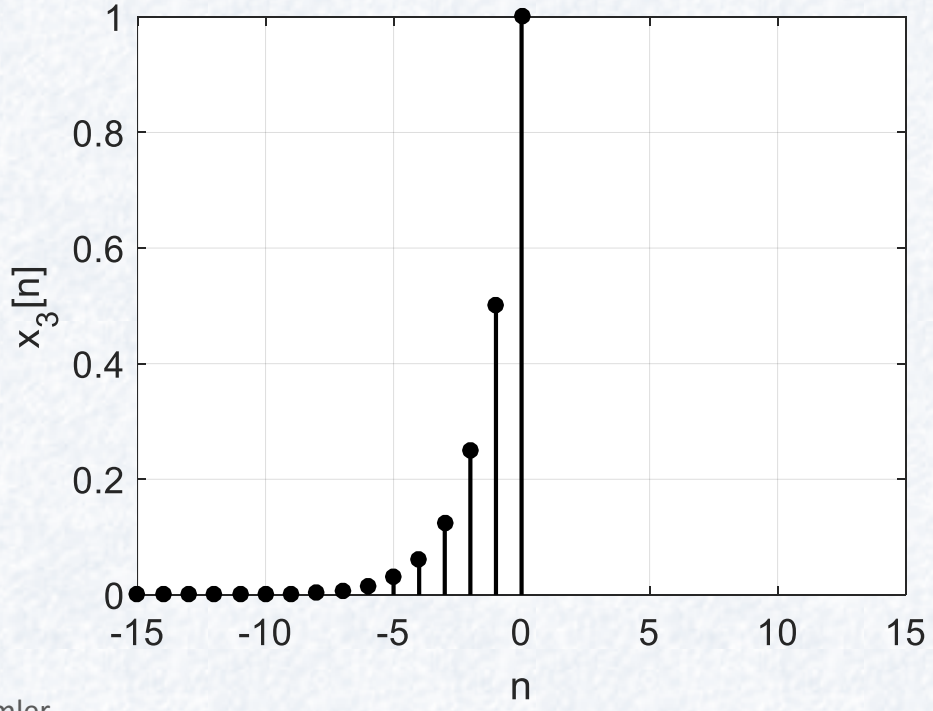
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = ?$



Örnek 3

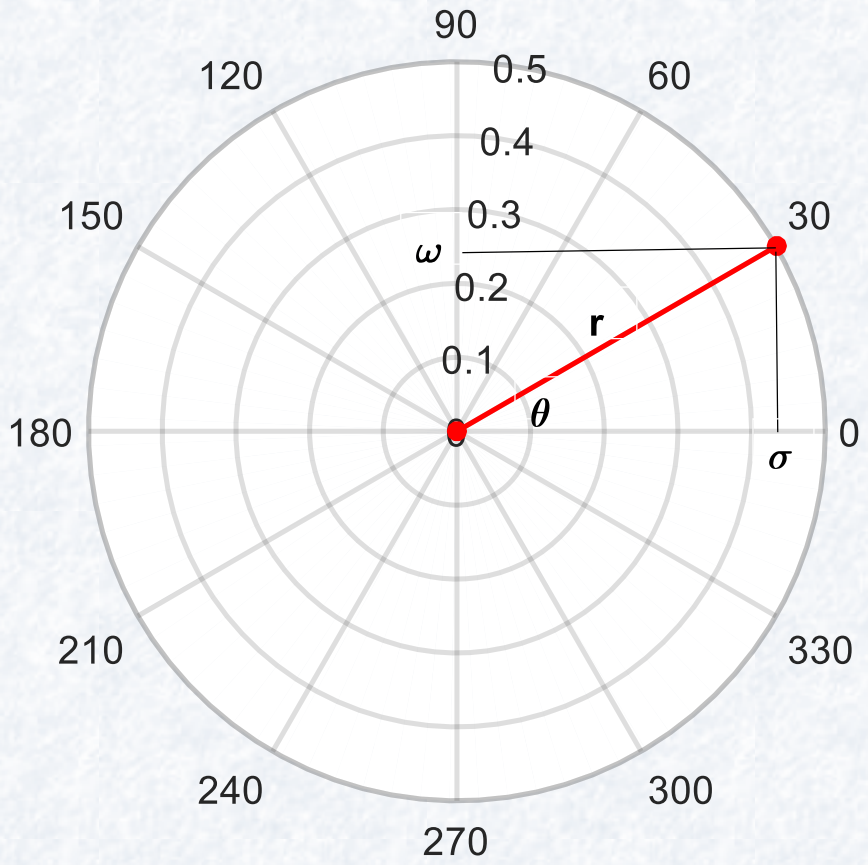
- $x[n] = \left(\frac{1}{2}\right)^n u(n)$
- $x_3[n] = x[-n] = 2^n u(-n)$





Karmaşık sayılar

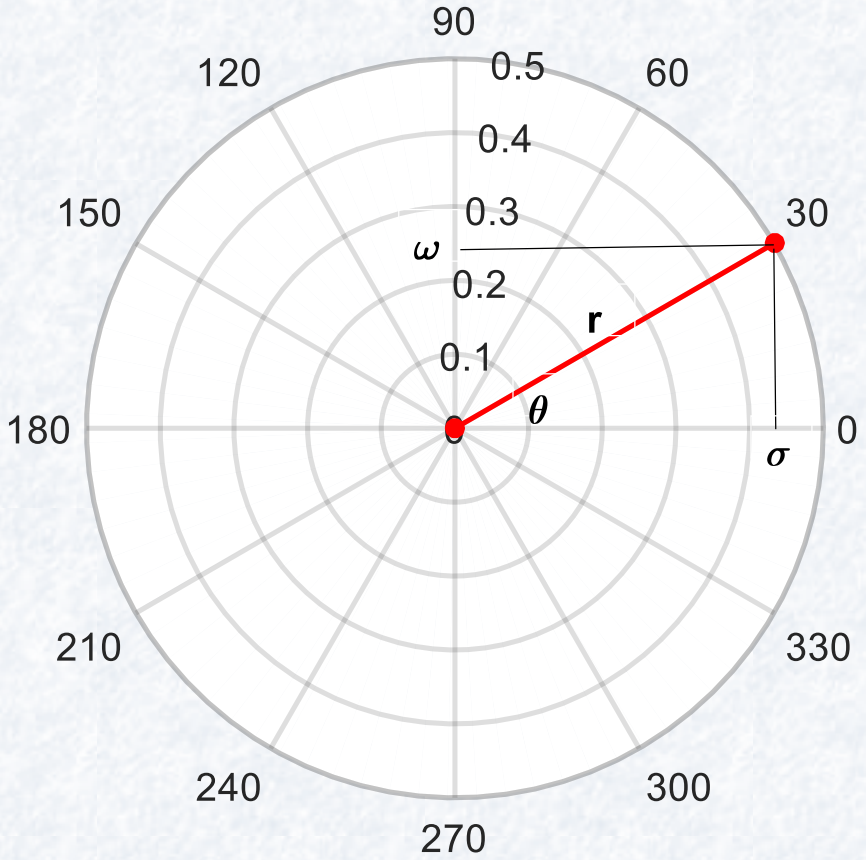
- $\sigma + j\omega$
  - $r = ?$
  - $\sigma = ?$
  - $\omega = ?$



