

$$T(n) = 1 + T(n-1)$$

$$T(n) = 1 + (1 + T(n-2))$$

$$T(n) = 2 + T(n-2)$$

$$T(n) = 3 + T(n-3)$$

$$T(n) = 4 + T(n-4)$$

.....

$$T(n) = k + T(n-k)$$

$$\text{For } k = (n-1)$$

$$T(n) = n - 1 + T(1)$$

$$T(n) = n$$

$$\text{i.e. } T(n) = \Theta(n)$$

$$T(n) = n + T(n-1)$$

$$T(n) = n + (n-1 + T(n-2))$$

$$T(n) = 2n - 1 + T(n-2)$$

$$T(n) = 2n - 1 + ((n-2) + T(n-3))$$

$$T(n) = 3n - 3 + T(n-3)$$

$$T(n) = 3n - 3 + n - 3 + T(n-4)$$

$$T(n) = 4n - 6 + T(n-4)$$

.....

$$T(n) = kn - c + T(n-k)$$

$$\text{For } k = (n-1)$$

$$T(n) = n^2 - 1 - c + T(1)$$

$$\text{i.e. } T(n) = \Theta(n^2)$$

$$T(n) = 1 + T(n/2)$$

$$T(n) = 1 + (1 + T(n/4))$$

$$T(n) = 2 + T(n/4)$$

$$T(n) = 3 + T(n/8)$$

$$T(n) = 3 + T(n/2^3)$$

.....

$$T(n) = k + T(n/2^k)$$

$$\text{For } k = \log n \Rightarrow n = 2^k$$

$$T(n) = \log n + T(1)$$

$$T(n) = 1 + \log(n)$$

$$\text{i.e. } T(n) = \Theta(\log n)$$

$$T(n) = n + T(n/2)$$

$$T(n) = n + (n/2 + T(n/4))$$

$$T(n) = n + n/2 + T(n/4)$$

$$T(n) = n + n/2 + n/4 + T(n/8)$$

$$T(n) = n + n/2 + n/4 + T(n/8).$$

$$T(n) = n(1 + 1/2 + 1/4) + T(n/8)$$

.....

$$T(n) = n(1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{k}) + T(n/2^k)$$

$$\text{For } k = n$$

$$T(n) = n(1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{n}) + T(1/2)$$

$$// \text{ Assume } T(1/2) = 1$$

$$T(n) = n \cdot (1 + 1) + 1$$

$$T(n) = \Theta(n)$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

From Wikipedia, the free encyclopedia

In [mathematics](#), the [infinite series](#)  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$  is an elementary example of a [geometric series](#) that [converges absolutely](#).

There are many expressions that can be shown to be equivalent to the problem, such as the form:  $2^{-1} + 2^{-2} + 2^{-3} \dots$

Its [sum](#) is

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}} = 1$$

$$T(n) = n + 2T(n/2)$$

$$T(n) = n + 2(n/2 + 2T(n/4))$$

$$T(n) = n + 2n/2 + 4T(n/4)$$

$$T(n) = n + 2n/2 + 4(n/4 + 2T(n/8))$$

$$T(n) = n + n + n + 8T(n/8)$$

$$T(n) = 3n + 8(n/8 + 2T(n/16))$$

$$T(n) = 4n + 2T(n/2^4)$$

.....

$$T(n) = kn + 2T(n/2^k)$$

$$\text{For } k = \log n \Rightarrow n = 2^k$$

$$T(n) = n \cdot \log n + T(1)$$

$$T(n) = O(n \log n)$$

$$T(n) = 2T(n-1)$$

$$T(n) = 2(2T(n-2))$$

$$T(n) = 4T(n-2)$$

$$T(n) = 4(2T(n-3))$$

$$T(n) = 8T(n-3)$$

$$T(n) = 8(2T(n-4))$$

$$T(n) = 16T(n-4)$$

$$T(n) = 2^4 T(n-4)$$

.....

$$T(n) = 2^k T(n-k)$$

$$\text{Let } k = n-1$$

$$T(n) = O(2^n)$$