Calculus-2	FAST-NUCES	Assignment-3
Instructor: Fareeha Sultan	Karachi Campus	Due on: Mid 2 of Cal-II

Submission instructions: This assignment should be done individually. Make sure that student name and I.D. are listed on the submission. If you receive help from someone outside your study group, excluding the course staff, you must also acknowledge them on your assignment.

Polar Curves

1. Find the area inside the inner loop of $r = 3 - 8\cos\theta$.
2. Find the area inside the graph of $r = 7 + 3\cos\theta$ and to the left of the y-axis.
3. Find the area that is inside $r = 3 + 3\sin\theta$ and outside $r = 2$.
4. Find the area that is inside $r = 2$ and outside $r = 3 + 3\sin\theta$.
5. Find the area that is inside $r = 4 - 2\cos\theta$ and outside $r = 6 + 2\cos\theta$.
6. Find the area that is inside both $r = 1 - \sin\theta$ and $r = 2 + \sin\theta$.

Dot and Cross Products

For problems 6 and 7 find the area of the parallelogram that has **u** and **v** as adjacent sides.

6. $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k},\ \ \mathbf{v} = 3\mathbf{j} + \mathbf{k}$
7. $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j},\ \ \mathbf{v} = -\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

For problems 8 and 9 find the area of the triangle with vertices *P*, *Q*, and *R*.

8. $P(1, 5, -2),\ \ Q(0, 0, 0),\ \ R(3, 5, 1)$
9. $P(2, 0, -3),\ \ Q(1, 4, 5),\ \ R(7, 2, 9)$

For problems 12 and 13 use a scalar triple product to find the volume of the parallelepiped that has **u**, **v**, and **w** as adjacent edges.

12. $\mathbf{u} = \langle 2, -6, 2\rangle,\ \ \mathbf{v} = \langle 0, 4, -2\rangle,\ \ \mathbf{w} = \langle 2, 2, -4\rangle$
13. $\mathbf{u} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k},\ \ \mathbf{v} = 4\mathbf{i} + 5\mathbf{j} + \mathbf{k},\ \ \mathbf{w} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

14. In each part, use a scalar triple product to determine whether the vectors lie in the same plane.

(a) $\mathbf{u} = \langle 1, -2, 1\rangle,\ \ \mathbf{v} = \langle 3, 0, -2\rangle,\ \ \mathbf{w} = \langle 5, -4, 0\rangle$
(b) $\mathbf{u} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k},\ \ \mathbf{v} = 4\mathbf{i} - \mathbf{j} + \mathbf{k},\ \ \mathbf{w} = \mathbf{i} - \mathbf{j}$
(c) $\mathbf{u} = \langle 4, -8, 1\rangle,\ \ \mathbf{v} = \langle 2, 1, -2\rangle,\ \ \mathbf{w} = \langle 3, -4, 12\rangle$

15. Suppose that $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = 3$. Find

(a) $\mathbf{u} \cdot (\mathbf{w} \times \mathbf{v})$ (b) $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u}$

(c) $\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})$ (d) $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w})$

(e) $(\mathbf{u} \times \mathbf{w}) \cdot \mathbf{v}$ (f) $\mathbf{v} \cdot (\mathbf{w} \times \mathbf{w})$.

Equations of Lines

For problems 1 – 4 give the equation of the line in vector form, parametric form and symmetric form.

1. The line through the points $(7,-3,1)$ and $(-2,1,4)$.

2. The line through the point $(1,-5,0)$ and parallel to the line given by $\vec{r}(t) = \langle 8-3t, -10+9t, -1-t \rangle$.

3. The line through the point $(1,-7,14)$ and parallel to the line given by $x = 6t$, $y = 9$, $z = 8-16t$.

4. The line through the point $(-7,2,4)$ and orthogonal to both $\vec{v} = \langle 0,-9,1 \rangle$ and $\vec{w} = 3\vec{i} + \vec{j} - 4\vec{k}$.

For problems 5–7 determine if the two lines are parallel, orthogonal or neither.

5. The line given by $\vec{r}(t) = \langle 4-7t, -10+5t, 21-4t \rangle$ and the line given by $\vec{r}(t) = \langle -2+3t, 7+5t, 5+t \rangle$.

