Artificial Intelligence (CS-401)

Chapter 6: Constraint Satisfaction Problem

Objectives

- Backtracking
- MRV and degree heuristic
- Forward checking
- Improvement in forward check (through heuristic combination)
- MAC (maintaining arc consistency)
- Back jumping
- Intelligent backtracking (constraint learning)

Constraint Satisfaction Problem

- So far in Search space, states evaluated by heuristics and goal.
- A constraint satisfaction problem (CSP) is a problem that requires its solution within some limitations/conditions also known as constraints.
- For which the typical problems can be converted to CSP and solved.

Converting problems to CSPs

- A problem to be converted to CSP requires the following steps:
- **Step 1:** Create a variable set.(Representing state)
- **Step 2:** Create a domain set.(values)
- **Step 3:** Create a constraint set with variables and domains after considering the constraints.(limitations)

CSP Example1-Sudoku

Rule: Fill all empty Squares so that the numbers 1 to 9 appear once in each row, column and 3x3 box

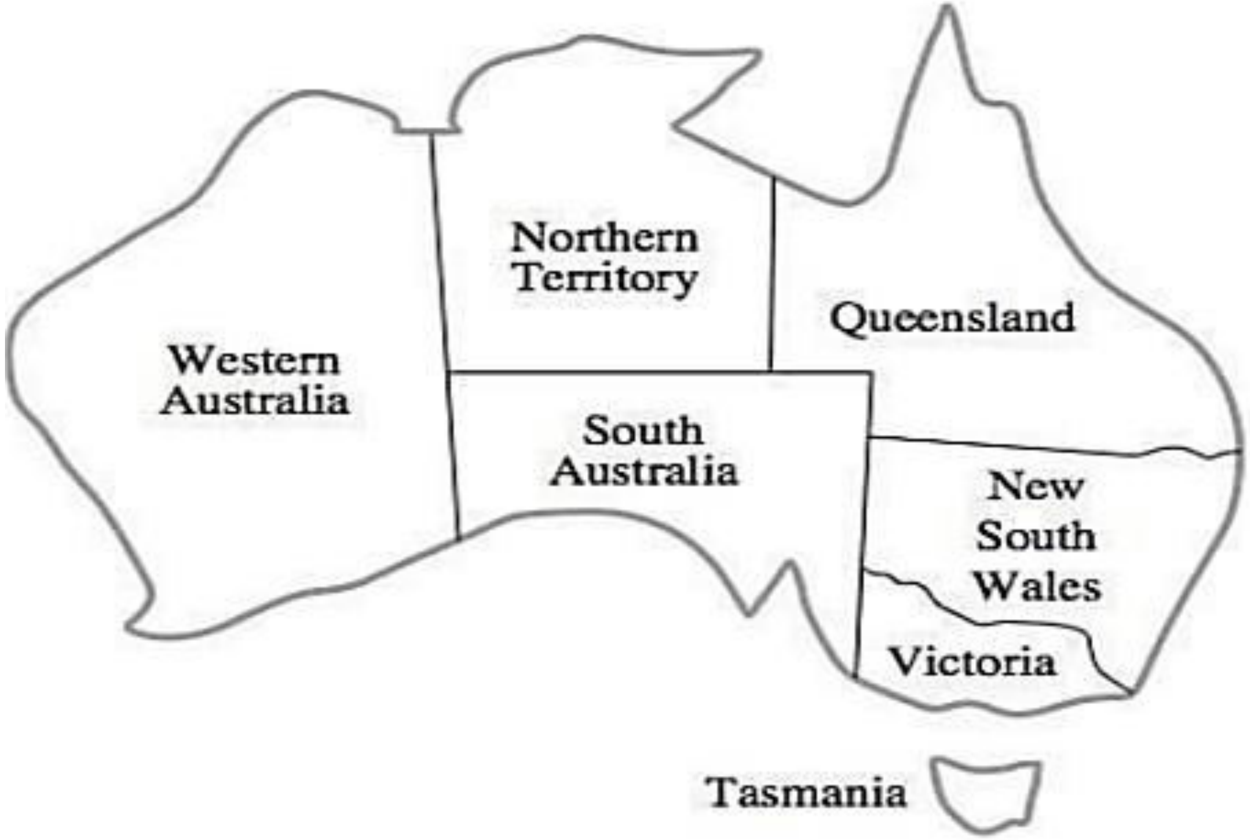
	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

VARIABLES – variables are cells (A1-I9)