Karmaşık sayılar

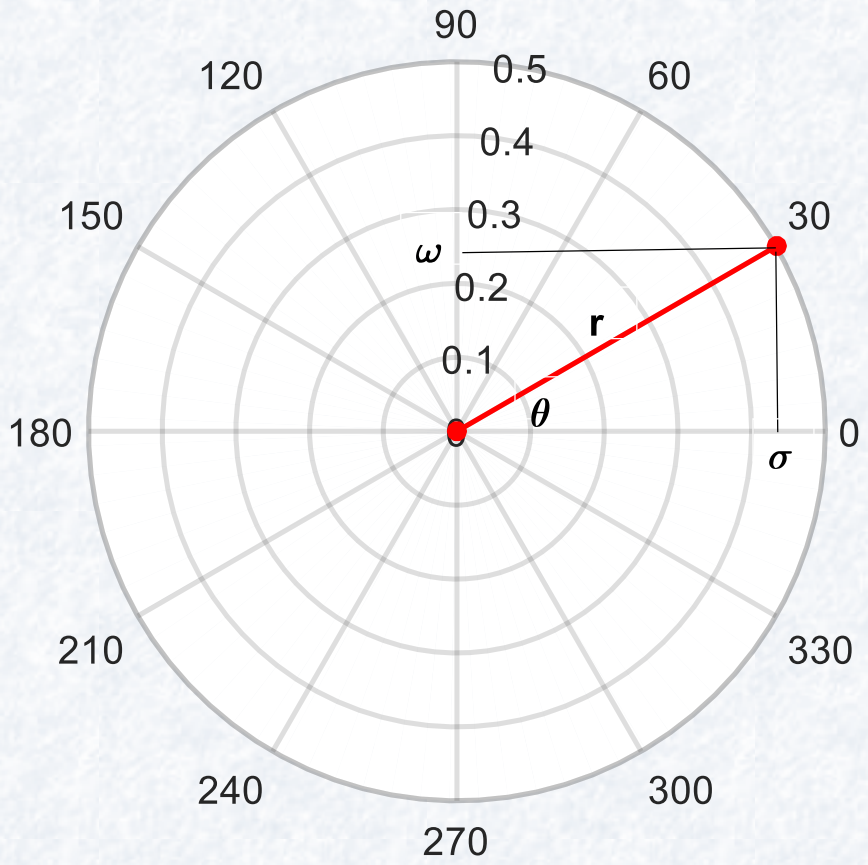
- $\sigma + j\omega$
  - $r = \sqrt{\sigma^2 + \omega^2}$
  - $\sigma = r\cos(\theta)$
  - $\omega = r\sin(\theta)$

- $\sigma + j\omega = r\cos(\theta) + jr\sin(\theta)$
- $\sigma + j\omega = r(\cos(\theta) + j\sin(\theta))$
- $\sigma - j\omega = r(\cos(-\theta) + j\sin(-\theta))$
- $\sigma - j\omega = r(\cos(\theta) - j\sin(\theta))$



Karmaşık sayılar

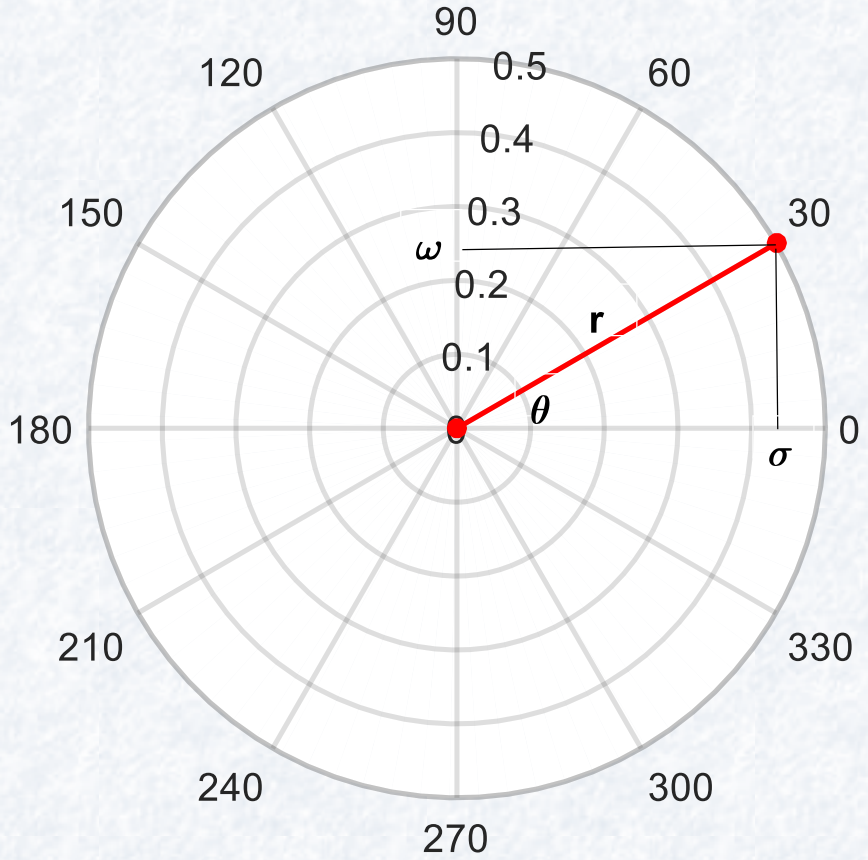
- $f(\theta) = \cos(\theta) + j\sin(\theta)$
- $f'(\theta) = ?$



Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j\sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j\cos(\theta)$

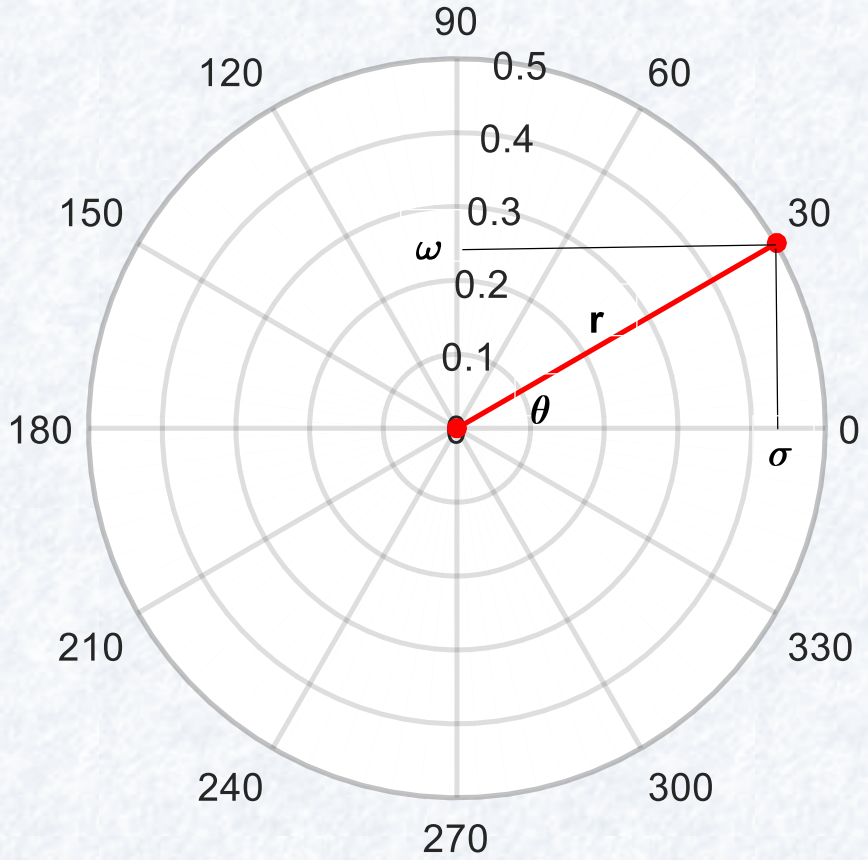
- $f'(\theta) = ?$



Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j\sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j\cos(\theta)$

- $f'(\theta) = jf(\theta)$
- $f(\theta) = ?$

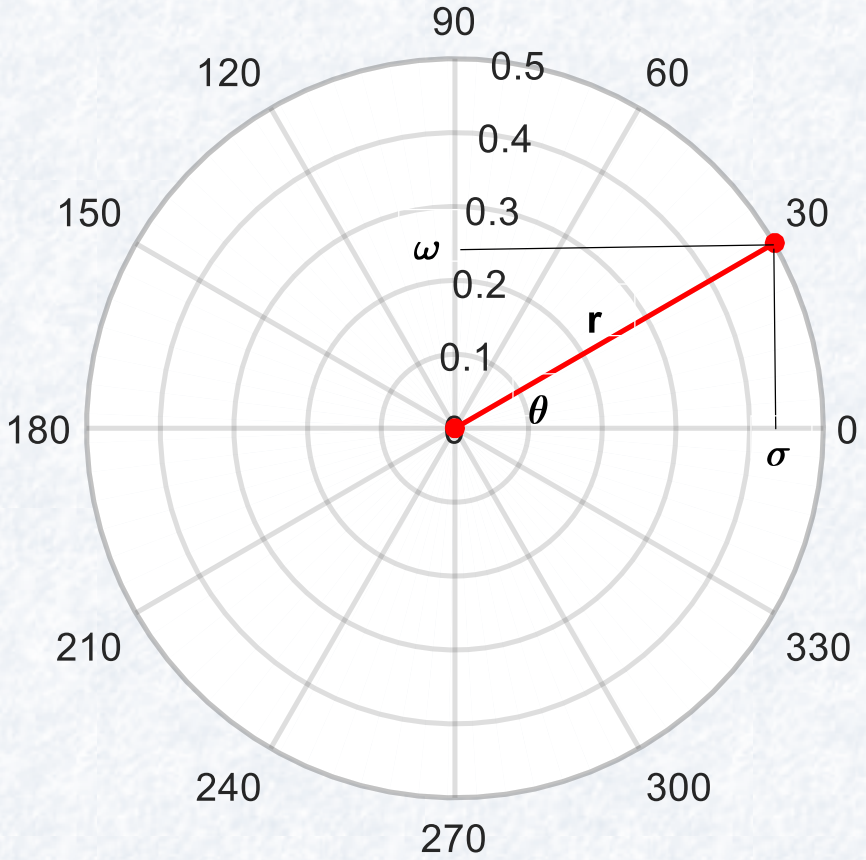


Karmaşık sayılar

- $f(\theta) = \cos(\theta) + j\sin(\theta)$
- $f'(\theta) = -\sin(\theta) + j\cos(\theta)$

