Non Deterministic Finite Automata

Lecture 7

Nondeterminism

- A *nondeterministic finite automaton* has the ability to be in several states at once.
- Transitions from a state on an input symbol can be to any set of states.
- Start in one start state.
- Accept if any sequence of choices leads to a final state.

Nondeterministic Finite Automata

- Same as a DFA, except:
  - On input a, state q may have more than one transition out, implying the possibility of multiple choices when processing an input symbol
  - On input a, state q may have no transition out, implying the possibility of "being stuck"
- A string w is acceptable as long as **there exists** an admissible state sequence for w

NFAs

- A nondeterministic finite automaton M is a five-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where:
  - Q is a finite set of states of M
  - Σ is the finite input alphabet of M
  - δ: $Q \times \Sigma \rightarrow$ power set of Q, is the state transition function mapping a state-symbol pair to a subset of Q
  - q_0 is the start state of M
  - $F \subseteq Q$ is the set of accepting states or final states of M

Nondeterministic Finite Automaton (NFA)

A **nondeterministic finite automaton (NFA)** is a 5-tuple (Q,Σ,Δ,s,F) where:

- Q is a finite set of elements called **states**
- Σ is a finite input alphabet
- $\Delta \subseteq \mathbf{Q} \times (\Sigma \cup \{e\}) \times Q$ — "the empty word"
- $s \in Q$ called the **start state**
- $F \subseteq Q$ called the **favorable states**

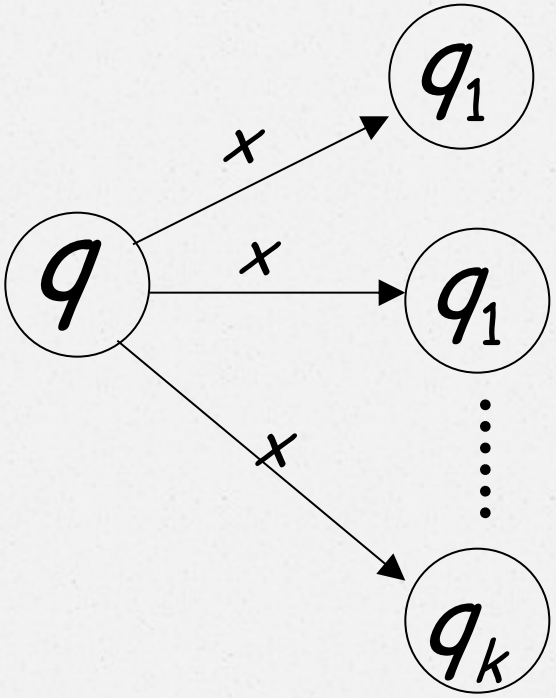
The crucial point is that Δ is a relation

Book says: (Q,Σ, δ,s,F) where:
δ is a transition function, $\mathbf{Q} \times (\Sigma \cup \{e\}) \times \wp(Q)$ — "power set"

Are δ and Δ representing the same transitions?

Transition Function δ

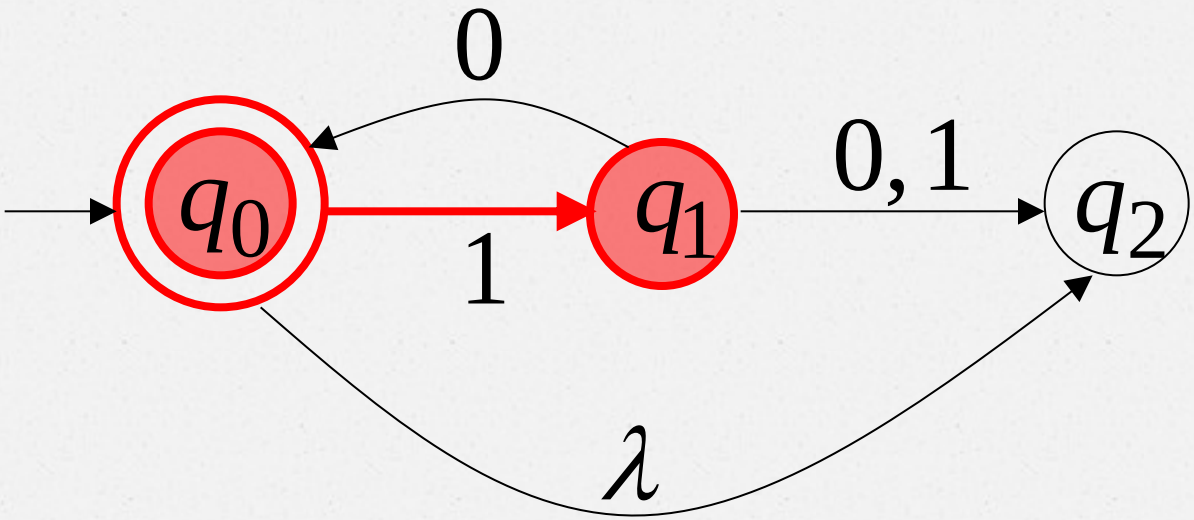
$$\delta(q, x) = \{q_1, q_2, \ldots, q_k\}$$