DOMAIN – domain of each variable is {1,2,3,4,5,6,7,8,9}

CONSTRAINT– constraints: rows, columns, boxes contain all different numbers

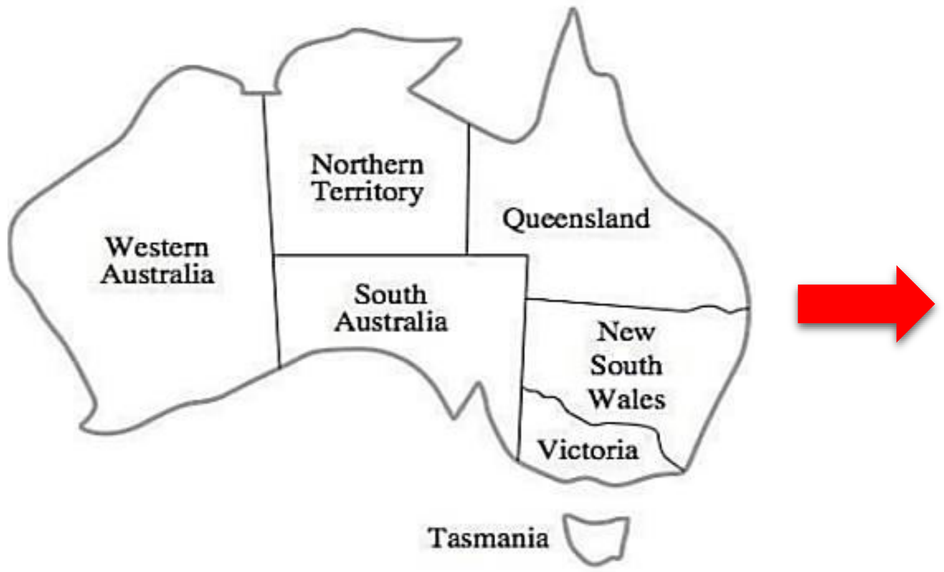
Example2-Map Coloring

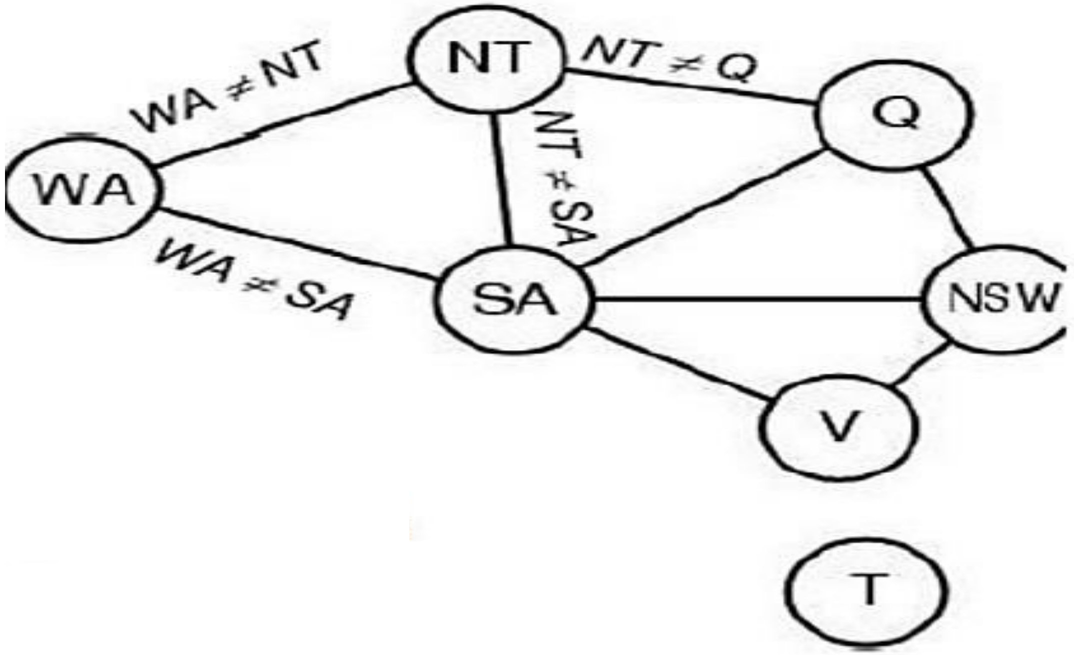


Rule - Color each region either red, green or blue

- No adjacent region can have the same color

CSP Representation of Map Coloring





Variables: WA, NT, Q, NSW, V, SA, T

Domains: Di = {red, green, blue}

Constraints: adjacent regions must have different colors

e.g., WA ≠ NT, or (WA,NT)

Constraint Graph

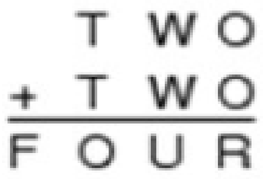
- **Nodes** are Variables
- **Arcs** are Constraint

Varieties of constraints

- Unary constraints involve a single variable, e.g. SA ≠ green
- Binary constraints involve pairs of variables (relates two variables),e.g., SA ≠ WA
- Higher-order constraints involve 3 or more variables, e.g. cryptarithmetic puzzles (column constraints)

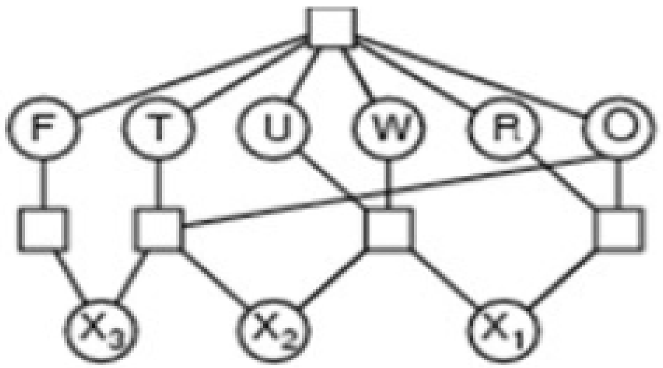
Example: Cryptarithmetic

Each letter stands for a distinct digit; the aim is to find a substitution of digits for letters such that the resulting sum is arithmetically correct, with the added restriction that no leading zeroes are allowed



- Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$
- Domains: $\{0,1,2,3,4,5,6,7,8,9\}$
- Constraints: *Alldiff (F,T,U,W,R,O)*
- where XI, X2, and X3 are auxiliary variables representing the digit (0 or 1) carried over into the next column