- $f'(\theta) = jf(\theta)$
- $f(\theta) = e^{j\theta}$

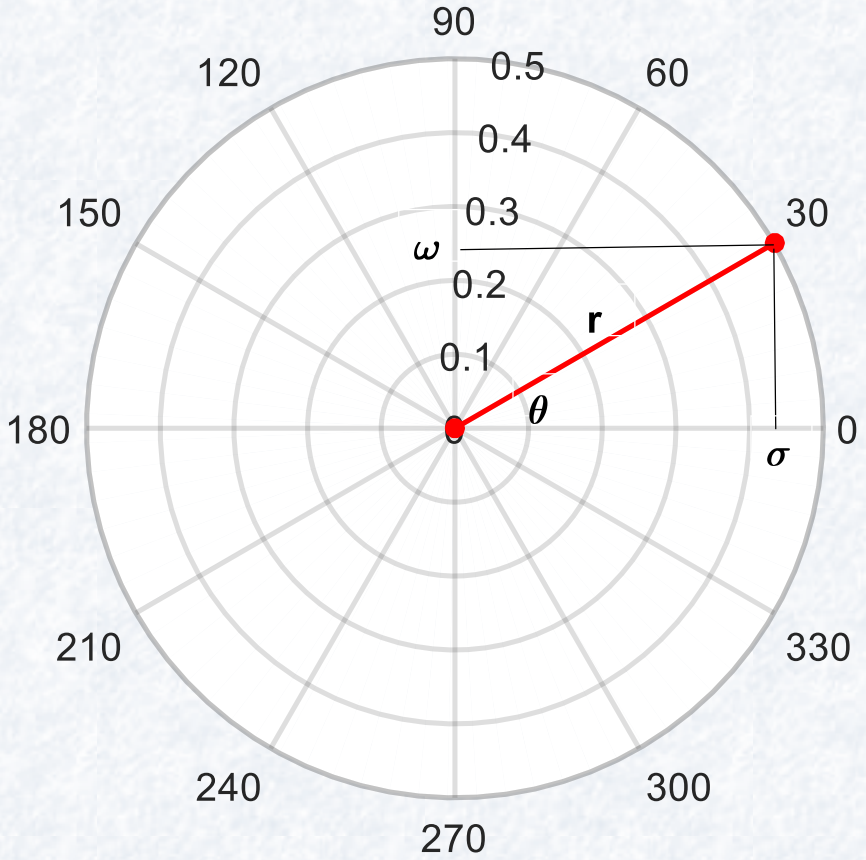
- $\boldsymbol{\sigma + j\omega = re^{j\theta}}$
- $\boldsymbol{\sigma - j\omega = re^{-j\theta}}$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

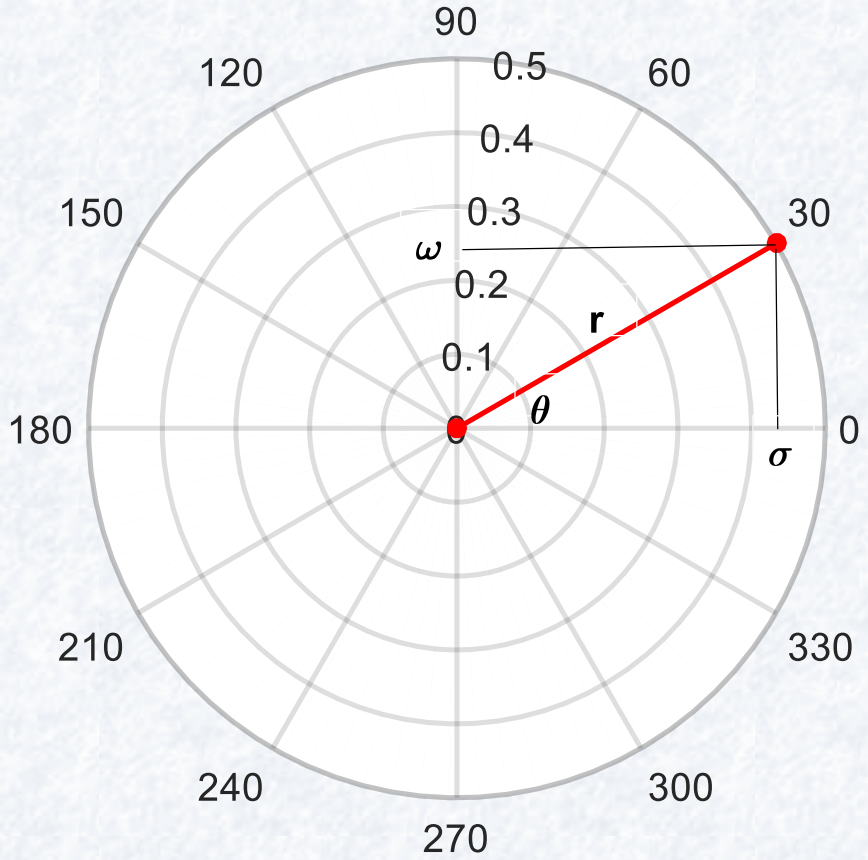
- $\cos(\theta) = ?$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

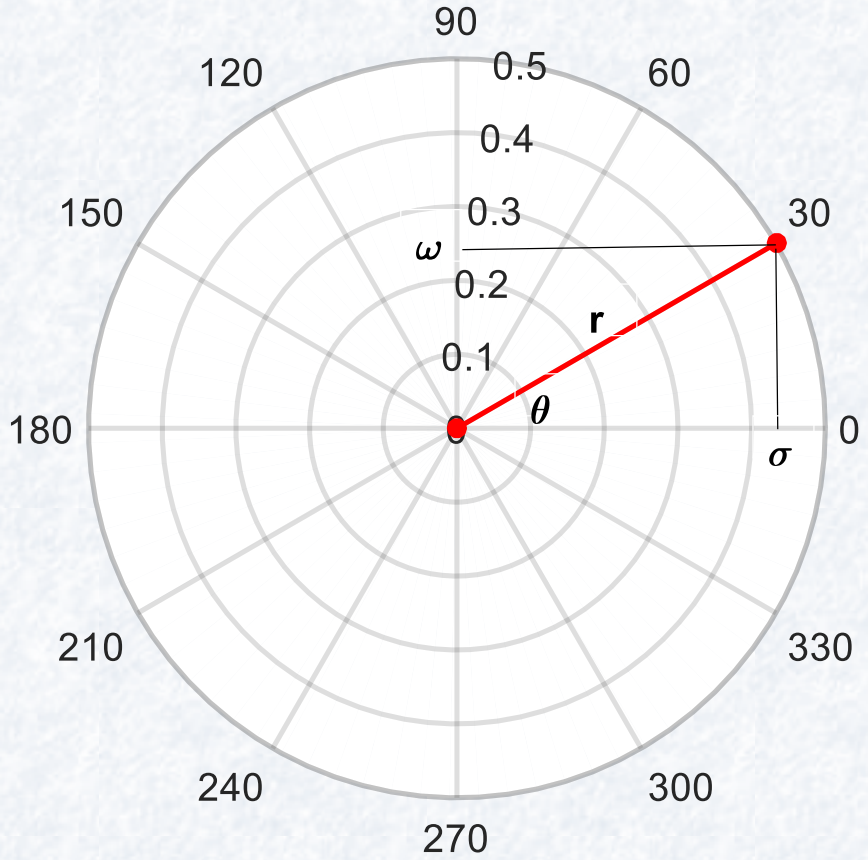
- $e^{j\theta} + e^{-j\theta} = ?$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

