Answer 1.

F_1	F_2	F_3	Class
1	2	3	A
2	3	4	A
7	6	4	B
8	7	3	B

$W = 0.5$

$b = 0.5$

$\hat{y}_1 = \sigma(w_1 F_1 + w_2 F_2 + w_3 F_3 + b)$

$= \sigma(3.5)$

$\hat{y}_1 = 0.9706 > 0.5$

Hence Class A

$\hat{y}_2 = \sigma((0.5 \times 2) + (0.5 \times 3) + (0.5 \times 4) + 0.5)$

$= \sigma(5)$

$\hat{y}_2 = 0.993 > 0.5$ hence Class A

$\hat{y}_3 = \sigma(9)$

$= 0.998 > 0.5$ hence Class A

$\hat{y}_4 = \sigma(9.5)$

$= 0.999 > 0.5$ hence class A

Now Calculate loss $L(Y, \hat{y}) = -y \log_2 \hat{y} - (1 - y) \log_2 (1 - \hat{y})$ ...

$$L(y_1, \hat{y}_1) = -1 \log_2 (0.97) - 0$$
$$= 0.0432$$

$$L(y_2, \hat{y}_2) = -1 \log_2 (0.99) - 0$$
$$= 0.0101$$

$$L(y_3, \hat{y}_3) = 0 - (1-0) \log_2 (0.998 - 1)$$
$$= 12.287$$

$$L(y_4, \hat{y}_4) = 0 - (1-0) \log_2 (1 - 0.994)$$
$$= 9.965$$

Now, back propagate to optimize weights;

$dw_1 = 0$, $dw_2 = 0$, $dw_3 = 0$, $db = 0$

$$\frac{dL(y, a)}{dz} = dz \frac{\delta L}{\delta a} \cdot \frac{\delta a}{\delta z} = \frac{\delta L}{\delta a}$$
$$[a(1-a)] = a - y$$

$(i = 1)$

$$dz^{(1)} = a^{(1)} - y^{(1)}$$
$$= 0.9706 - 1$$
$$dz^{(1)} = -0.0294$$

$$dw_1 = x_1^{(1)} dz^{(1)}$$
$$dw_1 = -0.0294$$

$$dw_2 \text{ +=} x_2^{(1)} d_2^{(1)}$$

$$\text{+= } 2(0.0294)$$

$$dw_2 \text{ +=} -0.058$$

$$dw_3 \text{ +=} x_3^{(1)} d_2^{(1)}$$

$$dw_3 \text{ +=} 3(-0.0294)$$

$$dw_3 \text{ +=} -0.0882$$

$$db = d_2^{(1)}$$

$$db = -0.0294$$

$(i = 2)$ $dw_1 = -0.0294$, $dw_2 = -0.058$

$$dw_3 = -0.0882.$$

$$db = -0.0294$$

$$d_2^{(2)} = a^{(2)} - y^{(2)}$$

$$= 0.993 - 1$$

$$d_2^{(1)} = -0.007$$

$$dw_1 = x_1^{(2)} d_2^{(2)} + dw_1$$

$$= 2(-0.007) + (-0.0294)$$

$$dw_1 = -0.0437$$

$$dw_2 = dw_2 + x_2^{(2)} d_3^{(2)}$$

$$= -0.0588 + 3(-0.007)$$

$$dw_2 = -0.0798$$

$$dw_2 = dw_3 + x_3^{(2)} d_3^{(1)}$$
$$= -0.0882 + (4 \times -0.007)$$
$$dw_3 = -0.1162$$

$$db = db + d_2^{(1)}$$
$$= -0.0294 - 0.007$$
$$= -0.0364.$$

$i = 3$

$$d_2^{(3)} = a^{(3)} - y^{(3)}$$
$$= 0.9998 - 0$$
$$d_2^{(3)} = 0.9998$$

$$dw_1 = dw_1 + x_1^{(3)} d_2^{(3)}$$
$$= -0.0434 + 7(0.9998)$$
$$dw_1 = 6.9552.$$