  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$

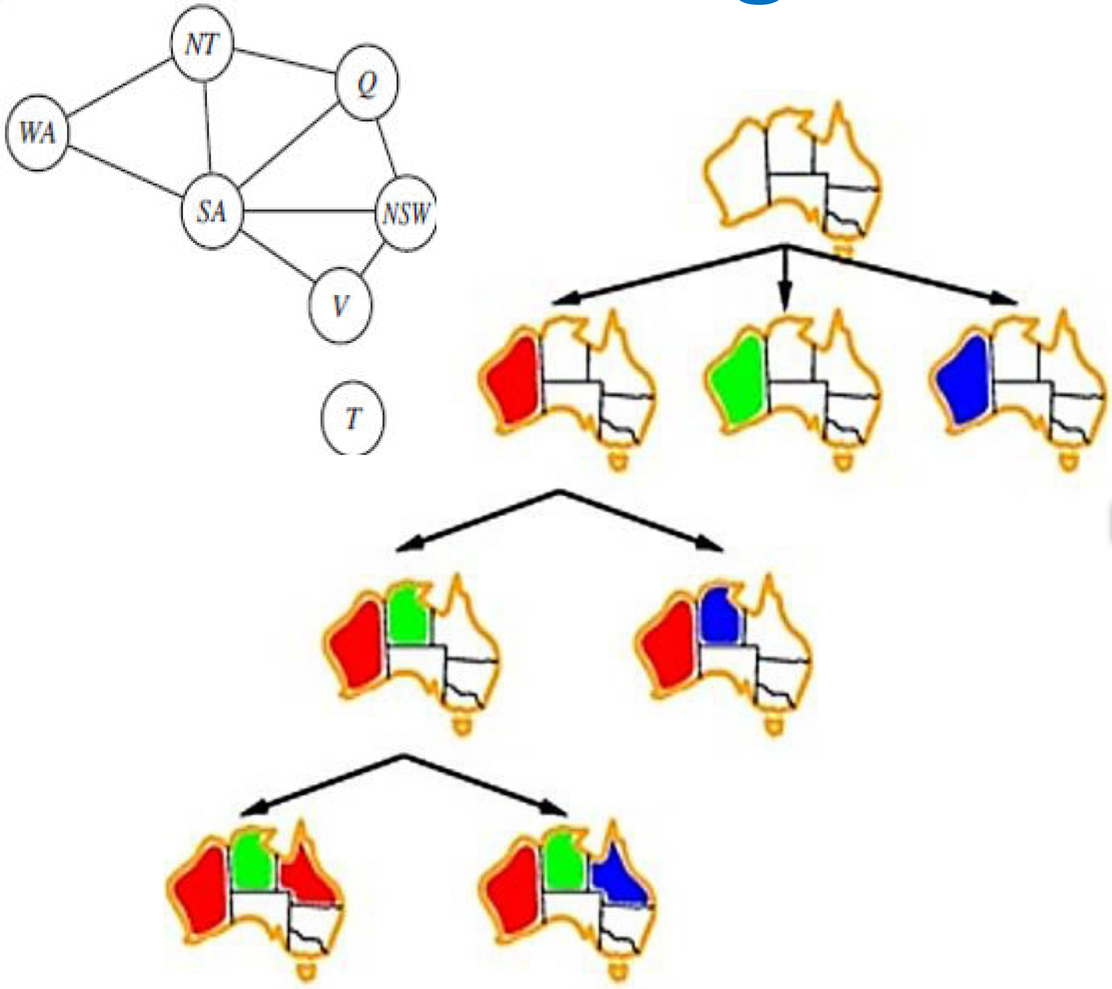


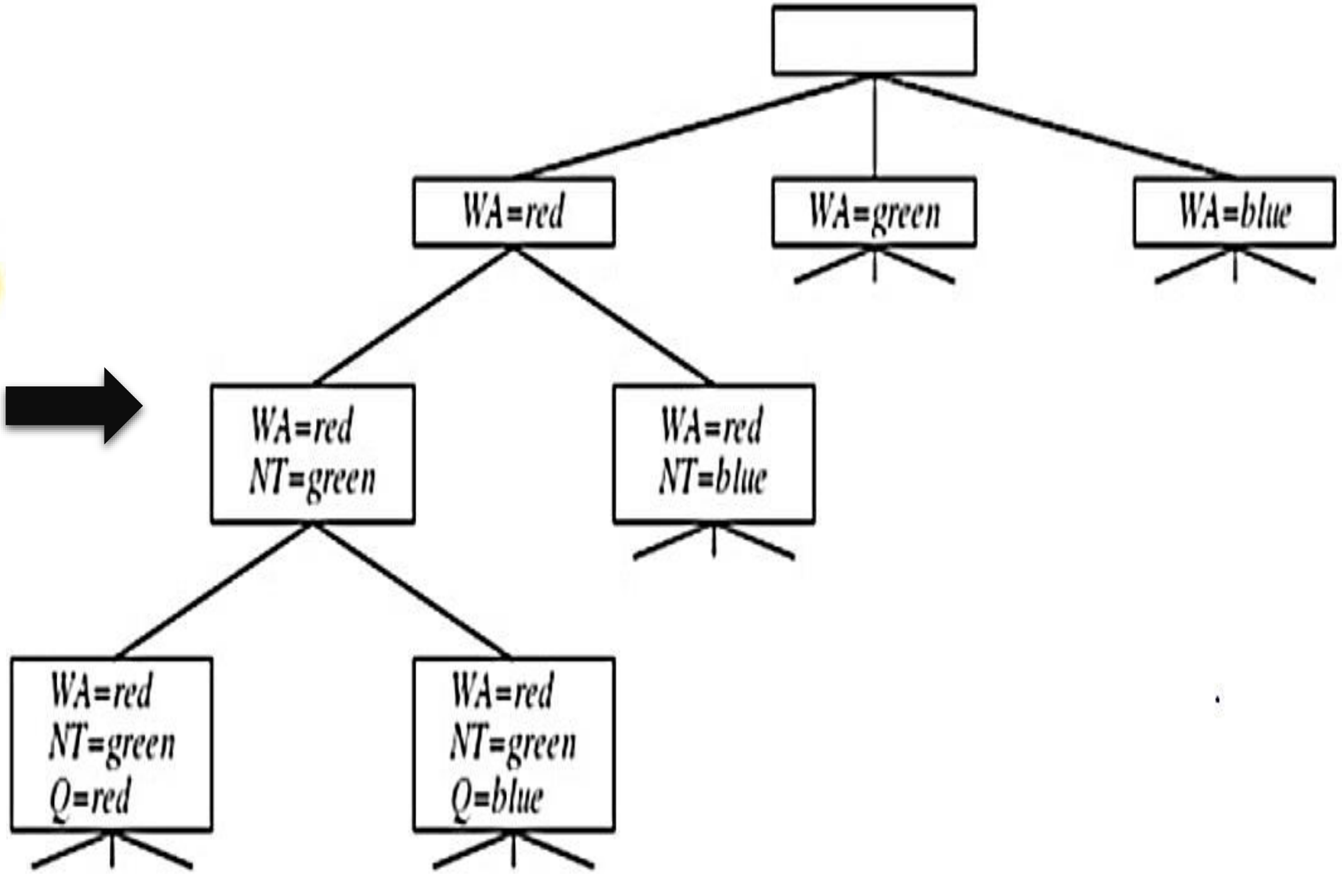
The constraint hyper graph for the cryptarithmetic problem, showing the *Alldzff constraint as well as the column addition* constraints