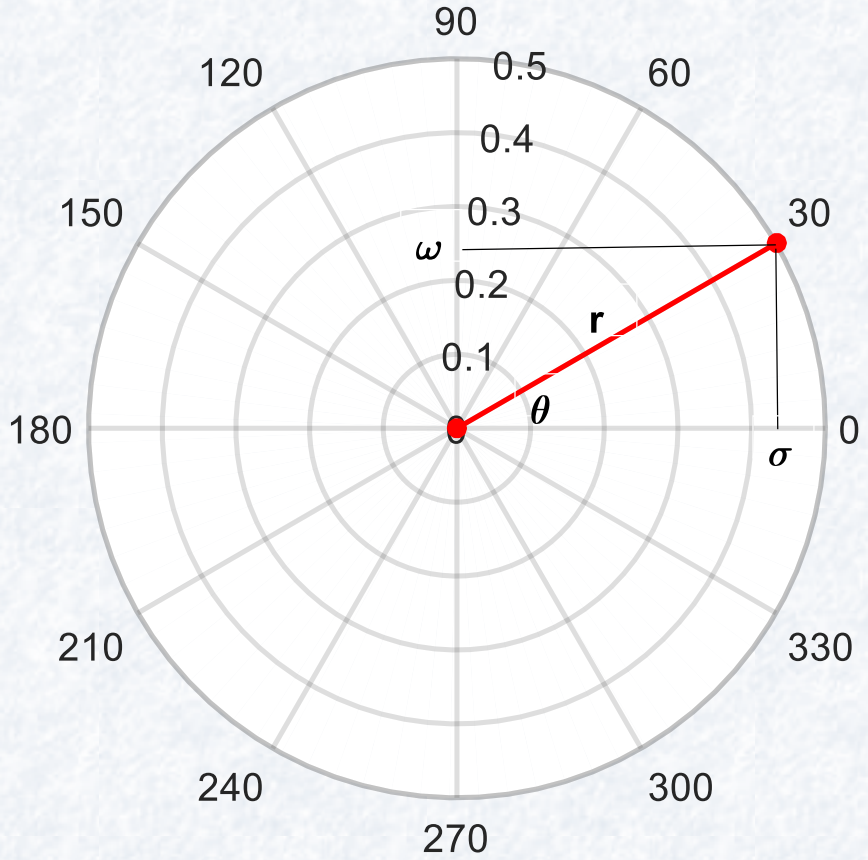
- $e^{j\theta} + e^{-j\theta} = 2\cos(\theta)$
- $\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

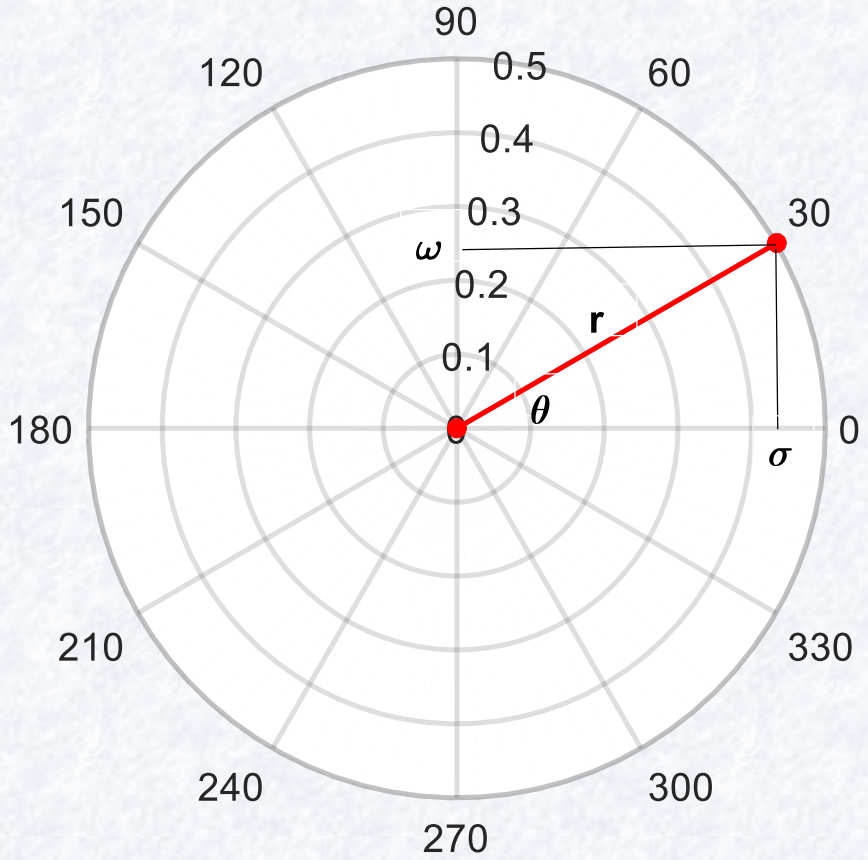
- $\sin(\theta) = ?$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

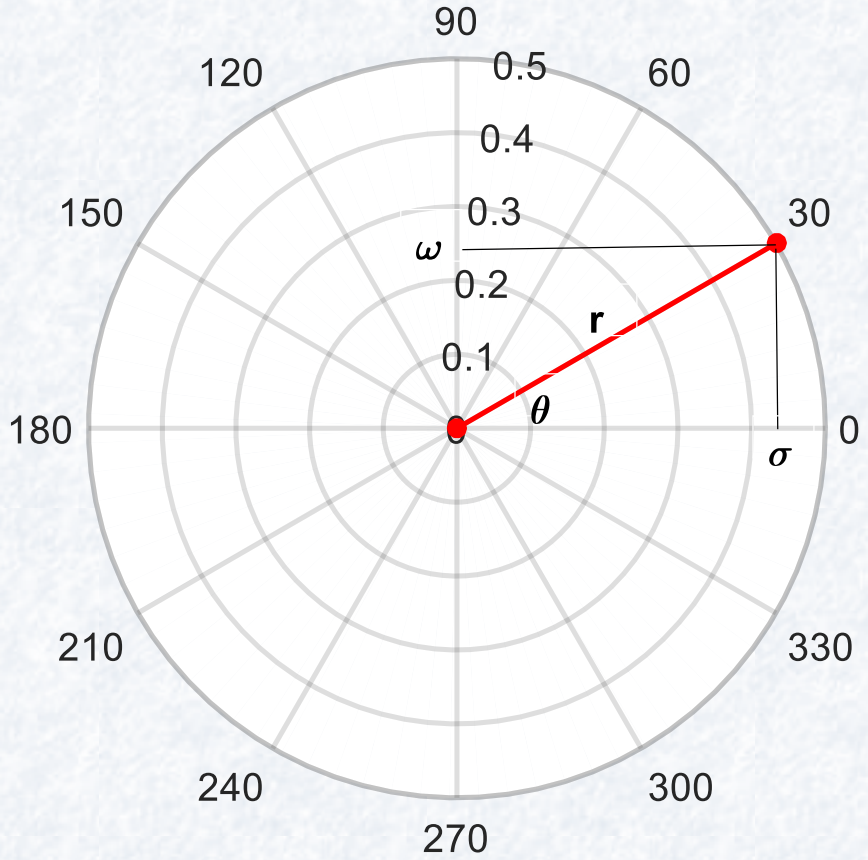
- $e^{j\theta} - e^{-j\theta} = ?$



Karmaşık sayılar

- $e^{j\theta} = \cos(\theta) + j\sin(\theta)$
- $e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

- $e^{j\theta} - e^{-j\theta} = 2j\sin(\theta)$
- $\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$



Periyodik İşaretler

- $x[n] = x[n+N] = x[n+kN]$
  - Tam sayı bir N>0 değeri var ise $x[n]$ periyodiktir.
  - N örnekte bir genlik tekrar eder.