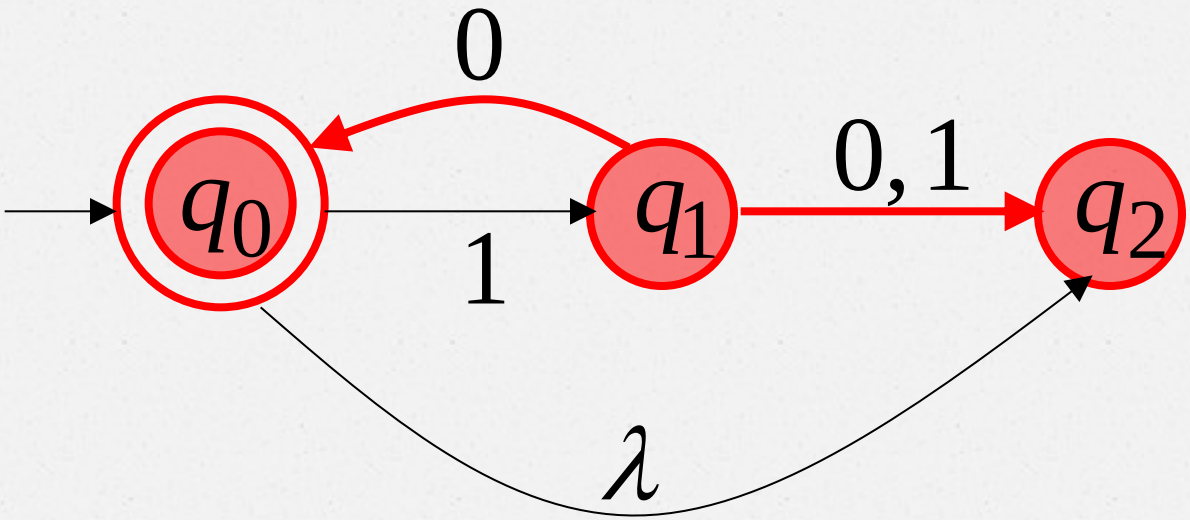
Resulting states with following one transition with symbol

