Available online at www.sciencedirect.com
SCIENCE DIRECT®

ELSEVIER

Journal of Empirical Finance 12 (2005) 576–611

Journal of EMPIRICAL FINANCE

www.elsevier.com/locate/econbase

Pricing American options when the underlying asset follows GARCH processes

Lars Stentoft *

School of Economics and Management, University of Aarhus, 322 University Park, DK-8000 Aarhus C, Denmark

Accepted 13 August 2004
Available online 31 August 2005

Abstract

As extensions to the Black–Scholes model with constant volatility, option pricing models with time-varying volatility have been suggested within the framework of generalized autoregressive conditional heteroskedasticity (GARCH). However, application of the GARCH option pricing model has been hampered by the lack of simulation techniques able to incorporate early exercise features. In the present paper, we show how new simulation techniques can be used to price options which have the possibility of early exercise in a GARCH framework. We report the results from an extensive Monte Carlo study, indicating that incorporating GARCH features in the option pricing model can potentially help explain some empirically well documented systematic pricing errors. Our empirical analysis of out-of-sample performance shows that GARCH effects are important when pricing options on individual stocks and lead to improvements over the constant volatility model. Specifications of the exponential GARCH-type generally have the smallest pricing errors.

JEL classification: C22; C53; G13
Keywords: Time-varying volatility; GARCH; American options; Least Squares Monte Carlo

* Tel.: +45 8942 1584; fax: +45 8613 6334.
E-mail address: lstentoft@econ.au.dk.

0927-5398/$ - see front matter
doi:10.1016/j.jempfin.2004.08.001

1. Introduction

In the seminal paper by Black and Scholes (1973) a closed form solution for the price of a European option is derived. The Black–Scholes formula has been celebrated as one of the major successes of modern financial economics, although empirical analysis has pointed towards several systematic pricing errors. At the same time, the assumptions underlying the model have been widely criticized, and much work effort has been put into extending the valuation framework. Apart from the assumption of continuous trade, a crucial assumption in the Black–Scholes model is that of log normality. However, the log normal model fails to explain a number of empirical regularities found in asset return series, the most important of which are leptokurtosis and the volatility clustering phenomenon (see Bollerslev et al., 1994). In the time series literature several stock return models have been developed which can accommodate these findings, and a large number of these have been within the framework of autoregressive conditional heteroskedastic (ARCH) processes suggested by Engle (1982). In the generalized ARCH (GARCH) models introduced by Bollerslev (1986), volatility is allowed to depend on past innovations and past volatilities, and the models could thus, in principle, explain both excess kurtosis and volatility clustering. These models have been successfully applied to financial data such as stock return data as seen from the survey article by Bollerslev et al. (1992) with reference to several hundred papers. The use of GARCH models to forecast volatility in financial markets has been surveyed recently in Poon and Granger (2003).

In terms of option pricing, a problem with the GARCH framework is that, in general, no preference free option valuation formula exists, when volatility is time-varying (see Amin and Ng, 1993b). However, some attempts have been made towards deriving option pricing models with GARCH type volatility specifications in an equilibrium framework. One of the first examples is the model of Amin and Ng (1993a), which assumes joint normality of shocks to the return process and to the state pricing density process. Duan (1995) provides a more comprehensive derivation using an extension to the risk-neutralization used in early work on discrete time option pricing models such as Brennan (1979). This model has been further extended in Duan (1999). However, actual empirical application of the GARCH option pricing model has been limited to that of pricing European style options or, as it was the case in the early research by Amin and Ng (1993a), American call options on stocks paying no dividend which are also essentially European style (see Hull, 1997). In particular, options on the Standard and Poor's 500 stock market index have been the focus of a large part of this research, and various GARCH specifications have been used by Heston and Nandi (2000), Hsieh and Ritchken (2000) and Christoffersen and Jacobs (2002). Also, Bollerslev and Mikkelsen (1996) and Bollerslev and Mikkelsen (1999) have successfully used the GARCH option pricing framework together with fractionally integrated GARCH processes to price long-term European style equity anticipation securities (LEAPS) on this particular index. Other applications include Myers and Hanson (1993) where evidence in favor of the GARCH model was found when pricing European commodity futures, and Duan and Zhang (2001) where the GARCH model was found to perform well for the Hang Seng Index options.

The main technical problem when using a GARCH option pricing model is that the distribution of future asset prices cannot be derived in closed form. Thus, generally no analytical formula for the option price exists and instead numerical methods have to be used, although an exception to this is the particular formulation in Heston and Nandi (2000). Simulation type methods are obvious candidates to be used with the GARCH models and, although other suggestions have appeared in recent papers by Duan and Simonato (2001) and Ritchken and Trevor (1999), in most of the empirical papers mentioned above this is the method used to price the options. However, in reality the majority of the exchange traded options are American style options and hence the optimal early exercise strategy has to be determined simultaneously with valuing the options. This complicates the pricing procedure and possibly explains why previous empirical work has focussed only on European style options. In particular, it has been the general belief that simulation methods can be used to price options with no early exercise possibility only (see Hull, 1997). However, recent work has shown otherwise (see e.g. Carriere, 1996; Longstaff and Schwartz, 2001; Tsitsiklis and Van Roy, 2001).

In this paper we show how simulation methods can be used to price American options in a GARCH framework by using the model of Duan (1995) together with the Least Squares Monte Carlo (LSM) method of Longstaff and Schwartz (2001). In Longstaff and Schwartz (2001) the conditional expectation of the payoff from continuing to hold the option is estimated from a cross-sectional regression on a set of simulated paths, and in the original paper the method was applied to various types of options. However, to our knowledge it has never been used with a GARCH option pricing model, and we report results substantiating that it can be used in this framework by comparing the method to the limited results from Duan and Simonato (2001) and Ritchken and Trevor (1999). We also report results from an extensive Monte Carlo simulation study illustrating how flexible the method is. The simulations indicate that the GARCH models could potentially explain some of the regularities found in the empirical option pricing literature. Furthermore, the results show that the early exercise feature of American options has an important impact on the characteristics of the pricing model and should not be neglected. In addition to the Monte Carlo study, an extensive empirical analysis is performed using option data from highly traded individual stocks as well as the major US stock index. This study indicates that allowing volatility to vary through time is very important when valuing individual stock options. Furthermore, the study shows that incorporating asymmetries in the option pricing model yields smaller pricing errors than the symmetric GARCH specification. In particular, models of the exponential type perform well. Interestingly, these models are also the ones with the best in sample performance. For the index options our results show that the models with time-varying volatility on average yield smaller pricing errors than the model with constant volatility.