6. The line through the points $(10,-4,18)$ and $(5,6,-7)$ and the line given by $x = 5+3t$, $y = -6t$, $z = 1+15t$.

7. The line given by $x = 29$, $y = -3-6t$, $z = 12-t$ and the line given by $\vec{r}(t) = \langle 12-14t, 2+7t, -10+3t \rangle$.

For problems 8 – 10 determine the intersection point of the two lines or show that they do not intersect.

8. The line passing through the points $(0,-9,-1)$ and $(1,6,-3)$ and the line given by $\vec{r}(t) = \langle -9-4t, 10+6t, 1-2t \rangle$.

9. The line given by $x = 1+6t$, $y = -1-3t$, $z = 4+12t$ and the line given by $x = 4+t$, $y = -10-8t$, $z = 3-5t$.

10. The line given by $\vec{r}(t) = \langle 14+5t, -3t, 1+7t \rangle$ and the line given by $\vec{r}(t) = \langle 3-3t, 5+2t, -2+4t \rangle$.

For problems 11 and 12 show that the lines L_1 and L_2 are parallel.

11. $L_1: x = 2 - t,\ y = 2t,\ z = 1 + t$
$L_2: x = 1 + 2t,\ y = 3 - 4t,\ z = 5 - 2t$

12. $L_1: x = 2t,\ y = 3 + 4t,\ z = 2 - 6t$
$L_2: x = 1 + 3t,\ y = 6t,\ z = -9t$

<u>Equations of Planes</u>

For problems 1 – 5 write down the equation of the plane.

1. The plane containing the points $(6,-3,1)$, $(5,-4,1)$ and $(3,-4,0)$.

2. The plane containing the point $(1,-5,8)$ and orthogonal to the line given by $x=-3+15t$, $y=14-t$, $z=9-3t$.

3. The plane containing the point $(-8,3,7)$ and parallel to the plane given by $4x+8y-2z=45$.

4. The plane containing the point $(2,0,-8)$ and containing the line given by $\vec{r}(t)=\langle 8t,-1-5t,4-t\rangle$.

5. The plane containing the two lines given by $\vec{r}(t)=\langle 7+5t,2+t,6t\rangle$ and $\vec{r}(t)=\langle 7-6t,2-2t,10t\rangle$.

For problems 6 – 8 determine if the two planes are parallel, orthogonal or neither.

6. The plane given by $-5x+3y+2z=-8$ and the plane given by $6x-8z=15$.

7. The plane given by $3x+9y+7z=-1$ and the plane containing the points $(1,-1,9)$, $(4,-1,2)$ and $(-2,3,4)$.

8. The plane given by $-x-8y+3z=6$ and the plane given by $2x+2y+6z=-91$.

For problems 9 – 11 determine where the line intersects the plane or show that it does not intersect the plane.

9. The line given by $\vec{r}(t)=\langle 9+t,-4+t,2+5t\rangle$ and the plane given by $4x-9y+z=6$.

10. The line given by $\vec{r}(t)=\langle 2-3t,1+t,-4-2t\rangle$ and the plane given by $x-7y-4z=-1$.

11. The line given by $x=8$, $y=-9t$, $z=1+10t$ and the plane given by $8x+9y+2z=17$.

<u>Partial Derivatives</u>

<u>13.3.</u> Q (1-14, 25-52, 81-92)

<u>13.5.</u> Q (1-10, 17-34)

<u>13.6.</u> Q (1-25, 33-46)

Complex Numbers

Let $z_1 = -2 + 11i$, $z_2 = 2 - i$. Showing the details of your work, find, in the form $x + iy$:

1. $z_1 z_2$, $\overline{(z_1 z_2)}$
2. $\text{Re}\,(1/z_2^2)$, $1/\text{Re}\,(z_2^2)$
3. $(z_1 - z_2)^2/16$, $(z_1/4 - z_2/4)^2$
4. z_1/z_2, z_2/z_1
5. $(z_1 + z_2)(z_1 - z_2)$, $z_1^2 - z_2^2$
6. $\bar{z}_1/\bar{z}_2$, $\overline{(z_1/z_2)}$
7. $4\,(z_1 + z_2)/(z_1 - z_2)$
8. $\text{Re}\,(z_1^2)$, $(\text{Re}\,z_1)^2$