Special Topics in Deep Learning

COMP 6211D & ELEC 6910T

Course website

https://course.cse.ust.hk/comp6211d

Who we are

Instructors: Qifeng Chen (cqf@ust.hk)

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Syllabus

Instructor:

Week 1-2: Overview of Deep Learning: Architecture, Losses, and Optimization

Student presentation:

Week 3-4: Convolutional Neural Networks: Dilated Convolutions, ResNet, Perceptual losses

Week 5: Deep 3D Vision: PointNet++, OctNet, Tangent convolutions

Week 6-7: Graph Convolutional Networks for Graph Processing and Optimization

Week 8-9: Sequential Modelling and Signal Processing: RNN, LSTM, TCN, and WaveNet

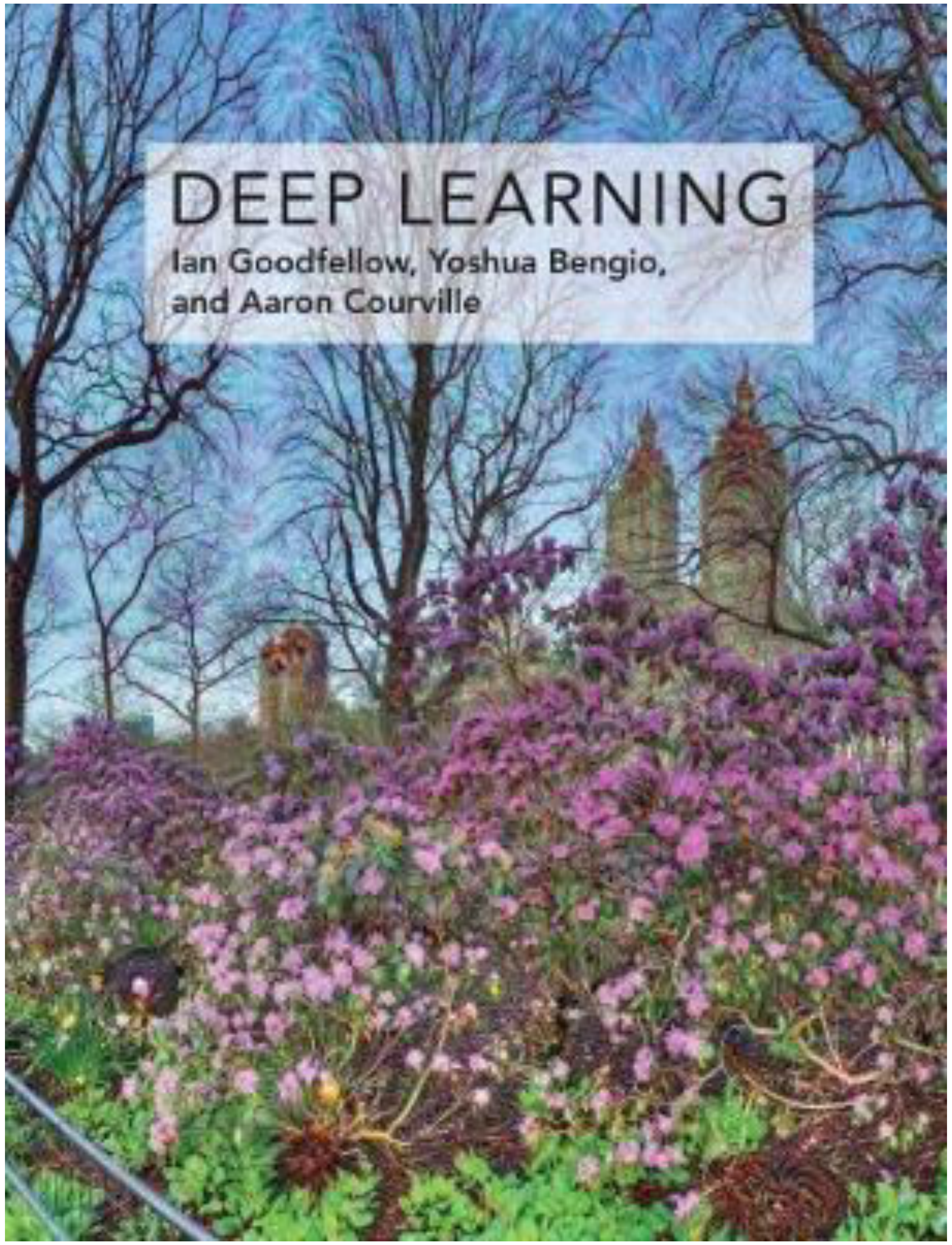
Week 10-11: Generative Models: GAN, Pix2pix, CycleGAN, CRN, VAE

Week 12-13: Final project presentation and project report due

Grading

Class participation:	10%
In-class presentation:	15%
Homework:	30%
Final project:	45%

Deep Learning Book



Representations Matter

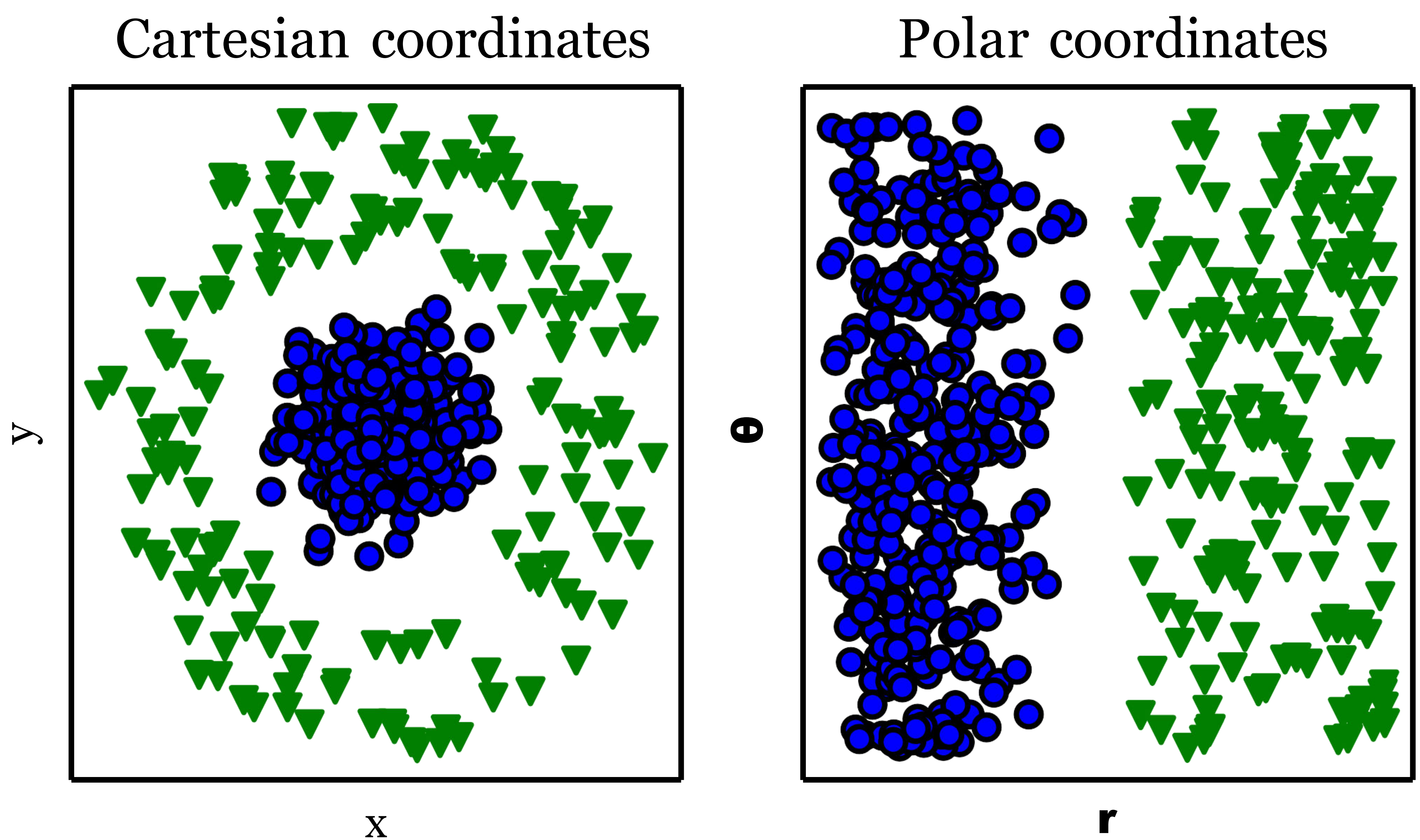


Figure 1.1

(Goodfellow 2016)

Depth: Repeated Composition

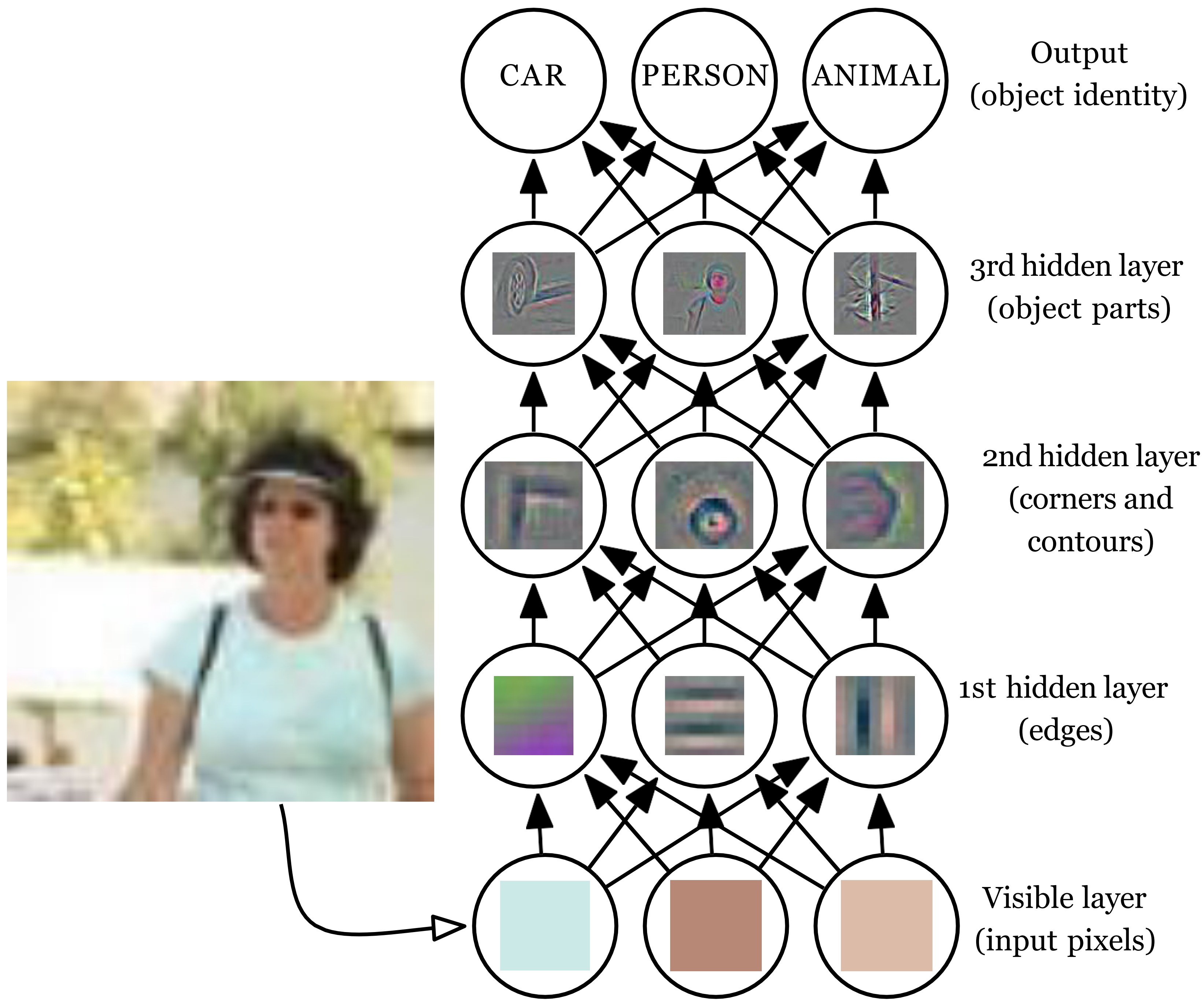


Figure 1.2

(Goodfellow 2016)