In addition to GARCH models, there are many other models of time-varying volatility in the literature. In particular, stochastic volatility (SV) models have been used in the context of option pricing. Hull and White (1987) and Wiggins (1987) were among the first to derive theoretical models, and more recent empirical studies include, to name a few, Andersen et al. (2002), Bakshi et al. (1997) and Bates (2000). However, in many cases GARCH models provide good approximations to SV models (see Duan, 1997; Nelson, 1990), and in practice, option pricing applications may well benefit from the relative parsimony of GARCH models and the fact that they are relatively easy to reestimate. In the GARCH models, the resulting option pricing formula has two state variables, the stock price and the most recent conditional volatility. Since the latter may be updated using only the most recent return, both variables are directly observed. Any SV model would, in addition, involve the unobserved volatility as a state variable. This is very complicated to predict, and may require e.g. the full reprojection machinery associated with the EMM procedure (see Gallant and Tauchen, 1998). Thus, it may not be realistic that this procedure is repeated, e.g. weekly, on a rolling basis in option pricing practice. In addition, since most relevant stock and index options are of American style, even the most convenient SV specification does not offer the advantage over GARCH that an analytical option pricing formula exists. For these reasons, we focus on the GARCH representation of the time-varying volatility feature in asset returns.

The rest of the paper is organized as follows: Section 2 motivates the use of GARCH specifications in terms of the return data and describes the estimation results. Section 3 describes how derivatives can be priced in discrete time in general and using the model in Duan (1995) in particular. We discuss how to implement the model using simulation and we provide evidence that the method can be used to price American options. Section 4 contains an extensive numerical study of the properties of the GARCH option pricing model. The results pertaining to the actual empirical option pricing performance are in Section 5. Section 6 concludes.

2. Return data and estimation results

For the empirical work we will use option data for General Motors (GM), International Business Machines (IBM), Merck and Company Inc. (MRK), as well as for the Standard and Poor's 100 index (OEX). The return series for the individual stocks can be obtained from the Center for Research in Security Prices (CRSP). We use return data beginning January 2, 1976 since this is as far back data on the individual stock return and dividend are available to us on a daily basis. Furthermore, our data on the corresponding options ends December 29, 1995 and for this reason this date marks the end of the sample which contains 5055 daily observations. Following the CRSP 1997 Stock File Guide (see CRSP, 1998), the time t return, $R_{t,\mathrm{CRSP}}$, from purchasing at time $t-1$ and selling at time t is specified as

$$R_{t,\mathrm{CRSP}} = \frac{S_t f_t + d_t}{S_{t-1}} - 1, \tag{1}$$

where S_t is the last sale price or closing bid-ask average at time t, d_t is the cash dividends per share for time t, and f_t is a price adjustment factor at time t for non cash dividends like spin-offs, mergers, exchanges, reorganizations, liquidations, and rights issues. To fit into the option pricing framework we work with the continuously compounded returns, and to limit potential numerical problems when estimating the different models we use returns in percentage terms. Thus, the series used for estimation for the individual returns are defined as

$$R_t = 100 * \ln\left(1 + R_{t,\mathrm{CRSP}}\right). \tag{2}$$

The continuously compounded return series in percentage terms for the Standard and Poor's 100 index was calculated from the return index, RI_t, supplied by Datastream as

$$R_t = 100 * \ln\left(\frac{\mathrm{RI}_t}{\mathrm{RI}_{t-1}}\right). \tag{3}$$

As it is the case for the individual returns this index includes dividends payments (for further information see the online documentation at http://product.datastream.com/navigator/dtdefns/en/3/RI.htm).

Table 1 shows sample statistics and selected diagnostic tests for the chosen continuously compounded return series. From row six in the table it is evident that the assumption of normality is rejected, and in general rows four and five show that the individual stock returns are negatively skewed and leptokurtic, although for MRK the skewness is insignificantly different from zero. The table also shows that the index return series is very negatively skewed and highly leptokurtic. Furthermore, the time plots of R_t shown in Fig. 1 clearly indicate that the returns are not independently and identically distributed through time. On the contrary it seems clear that periods of low volatility are followed by high volatility periods and vice versa. This is known as volatility clustering.

Table 1
Sample statistics for return series

Ticker	GM	IBM	MRK	OEX
Mean	0.0366	0.0250	0.0669	0.0531
S.D.	1.6455	1.4677	1.4469	0.9896
Skewness statistic	−0.344 [0.0000]	−0.940 [0.0000]	−0.001 [0.9879]	−2.428 [0.0000]
Ex. kurt. statistic	10.761 [0.0000]	23.931 [0.0000]	3.417 [0.0000]	60.987 [0.0000]
Normality statistic	24 490 [0.0000]	121 370 [0.0000]	2460 [0.0000]	788 380 [0.0000]
R_t ACF(1)	0.022	−0.041	0.041	0.006
R_t ACF(2)	−0.040	−0.012	−0.016	−0.032
R_t ACF(3)	−0.019	0.008	−0.041	−0.027
R_t ACF(4)	−0.048	−0.030	−0.013	−0.048
R_t ACF(5)	−0.004	0.045	0.008	0.033
$\|R_t\|$ ACF(1)	0.130	0.124	0.124	0.128
$\|R_t\|$ ACF(2)	0.077	0.064	0.048	0.083
$\|R_t\|$ ACF(3)	0.088	0.026	0.054	0.102
$\|R_t\|$ ACF(4)	0.075	0.084	0.069	0.071
$\|R_t\|$ ACF(5)	0.048	0.097	0.041	0.121
$Q(20)$ statistic	41.705 [0.0030]	48.405 [0.0004]	38.682 [0.0073]	47.193 [0.0006]
$Q^2(20)$ statistic	637.215 [0.0000]	208.389 [0.0000]	497.691 [0.0000]	439.287 [0.0000]