$$dw_2 = dw_2 + x_2^{(3)} d_2^{(3)}$$
$$= -0.0798 + 6(0.9998)$$
$$dw_2 = 5.919$$

$$dw_3 = dw_3 + x_3^{(3)} d_3^{(3)}$$
$$= -0.1162 + 4(0.998)$$
$$dw_3 = 3.883$$

$$db = db + d_2^{(3)}$$
$$= -0.0364 + 0.998 = 0.9634$$

$(i = 4)$

$$d_2^{(4)} = a^4 - y^4$$
$$= 0.999 - 0 = 0.999$$

$$dw_1 = 6.9552 + 8(0.99)$$
$$= 14.947$$

$$dw_2 = 5.919 + 7(0.999)$$
$$= 12.912$$

$$dw_3 = dw_3 + x_3 d_2^{(4)}$$

$$dw_3 = 3.883 + 3(0.999)$$
$$= 6.88$$

$$db = db + d_2^{(4)}$$
$$= 0.9634 + 0.999 = 1.9624$$

$$dw_1 = \frac{dw_1}{m} = \frac{14.9472}{4} = 3.736$$

$$dw_2 = \frac{dw_2}{m} = \frac{12.912}{4} = 3.228$$

$$db = \frac{db}{m} = \frac{6.88}{4} = 1.72$$

$\alpha = 0.001$

$w_1 = w_1 - \alpha \frac{\delta \bar{J}(w,b)}{\delta w}$

$w_1 = w_1 - \alpha dw_1$

$= 0.5 - (0.001)(3.73)$

$w_1 = 0.4962$

$w_2 = w_2 - \alpha dw_2$

$= 0.5 - 0.001(3.223)$

$w_2 = 0.496$

$w_3 = w_3 - \alpha dw_3$

$= 0.5 - 0.001(1.72)$

$w_3 = 0.4982$

$b = b - \alpha db$

$= 0.5 - 0.001(0.4906)$

$b = 0.499$

Question No 3.

Cluster One. A1(2,10) = C1

Cluster 2 A4(5,8) = C2

Cluster 3 A7(1,2) = C3

(a)

~~for A1:~~

~~d(A1, C1) = 0~~

~~d(A1, C2)~~

~~d(A1, C3)~~

	x	y	C_1	C_2	C_3	Cluster
A_1	2	10	0	3.6	8.06	C_1
A_2	2	5	5	4.25	3.15	C_3
A_3	8	4	8.5	5	7.3	C_2
A_4	5	8	3.6	0	7.2	C_2
A_5	7	5	7.1	3.6	6.7	C_2
A_6	6	4	7.21	4.12	5.4	C_2
A_7	1	2	8.04	7.21	0	C_3
A_8	4	9	2.24	1.42	7.6	C_2

New Clusters & Centroid.

$C_1 \rightarrow \{A1\}$

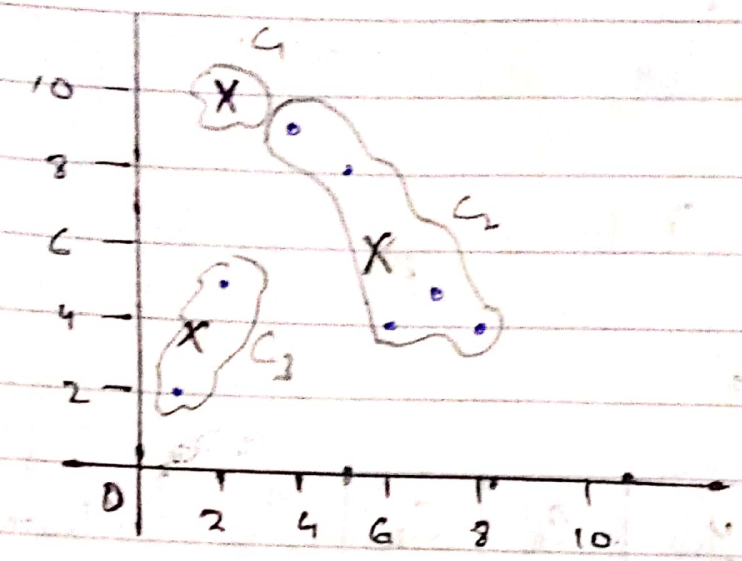
$C_2 \rightarrow \{A_3, A_4, A_5, A_6, A_8\}$