Computational Graphs

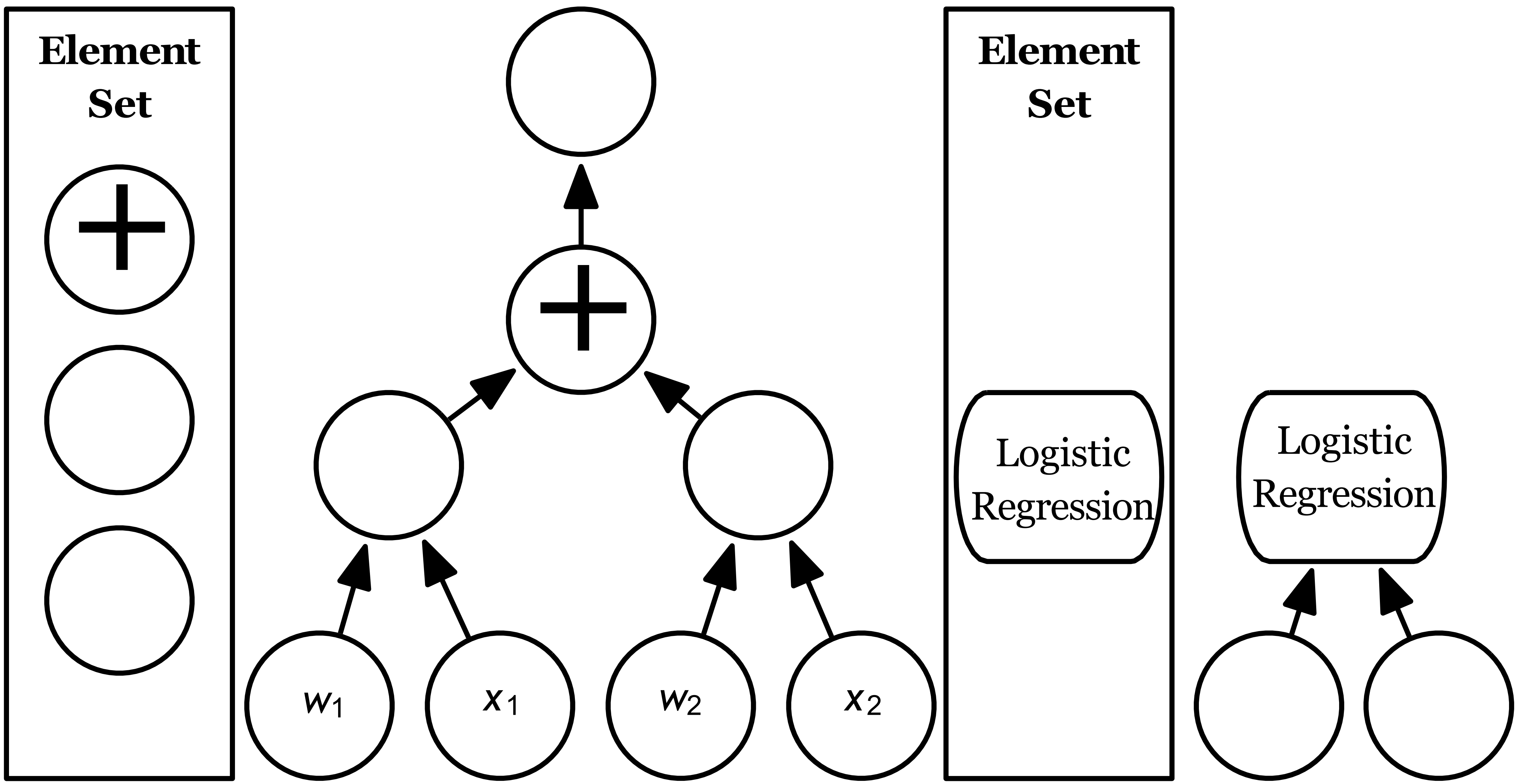


Figure 1.3

(Goodfellow 2016)

Machine Learning and AI

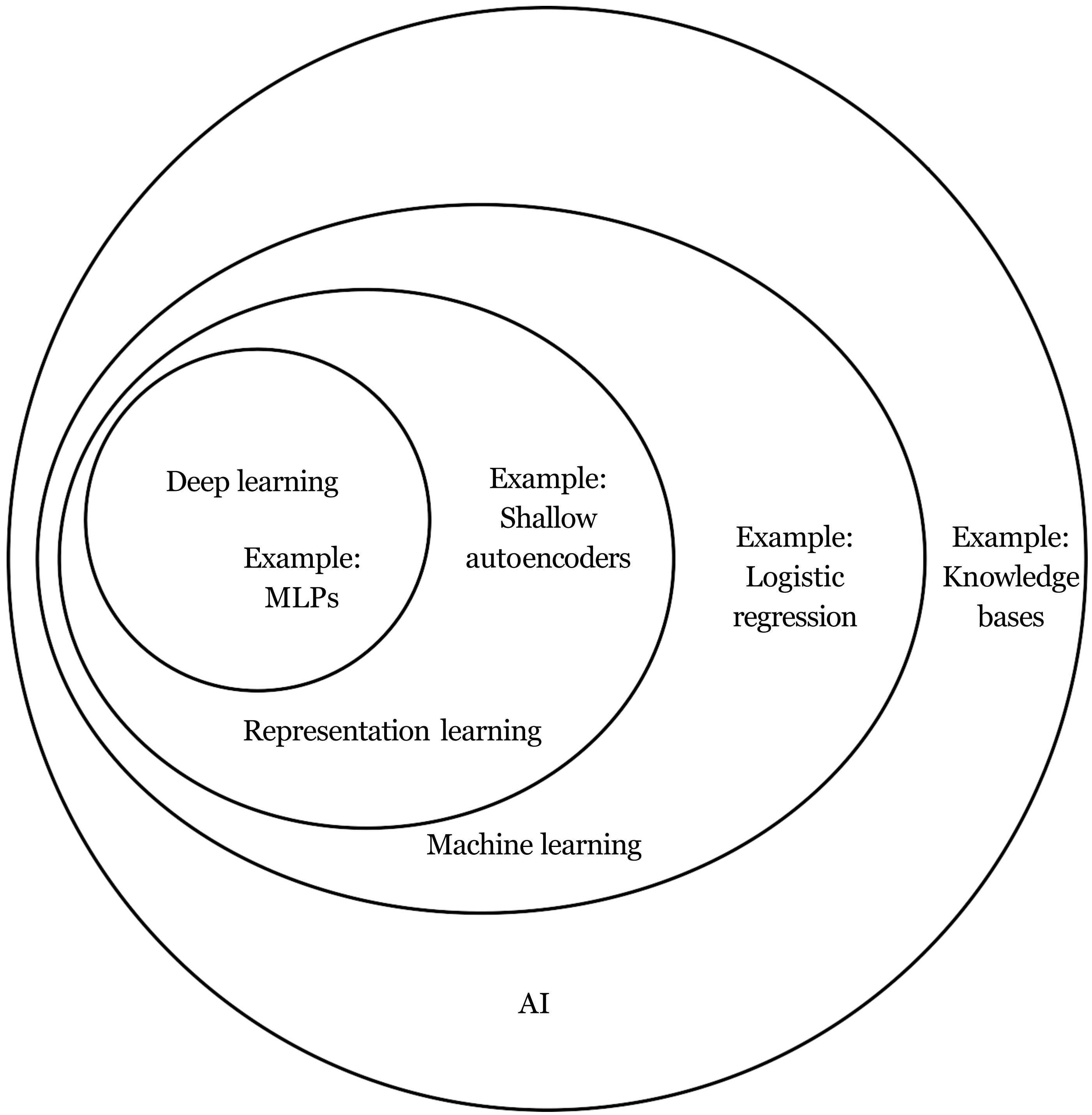
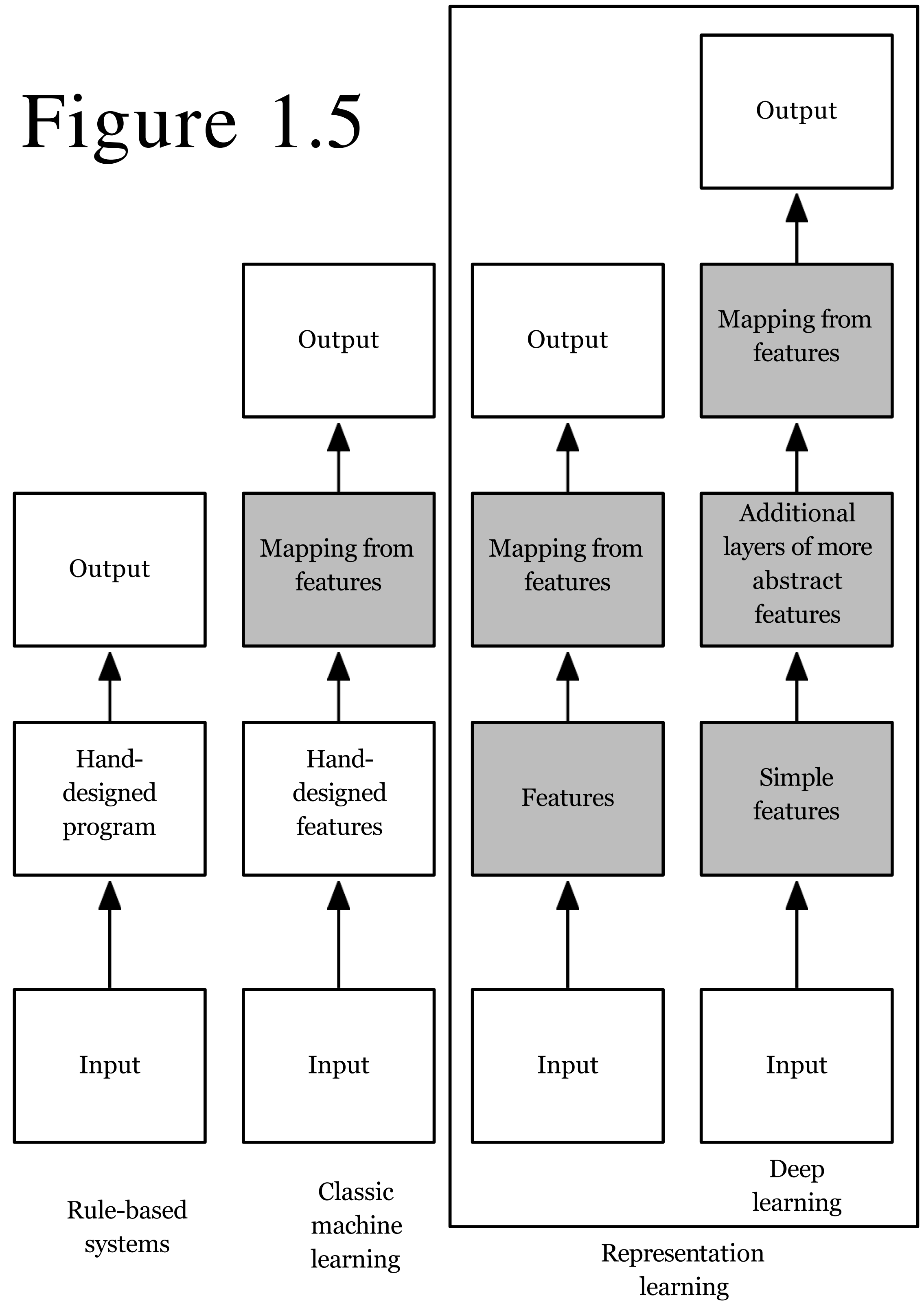


Figure 1.4

(Goodfellow 2016)