This table shows sample statistics for the continuously compounded returns, R_t, for the individual stocks and the index considered. The sample period is January 2, 1976 to December 29, 1995 for a total of $N=5055$ observations. For the skewness (Skewness) and excess kurtosis (Ex. kurt.) statistics, the brackets next to the statistics report the p-values from testing the significance of the difference between the empirical values and the theoretical values from the Normal distribution using a t-test. For the normality (Normality) statistic, the p-value of a t-version of the well-known Jarque–Bera test for normality is reported in brackets. Under the heading ACF, the autocorrelation functions of lags 1 through 5 are reported for the returns respectively the absolute returns. The asymptotic variance of the autocorrelations is $1/N$. Finally, $Q(20)$ is the Ljung–Box portmanteau test for up to 20th order serial correlation in the returns, whereas $Q^2(20)$ is for up to 20th order serial correlation in the squared returns. In square brackets next to these test statistics, p-values are reported.

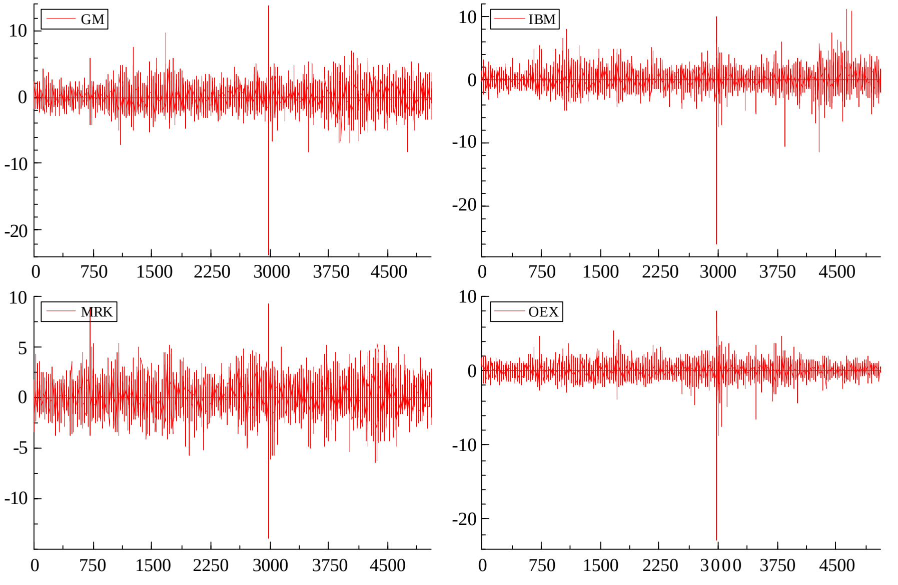

Fig. 1. Time series plots of the continuously compounded percentage return, R_t.

Table 1 also shows that some of the return autocorrelations of lags between one and five in rows seven through eleven are statistically significant. However, for the absolute returns the table shows that the problem of serial correlation is much more severe since all but one of the autocorrelations of lags between one and five are statistically different form zero. Fig. 2 indicates that the absolute return autocorrelations remain significant for long lags. Finally, the Ljung–Box portmanteau statistics for up to 20th order serial correlation in the returns and the squared returns in rows seventeen and eighteen are also significant with the latter much more so.

In the literature, findings of asymmetries, leptokurtosis and volatility clustering are well documented for many other series than the ones in the present paper. The literature also presents an abundance of explanations for these empirical regularities. Although we recognize the large literature examining and seeking explanations of these regularities, the purpose of this paper is not to give yet another explanation. For the present we simply take them as given, and merely attempt to find a model able to accommodate the empirical regularities. We do this using the framework of generalized autoregressive conditional heteroskedastic processes as do many others (see Bollerslev et al., 1992).

2.1. Specification of the GARCH models

The generalized ARCH (GARCH) process of Bollerslev (1986) restricts the conditional variance of a time series to depend upon past innovations and lagged values of the variance, and when put together with a mean equation we talk about a GARCH (regression) model. With the particular mean equation to be used in Section 3, a simple

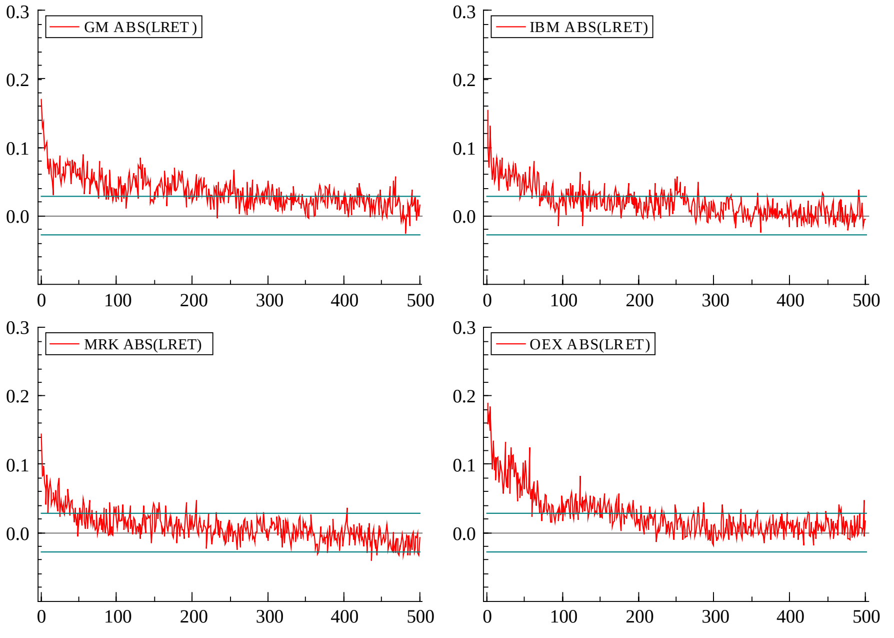

Fig. 2. Plots of the autocorrelations for the first 500 lags of the absolute value of the continuously compounded return, $|R_t|$.