Backtracking search

- Depth-first search for CSPs with single-variable assignments is called backtracking search
- Variable assignments are commutative: [WA = red then NT = green] same as [NT = green then WA = red]
- Only need to consider assignments to a single variable at each node
- Backtracks when variable has no legal value

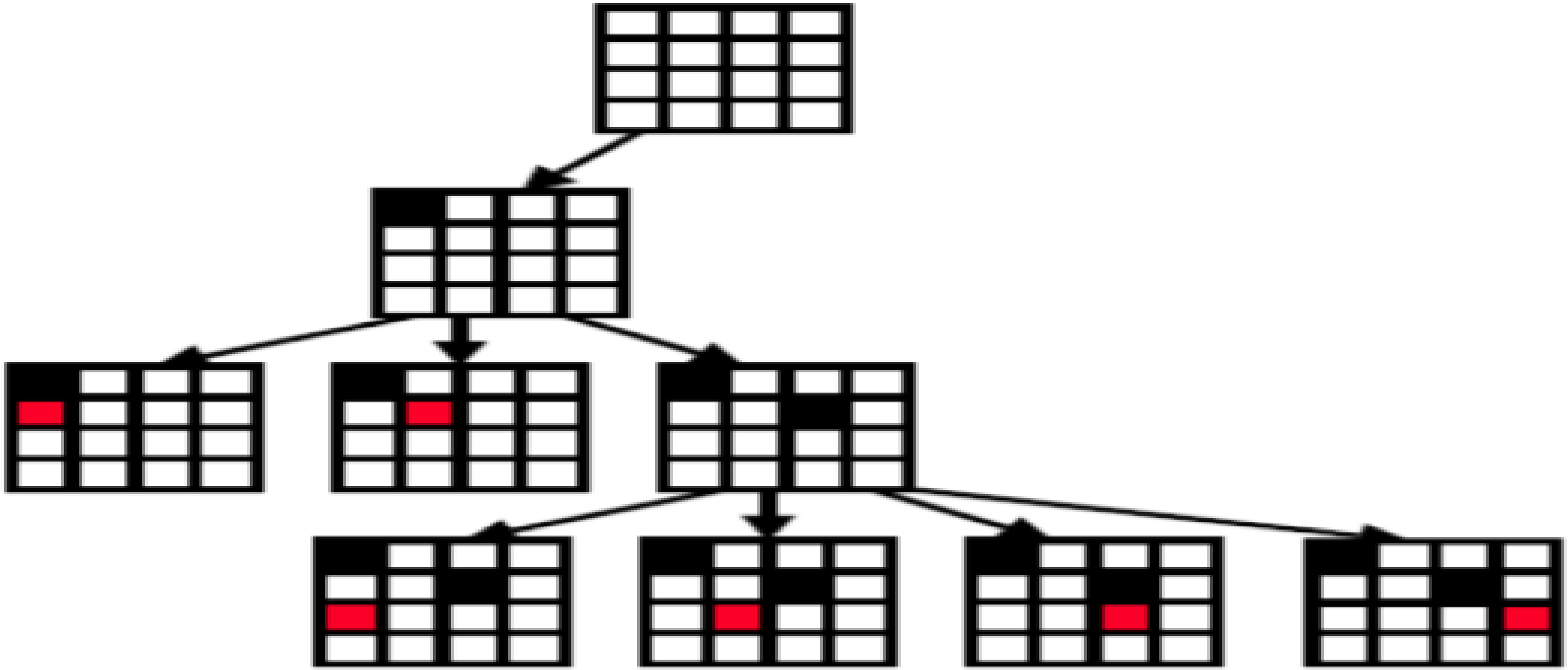
Backtracking for the map-coloring problem