Learning Multiple Components

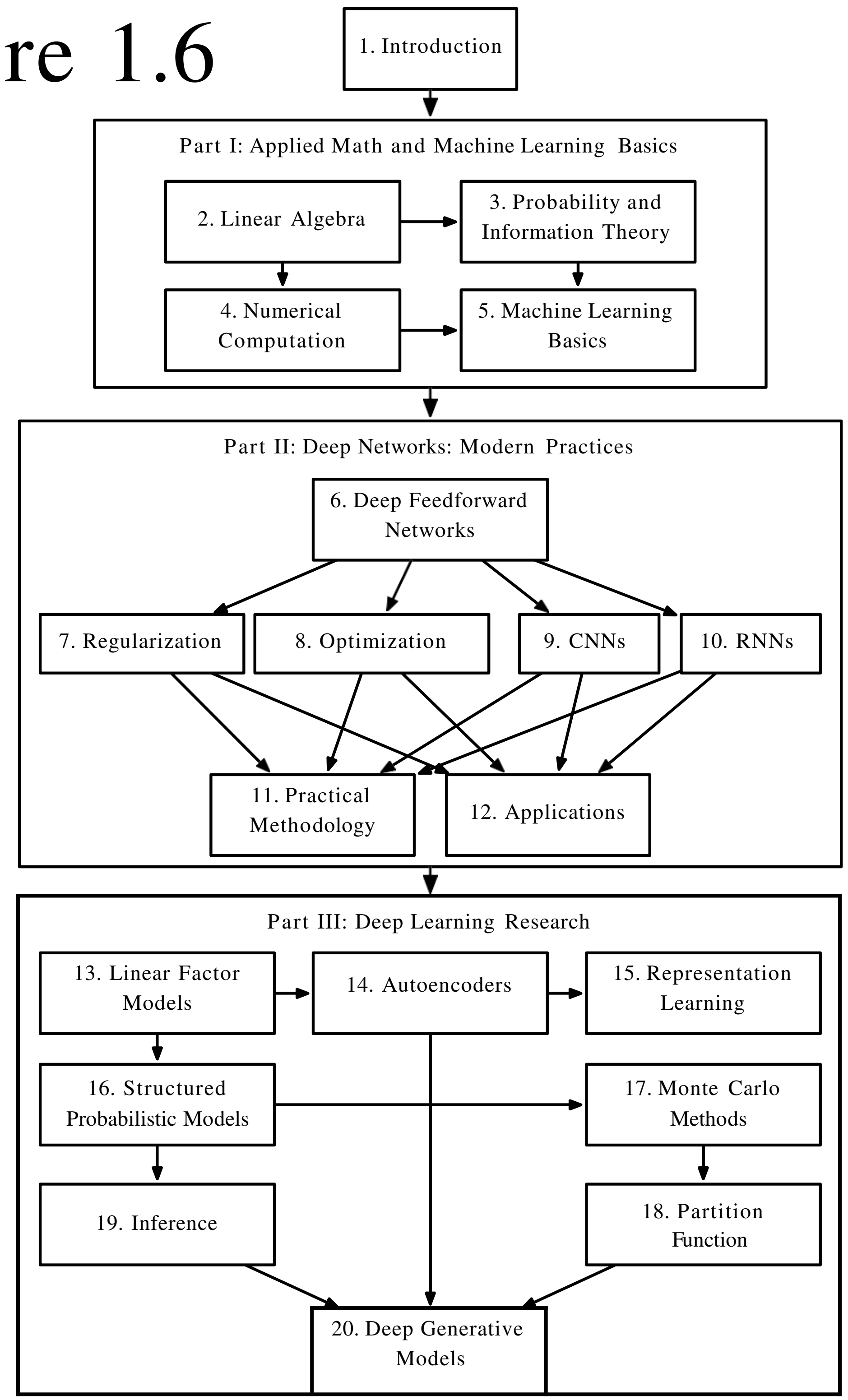
Figure 1.5



(Goodfellow 2016)

Organization of the Book

Figure 1.6



(Goodfellow 2016)

Historical Waves

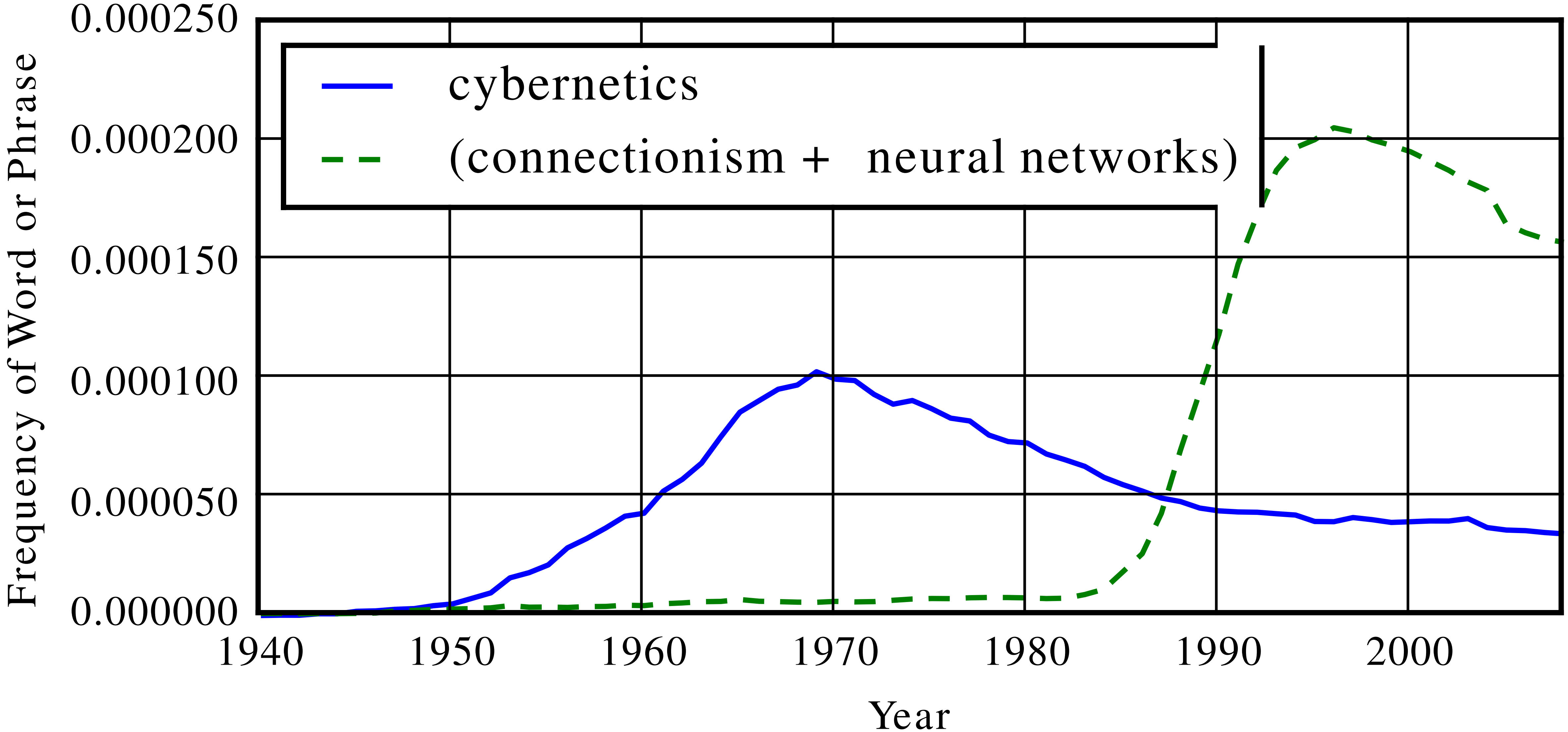


Figure 1.7

(Goodfellow 2016)

Historical Trends: Growing Datasets

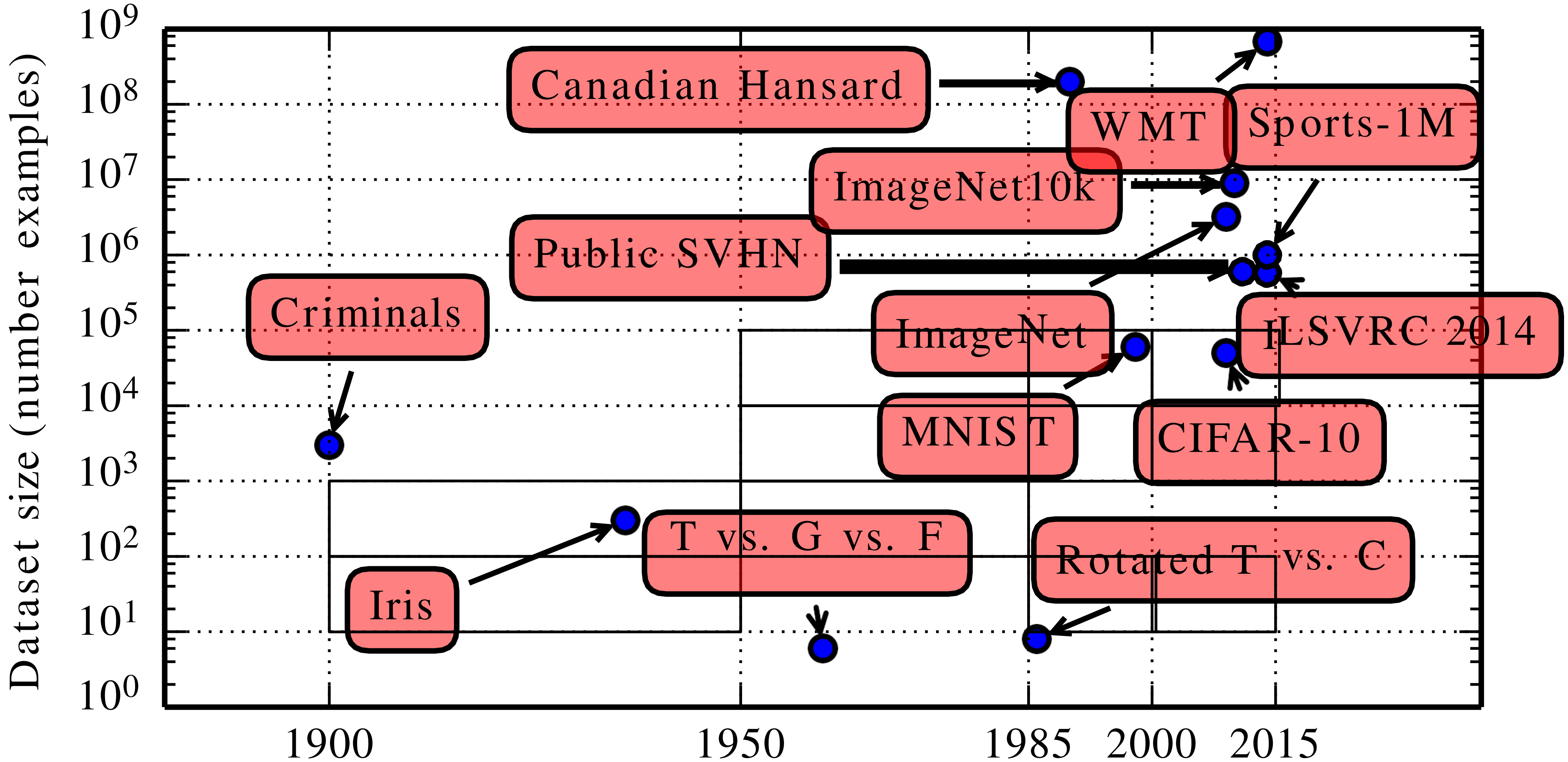


Figure 1.8

(Goodfellow 2016)

The MNIST Dataset

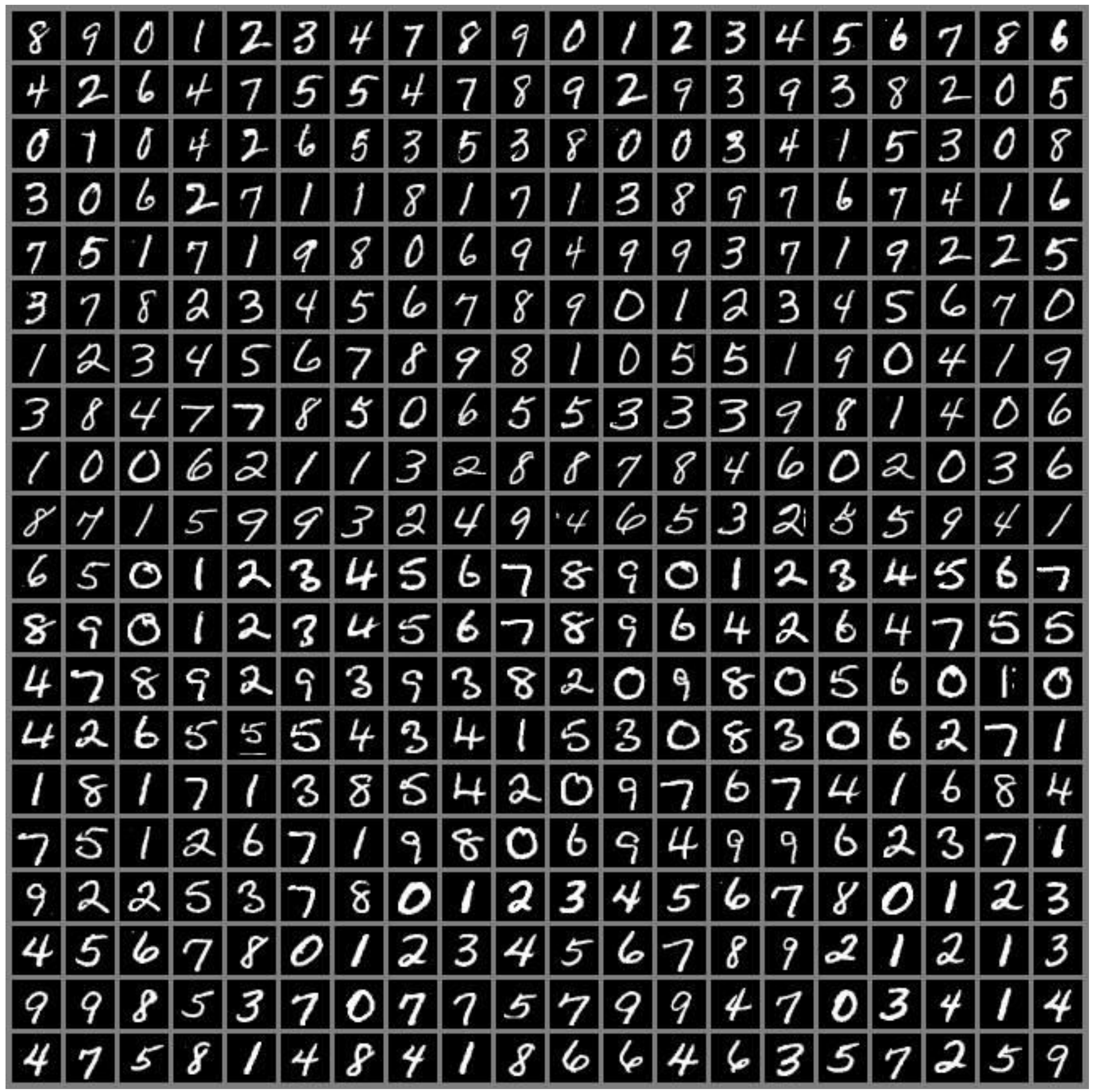


Figure 1.9

(Goodfellow 2016)