GARCH model for R_t depending on only the past innovation and the variance lagged once can be formulated as

$$R_t = r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t, \quad \text{where } \varepsilon_t|\mathcal{F}_{t-1} \sim N(0,\, h_t), \tag{4}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{5}$$

where $\mathcal{F}_{t-1}$ is the time $t-1$ information set, r is the risk-free rate of return, and ω, α and β should be assumed positive in order to avoid any non-negativity-related problems. Although more general versions of this so-called GARCH(1,1) model in (4) and (5) could be considered, also denoted GARCH(p,q) models, we will limit attention to the specification with $p=q=1$. For this reason we suppress the "(1,1)" attached to the name. Furthermore, to be precise we should denote the model as a GARCH in mean or GARCH-M model due to the presence of the conditional variance in the mean equation. However, for ease of notation this will be suppressed as well in what follows.

The parameters in the model in (4) and (5) can be estimated using maximum likelihood (ML). Furthermore, even if the assumption of normality is violated, which is the case for the returns under consideration here, consistent estimates can still be obtained and inference made using the robust standard errors from the QML estimation procedure of Bollerslev and Wooldridge (1992). With some care, the model can be considered an ARMA model for the variance (see Bollerslev, 1986), and therefore it has the potential to explain the volatility clustering often found in asset returns. Furthermore, ε_t can be expressed as the product of the volatility term $\sqrt{h_t}$ and a standardized innovation, z_t, with conditional mean zero and time invariant variance equal to unity. Thus, even if z_t is normally distributed the GARCH model could potentially explain the excess kurtosis found in the return series if h_t varies. In addition to the GARCH specification we will use a number of different volatility specifications which can potentially accommodate asymmetric responses to negative and positive return innovations. Thus, these models allow for a leverage effects, which refers to the tendency for changes in stock prices to be negatively correlated with volatility.

The first asymmetric GARCH model we consider is the non-linear asymmetric GARCH model, or NGARCH for short, of Engle and Ng (1993). The particular specification of the variance process for this model is given by

$$h_t = \omega + \beta h_{t-1} + \alpha\left(\varepsilon_{t-1} + \gamma\sqrt{h_{t-1}}\right)^2. \tag{6}$$

The second asymmetric model considered is the GJR specification of Glosten et al. (1993), sometimes referred to as a threshold GARCH model, where the variance process is specified as

$$h_t = \omega + \beta h_{t-1} + \alpha\left(\varepsilon_{t-1}^2 + \gamma(\max[0, {}_{t-1}])^2\right). \tag{7}$$

In the NGARCH model and the GJR model the leverage effects is modelled through the parameter γ, and if $\gamma < 0$ leverage effects are found. It is clear that both models nest the ordinary GARCH specification, which obtains when $\gamma = 0$. In order to avoid problems with the variance becoming negative ω, $\alpha(1+\gamma)$ and β should be positive when the GJR model is estimated, whereas for the NGARCH model the parameter restrictions ensuring positivity of the variance are the same as for the GARCH model.

The next asymmetric process we consider is a version of the exponential GARCH model, or EGARCH model for short, of Nelson (1991). Instead of (5) the volatility process is now specified as

$$\ln(h_t) = \omega + \beta\ln(h_{t-1}) + \alpha(|v_{t-1}| - E[|v_{t-1}|] + \theta v_{t-1}), \tag{8}$$

where $v_t = \frac{\varepsilon_t}{\sqrt{h_t}}$. In the EGARCH model the term $|v_{t-1}| - E[|v_{t-1}|]$ is interpreted as the magnitude effects and θv_{t-1} is called the sign effects. If $-1 < \theta < 0$ a positive surprise increases volatility less than a negative one, and we say that leverage effects are present. We note that the EGARCH model does not nest the GARCH model. However, it has the nice feature that no restrictions need to be put on the parameters to ensure non-negativity of the variance.

The last volatility specification considered is motivated by the fact that empirically it is often found that the sum of the ARCH and GARCH parameters is very close to one in the simple GARCH model. This has been linked to the persistence in the auto correlations for absolute returns, which is evident from Fig. 2, and led to the formulation of models with integrated volatility. To motivate these models we rewrite (5) as

$$(1 - \alpha L - \beta L)\varepsilon_t^2 = \omega + (1 - \beta L)\left(\varepsilon_t^2 - h_t\right), \tag{9}$$

where L denotes the lag operator, i.e. $Lh_t = h_{t-1}$. If the polynomial $(1-\alpha L-\beta L)$ contains a unit root, i.e. when $\alpha+\beta=1$, we have the IGARCH model of Engle and Bollerslev (1986). However, the knife-edge distinction between GARCH and IGARCH models has been criticized, and an obvious generalization is to allow for fractional orders of integration, such that $(1-\alpha L-\beta L)=\phi(L)(1-L)^d$, leading to the Fractionally Integrated GARCH models suggested by Baillie et al. (1996). In the present paper we will consider the FIGARCH model, which can be specified as

$$\phi(L)(1-L)^d\varepsilon_t^2 = \omega + (1-\beta L)\left(\varepsilon_t^2 - h_t\right), \tag{10}$$

meaning that the conditional variance, h_t, can be written as

$$h_t = [1-\beta]^{-1}\omega + \left[1-[1-\beta L]^{-1}\phi(L)(1-L)^d\right]\varepsilon_t^2. \tag{11}$$

Conditions ensuring positivity of the conditional variance almost surely can be derived relatively easy and following Laurent and Peters (2002) these are $\omega>0$, $\beta-d\leq\phi\leq(2-d)/(2)$ and $d(\phi-(1-d)/(2))\leq\beta(\phi-\beta+d)$ for the FIGARCH in (11). For estimation purposes the unobserved values of ε_t^2 are set equal to their unconditional expectation and $(1-L)^d$ is truncated.[1] Baillie et al. (1996) suggested that the order of truncation be at least 1000 lags and we follow this recommendation.

2.2. Estimation of the GARCH models

Tables 2 and 3 reports the QML estimation results for the return series with all computational work performed using Ox (see Doornik, 2001). We start by looking at column two in both tables containing the results for the constant volatility (CV) model. This model is obviously a special case nested by all the volatility specifications above and is specified as

$$h_t = \omega. \tag{12}$$

Furthermore, it corresponds to the distributional assumption in the Black–Scholes model and thus serves as a natural benchmark.