$C_3 \rightarrow \{A_2, A_7\}$

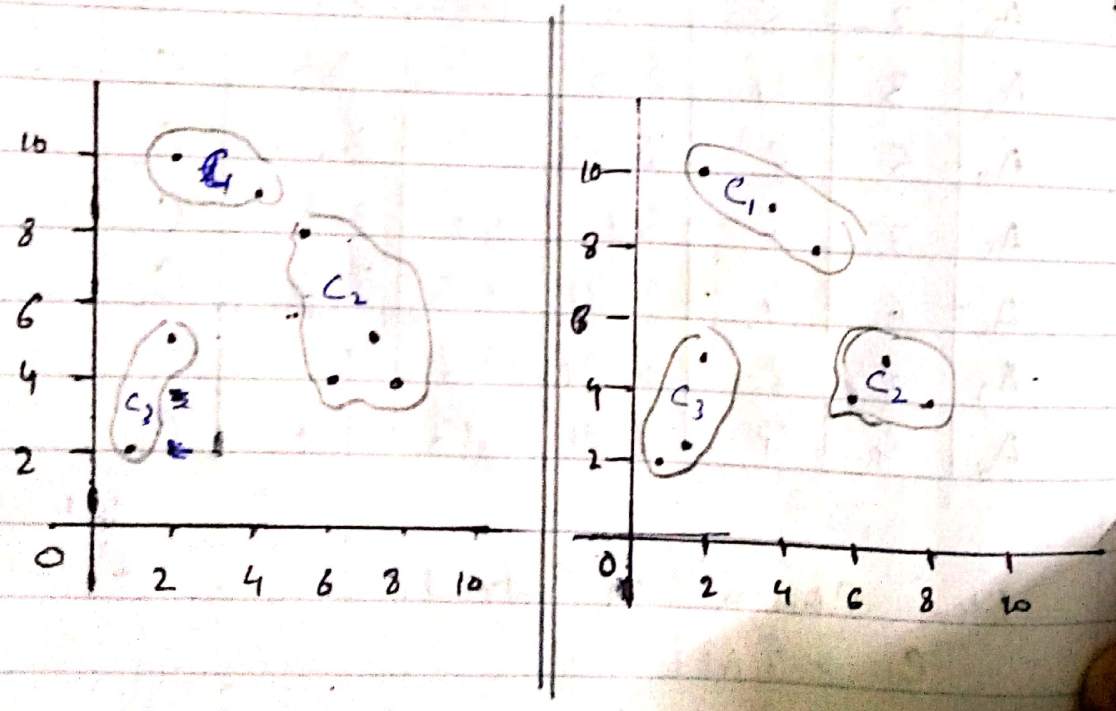
Centroid 1 = (2, 10)

Centroid 2 = (8+5+7+6+4/5, 4+8+5+4+9/5) = (6,

Centroid 3 = (2+1/2, 5+2/2) = (2.5, 3.5)



10 × 10 (Space) After Interation No. 1



Total 3 Iteration Required.

Answer #4. (i) Single linkage.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
A_1	0	5	8.5	3.6	7.1	7.2	8.1	2.2
A_2	5	0	6.01	4.3	5	4.1	3.1	4.5
A_3	8.5	6.1	0	5	1.4	2	7.3	6.4
A_4	3.6	4.3	5	0	3.6	4.1	7.2	1.4
A_5	7.1	5	1.5	3.6	0	1.4	6.7	5
A_6	7.2	4.1	2	4.1	1.4	0	5.4	5.4
A_7	8.1	3.1	7.3	7.2	6.7	5.4	0	7.6
A_8	2.2	4.5	6.4	1.4	5	5.4	7.6	0

① Select Min Value from Upper Triangle

(A_3, A_5) U (A_5, A_6) (A_4, A_8)