Connections per Neuron

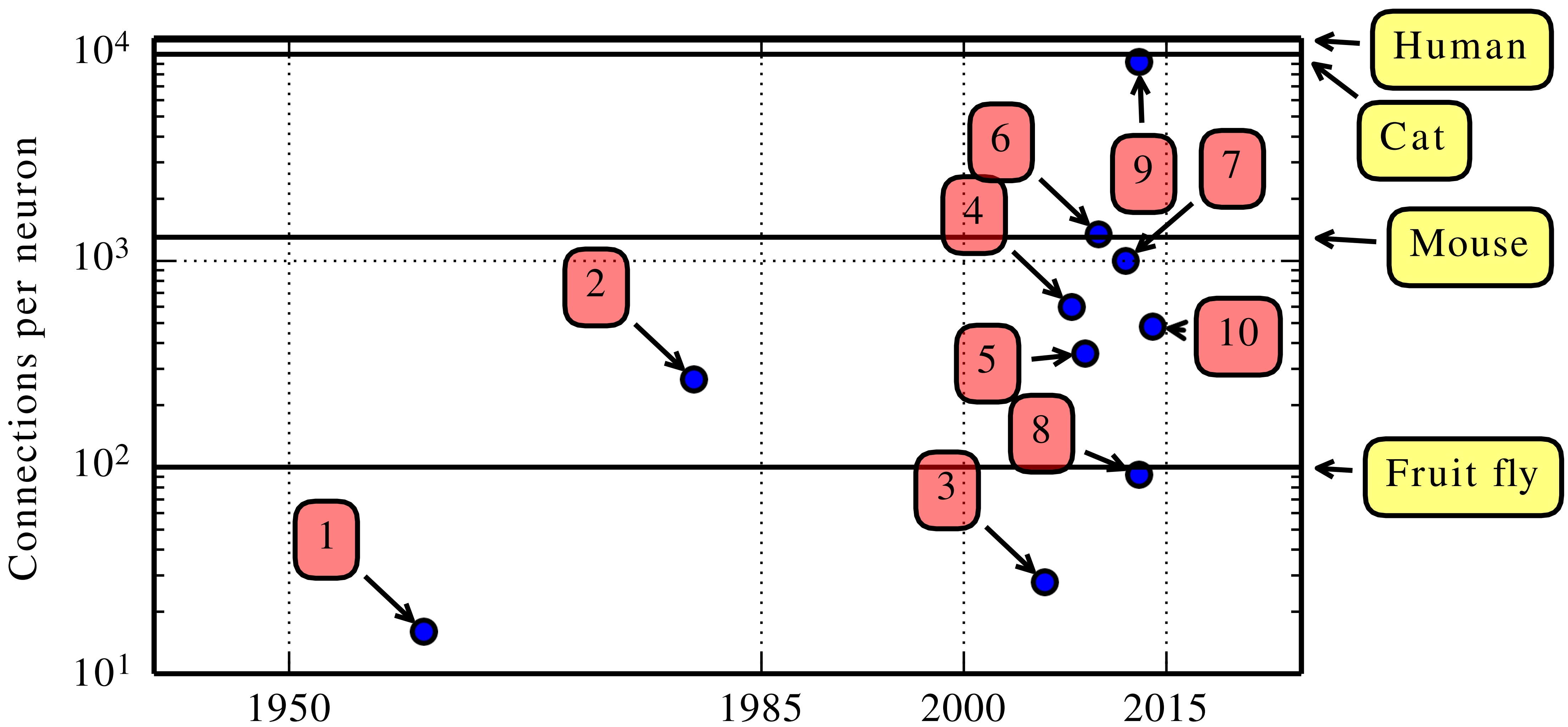


Figure 1.10

(Goodfellow 2016)

Number of Neurons

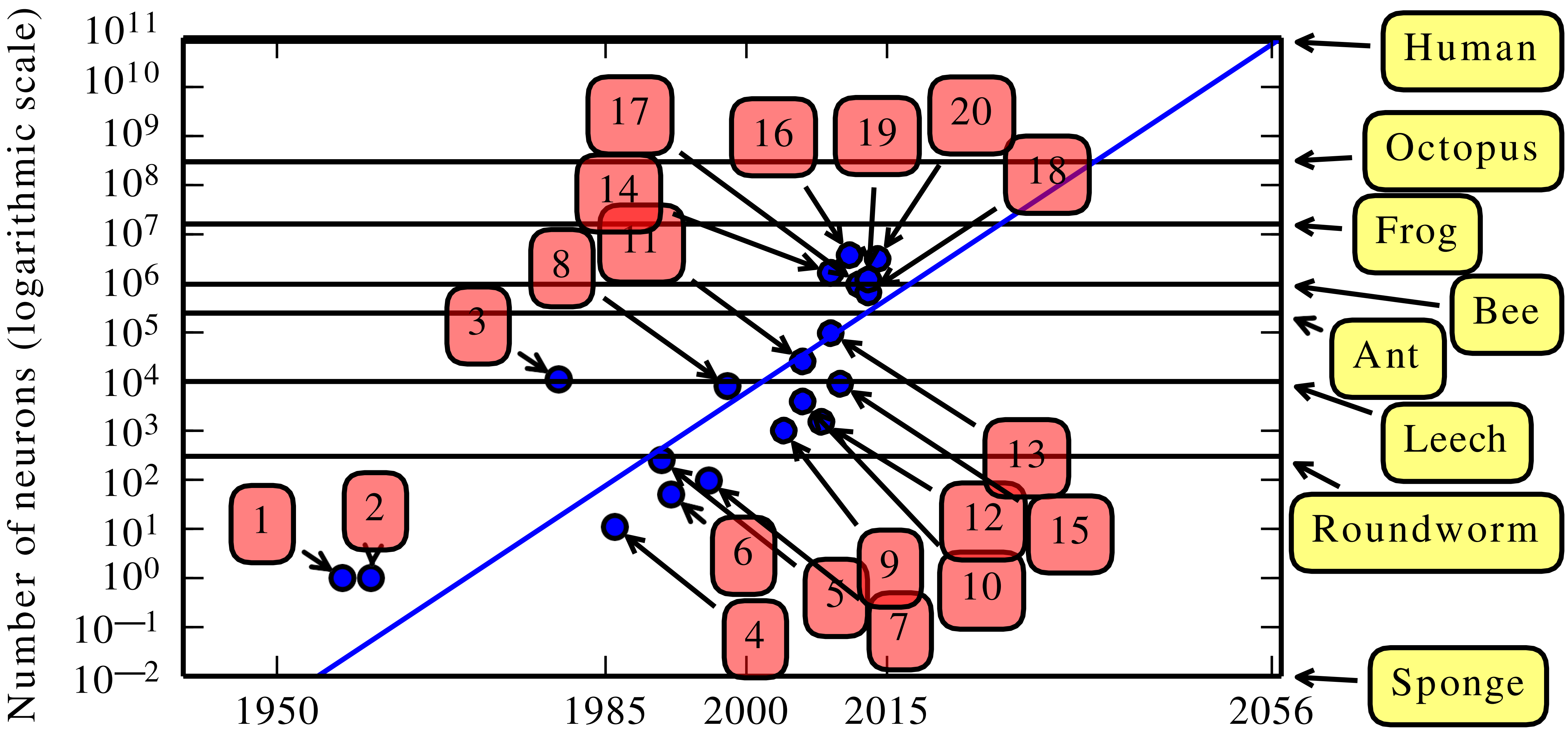


Figure 1.11

(Goodfellow 2016)

Solving Object Recognition

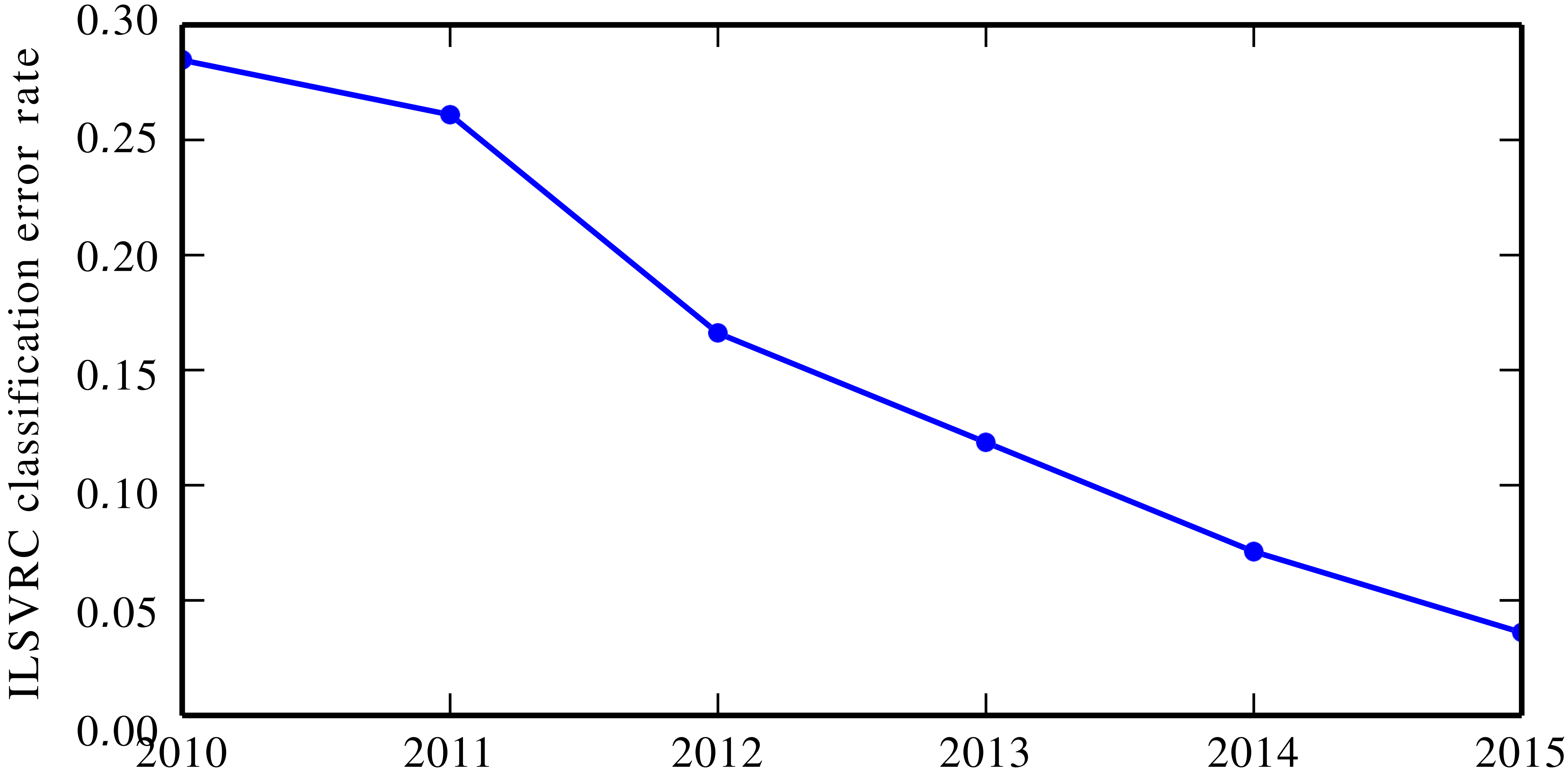


Figure 1.12

(Goodfellow 2016)

Linear Algebra

Lecture slides for Chapter 2 of *Deep Learning*

Ian Goodfellow

2016-06-24

About this chapter

- Not a comprehensive survey of all of linear algebra
- Focused on the subset most relevant to deep learning
- Larger subset: e.g., *Linear Algebra* by Georgi Shilov

Scalars

- A scalar is a single number
- Integers, real numbers, rational numbers, etc.
- We denote it with italic font:

$$a, n, x$$

(Goodfellow 2016)

Vectors

- A vector is a 1-D array of numbers:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}. \tag{2.1}$$

- Can be real, binary, integer, etc.