The diagnostic tests for the CV model, however, indicate that the non-normality found in Table 1 is not the only problem this model faces. Another problem is the finding of ARCH type errors. This is indicated by the $Q^2(20)$ statistics which are significant at a 5% level. The null hypothesis of this test is that of no serial correlation of up to order twenty in the squared standardized residuals and it has been shown to be applicable by Bollerslev and Mikkelsen (1996) in the GARCH framework when the degrees of freedom are corrected by the number of estimated ARCH parameters. The $Q(20)$ tests for serial correlation in the mean are also significant. However, one should note that the presence of

[1] In all cases the fractional differencing operator is defined by its Maclaurian series expansion

$$(1-L)^d = 1-\sum_{k=1}^{\infty} d*\Gamma(k-d)\Gamma(1-d)^{-1}\Gamma(k+1)^{-1}L^k,$$

where $\Gamma(\cdot)$ denotes the gamma function.

Table 2
Estimation results for individual stocks

	CV	GARCH	NGARCH	GJR	EGARCH	FIGARCH
Panel A: General Motors (GM)						
λ	0.0174	0.0322	0.0134	0.0183	0.0187	0.0255
	(0.0141)	(0.0140)	(0.0140)	(0.0141)	(0.0139)	(0.0140)
ω	2.7077	0.0305	0.0191	0.0242	0.0128	0.1348
	(0.1360)	(0.0207)	(0.0102)	(0.0128)	(0.0066)	(0.0535)
β		0.9330	0.9392	0.9433	0.9893	0.6043
		(0.0303)	(0.0196)	(0.0191)	(0.0057)	(0.0983)
α		0.0574	0.0417	0.0703	0.1013	0.3956
		(0.0258)	(0.0149)	(0.0263)	(0.0334)	(0.0977)
$\gamma/\theta/d$			-0.5668	-0.5878	-0.4175	0.3129
			(0.1233)	(0.0992)	(0.0969)	(0.0482)
Log-likelihood	-9690.36	-9360.67	-9337.06	-9344.91	-9338.45	-9342.35
J–B	24490.12	1815.68	1262.57	1236.00	1229.00	1129.59
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(20)$	41.62	34.20	32.80	33.18	30.90	35.89
	[0.0031]	[0.0248]	[0.0355]	[0.0323]	[0.0565]	[0.0158]
$Q^2(20)$	635.02	37.66	36.89	32.22	43.76	16.62
	[0.0000]	[0.0043]	[0.0054]	[0.0207]	[0.0006]	[0.5493]
SIC	3.8342	3.7041	3.6949(*)	3.6979	3.6954	3.6969
Panel B: International Business Machines (IBM)						
λ	0.0098	0.0316	0.0100	0.0170	0.0137	0.0303
	(0.0140)	(0.0171)	(0.0140)	(0.0141)	(0.0150)	(0.0155)
ω	2.1542	0.0246	0.0270	0.0324	0.0131	0.1113
	(0.1543)	(0.0143)	(0.0131)	(0.0175)	(0.0056)	(0.0603)
β		0.9391	0.9265	0.9327	0.9877	0.6242
		(0.0266)	(0.0271)	(0.0245)	(0.0054)	(0.1426)
α		0.0522	0.0487	0.0830	0.1104	0.3931
		(0.0258)	(0.0180)	(0.0392)	(0.0306)	(0.1310)
$\gamma/\theta/d$			-0.5433	-0.6651	-0.3984	0.3471
			(0.1230)	(0.1280)	(0.0936)	(0.0940)
Log-likelihood	-9112.36	-8798.75	-8769.71	-8773.55	-8751.84	-8793.21
J–B	121373.13	13673.47	7077.48	6702.54	6122.56	10736.07
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(20)$	48.30	24.82	24.90	24.46	26.11	24.82
	[0.0004]	[0.2083]	[0.2053]	[0.2228]	[0.1623]	[0.2084]
$Q^2(20)$	207.82	9.16	10.16	10.13	12.18	8.03
	[0.0000]	[0.9559]	[0.9264]	[0.9275]	[0.8381]	[0.9783]
SIC	3.6056	3.4818	3.4704	3.4719	3.4633(*)	3.4797

(continued on next page)

Table 2 (*continued*)

	CV	GARCH	NGARCH	GJR	EGARCH	FIGARCH
Panel C: Merck and Company Inc. (MRK)						
λ	0.0386	0.0512	0.0407	0.0444	0.0415	0.0474
	(0.0141)	(0.0139)	(0.0141)	(0.0140)	(0.0142)	(0.0138)
ω	2.0936	0.0692	0.0628	0.0678	0.0252	0.1151
	(0.0685)	(0.0281)	(0.0198)	(0.0246)	(0.0078)	(0.0518)
β		0.9072	0.9100	0.9115	0.9690	0.6708
		(0.0276)	(0.0205)	(0.0240)	(0.0097)	(0.0893)
α		0.0600	0.0535	0.0735	0.1253	0.5263
		(0.0169)	(0.0128)	(0.0207)	(0.0256)	(0.0838)
$\gamma/\theta/d$			−0.3505	−0.4577	−0.2621	0.2838
			(0.1211)	(0.1440)	(0.0894)	(0.0601)
Log-likelihood	−9040.23	−8858.53	−8849.72	−8852.31	−8847.33	−8845.64
J–B	2459.82	523.78	431.80	438.09	429.68	428.36
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(20)$	38.62	38.85	39.01	38.51	37.89	39.46
	[0.0074]	[0.0070]	[0.0067]	[0.0077]	[0.0091]	[0.0058]
$Q^2(20)$	507.39	20.02	22.85	22.86	23.24	12.13
	[0.0000]	[0.3315]	[0.1966]	[0.1959]	[0.1814]	[0.8406]
SIC	3.5770	3.5054	3.5020	3.5030	3.5011	3.5004(*)

This table reports quasi-maximum likelihood estimates (QMLE) for the daily returns assuming a risk-free interest rate of 5.4% corresponding to the value on December 29, 1995. Robust standard errors are reported in parentheses. J–B is the value of the usual Jarque–Bera normality test for the standardized residuals. $Q(20)$ is the Ljung–Box portmanteau test for up to 20th order serial correlation in the standardized residuals, whereas $Q^2(20)$ is for up to 20th order serial correlation in the squared standardized residuals. In square brackets below all test statistics, p-values are reported. The last row reports the Schwarz Information Criteria, with an asterisk denoting the minimum value.