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n+N]$
- $e^{j\omega_0 n} =$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n+N]$
- $e^{j\omega_0 n} = e^{j\omega_0 (n+N)}$
- $e^{j\omega_0 n} =$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n+N]$
- $e^{j\omega_0 n} = e^{j\omega_0 (n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n+N]$
- $e^{j\omega_0 n} = e^{j\omega_0 (n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$
  - $r = 1$
  - $\theta = 0 = \cdots$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $x[n] = x[n+N]$
- $e^{j\omega_0 n} = e^{j\omega_0 (n+N)}$
- $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 N}$
- $1 = e^{j\omega_0 N}$
- $1 + j0 = ?$
  - $r = 1$
  - $\theta = 0 = 2\pi = 4\pi = \cdots = 2\pi k$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $N = ?$

Periyodik İşaretler

- $x[n] = e^{j\omega_0 n}$
- $1 = e^{j\omega_0 N}$
- $1 = 1e^{j2\pi k}$
- $1e^{j2\pi k} = e^{j\omega_0 N}$
- $\omega_0 N = 2\pi k$
- $N = \frac{2\pi}{\omega_0} k$
  - ♦ k>0, olabilecek en küçük tam sayı

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$

Örnek 4

- $x[n] = \cos\left(\frac{2\pi}{12}n\right)$ periyodik midir?
- $N = \frac{2\pi}{2\pi/12}k$
- $N = 12k$
- $N = 12$

- $x[0] = x[12] = x[24] = \cdots$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = ?$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N =$

Örnek 5

- $x[n] = \sin\left(\frac{8\pi}{25}n\right)$ periyodik midir?
- $N = \frac{2\pi}{8\pi/25}k$
- $N = \frac{25}{4}k$
- $N = 25$

- $x[0] = x[25] = x[50] = \cdots$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = ?$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- $N =$

Örnek 6

- $x[n] = \cos\left(\frac{n}{6}\right)$ periyodik midir?
- $N = \frac{2\pi}{1/6} k$
- $N = 12\pi k$
- Geçerli bir N değeri yok. Periyodik değil.

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x[n] = x[n + N]$
- $x_1[n] = ?$
- $x_2[n] = ?$

Örnek 7

- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_1[n + mN_2]$
- $x[n] = x[n + N]$

Örnek 7

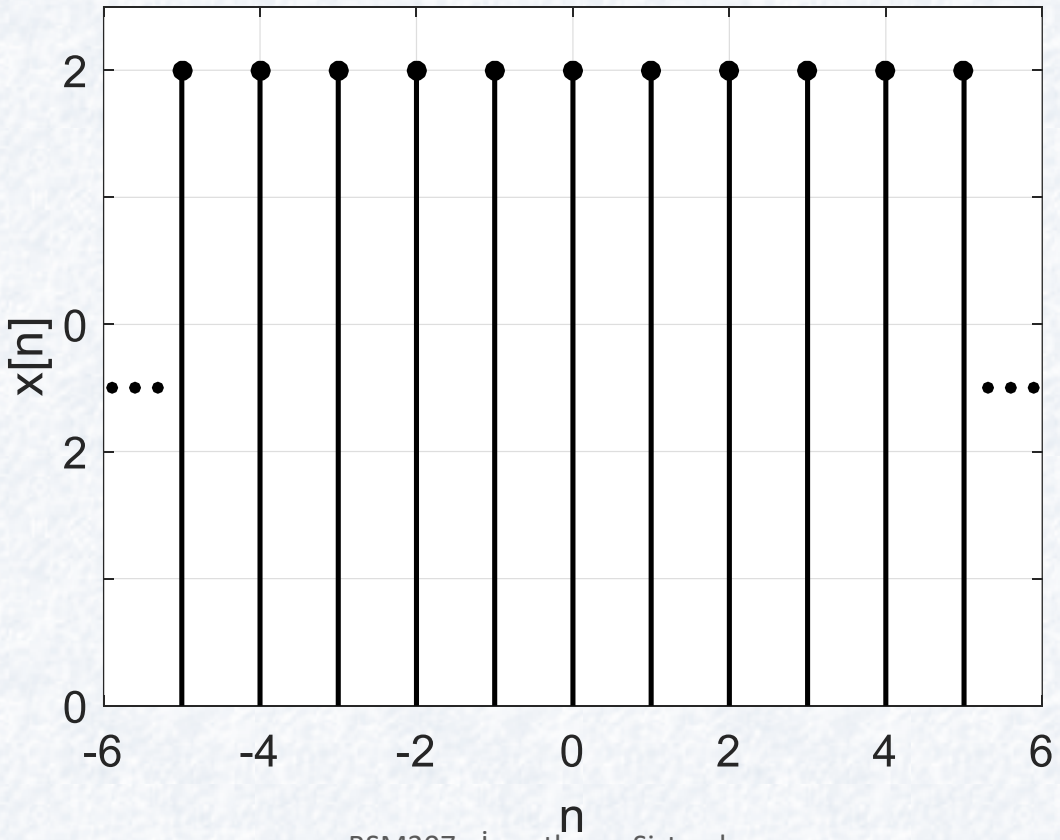
- $x_1[n]$, periyot: N_1
- $x_2[n]$, periyot: N_2
- $x[n] = x_1[n] + x_2[n]$, periyodik midir?
- $x_1[n] = x_1[n + N_1] = x_1[n + kN_1]$
- $x_2[n] = x_2[n + N_2] = x_2[n + mN_2]$
- $x[n] = x[n + N]$
- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$

Örnek 7

- $x_1[n + kN_1] + x_2[n + mN_2] = x_1[n + N] + x_2[n + N]$
- $N = kN_1 = mN_2$
  - EKOK

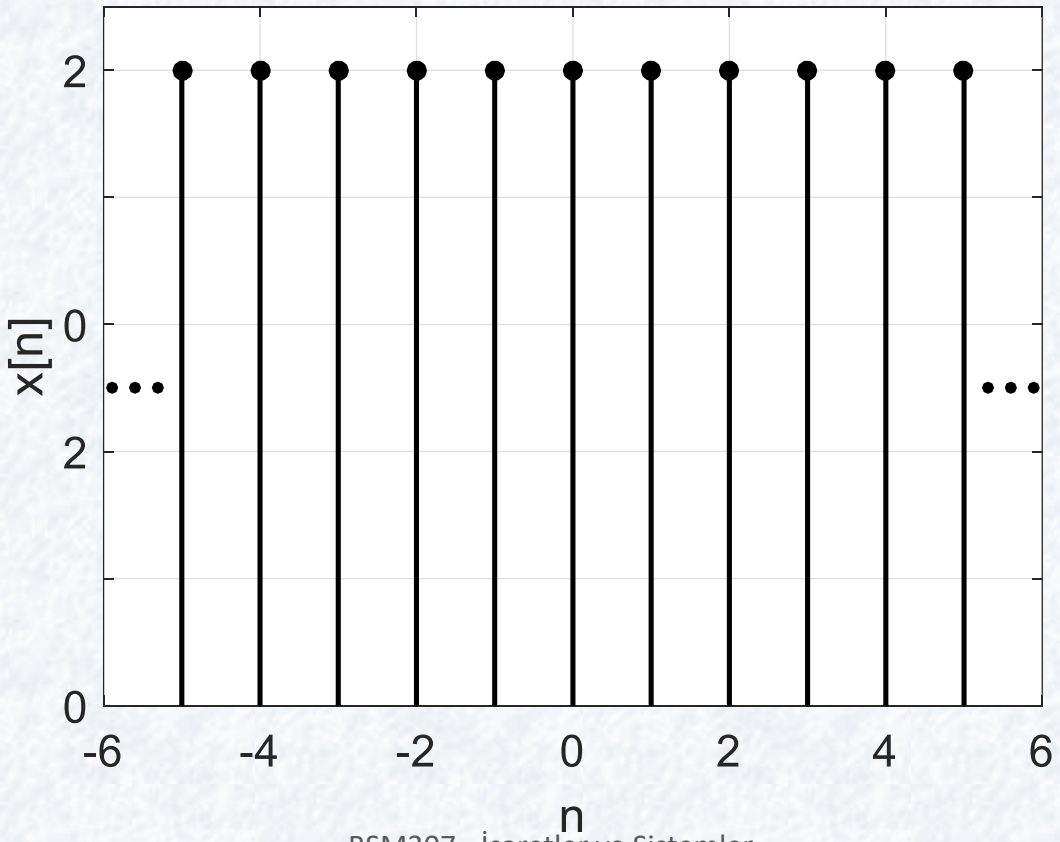
Örnek 8

- $x[n] = 2$, periyodik midir?



Örnek 8

- $x[n] = 2$, periyodik midir?
- $N = 1$



Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] =$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{4}n\right)$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n]$, periyodik midir?