- Example notation for type and size:

$$\mathbb{R}^n$$

(Goodfellow 2016)

Matrices

- A matrix is a 2-D array of numbers:

Row

$$\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}. \tag{2.2}$$

Column

- Example notation for type and shape:

$$\boldsymbol{A} \in \mathbb{R}^{m \times n}$$

(Goodfellow 2016)

Tensors

- A tensor is an array of numbers, that may have
  - zero dimensions, and be a scalar
  - one dimension, and be a vector
  - two dimensions, and be a matrix
  - or more dimensions.

(Goodfellow 2016)

Matrix Transpose

$$(\boldsymbol{A}^\top)_{i,j} = A_{j,i}. \tag{2.3}$$

$$\boldsymbol{A} = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \\ A_{3,1} & A_{3,2} \end{bmatrix} \Rightarrow \boldsymbol{A}^\top = \begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} \\ A_{1,2} & A_{2,2} & A_{3,2} \end{bmatrix}$$

Figure 2.1: The transpose of the matrix can be thought of as a mirror image across the main diagonal.

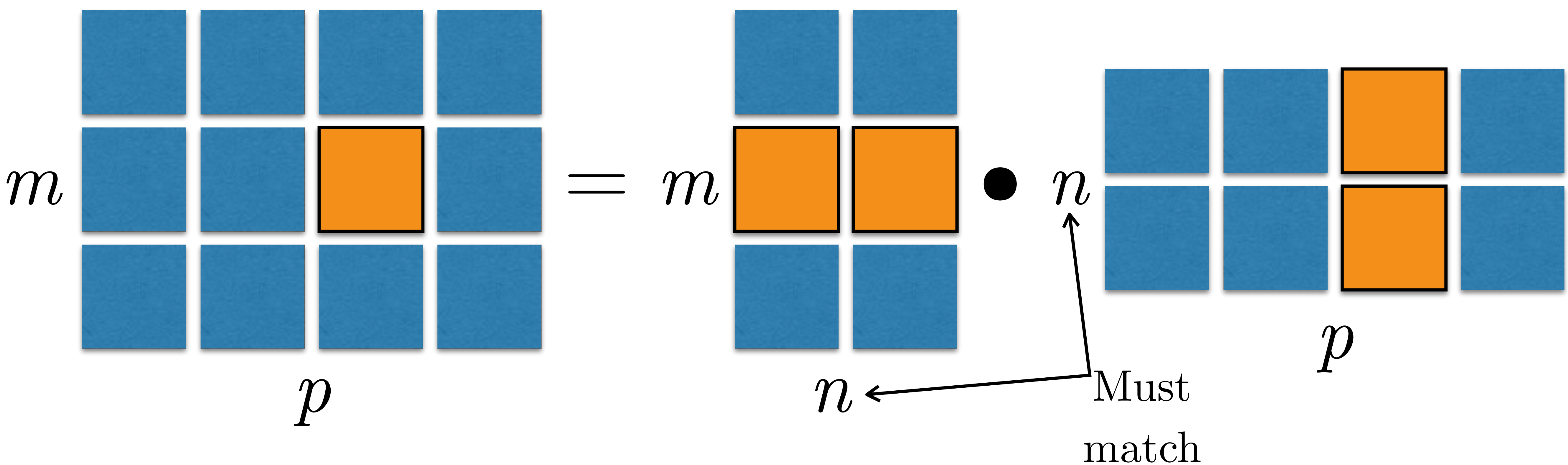
$$(\boldsymbol{AB})^\top = \boldsymbol{B}^\top \boldsymbol{A}^\top. \tag{2.9}$$

(Goodfellow 2016)

Matrix (Dot) Product

$$\boldsymbol{C} = \boldsymbol{A}\boldsymbol{B}. \tag{2.4}$$

$$C_{i,j} = \sum_k A_{i,k} B_{k,j}. \tag{2.5}$$



(Goodfellow 2016)

Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 2.2: *Example identity matrix*: This is $\boldsymbol{I}_3$.

$$\forall \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{I}_n \boldsymbol{x} = \boldsymbol{x}. \tag{2.20}$$

(Goodfellow 2016)

Systems of Equations

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.11}$$

expands to

$$\boldsymbol{A}_{1,:}\boldsymbol{x} = b_1 \tag{2.12}$$

$$\boldsymbol{A}_{2,:}\boldsymbol{x} = b_2 \tag{2.13}$$

$$\dots \tag{2.14}$$

$$\boldsymbol{A}_{m,:}\boldsymbol{x} = b_m \tag{2.15}$$

(Goodfellow 2016)

Solving Systems of Equations

- A linear system of equations can have:
  - No solution
  - Many solutions
  - Exactly one solution: this means multiplication by the matrix is an invertible function

Matrix Inversion

- Matrix inverse:

$$\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}_n. \tag{2.21}$$

- Solving a system using an inverse:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.22}$$

$$\boldsymbol{A}^{-1}\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{2.23}$$

$$\boldsymbol{I}_n\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{2.24}$$

- Numerically unstable, but useful for abstract analysis

Invertibility

- Matrix can't be inverted if…
  - More rows than columns
  - More columns than rows
  - Redundant rows/columns ("linearly dependent", "low rank")

Norms

- Functions that measure how "large" a vector is
- Similar to a distance between zero and the point represented by the vector
    - $f(\boldsymbol{x}) = 0 \Rightarrow \boldsymbol{x} = \boldsymbol{0}$
    - $f(\boldsymbol{x} + \boldsymbol{y}) \leq f(\boldsymbol{x}) + f(\boldsymbol{y})$ (the *triangle inequality*)
    - $\forall \alpha \in \mathbb{R}, f(\alpha \boldsymbol{x}) = |\alpha| f(\boldsymbol{x})$

(Goodfellow 2016)

Norms

- L^p norm

$$||\boldsymbol{x}||_p = \left(\sum_i |x_i|^p \right)^{\frac{1}{p}}$$

- Most popular norm: L2 norm, p=2
- L1 norm, p=1: $$||\boldsymbol{x}||_1 = \sum_i |x_i|. \quad (2.31)$$
- Max norm, infinite p: $$||\boldsymbol{x}||_\infty = \max_i |x_i|. \quad (2.32)$$

(Goodfellow 2016)

Special Matrices and Vectors

- Unit vector:

$$||\boldsymbol{x}||_2 = 1. \tag{2.36}$$

- Symmetric Matrix:

$$\boldsymbol{A} = \boldsymbol{A}^\top. \tag{2.35}$$

- Orthogonal matrix:

$$\boldsymbol{A}^\top \boldsymbol{A} = \boldsymbol{A}\boldsymbol{A}^\top = \boldsymbol{I}. \tag{2.37}$$

$$\boldsymbol{A}^{-1} = \boldsymbol{A}^\top$$

(Goodfellow 2016)

Eigendecomposition

- Eigenvector and eigenvalue:

$$\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{v}. \tag{2.39}$$

- Eigendecomposition of a diagonalizable matrix:

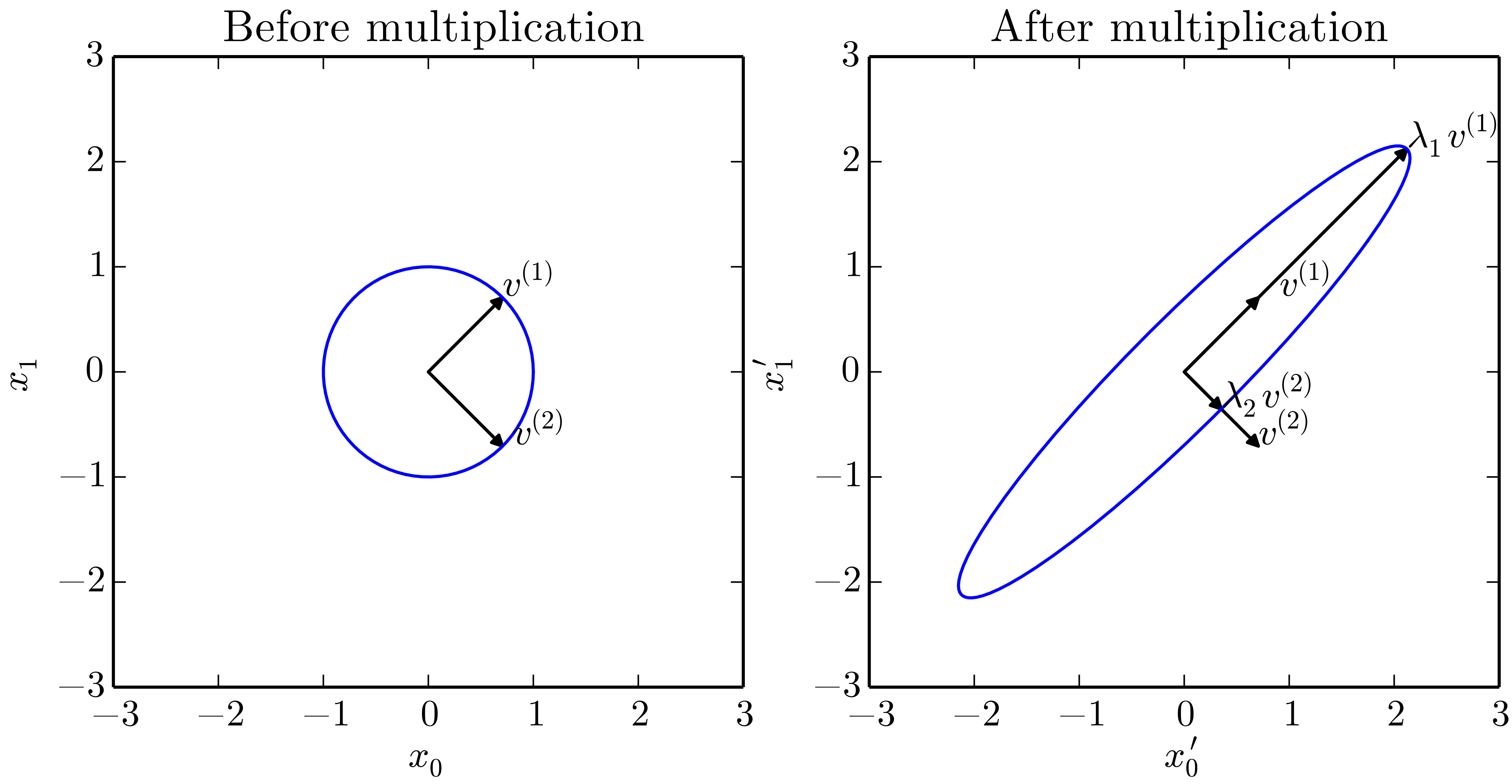
$$\boldsymbol{A} = \boldsymbol{V} \operatorname{diag}(\boldsymbol{\lambda}) \boldsymbol{V}^{-1}. \tag{2.40}$$

- Every real symmetric matrix has a real, orthogonal eigendecomposition:

$$\boldsymbol{A} = \boldsymbol{Q} \boldsymbol{\Lambda} \boldsymbol{Q}^{\top} \tag{2.41}$$

(Goodfellow 2016)

Effect of Eigenvalues



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Singular Value Decomposition

- Similar to eigendecomposition
- More general; matrix need not be square

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^\top. \tag{2.43}$$

(Goodfellow 2016)

Moore-Penrose Pseudoinverse

$$\boldsymbol{x} = \boldsymbol{A}^{+}\boldsymbol{y}$$

- If the equation has:
  - Exactly one solution: this is the same as the inverse.
  - No solution: this gives us the solution with the smallest error $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}||_2$.
  - Many solutions: this gives us the solution with the smallest norm of $\boldsymbol{x}$.