ARCH may give rise to spurious significance of this type of tests for serial correlation in the mean (see Bollerslev and Mikkelsen, 1996).

The results from estimating the GARCH models are shown in column three in the tables. Compared to these the first thing to note is that the simpler CV model is easily rejected using a likelihood ratio test. Furthermore, for all series, both extra parameters are significant and the estimates are in line with what is usually found in the literature. In terms of serial correlation in the squared standardized residuals, the $Q^2(20)$ statistics show that for all but GM the null of no correlation cannot be rejected at a 5% or even at a 1% level. Thus, it seems that modelling volatility as a GARCH process goes quite a way in terms of eliminating the ARCH effects for IBM, MRK and OEX. Furthermore, for three of the four series the $Q(20)$ statistics are now insignificant at a 1% level.

Columns four and five present estimation results for the NGARCH and the GJR models. Comparing the GARCH model to these models the tables show that in all cases adding the leverage parameter increases the log-likelihood value significantly. Furthermore, for all the return series the estimated leverage parameter, γ, is significantly different from zero and has the expected sign. The same holds for the EGARCH model for which the estimation results are shown in column six. Unfortunately, changes in log-likelihood values cannot be used to compare the exponential GARCH models to the non-exponential GARCH models. Instead, in the literature comparisons between such models are made by minimizing various information criteria. This procedure is used even though little is

Table 3
Standard and Poor's 100 Index (OEX) estimation results

	CV	GARCH	NGARCH	GJR	EGARCH	FIGARCH
λ	0.0370	0.0529	0.0380	0.0409	0.0396	0.0522
	(0.0146)	(0.0150)	(0.0142)	(0.0146)	(0.0145)	(0.0143)
ω	0.9793	0.0112	0.0124	0.0131	0.0020	0.0265
	(0.1093)	(0.0065)	(0.0070)	(0.0078)	(0.0029)	(0.0130)
β		0.9364	0.9286	0.9361	0.9848	0.7233
		(0.0274)	(0.0299)	(0.0249)	(0.0078)	(0.0719)
α		0.0523	0.0488	0.0696	0.1171	0.4797
		(0.0246)	(0.0192)	(0.0341)	(0.0391)	(0.0800)
$\gamma/\theta/d$			−0.4481	−0.5613	−0.3555	0.3535
			(0.1188)	(0.1591)	(0.0903)	(0.0934)
Log-likelihood	−7119.99	−6519.31	−6503.83	−6505.93	−6516.44	−6499.39
J–B	788375.31	9200.32	7087.99	6330.47	8589.17	4139.46
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
$Q(20)$	47.09	32.41	29.01	29.24	27.39	32.77
	[0.0006]	[0.0391]	[0.0875]	[0.0832]	[0.1248]	[0.0357]
$Q^2(20)$	435.65	16.78	12.70	11.96	16.92	13.00
	[0.0000]	[0.5386]	[0.8093]	[0.8494]	[0.5288]	[0.7913]
SIC	2.8173	2.5799	2.5739	2.5747	2.5789	2.5721(*)

See the notes to Table 2.

known about the statistical properties of these information criteria in a GARCH context. However, Bollerslev and Mikkelsen (1996) do show that the Schwarz Information Criteria, $SIC = -2\frac{\mathcal{L}}{N} + 2\frac{\ln(j)}{N}$, where $\mathcal{L}$ is the log-likelihood value, N is the number of observations, and j is the number of parameters, can be used to discriminate between the alternatives with good results. For the individual stock returns in Table 2 the SIC value is smaller for the asymmetric models than for the symmetric GARCH model, which indicates that asymmetries in the volatility specification are important features of the individual return data under consideration.

The last column of the tables, column seven, reports the estimation results for the FIGARCH model and shows that for all the return series the fractionally integration parameter, d, is estimated significantly different from zero. For the individual return series, in particular for GM, a major improvement is found in terms of the test statistics for the squared returns. However, it is only for the index series in Table 3 that the FIGARCH specification has a much larger log-likelihood value than any of the other models. In terms of the SIC the FIGARCH model has the smallest value for MRK as well as for OEX.

Thus, in conclusion, the estimation results indicates that asymmetric volatility specifications are particularly important for the individual returns series whereas long memory features are important for the index series under consideration. However, at the same time it is obvious that the simple GARCH models do not capture the observed non-normality in the return series. This is immediately clear from the Jarque–Bera test for normality, which is significant for all return series.[2]

[2] An alternative way to generate thick tails and skewness in the GARCH models is to assume that z_t follows a fat tailed distribution instead of a normal (see Bollerslev et al., 1994). However, since this complicates the option pricing framework to follow we do not pursue this idea at present.

3. Option pricing with GARCH type volatility

A major advantage with discrete time return models is that empirical data are readily available for estimation. However, when deriving a theoretical option pricing model, discreteness poses a potential problem as the asset market models easily become incomplete. This is indeed the case for the GARCH type models. Heuristically, the reason is that, unlike in the binomial model where the stock can take on only two different values next period conditional on the price today, in a GARCH model the number of future possible values is infinite. In some of the first attempts to derive a GARCH type option pricing model, e.g. Amin and Ng (1993a), this problem was dealt with by simply assuming the existence and uniqueness of a state pricing density process Π_t, where $\Pi_0=1$, such that the price of any time t claim, C_t, to a cash flow C_{t+1} at time $t+1$ is equal to

$$C_t = E_t\left[\frac{C_{t+1}\Pi_{t+1}}{\Pi_t}\right]. \tag{13}$$

Since the existence and uniqueness of the state pricing density process is equivalent to that of the existence of a unique equivalent martingale, following Harrison and Kreps (1979) we can price all securities as their expected future payoff discounted using the risk-free rate of interest. Thus, using the law of iterated expectations in (13) a European put option with terminal payoff $Z_T(S_T)=\max[0, (K-S_T)]$, where K is the strike price and S_t the price of the underlying asset at time t, should have a time 0 price of