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = ?$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = ?$

Örnek 8

- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$

Örnek 8

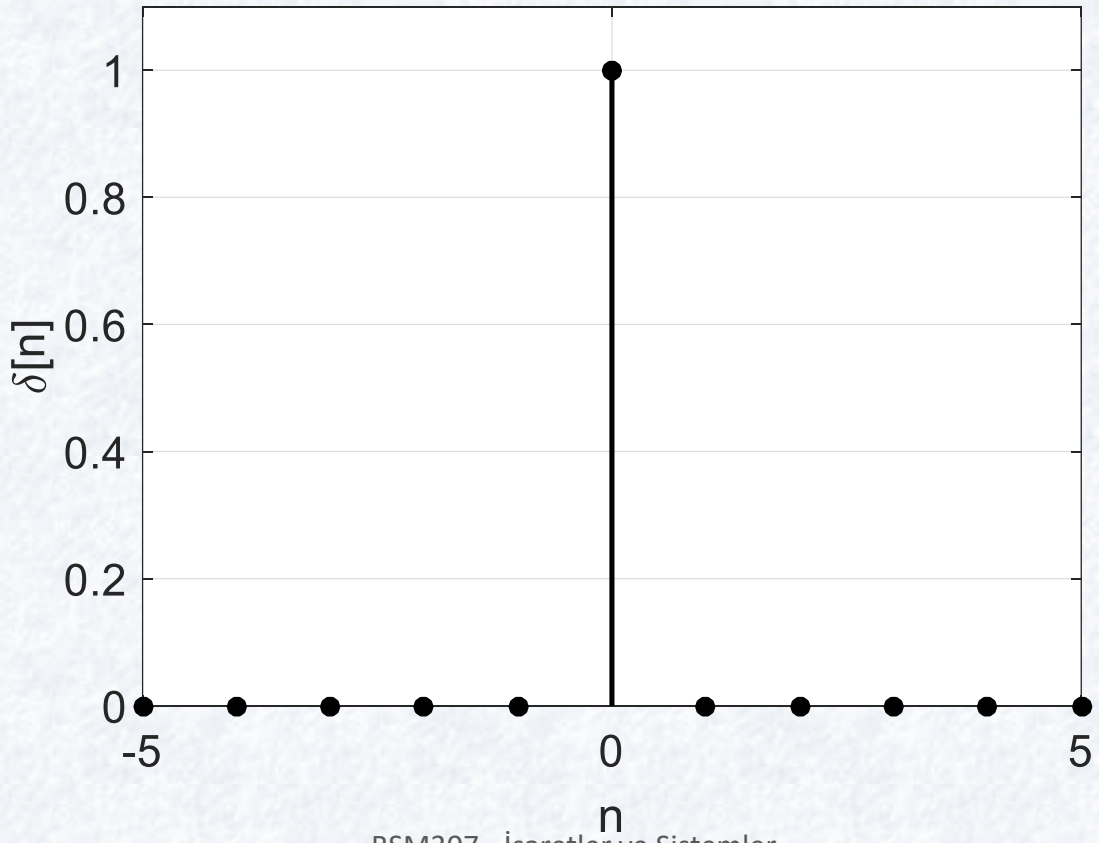
- $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$, periyodik midir?
- $x[n] = \frac{1+\cos\left(\frac{\pi}{4}n\right)}{2} = \underbrace{\frac{1}{2}}_{x_1[n]} + \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{4}n\right)}_{x_2[n]}$
- $x_1[n], N_1 = 1$
- $x_2[n], N_2 = 8$
- $N = k1 = m8 = 8$

Örnek 8

- $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$, periyodik midir?