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Computing the Pseudoinverse

The SVD allows the computation of the pseudoinverse:

$$\boldsymbol{A}^{+} = \boldsymbol{V}\boldsymbol{D}^{+}\boldsymbol{U}^{\top}, \tag{2.47}$$

Take reciprocal of non-zero entries

(Goodfellow 2016)

Trace

$$\mathrm{Tr}(\boldsymbol{A}) = \sum_i \boldsymbol{A}_{i,i}. \tag{2.48}$$

$$\mathrm{Tr}(\boldsymbol{ABC}) = \mathrm{Tr}(\boldsymbol{CAB}) = \mathrm{Tr}(\boldsymbol{BCA}) \tag{2.51}$$

(Goodfellow 2016)

Learning linear algebra

- Do a lot of practice problems
- Start out with lots of summation signs and indexing into individual entries
- Eventually you will be able to mostly use matrix and vector product notation quickly and easily

Probability and Information Theory

Lecture slides for Chapter 3 of *Deep Learning*

www.deeplearningbook.org

Ian Goodfellow

2016-09-26

Probability Mass Function

- The domain of P must be the set of all possible states of x.
- $\forall x \in \mathrm{x}, 0 \leq P(x) \leq 1$. An impossible event has probability 0 and no state can be less probable than that. Likewise, an event that is guaranteed to happen has probability 1, and no state can have a greater chance of occurring.
- $\sum_{x \in \mathrm{x}} P(x) = 1$. We refer to this property as being **normalized**. Without this property, we could obtain probabilities greater than one by computing the probability of one of many events occurring.

Example: uniform distribution: $P(\mathrm{x} = x_i) = \frac{1}{k}$

(Goodfellow 2016)

Probability Density Function

- The domain of p must be the set of all possible states of x.
- $\forall x \in \mathrm{x}, p(x) \geq 0$. Note that we do not require $p(x) \leq 1$.
- $\int p(x)dx = 1$.

Example: uniform distribution: $u(x; a, b) = \frac{1}{b-a}$.

(Goodfellow 2016)

Computing Marginal Probability with the Sum Rule

$$\forall x \in \mathrm{x}, P(\mathrm{x} = x) = \sum_y P(\mathrm{x} = x, \mathrm{y} = y). \tag{3.3}$$

$$p(x) = \int p(x, y) dy. \tag{3.4}$$

(Goodfellow 2016)

Conditional Probability

$$P(\mathrm{y} = y \mid \mathrm{x} = x) = \frac{P(\mathrm{y} = y, \mathrm{x} = x)}{P(\mathrm{x} = x)}. \tag{3.5}$$

(Goodfellow 2016)

Chain Rule of Probability

$$P(\mathrm{x}^{(1)}, \ldots, \mathrm{x}^{(n)}) = P(\mathrm{x}^{(1)})\Pi_{i=2}^{n} P(\mathrm{x}^{(i)} \mid \mathrm{x}^{(1)}, \ldots, \mathrm{x}^{(i-1)}). \tag{3.6}$$

(Goodfellow 2016)

Independence

$$\forall x \in \mathrm{x}, y \in \mathrm{y},\ p(\mathrm{x} = x, \mathrm{y} = y) = p(\mathrm{x} = x)p(\mathrm{y} = y). \tag{3.7}$$

(Goodfellow 2016)

Conditional Independence

$$\forall x \in \mathrm{x}, y \in \mathrm{y}, z \in \mathrm{z},\ p(\mathrm{x}=x, \mathrm{y}=y \mid \mathrm{z}=z) = p(\mathrm{x}=x \mid \mathrm{z}=z)p(\mathrm{y}=y \mid \mathrm{z}=z). \tag{3.8}$$

(Goodfellow 2016)

Expectation

$$\mathbb{E}_{\mathrm{x}\sim P}[f(x)] = \sum_x P(x)f(x), \tag{3.9}$$

$$\mathbb{E}_{\mathrm{x}\sim p}[f(x)] = \int p(x)f(x)dx. \tag{3.10}$$

linearity of expectations:

$$\mathbb{E}_{\mathrm{x}}[\alpha f(x) + \beta g(x)] = \alpha\mathbb{E}_{\mathrm{x}}[f(x)] + \beta\mathbb{E}_{\mathrm{x}}[g(x)], \tag{3.11}$$

(Goodfellow 2016)

Variance and Covariance

$$\mathrm{Var}(f(x)) = \mathbb{E}\left[(f(x) - \mathbb{E}[f(x)])^2\right]. \tag{3.12}$$

$$\mathrm{Cov}(f(x), g(y)) = \mathbb{E}\left[(f(x) - \mathbb{E}\left[f(x)\right])\left(g(y) - \mathbb{E}\left[g(y)\right]\right)\right]. \tag{3.13}$$

Covariance matrix:

$$\mathrm{Cov}(\mathbf{x})_{i,j} = \mathrm{Cov}(\mathrm{x}_i, \mathrm{x}_j). \tag{3.14}$$

(Goodfellow 2016)

Bernoulli Distribution

$$P(\mathrm{x} = 1) = \phi \tag{3.16}$$

$$P(\mathrm{x} = 0) = 1 - \phi \tag{3.17}$$

$$P(\mathrm{x} = x) = \phi^x (1 - \phi)^{1-x} \tag{3.18}$$

$$\mathbb{E}_{\mathrm{x}}[\mathrm{x}] = \phi \tag{3.19}$$

$$\mathrm{Var}_{\mathrm{x}}(\mathrm{x}) = \phi(1 - \phi) \tag{3.20}$$

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Gaussian Distribution

Parametrized by variance:

$$\mathcal{N}(x; \mu, \sigma^2) = \sqrt{\frac{1}{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right). \tag{3.21}$$

Parametrized by precision:

$$\mathcal{N}(x; \mu, \beta^{-1}) = \sqrt{\frac{\beta}{2\pi}} \exp\left(-\frac{1}{2}\beta(x-\mu)^2\right). \tag{3.22}$$

Gaussian Distribution

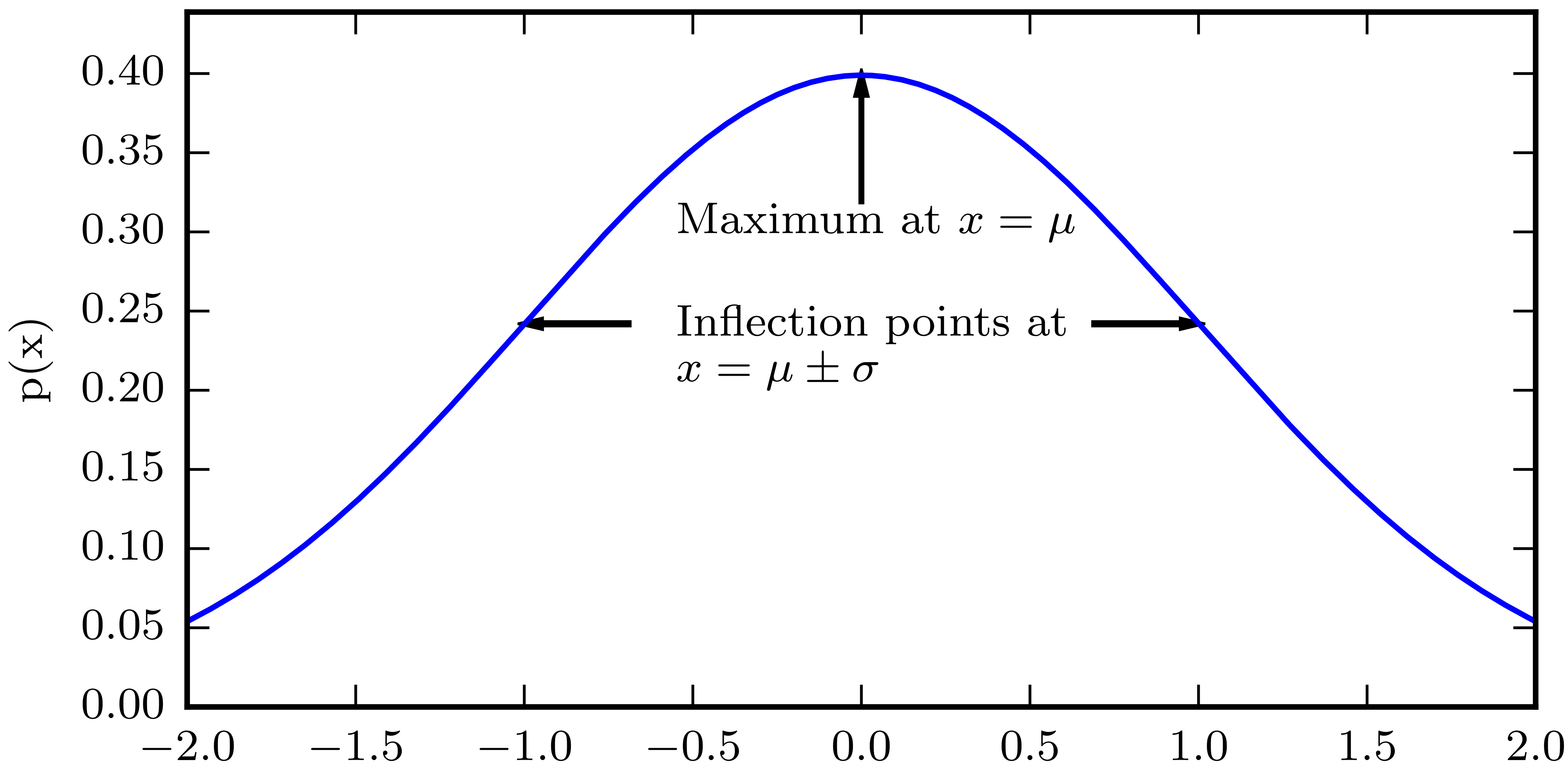


Figure 3.1

(Goodfellow 2016)

Multivariate Gaussian

Parametrized by covariance matrix:

$$\mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \sqrt{\frac{1}{(2\pi)^n \det(\boldsymbol{\Sigma})}} \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{\mu})\right). \tag{3.23}$$

Parametrized by precision matrix:

$$\mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\beta}^{-1}) = \sqrt{\frac{\det(\boldsymbol{\beta})}{(2\pi)^n}} \exp\left(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu})^\top \boldsymbol{\beta}(\boldsymbol{x} - \boldsymbol{\mu})\right). \tag{3.24}$$

(Goodfellow 2016)

More Distributions

Exponential:

$$p(x;\lambda) = \lambda \mathbf{1}_{x \geq 0} \exp\left(-\lambda x\right). \tag{3.25}$$

Laplace:

$$\text{Laplace}(x;\mu,\gamma) = \frac{1}{2\gamma} \exp\left(-\frac{|x-\mu|}{\gamma}\right). \tag{3.26}$$

Dirac:

$$p(x) = \delta(x-\mu). \tag{3.27}$$

(Goodfellow 2016)

Empirical Distribution

$$\hat{p}(\boldsymbol{x}) = \frac{1}{m} \sum_{i=1}^{m} \delta(\boldsymbol{x} - \boldsymbol{x}^{(i)}) \tag{3.28}$$

Mixture Distributions

$$P(\mathrm{x}) = \sum_i P(\mathrm{c} = i) P(\mathrm{x} \mid \mathrm{c} = i) \tag{3.29}$$