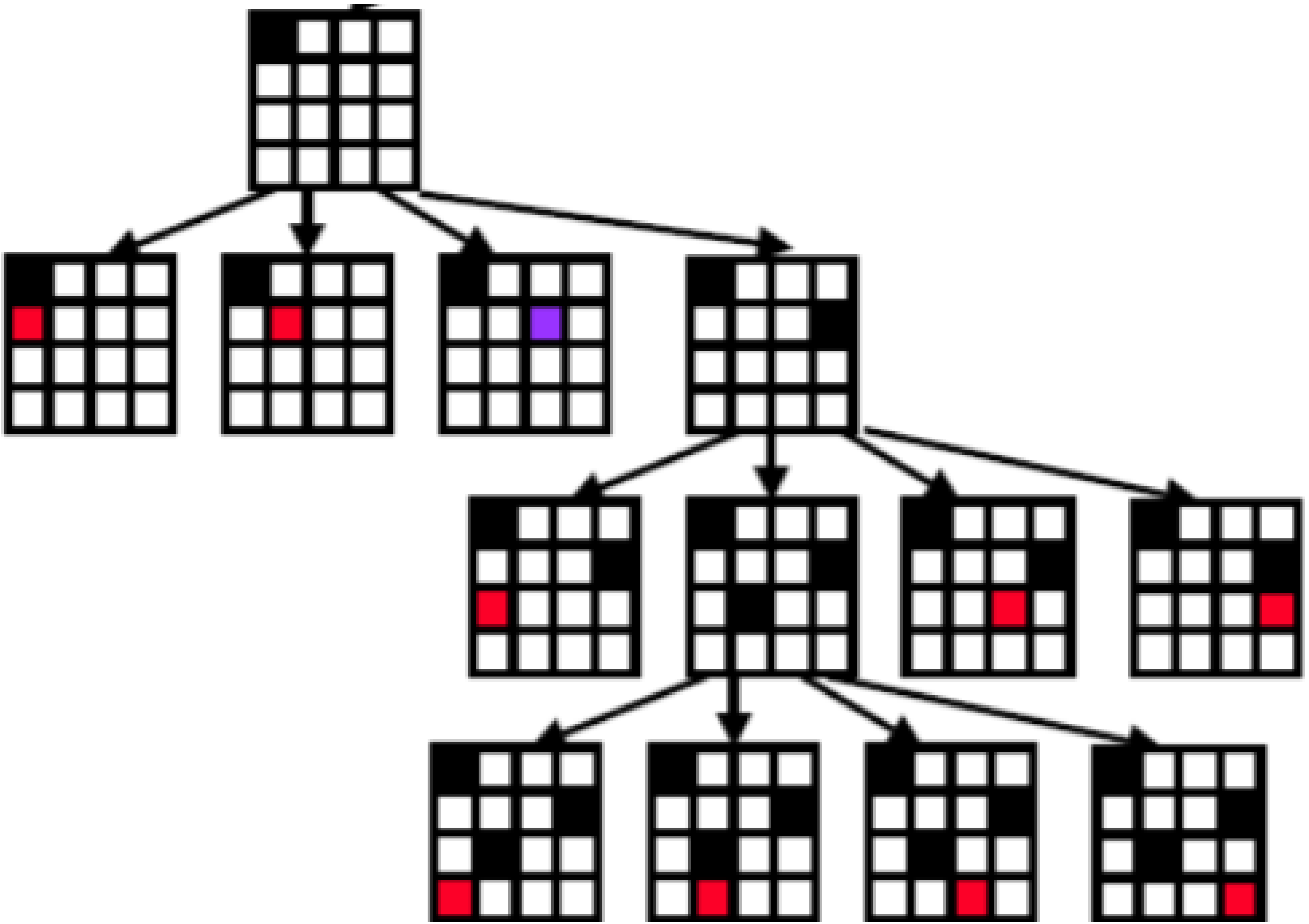




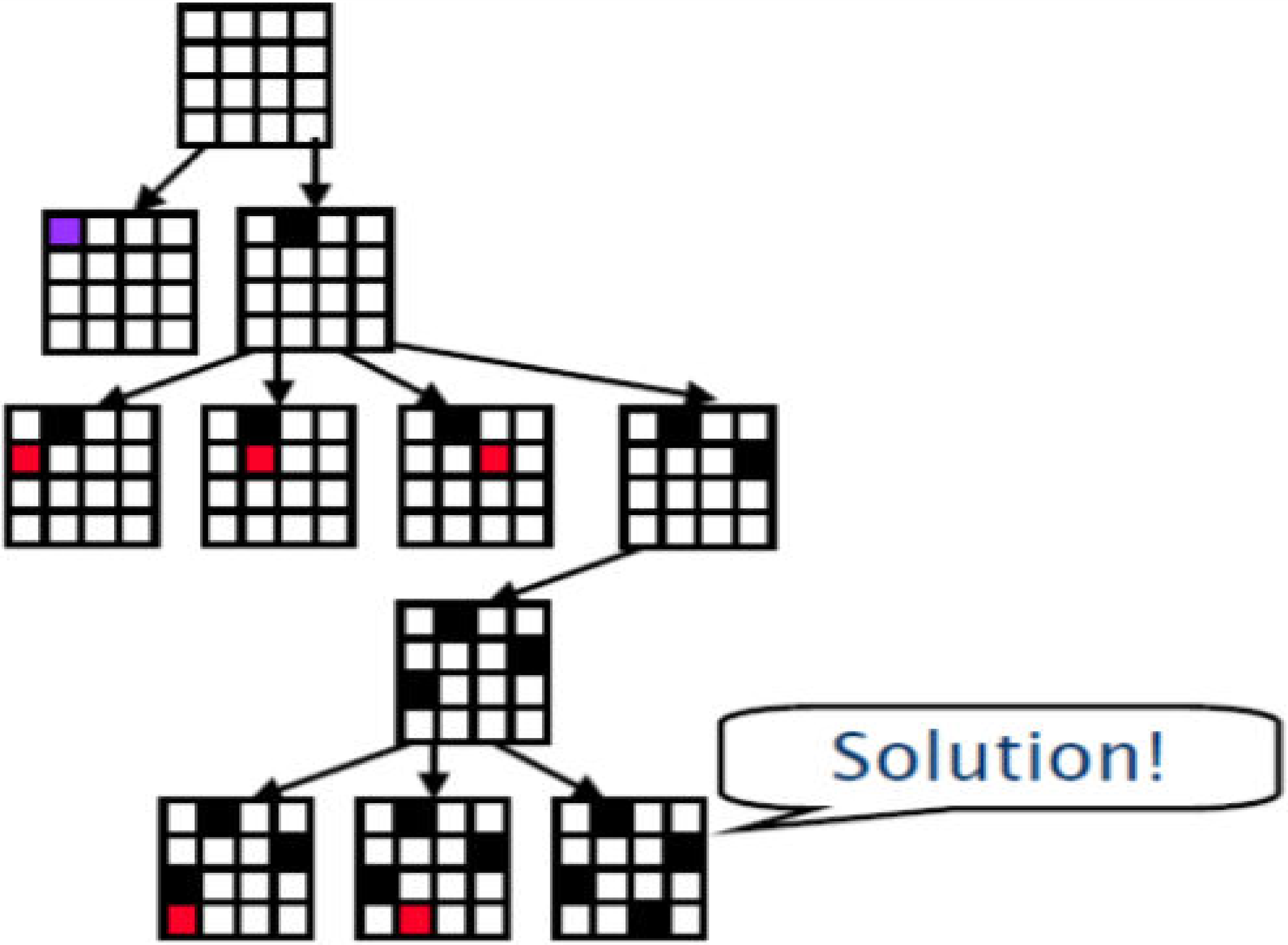
Backtracking Example:2

- 4X4 Queens





- 4X4 Queen



Improving backtracking efficiency (Variable and Value Ordering)