$$p_0 = E^{\mathcal{Q}}\left[\exp\left\{-\sum_{t=1}^{T} r_{t-1}\right\} Z_T(S_T)\middle|\mathcal{F}_0\right], \tag{14}$$

where $E^{\mathcal{Q}}[\cdot|\mathcal{F}_0]$ means expectation under the equivalent martingale measure $\mathcal{Q}$ conditional on the time $t=0$ information set. In the same manner we can express the time 0 price of an American put option with payoff $Z_t(S_t)=\max[0, (K-S_t)]$ if exercised at time t, as

$$P_0 = \sup_{\tau} E^{\mathcal{Q}}\left[\exp\left\{-\sum_{t=1}^{\tau} r_{t-1}\right\} Z_\tau(S_\tau)\middle|\mathcal{F}_0\right], \tag{15}$$

where the supremum is over all stopping times τ with $0\leq\tau\leq T$.

3.1. The GARCH option pricing model of Duan (1995)

As an alternative to simply assuming the existence and uniqueness of a state pricing density, further assumptions about preferences, apart from non-satiation, or about the risk premium have to be made in order to get to a risk-neutral valuation relationship. In Duan (1995) an extension of the risk-neutralization principle in Brennan (1979), referred to as the locally risk-neutral valuation relationship (LRNVR), is used. The LRNVR can be shown to hold under some familiar assumptions on preferences and assumed conditional lognormality. Although the assumption of conditional lognormality can be relaxed (see Duan, 1999), this extension complicates the pricing system as well as the actual estimation procedure and we do not pursue this at the present (see also Footnote 2).

To be specific, the model we use is a discrete time economy with an asset price denoted S_t and with dividends denoted d_t, in which the one-period rate of return is lognormally distributed under measure $\mathcal{P}$ conditional on the variance. In particular, in the most general form with the mean equation used in Duan (1995) the model can be specified as

$$\ln\left(\frac{S_t + d_t}{S_{t-1}}\right) = r + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t, \quad \text{where } \varepsilon_t|\mathcal{F}_{t-1} \sim N(0, h_t) \text{ under measure } \mathcal{P}, \tag{16}$$

and where the variance process is specified as

$$h_t = g(h_{t'}, \varepsilon_{t'}; t' \leq t-1). \tag{17}$$

From (17) it is immediately clear that all the variance specifications of the previous section can be accommodated in this framework. Furthermore, since it follows from lognormality that one plus the conditional expected rate of return equals $\exp\left(r + \lambda\sqrt{h_t}\right)$, λ is readily interpreted as the unit risk premium when r in (16) is the constant one-period continuously compounded risk-free rate of return.[3]

Following Duan (1995), the LRNVR stipulates that under the risk-neutralized pricing measure $\mathcal{Q}$ $(S_t + d_t)/S_{t-1}$ distributes lognormally conditionally on F_{t-1} with $E^{\mathcal{Q}}[(S_t + d_t)/S_{t-1}|\mathcal{F}_{t-1}] = \exp(r)$ and that the one-period ahead conditional variance is invariant with respect to the change to $\mathcal{Q}$ such that

$$\mathrm{Var}\left[\ln\left(\frac{S_t + d_t}{S_{t-1}}\right)\middle|\mathcal{F}_{t-1}\right] = \mathrm{Var}^{\mathcal{Q}}\left[\ln\left(\frac{S_t + d_t}{S_{t-1}}\right)\middle|\mathcal{F}_{t-1}\right]. \tag{18}$$

Invoking the LRNVR, it is relatively easy to derive the dynamics under the equivalent martingale measure. First of all, the restriction on the mean return in the lognormal distribution of returns results in the following dynamics

$$\ln\left(\frac{S_t + d_t}{S_{t-1}}\right) = r - \frac{1}{2}h_t + \tilde{\varepsilon}_t, \quad \text{where } \tilde{\varepsilon}|\mathcal{F}_{t-1} \sim N(0, h_t) \text{ under measure } \mathcal{Q}. \tag{19}$$

Secondly, to ensure (18) the variance process has to be specified as

$$h_t = g\left(h_{t'}, \tilde{\varepsilon}_t - \lambda\sqrt{h_{t'}}; t' \leq t-1\right). \tag{20}$$

[3] We note that more general formulations for the mean can be used as long as these are measurable with respect to the information set $\mathcal{F}_{t-1}$. However, changing the mean specification has generally little effects on the parameters of the volatility process and thus on the dynamics under the equivalent martingale measure (see also the discussion in Section 5.2).

Thus, while we may be lured into believing that we have successfully eliminated all preference related parameters this is not the case. However, the LRNVR is sufficient to reduce the preference considerations to the constant unit risk premium present in the variance equation.

3.2. Implementing the GARCH option pricing model

An actual application of the pricing system in (19) and (20) is hampered by the fact that even for very simple time-varying volatility specifications no analytical expression exists for the time T values. Instead numerical methods have to be used and the most obvious procedure is to use (19) and (20) to simulate paths of future values of the risk-neutralized stock price. With these, an estimate of (14) can be calculated as

$$\bar{p}_0^M = \frac{1}{M}\sum_{m=1}^{M} e^{-rT}\max[0,\ (K - S_T(m))], \tag{21}$$

where M is the number of simulated paths and $S_T(m)$ is the risk-neutralized value of the underlying stock at expiration of the option for path number m. Stemming back from Boyle (1977) estimates like (21) have been used to price European options. Unfortunately, things are not quite as simple when American options are considered. The problem is the need to simultaneously determine the optimal exercise strategy.

In the present paper we will use the method proposed by Longstaff and Schwartz (2001) in which expressions like (15) are evaluated using the cross-sectional information available at each step in the simulation. In particular, working backwards from the expiration point, at any point in time where exercise should be considered we will estimate the conditional expectation of future pathwise payoffs by regressing the payoffs on functions of the current stock prices and volatility levels. With this estimated conditional expectation an optimal early exercise strategy can be derived and the option can be priced. The Least Squares Monte Carlo method, or LSM method for short, has been examined in some detail in Moreno and Navas (2003) and Stentoft (2004a), whereas the mathematical foundation for the use of the method in derivatives research is provided in Stentoft (2004b).