Gaussian mixture
with three
components

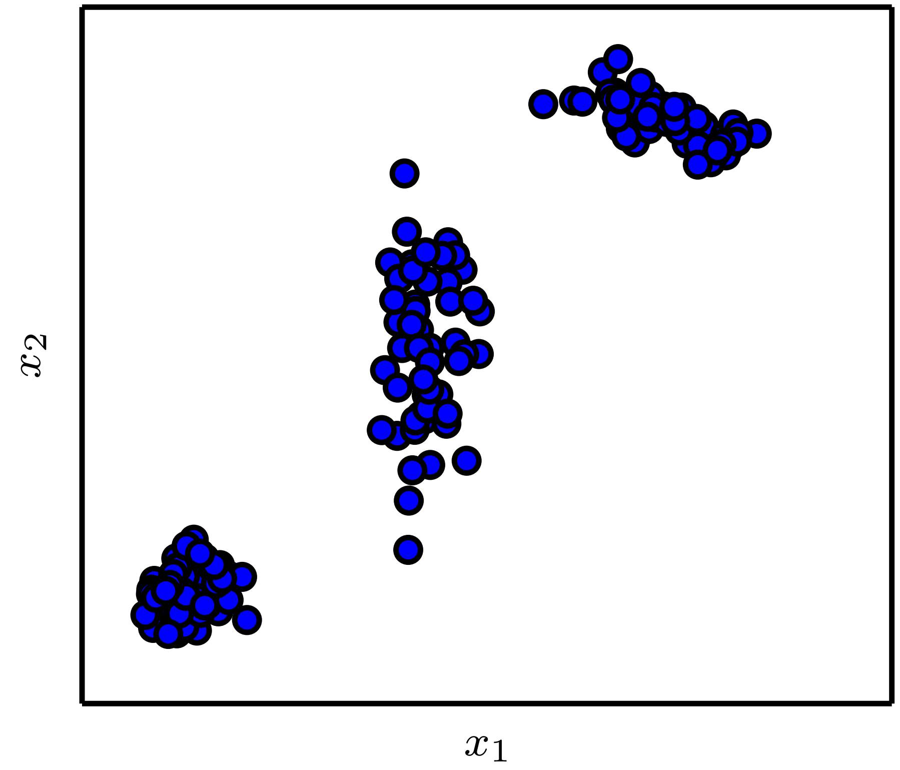


Figure 3.2

(Goodfellow 2016)

Logistic Sigmoid

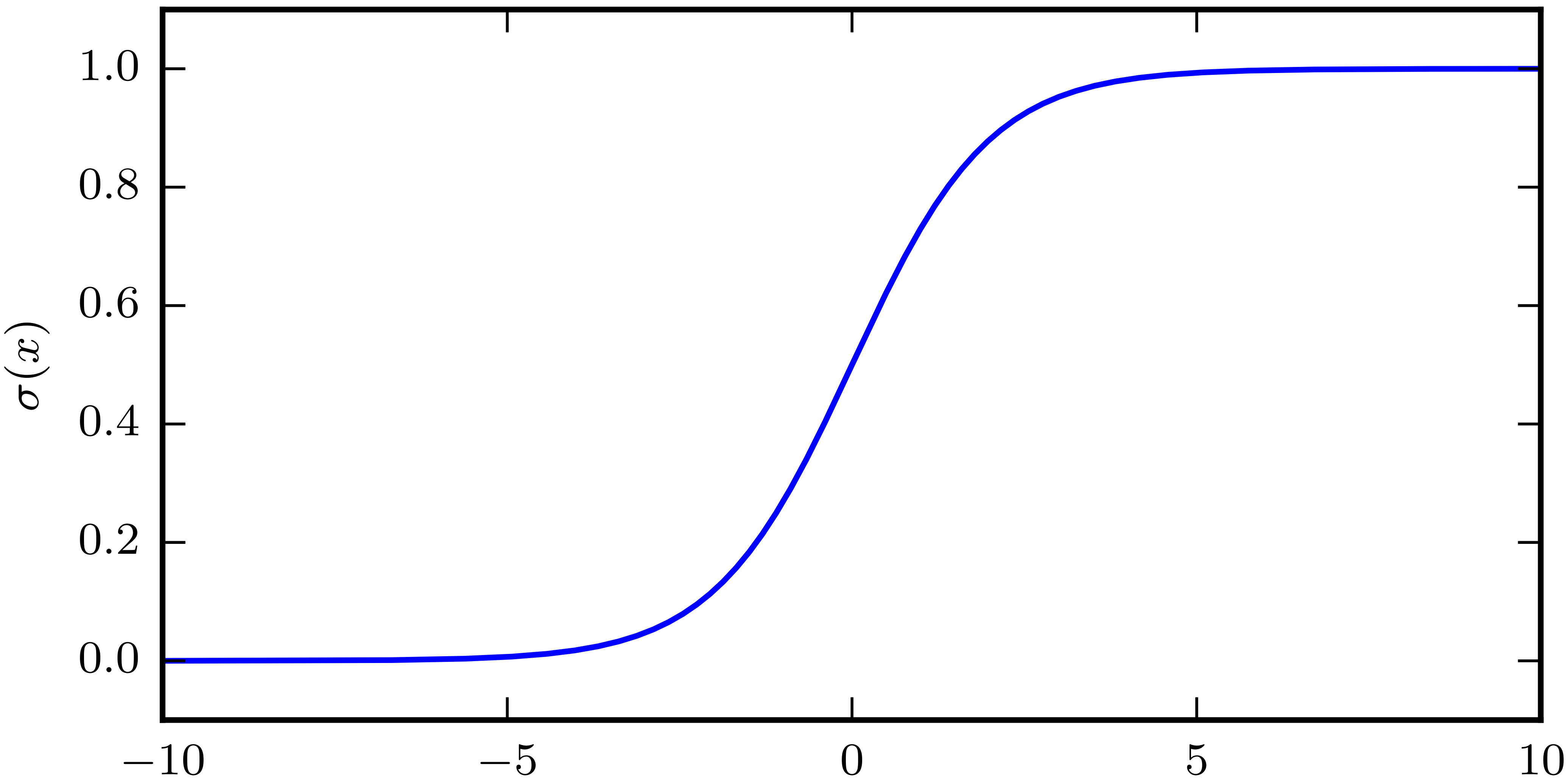


Figure 3.3: The logistic sigmoid function.

Commonly used to parametrize Bernoulli distributions

(Goodfellow 2016)

Softplus Function

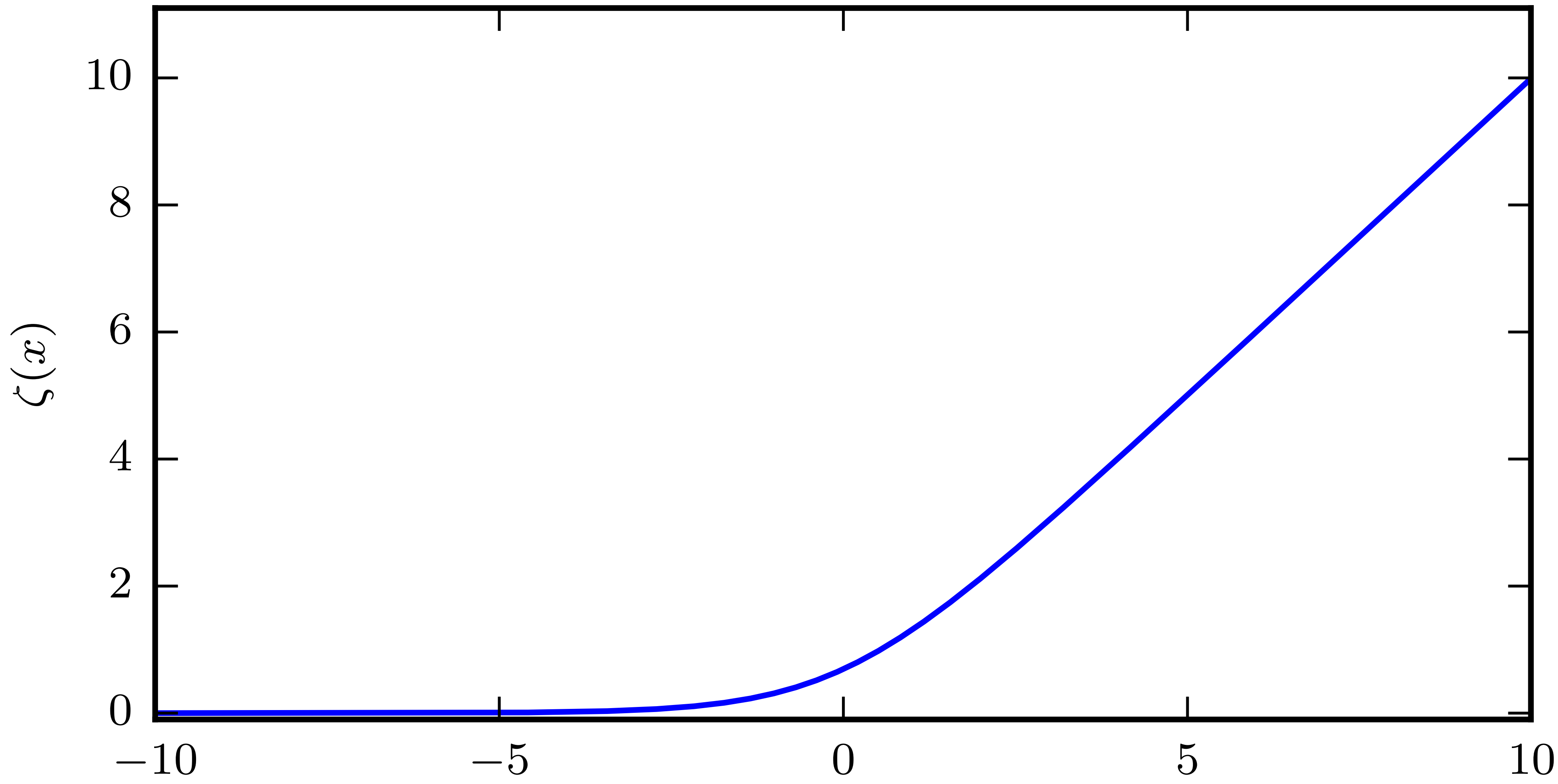


Figure 3.4: The softplus function.

(Goodfellow 2016)

Bayes' Rule

$$P(\mathrm{x} \mid \mathrm{y}) = \frac{P(\mathrm{x})P(\mathrm{y} \mid \mathrm{x})}{P(\mathrm{y})}. \tag{3.42}$$

Change of Variables

$$p_x(\boldsymbol{x}) = p_y(g(\boldsymbol{x})) \left| \det \left(\frac{\partial g(\boldsymbol{x})}{\partial \boldsymbol{x}} \right) \right|. \tag{3.47}$$

(Goodfellow 2016)

Information Theory

Information:

$$I(x) = -\log P(x). \tag{3.48}$$

Entropy:

$$H(\mathrm{x}) = \mathbb{E}_{\mathrm{x}\sim P}[I(x)] = -\mathbb{E}_{\mathrm{x}\sim P}[\log P(x)]. \tag{3.49}$$

KL divergence:

$$D_{\mathrm{KL}}(P\|Q) = \mathbb{E}_{\mathrm{x}\sim P}\left[\log \frac{P(x)}{Q(x)}\right] = \mathbb{E}_{\mathrm{x}\sim P}\left[\log P(x) - \log Q(x)\right]. \tag{3.50}$$

(Goodfellow 2016)

Entropy of a Bernoulli Variable

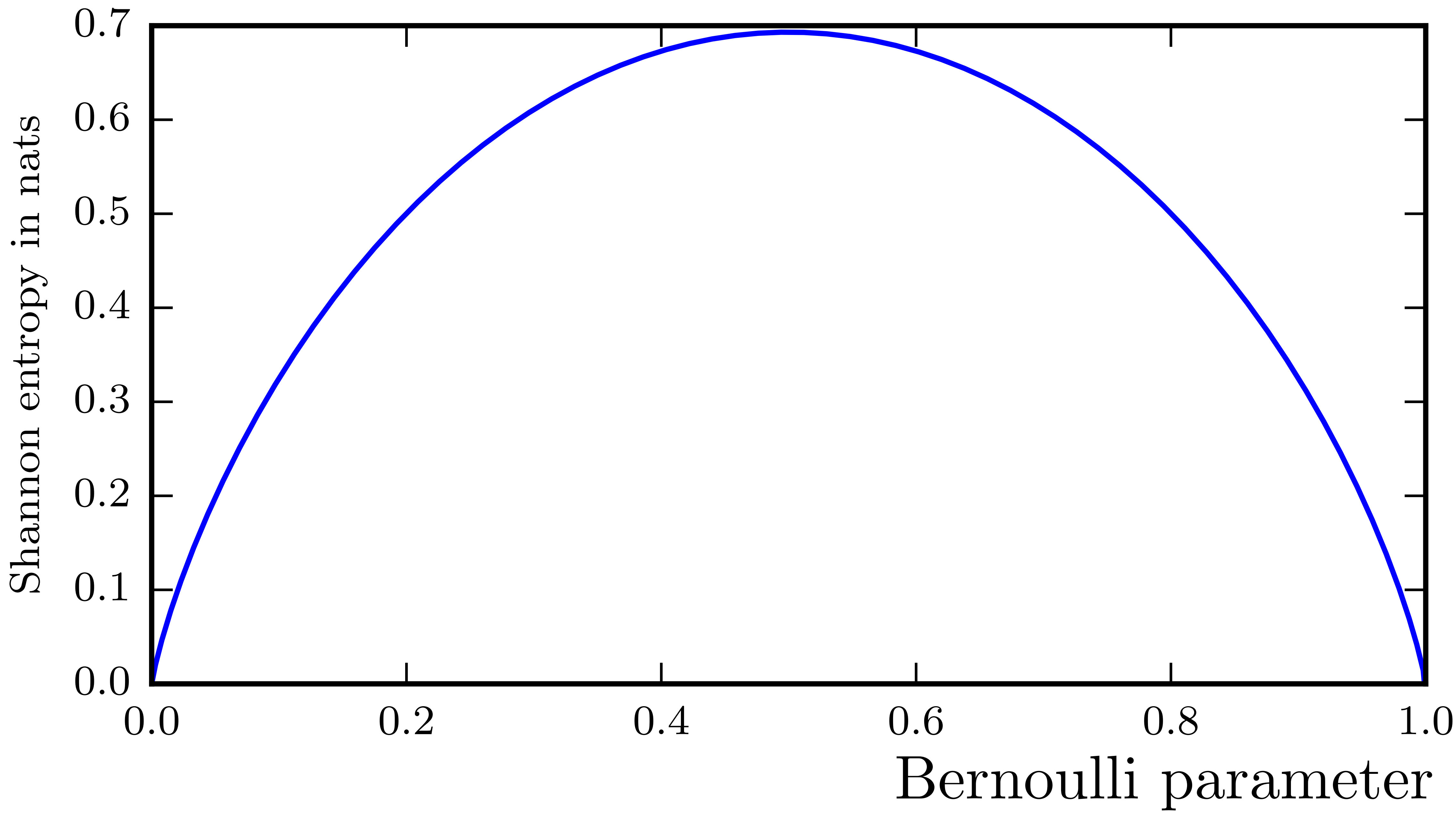


Figure 3.5

(Goodfellow 2016)