$\min\left((A_3, A_6), (A_5, A_6)\right)$

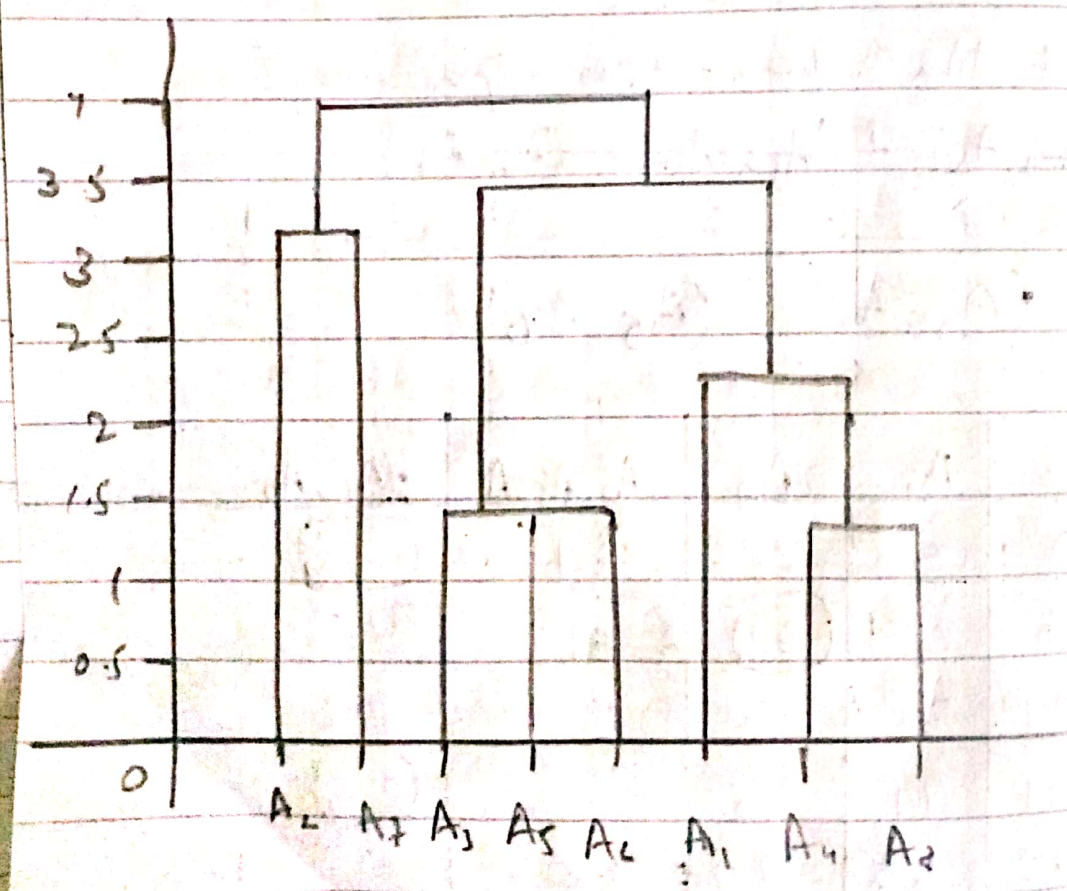
	A_1	A_2	A_7	A_3, A_5, A_6	A_4, A_8
A_1	0	5	8.1	7.1	2.2
A_2	5	0	3.1	4.1	4.2
A_7	8.1	3.1	0	5	7.2
A_3, A_5, A_6	7.1	4.1	5	0	3.6
A_4, A_8	2.2	4.2	7.2	3.6	0