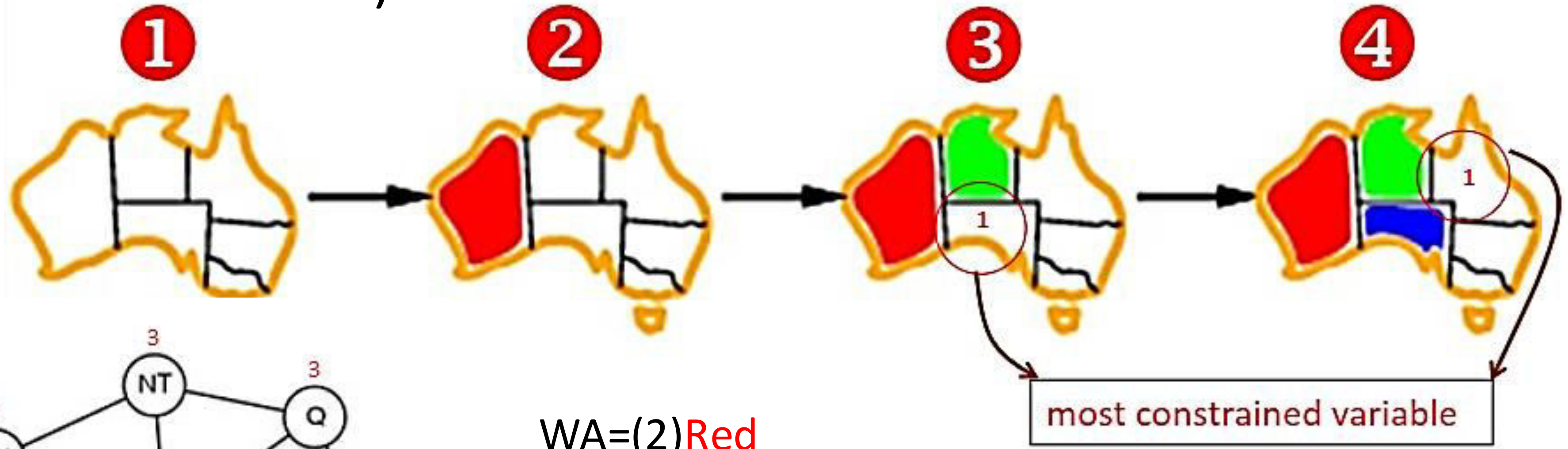
By default, Backtrack algorithm simply selects the next unassigned variable in the order given by the list VARIABLES[csp].

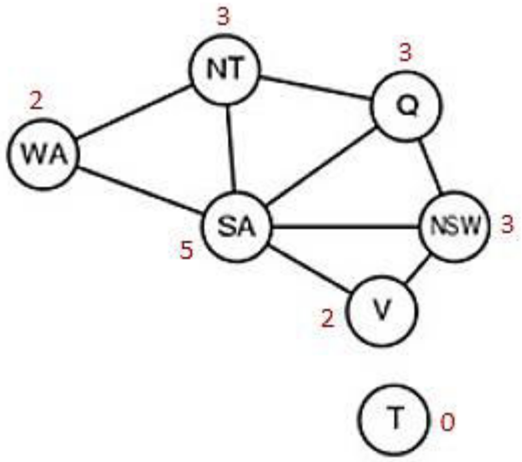
General-purpose methods can give huge gains in speed:

- Which variable should be assigned next?
- In what order should its values be tried ?
- Can we detect inevitable failure early?

Minimum remaining values (MRV)

- This intuitive idea-choosing the variable with the fewest "legal" values is called the minimum remaining values (MRV) heuristic (also known most constrained variable)



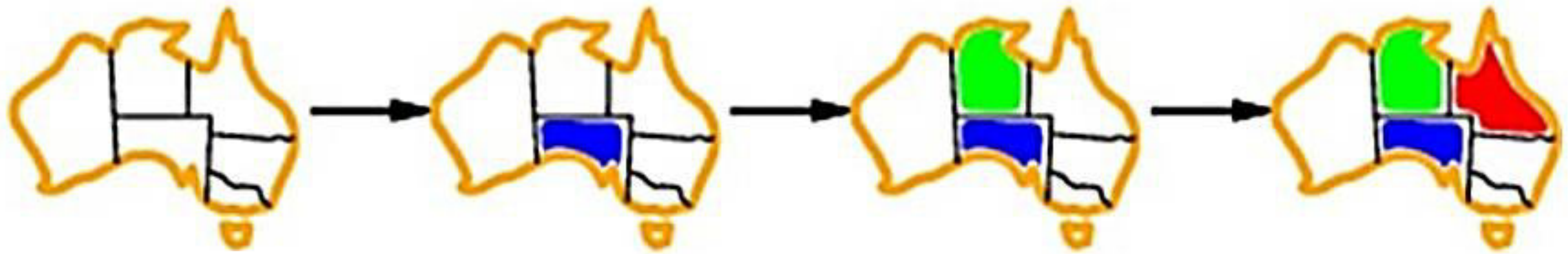


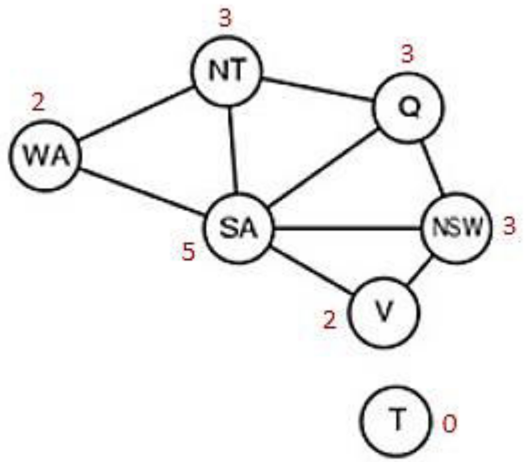
WA=(2)Red
NT(Remaining values)=2(Blue , Green)
while Q and NSW have 3 remaining values so, go for NT
SA(Remaining values)= 1(Blue)

after the assignments for WA = red and NT = green, there is only one possible value for SA, so it makes sense to assign **SA** = blue next rather than assigning Q, after SA is assigned, the choices for Q, NS W, and V are all forced.