The KL Divergence is Asymmetric

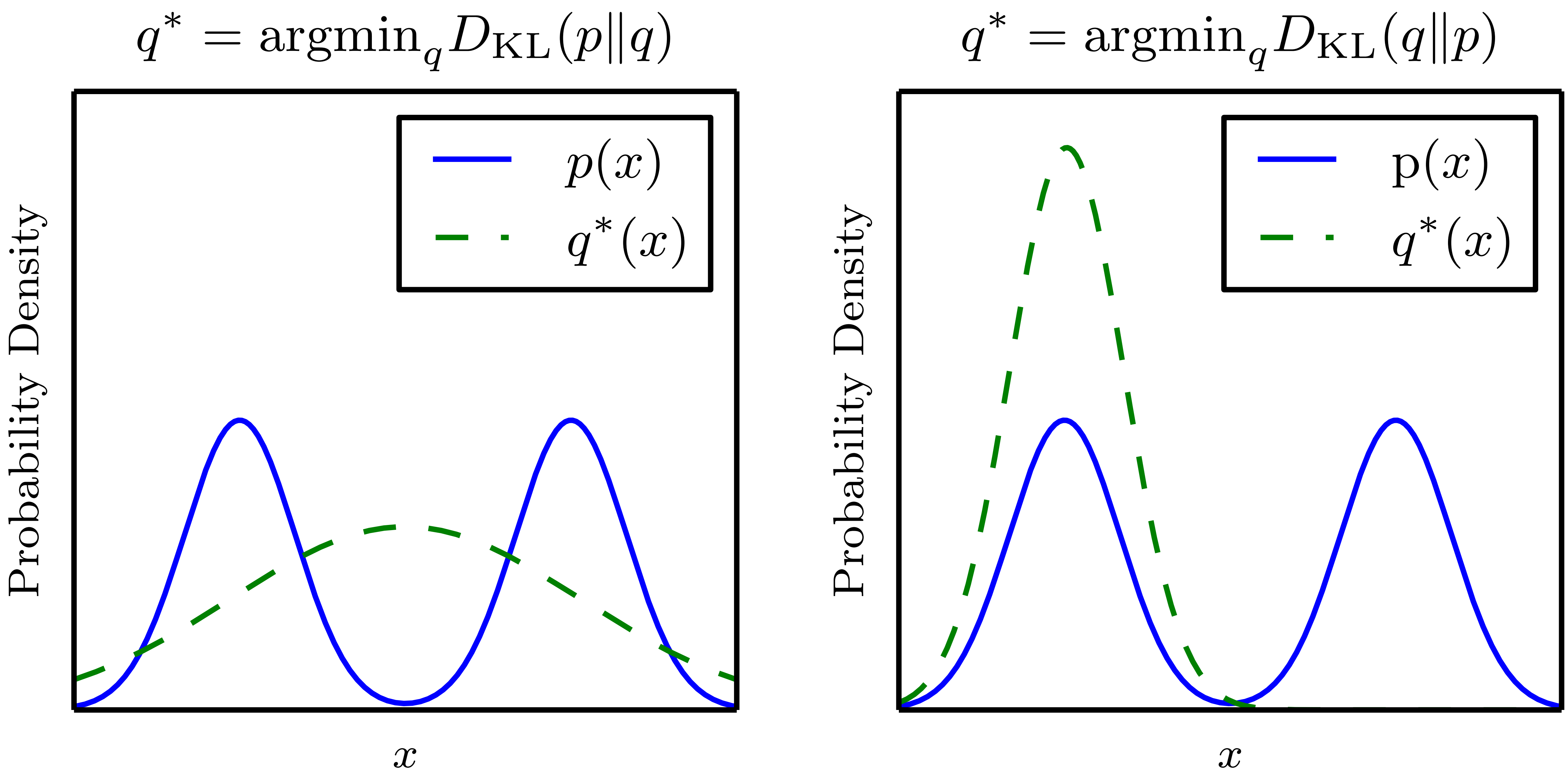
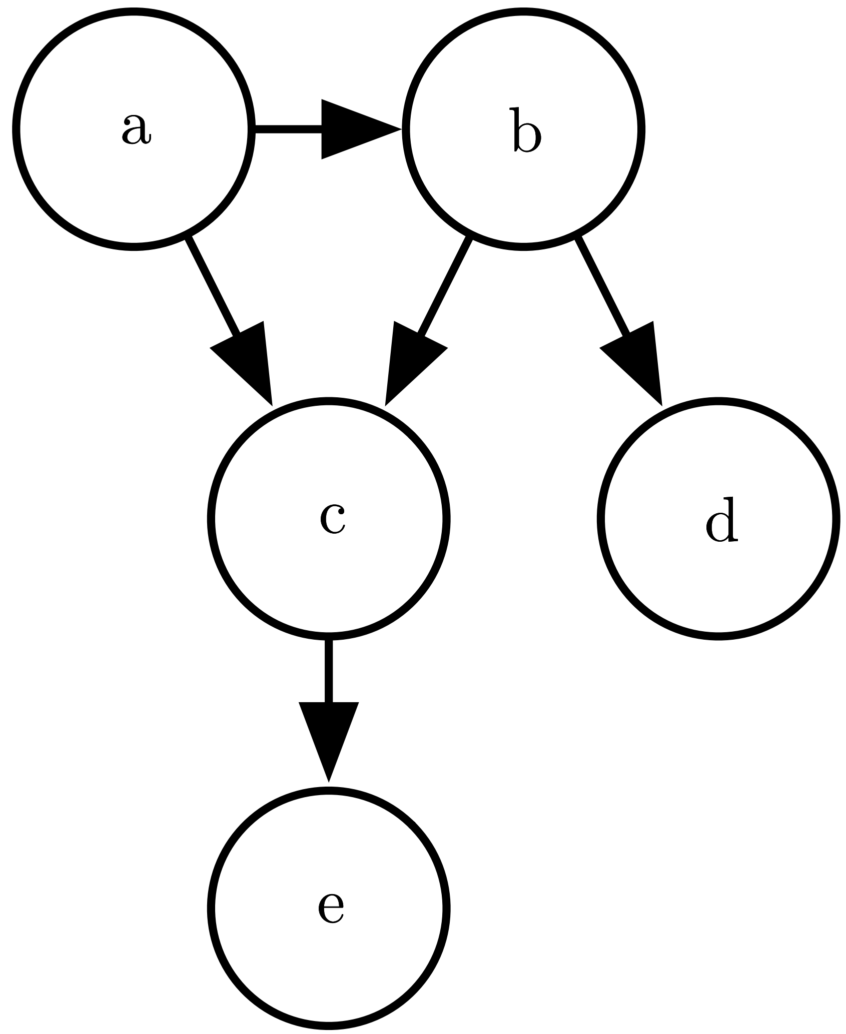


Figure 3.6

(Goodfellow 2016)

Directed Model

Figure 3.7



$$p(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}) = p(\mathrm{a})p(\mathrm{b} \mid \mathrm{a})p(\mathrm{c} \mid \mathrm{a}, \mathrm{b})p(\mathrm{d} \mid \mathrm{b})p(\mathrm{e} \mid \mathrm{c}). \tag{3.54}$$

(Goodfellow 2016)

Undirected Model

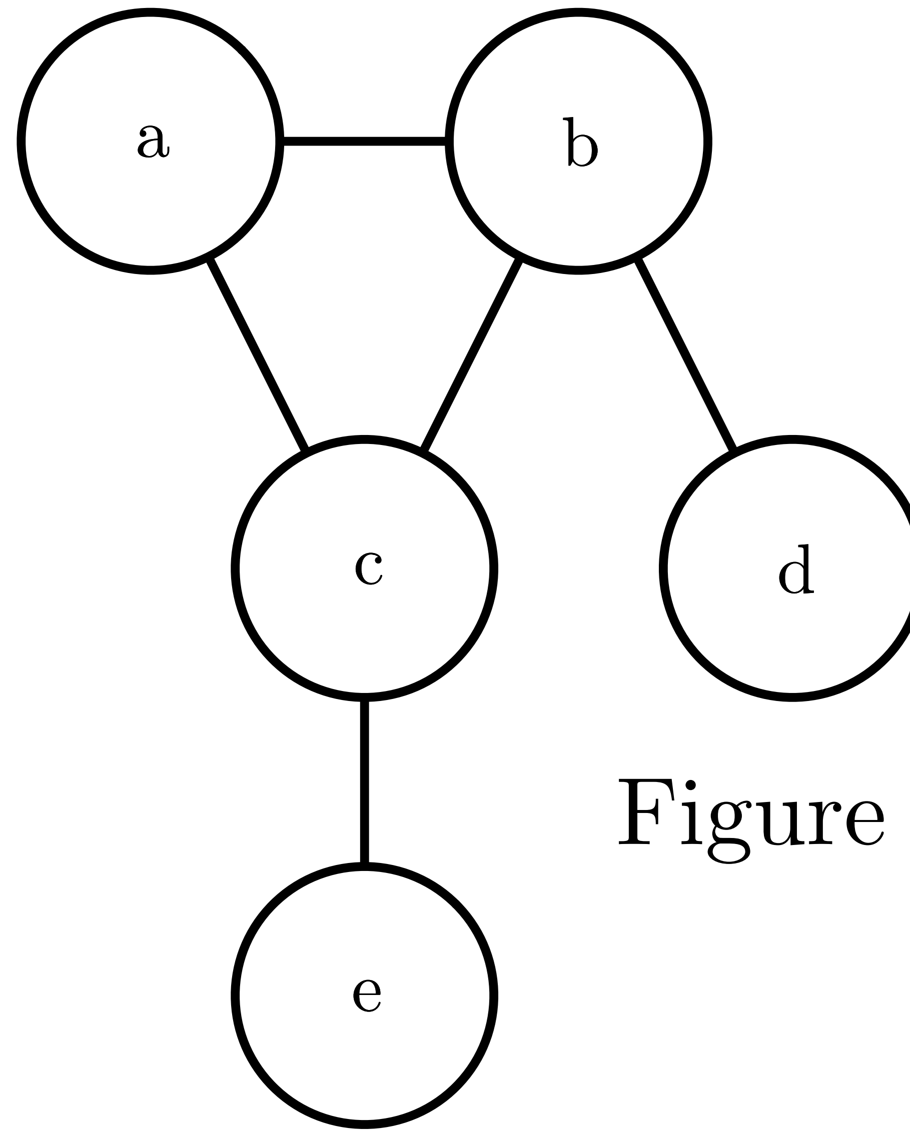


Figure 3.8

$$p(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}) = \frac{1}{Z}\phi^{(1)}(\mathrm{a}, \mathrm{b}, \mathrm{c})\phi^{(2)}(\mathrm{b}, \mathrm{d})\phi^{(3)}(\mathrm{c}, \mathrm{e}). \tag{3.56}$$

(Goodfellow 2016)