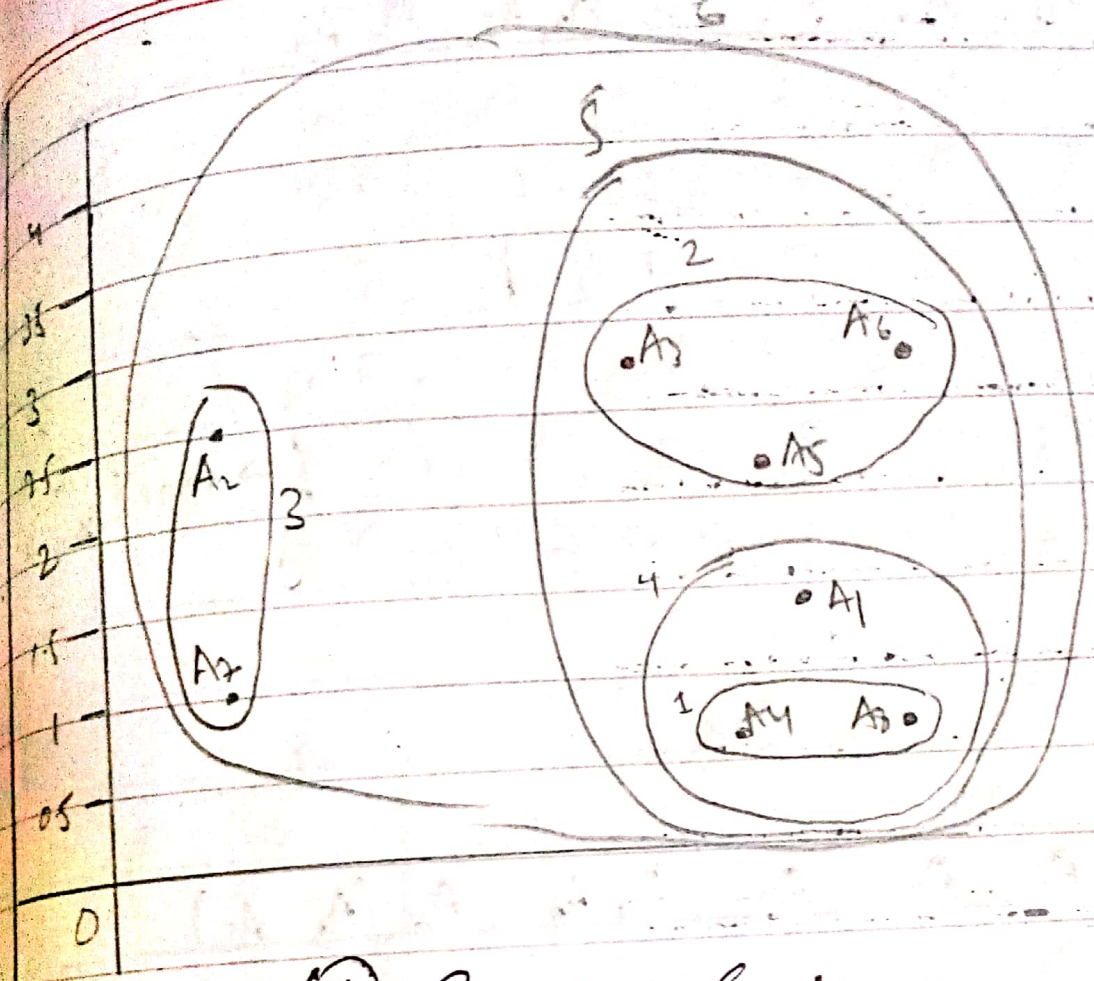
(A_2, A_7)

$\min\{dis(A_1, A_4), dis(A_1, A_8)\}$

	A_1	A_4, A_8	A_3, A_5, A_6	A_2, A_7
A_1	0	2.2	7.1	5
A_4, A_8	2.2	0	3.6	4.2
A_3, A_5, A_6	7.1	3.6	0	4.1
A_2, A_7	5	4.2	4.1	0

	$A_1 (A_4, A_8)$	$A_3 A_5 A_6$	$A_2 A_7$
$A_1 (A_4, A_8)$	0	3.6	4.2
$A_3 A_5 A_6$	3.6	0	4.1
A_2, A_7	4.2	4.1	0





(ii) Complete Linkage

	A1	A2	A3	A4	A5	A6	A7	A8
A1	0	5	8.5	3.6	7.1	7.2	8.1	2.2
A2	5	0	6.1	4.2	5	4.1	3.1	4.5
A3	8.5	6.1	0	5	1.4	2	7.3	6.4
A4	3.6	4.2	5	0	3.6	4.1	7.2	1.4
A5	7.1	5	1.4	3.6	0	1.41	6.4	5
A6	7.2	4.1	2	4.1	1.4	0	5.4	5.4
A7	8.1	3.1	7.3	7.2	6.7	5.4	0	7.6
A8	2.2	4.5	6.4	1.4	5	5.4	7.6	0