x

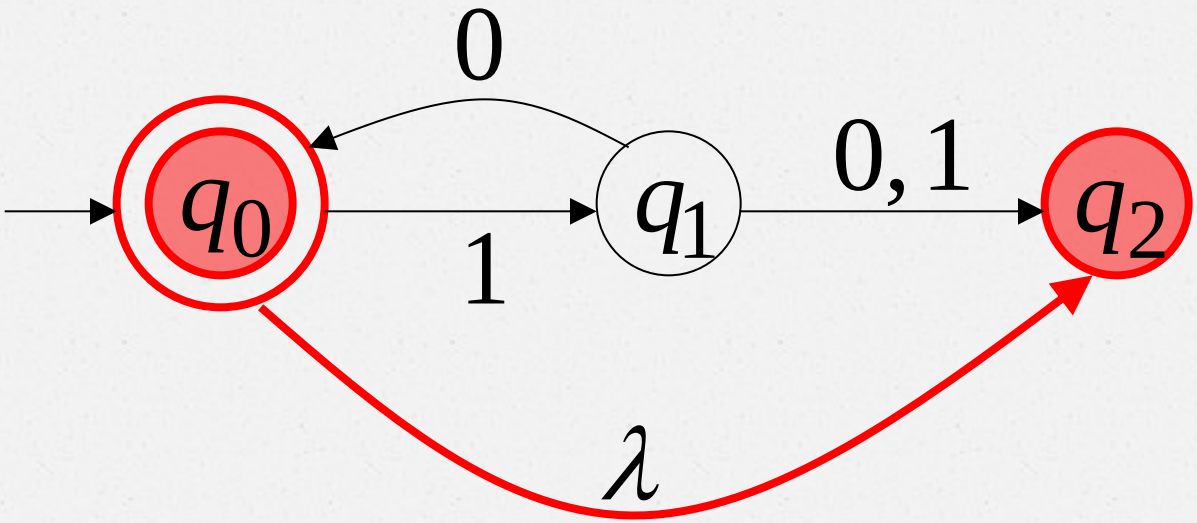
$$\delta(q_0, 1) = \{q_1\}$$



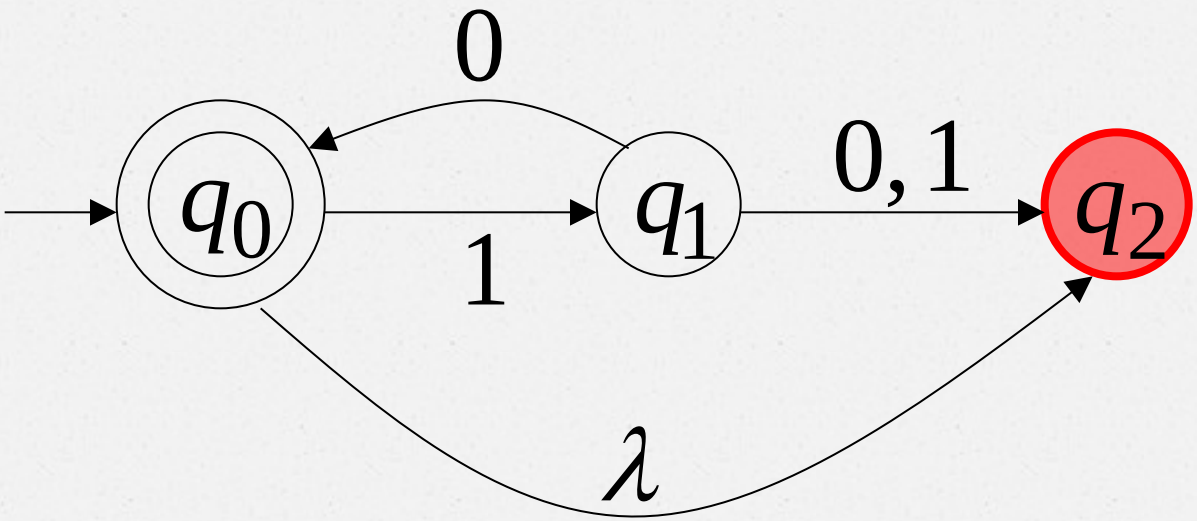
$$\delta(q_1,0) = \{q_0, q_2\}$$



$$\delta(q_0, \lambda) = \{q_2\}$$

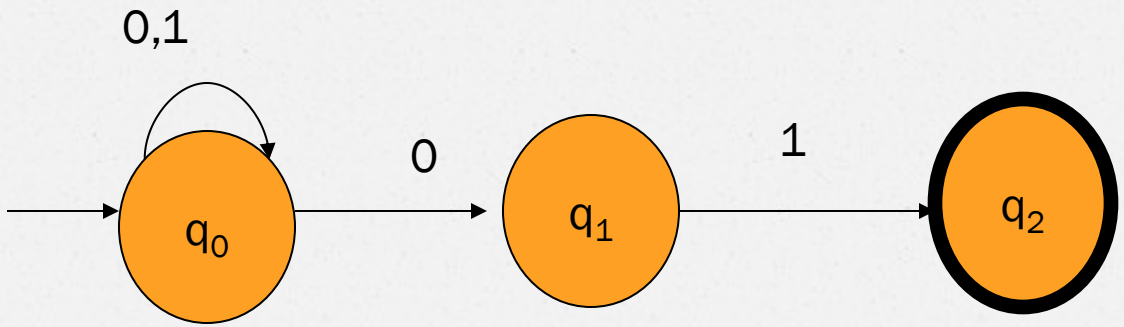


$$\delta(q_2,1)=\varnothing$$



Example NFA

- NFA that recognizes the language of strings that end in 01



note: $\delta(q_0,0) = \{q_0,q_1\}$
$\delta(q_1,0) = \{\}$

Exercise: draw the complete transition table for this NFA