Birim Darbe İşareti

- $\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$

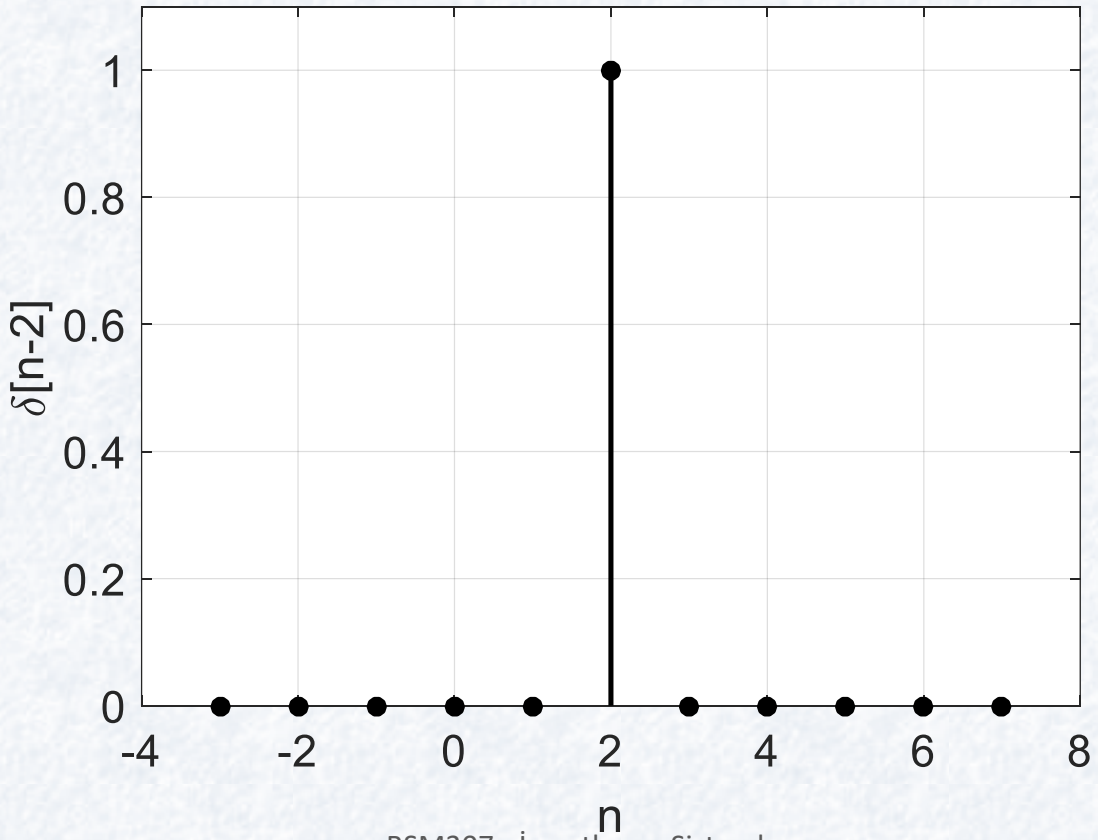


Birim Darbe İşareti

- $\delta[n-2] = ?$

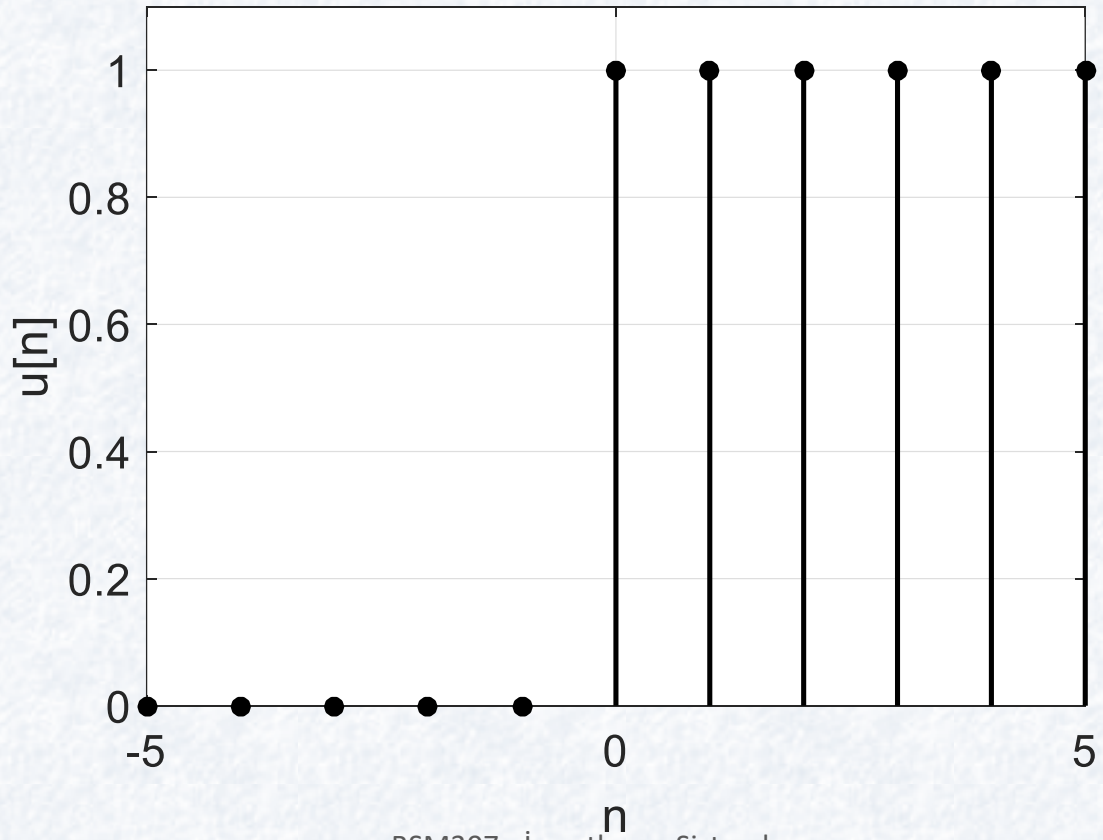
Birim Darbe İşareti

- $\delta[n-2]$



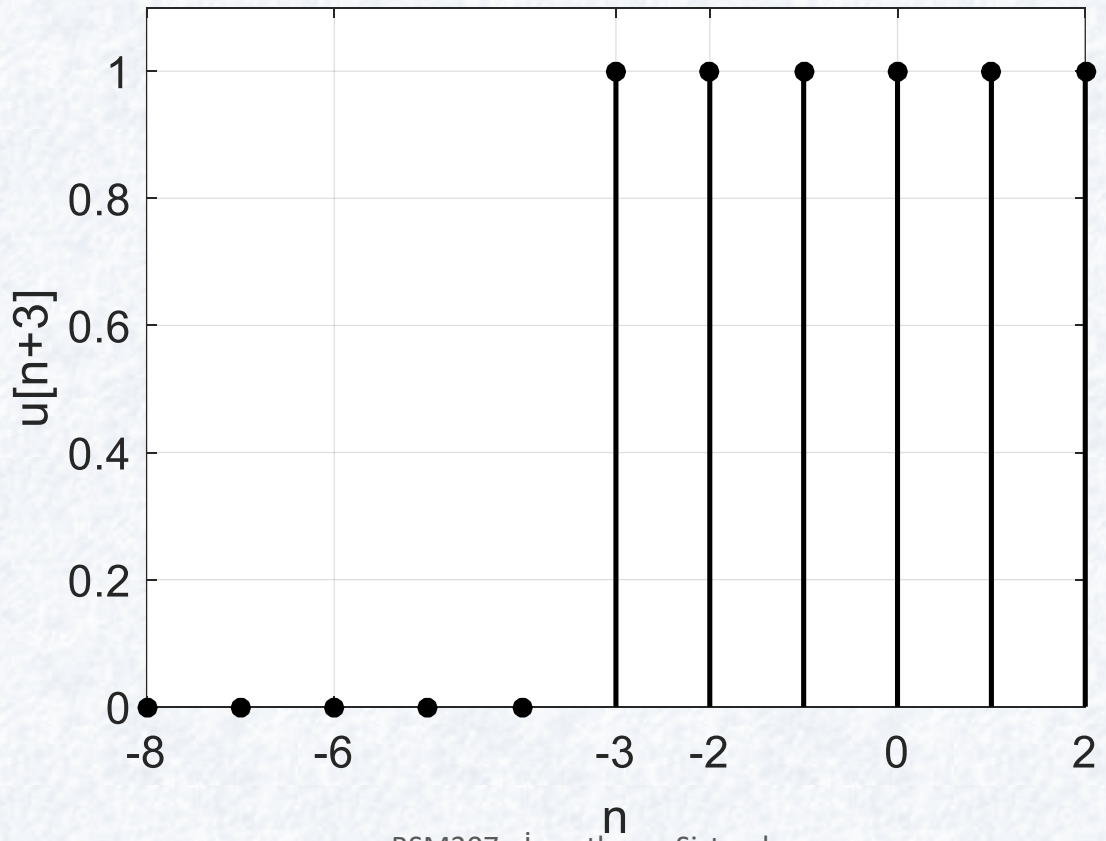
Birim Basamak İşareti

- $u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$



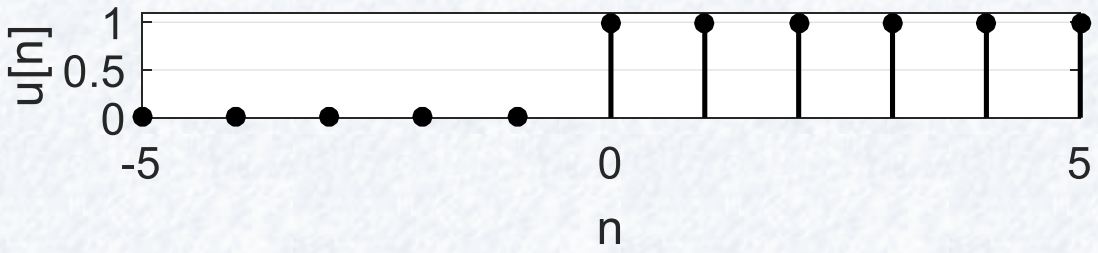
Birim Basamak İşareti

- $u[n+3] = ?$



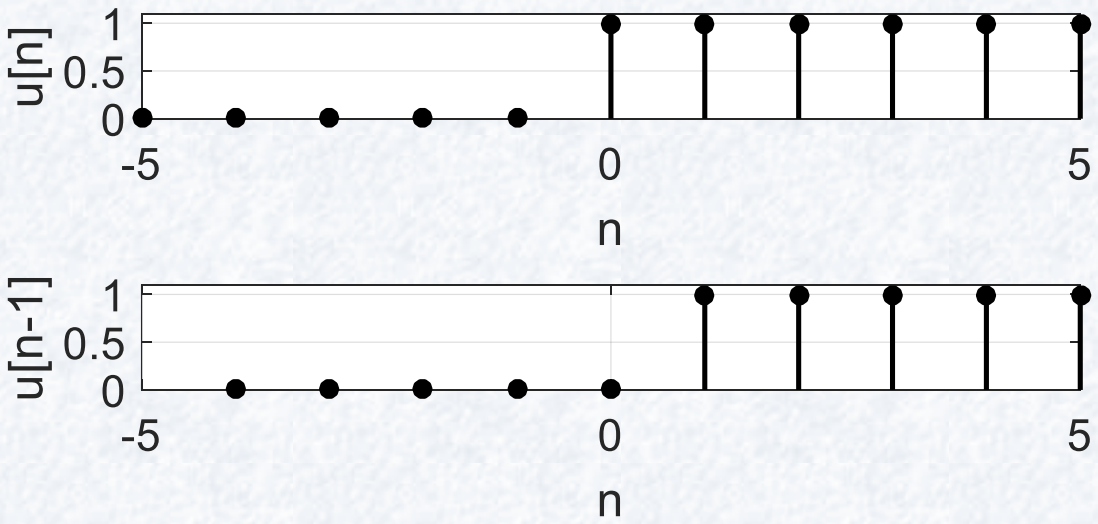
Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