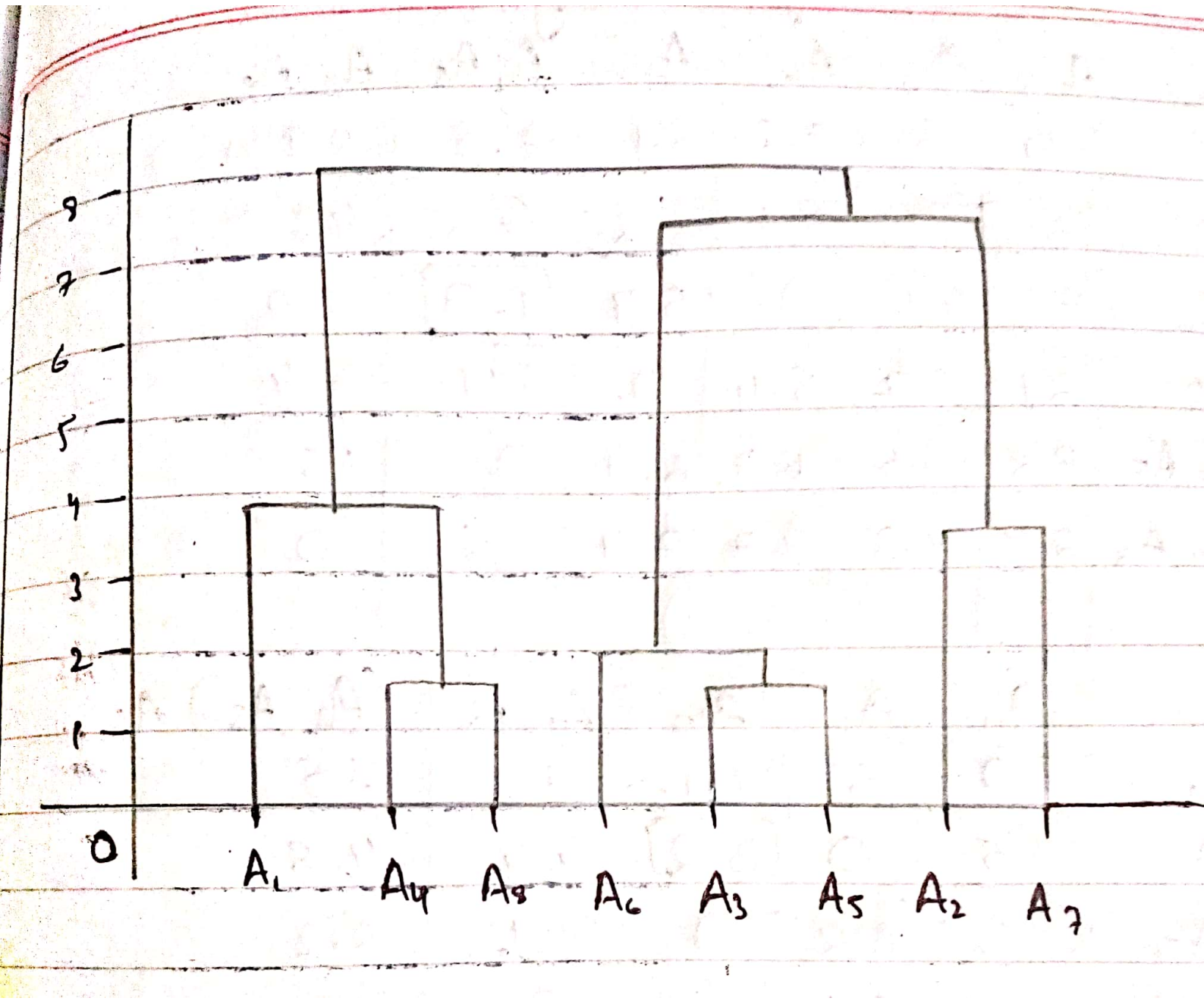
$(A_3, A_5), A_6$

$= (A_3, A_5)$ $\max\{(A_3, A_8)(A_i, A_8)\}$

	A_1	A_2	A_6	A_7	A_3, A_5	A_4, A_8
A_1	0	5	7.2	8.1	8.5	3.6
A_2	5	0	4.1	3.2	6.1	4.5
A_6	7.2	4.1	0	5.4	2	5
A_7	8.1	3.2	5.4	0	7.3	7.6
$A_3\ A_5$	8.5	6.1	2	7.3	0	6.4 → max{(A2, A4)/(A3, A4)/(A
A_4, A_8	3.6	4.5	5	7.6	6.4	0

	A_1	A_2	A_3	A_4, A_8	$A_6, (A_3, A_5)$
A_1	0	8.1	3.6	3.6	8.5
A_2	5	0	3.2	4.5	6.1
A_7	8.1	3.2	0	7.6	6.1
A_4, A_8	3.6	4.5	7.6	0	6.4
$A_6(A_3, A_5)$	8.5	6.1	7.3	6.4	0

	A_1	A_4, A_8	$A_6(A_3, A_5)$	(A_2, A_7)
A_1	0	3.6	8.5	8.1
A_4, A_8	3.6	0	6.4	7.6
$A_1, (A_3, A_5)$	8.5	6.4	0	7.3
(A_2, A_7)	8.1	7.6	7.3	0