Most-Constraining Variable (Degree Heuristics)

- Most constraining variable: choose the variable with the most constraints on remaining variables

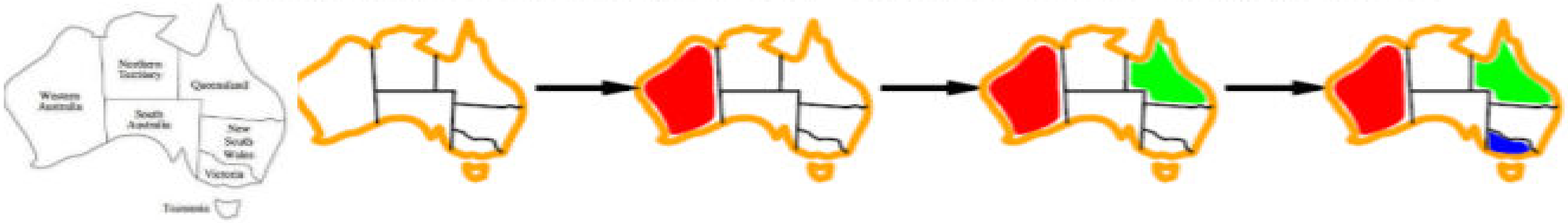




SA=5 (Most Constraints) BLUE
NT, Q, NSW = 3 **TIE** b/w them break it by choosing the first choice either NT or NSW.
So, choose NT Then Q, NSW OR NSW, Then Q and NT

Forward Checking

Idea: Keep track of remaining legal values for unassigned variables
Terminate search when any variable has no legal values

