Problem No 1:

a) Cumulative Gain (CG).

for q_1:

$CG = 2 + 0 + 2 + 2 + 1 + 1 + 1 = 9$

for q_2:

$CG = 2 + 2 + 2 + 1 + 1 + 0 + 0 = 8$

(b) Discount Cumulative Gain (DCG)

for q_1:

i	doc	rel_i	$\log_2(i+1)$	$rel/\log_2(i+1)$
1	4	2	1	2
2	5	0	1.585	0
3	6	2	2	1
4	7	2	2.322	0.861
5	1	1	2.585	0.3868
6	8	1	2.807	0.3620
7	9	1	3	0.333

$$DCG = \sum_{i=1}^{7} \frac{rel}{\log_2(i+1)} = 2 + 0 + 1 + 0.861 + 0.3868 + 0.3562 + 0.333$$

$$= 4.937 \cong 5$$

Ans.

for q_2's

i	doc No	rel$_i$	$\log_2(i+1)$	rel$_i$ / $\log_2(i+1)$
1	2	2	1	2
2	3	2	1.585	1.261
3	8	2	2	1
4	4	1	2.322	0.430
5	5	1	2.585	0.3868
6	9	0	2.807	0
7	1	0	3	0

$$DCG \sum_{i=1}^{7} \frac{rel_i}{\log_2(i+1)} = 2 + 1.261 + 1 + 0.430 + 0.3868 = 5.0778$$

Ans/

(c) Normalized DCG's:

for q_1:

$$IDCG_7 = \frac{2}{1} + \frac{2}{1.585} + \frac{2}{2} + \frac{1}{2.322} + \frac{1}{2.585} + \frac{1}{2.807} + \frac{0}{3.} = 5.435$$

$$\therefore DGG_7 = \frac{DCG_7}{IDCG_7} = \frac{4.937}{5.435} = 0.909$$

for q_2

$$IDCG_7 = \frac{2}{1} + \frac{2}{1.585} + \frac{2}{2} + \frac{1}{2.322} + \frac{1}{2.585}$$

$$\frac{0}{2.807} + \frac{0}{3} = 5.0778$$

$$nDCG_7 = \frac{IDCG_7}{DCG_7} = 1 \quad \text{Ans!}$$

Problem No 2:

		Judge 2.		
	X	Yes ← 1	No ← 0	Total
Judge One	Yes ← 1	2	2	4
	No ← 0	2	2	4
	Total	4	4	8

$$K = \frac{P(A) - P(E)}{1 - P(E)}$$

$$= \frac{4/8 - \left((8)^2/16 + (8)^2/16\right)}{1 - \left((8)^2/16 + (8)^2/16\right)}$$

$$P(E) = P(\text{relevant})^2 + P(\text{non rel})^2$$

$$P(\text{relev}) = (4+4)/16 = 0.5$$

$$P(\text{non rel}) = (4+4)/16 = 0.5$$

$P(E) = 0.5$

$K = \frac{4/8 - 0.5}{1 - 0.5} = 0.$

(b) relevant ← if the two judges agree:

Precision: $\frac{5 - 2}{8 \leftarrow \text{total}} = 0.5$

Recall: $\frac{8 - 2}{2} = 1$

F1:

Precision: $\frac{2}{5} = 0.4$

Recall: $\frac{2}{2} = 1$

F1: $(1+1) * 0.4 * 1 / 1^2 * 0.4 + 1$

$= 0.5714$ Ans!

(a) precision : $\frac{4}{5} = 0.8$

recall : $\frac{4}{6} = 0.67$

$F1 = (2 * 0.8 * 0.67) / 1^2 * (0.8 + 0.67)$

$F1 = 0.729$

Ans

Problem No 3:

(a)

$$\vec{q}_m = \alpha \vec{q}_0 + \beta \frac{1}{|D_r|} \sum_{d_i \in D_r} d_i - \gamma \frac{1}{|D_{nr}|} \sum_{d_i \in D_{nr}} \vec{d_i}$$

$$= (0.1)\langle 0.01, 0.22, 0.11, 0.01, 0.01, 0.22, 0.1\rangle$$

$$+ (0.2)\frac{1}{3}\langle 0.61, 0.71, 0.31, 0.21, 0.55, 0.81, 0.41\rangle$$

$$- (0.4)\frac{1}{1}\langle 0.11, 0.21, 0.11, 0.11, 0.12, 0.12, 0.01\rangle$$

$$\vec{q}_m = \langle 1\times10^{-3}, 0.022, 0.011, 1\times10^{-3}, 1\times10^{-3}, 0.022, 0.01\rangle + \langle 0.040, 0.047, 0.020, 0.014, 0.0367, 0.0541, 0.0274\rangle - \langle 0.044, 0.084, 0.044, 0.044, 0.048, 0.048, 4\times10^{-3}\rangle$$

$$\vec{q}_m = \langle -3\times10^{-3}, -0.015, -0.013, -0.029, -0.0103, 0.028, 0.0334\rangle$$

(b) $\alpha = 0.1$, $\beta = 0.2$ & $\gamma = 0$

Because we need only relevent feedback. So, $\gamma = 0$ & $\beta > \gamma$ i.e. $0.2 > 0$

$$\vec{q}_1 = (0.1)\,\vec{q}_0 + (0.2)\,\frac{1}{3} \sum_{d_j \in D_r} d_j - 0$$

$$= \langle 1\times10^{-3}, 0.022, 0.011, 1\times10^{-3}, 1\times10^{-3}, 0.022, 0.01 \rangle + \langle 0.040, 0.047, 0.020, 0.014, 0.0367, 0.0548, 0.0274 \rangle$$

$$= \langle 0.041, 0.069, 0.031, 0.015, 0.0377, 0.072, 0.0374 \rangle$$

Ans!

(L) When $\alpha = 1$, $\beta \frac{1}{|D_r|} \sum_{d_j \in D_r} d_j - \gamma \frac{1}{|D_{nr}|} \sum_{d_j \in D_{nr}} d_j$

No, if β is very small & γ is very large. q_0 may be closer to centroid of the relevant document.