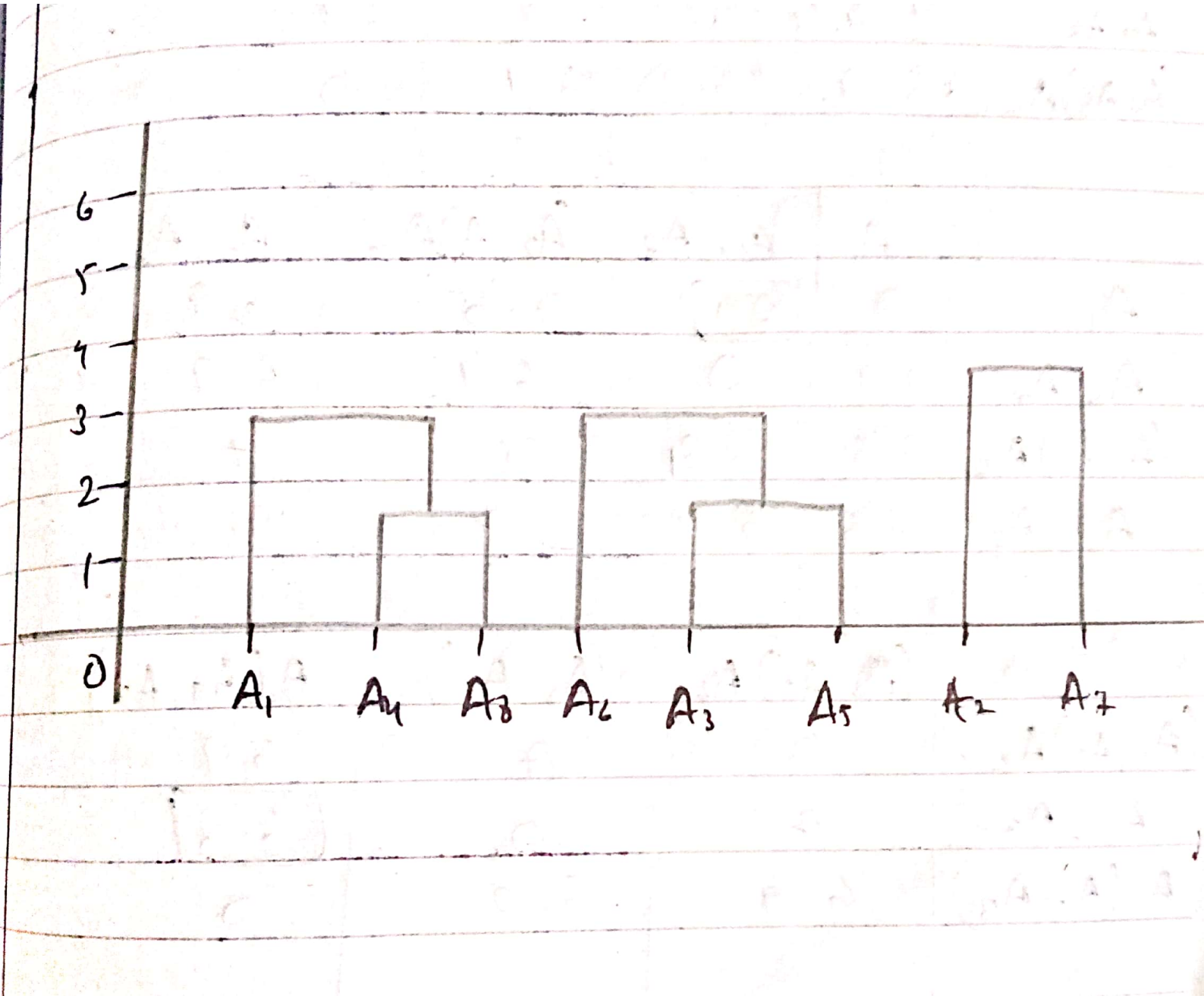
(iii) Group Average

	A_1	A_2	A_6	A_7	A_3 A_5	A_4, A_8
A_1	0	5	7.2	8.1	7.8	2.9
A_2	5	0	4.1	3.2	5.5	4.3
A_6	7.2	4.1	0	5.4	**1.7**	4.7
A_7	8.1	3.2	5.4	0	6.9	7.4
A_3, A_5	7.8	5.5	1.7	6.9	0	5
A_4, A_8	2.9	4.3	4.7	7.4	5	0

	A_1	A_2	A_7	A_4, A_8	(A_3, A_5) A_6
A_1	0	5	8.1	2.9	7.5
A_2	5	0	**3.2**	4.4	4.8
A_7	8.1	3.2	0	7.4	6.2
A_4 A_8	2.9	4.4	7.5	0	6.4
$(A_3$ $A_5)A_6$	7.5	4.8	6.2	6.4	0

	A_1	A_4, A_8	$(A_3, A_5)A_6$	A_2, A_7
A_1	0	**2.9**	7.5	6.5
A_4, A_8	2.9	0	6.4	5.9
$(A_3, A_5)A_6$	7.5	6.4	0	7
A_2, A_7	6.5	5.9	7	0

	$(A_3, A_5)A_6$	(A_2, A_7)	$A_1(A_4, A_8)$
$(A_3, A_5)A_6$	0	7	6.9