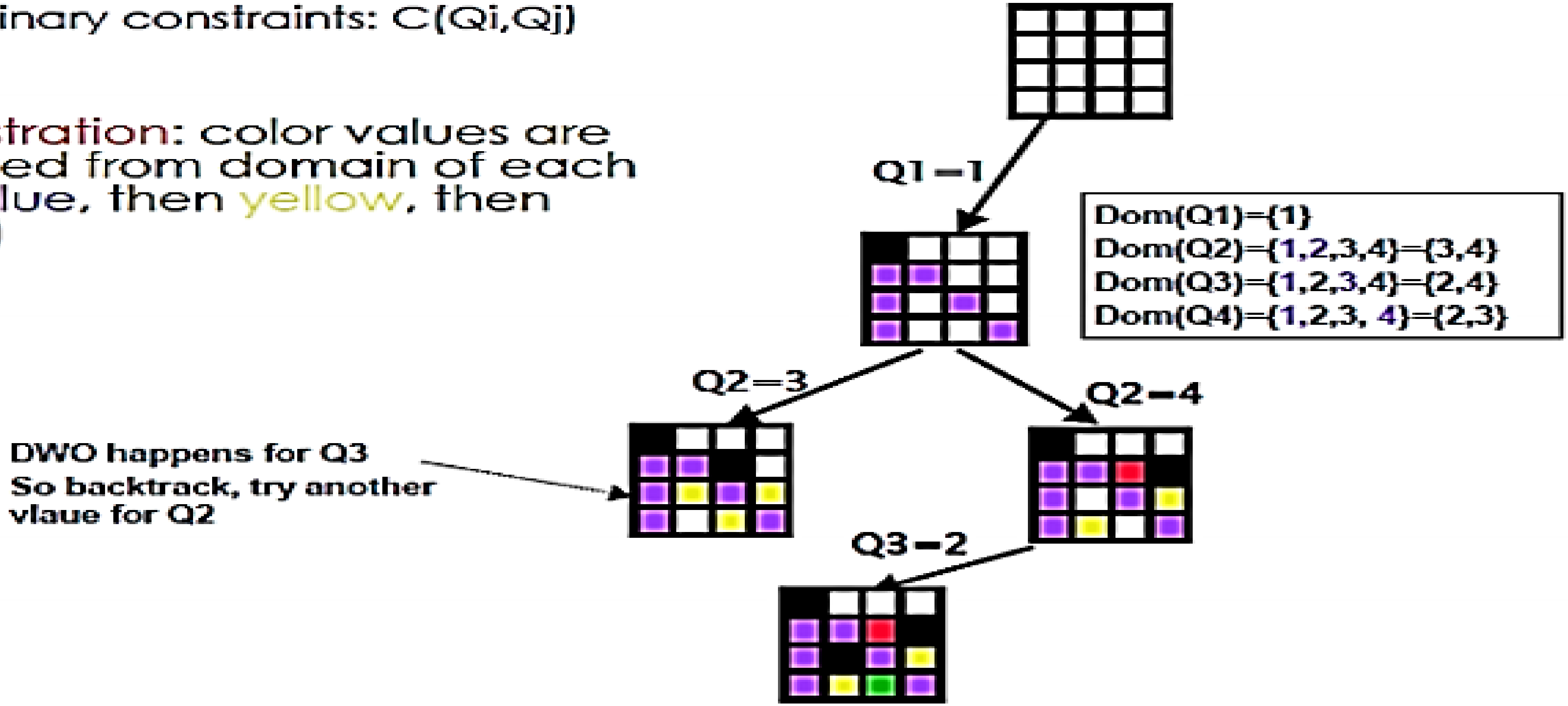


No possible assignments for SA, we try other assignments

Forward Checking Example:2

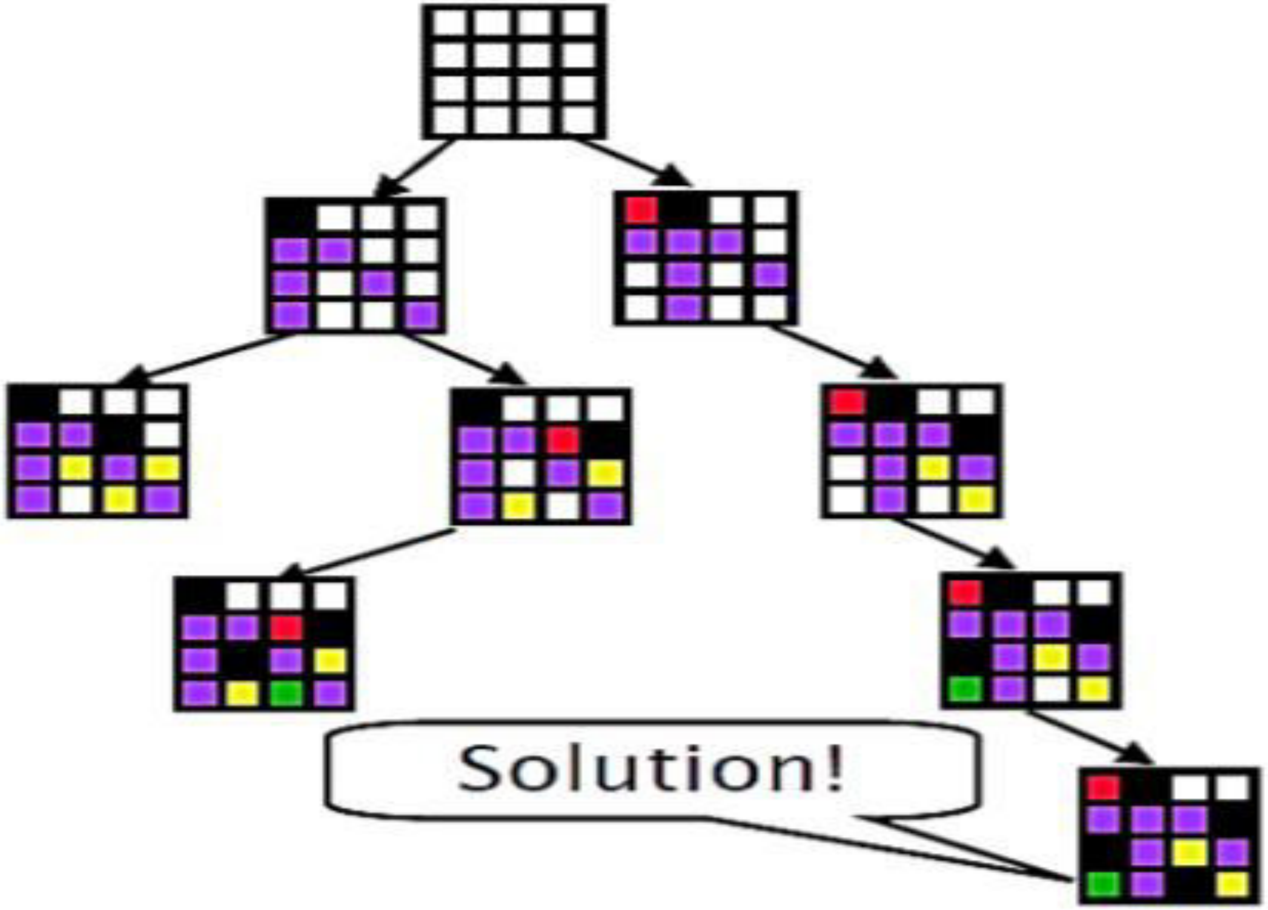
- 4X4 Queens
  - Q1,Q2,Q3,Q4 with domain {1..4}
  - All binary constraints: C(Qi,Qj)

- FC illustration: color values are removed from domain of each row (blue, then yellow, then green)



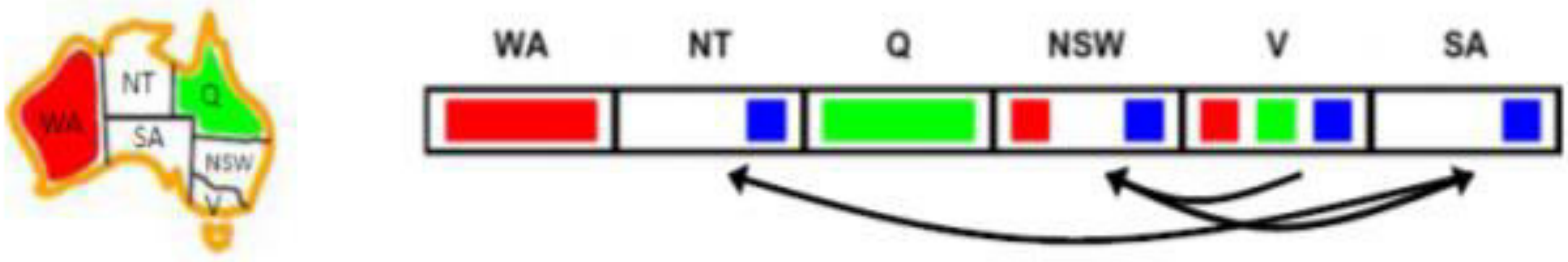
Forward Checking Example:2……

- 4X4 Queens continue…



ARC Consistency

- A simple form of propagation makes sure all arcs are simultaneously consistent:



- Arc consistency detects failure earlier than forward checking
- Important: If X loses a value, neighbors of X need to be rechecked!
- Must rerun after each assignment!

Remember: Delete from the tail!

Intelligent Backtracking: Looking Backward

- Consider a fixed variable ordering Q, NSW, V , T, SA, WA, NT. Suppose we have generated the partial assignment {Q=red, NSW =green, V =blue, T =red}. When we try the next variable,
- SA, we see that every value violates a constraint(chronological backtracking)
- Recoloring Tasmania cannot possibly resolve the problem with South Australia.
- A more intelligent approach to backtracking is to backtrack to a variable that might fix the problem
- The set (in this case {Q=red ,NSW =green, V =blue, }), is called the conflict set for SA.
- In this case, back jumping would jump over Tasmania and try a new value for V .