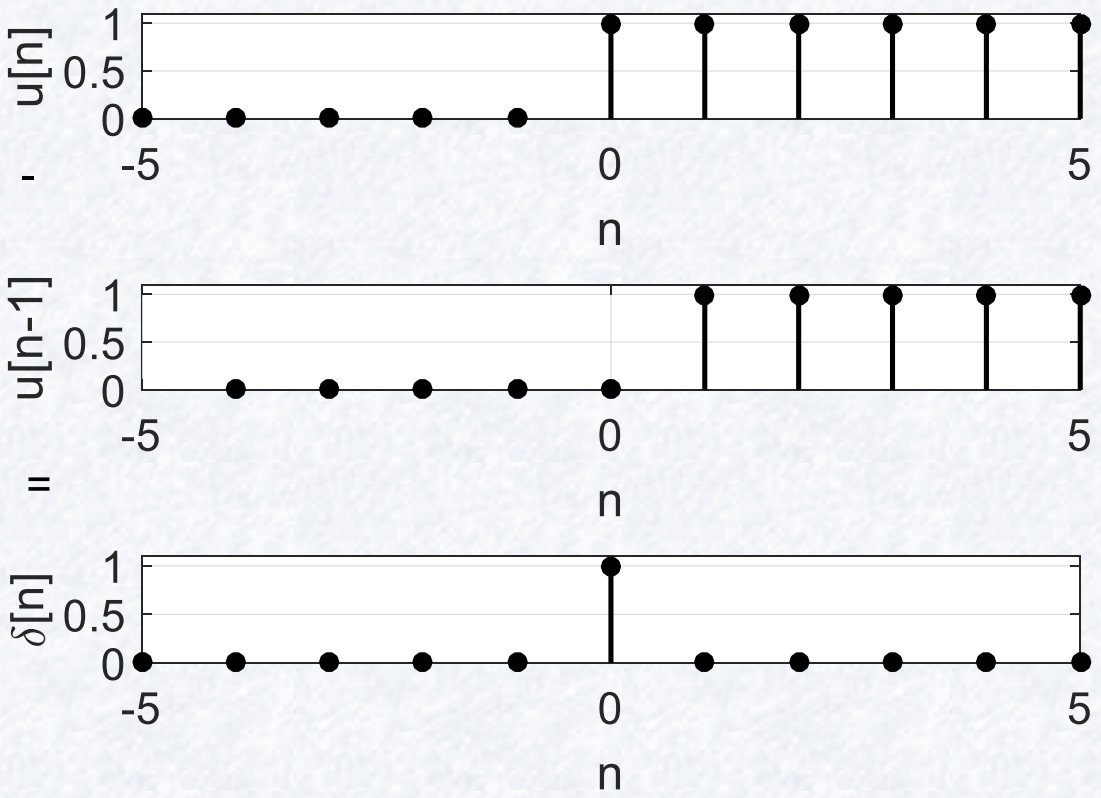
Birim Basamaktan Birim Darbe

- $\delta[n] = ?$



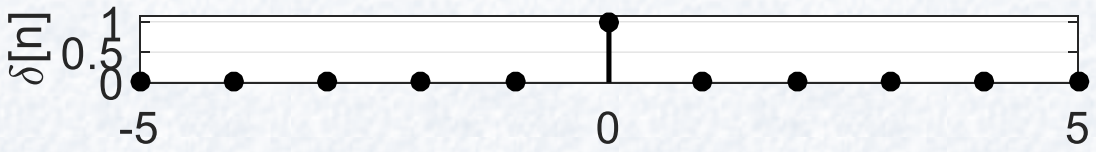
Birim Basamaktan Birim Darbe

- $\delta[n] = u[n] - u[n-1]$



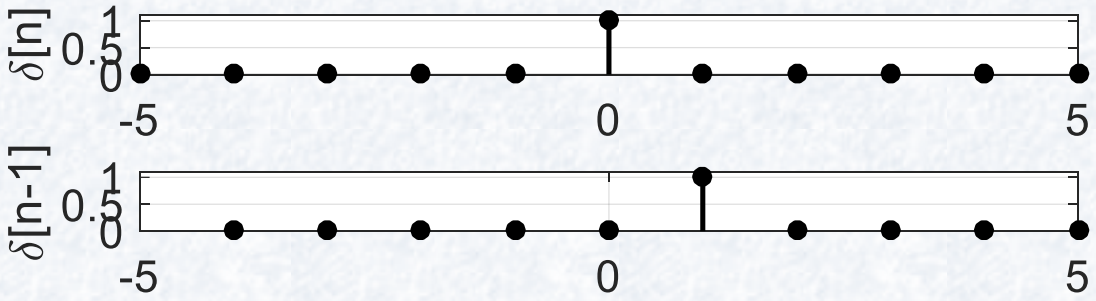
Birim Darbeden Birim Basamak

- $u[n] = ?$



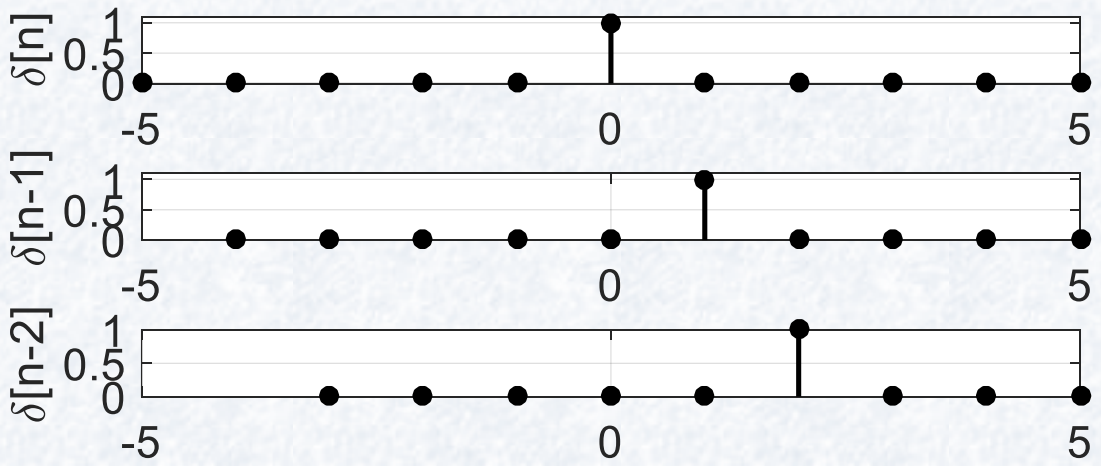
Birim Darbeden Birim Basamak

- $u[n] = ?$



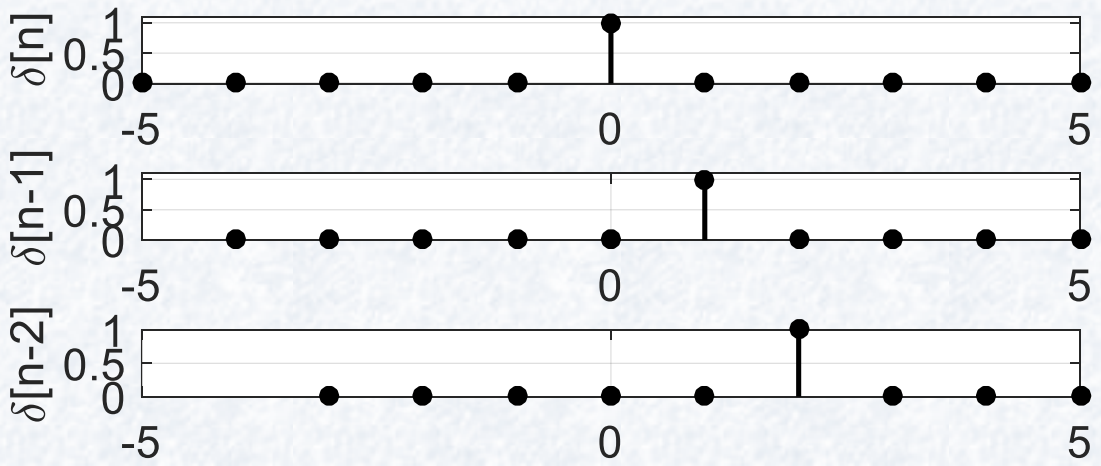
Birim Darbeden Birim Basamak

- $u[n] = ?$



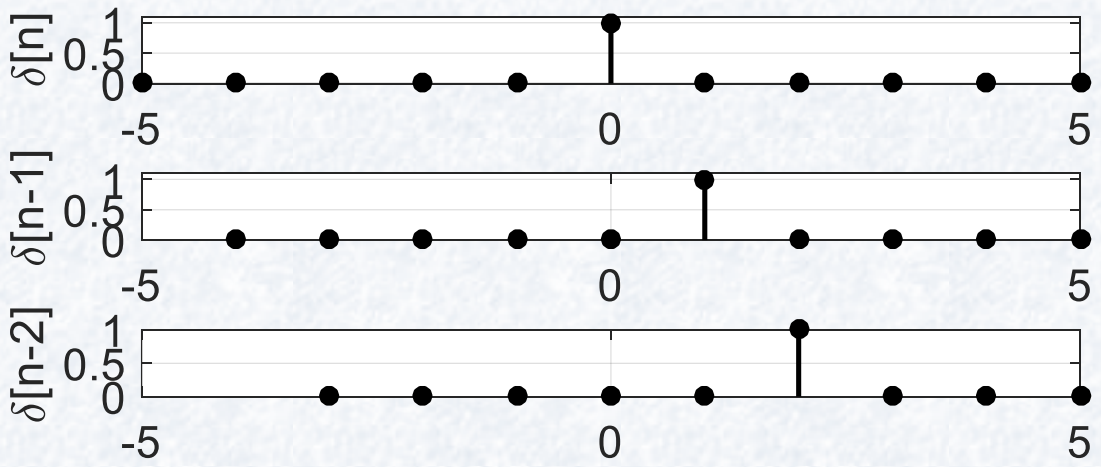
Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots$



Birim Darbeden Birim Basamak

- $u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \cdots = \sum_{k=0}^{\infty} \delta(n-k)$



Birim Darbeden Birim Basamak

- $u[n] = \sum_{k=-\infty}^{n} \delta[k]$