A_2, A_7	7	0	**6.2**
A_1 (A_4, A_8)	6.9	6.2	0



(4) Distance Between Centroid.

	A_1	A_2	A_6	A_7	A_3 A_5	A_4A_8
A_1	0					
A_2	5	0				
A_6	7.2	4.2	0			
A_7	8.1	3.1	5.21	0		
A_3, A_5	7.8	5.5	(1.7)	6.9	0	
A_4, A_8	2.9	4.3	4.75	7.4	5	0

	A_1	A_2	A_7	A_4, A_8	$(A_3, A_5) A_6$
A_1	0				
A_2	5	0			
A_7	8.1	3.1	0		
A_4, A_8	(2.9)	4.3	7.4	0	
$(A_3, A_5) A_6$	7.1	4.1	5.3	4.47	0

	$A_1 (A_4, A_8)$	A_2	A_7	$(A_3 A_5) A_6$
$A_1 (A_4, A_8)$	0			
A_2	4.3	0		
A_7	7.4	(3.16)	0	
$(A_3, A_5) A_6$	4.9	4.12	5.4	0

	$A_1(A_4, A_8)$	A_2, A_7	$(A_3 A_5) A_6$
$A_1(A_4, A_8)$	0		
(A_2, A_7)	4.33	0	
$(A_3, A_5) A_6$	4.85	5.04	0