3.3. Comparison with existing methods

Although the existing literature on the pricing of American options in a GARCH framework is rather limited results do exist. In Table 4 we compare the results for the LSM method to results reported in the paper by Duan and Simonato (2001) and in the paper by Ritchken and Trevor (1999). For the LSM estimates the reported numbers are averages of 100 estimates using 20,000 paths and different seeds in the random number generator, “rann”, in Ox. In parenthesis the standard errors of the estimates are reported. For the American price estimates products and cross-products between the stock level and the volatility of total order less than or equal to two were used in the cross-sectional regressions. Furthermore, exercise is considered once every day.

Table 4
Comparison with estimates from Duan and Simonato (2001) and Ritchken and Trevor (1999)

Panel A: Comparison with estimates from Duan and Simonato (2001)

T	K	American prices			European prices			Early exercise premiums		
		DS	LSM	Δ	DS	LSM	Δ	DS	LSM	Δ
30	55	5.0000	5.0009 (0.0023)	−0.0009	4.8377	4.8387 (0.0216)	−0.0010	0.1623	0.1622 (0.0209)	0.0001
30	50	1.1026	1.0971 (0.0105)	0.0055	1.0884	1.0870 (0.0131)	0.0014	0.0142	0.0100 (0.0080)	0.0042
30	45	0.0742	0.0797 (0.0035)	−0.0055	0.0715	0.0774 (0.0037)	−0.0059	0.0027	0.0023 (0.0013)	0.0004
90	55	5.1861	5.1766 (0.0198)	0.0095	4.9550	4.9520 (0.0292)	0.0030	0.2311	0.2246 (0.0256)	0.0065
90	50	1.8737	1.8682 (0.0161)	0.0055	1.8197	1.8191 (0.0190)	0.0006	0.0540	0.0491 (0.0118)	0.0049
90	45	0.4132	0.4261 (0.0083)	−0.0129	0.4036	0.4143 (0.0091)	−0.0107	0.0096	0.0118 (0.0048)	−0.0022
270	55	5.9800	5.9424 (0.0337)	0.0376	5.4899	5.4744 (0.0437)	0.0155	0.4901	0.4680 (0.0323)	0.0221
270	50	3.0463	3.0315 (0.0238)	0.0148	2.8471	2.8400 (0.0303)	0.0071	0.1992	0.1915 (0.0214)	0.0077
270	45	1.2524	1.2636 (0.0159)	−0.0112	1.1867	1.1928 (0.0182)	−0.0061	0.0657	0.0707 (0.0112)	−0.0050

Panel B: Comparison with estimates from Ritchken and Trevor (1999)

T	K	American prices			European prices			Early exercise premiums		
		RT	LSM	Δ	RT	LSM	Δ	RT	LSM	Δ
2	100	0.556	0.5589 (0.0065)	−0.003	0.556	0.5589 (0.0065)	−0.003	0.000	0.0000 (0.0001)	0.000
10	100	1.192	1.1930 (0.0110)	−0.001	1.175	1.1760 (0.0122)	−0.001	0.017	0.0170 (0.0068)	0.000
50	100	2.398	2.3984 (0.0234)	0.000	2.281	2.2842 (0.0263)	−0.003	0.117	0.1142 (0.0144)	0.003
100	100	3.143	3.1443 (0.0273)	−0.001	2.882	2.8906 (0.0328)	−0.009	0.261	0.2537 (0.0207)	0.007

In this table price estimates from the LSM method are compared to the results of Duan and Simonato (2001) and Ritchken and Trevor (1999), both of which use the NGARCH(1,1) specification. For the LSM estimates, the reported numbers are averages of 100 estimates using 20,000 paths and different seeds in the random number generator, "rann", in Ox. For the American price estimates products and cross-products between the stock level and the volatility of total order less than or equal to two were used. Exercise is considered once every day. In parentheses the standard errors of the prices are reported. In Panel A, the interest rate is fixed at 5% (annualized using 365 days a year), and the stock price is 50. The parameter values are the ones from Duan and Simonato (2001), that is $\omega=0.1$ (as we are working with returns in percentage terms), $\beta=0.80$, $\alpha=0.10$, $\gamma=-0.30$, and $\lambda=0.20$. Δ reports the difference between the two estimates. In Panel B, the interest rate is fixed at 10% (annualized using 365 days a year), and the stock price is 100. The parameter values are the ones from Ritchken and Trevor (1999), that is $\omega=0.06575$ (as we are working with returns in percentage terms), $\beta=0.90$, $\alpha=0.04$, $\gamma=0$, and $\lambda=0$. Thus we are essentially working with a GARCH(1,1) model. Again, Δ reports the difference between the two estimates.

In the paper by Duan and Simonato (2001) a Markov Chain approximation to the underlying asset price process for a NGARCH specification is used. Panel A of Table 4 shows the results from their paper together with our results using the LSM method. From the panel it is clear that the LSM method with the chosen specification produces estimates which are very close to those reported in the paper by Duan and Simonato (2001). The same result obtains for the early exercise premiums. In general, the differences are small and both negative and positive ones occur. For the early exercise premiums the differences are less than one cent with the exception of the in the money long-term option. However, as noted in Duan and Simonato (2001), the convergence of the Markov Chain approximation is slowest for exactly this option.

The paper by Ritchken and Trevor (1999) develops a lattice algorithm to price options under general GARCH processes along the lines of the well known binomial tree, and although the results reported in that paper are mostly on European options the method is used to price four American at the money put options with different terms to expiration using what is essentially a GARCH specification. Panel B of Table 4 compares their results to the estimates from the LSM algorithm, and again we see that the differences are very small except possibly for the very short-termed options. For all but one of the reported estimates the differences are well below one tenth of a cent. The exception to this is the long-term European price. Unfortunately, no Monte Carlo results are reported for these European price estimates in the paper by Ritchken and Trevor (1999). However, using our algorithm on the other European options in that paper we find that in all cases our estimates are within the confidence intervals reported. Thus, we believe the European price reported in Ritchken and Trevor (1999, Table IV) to be erroneous.

In conclusion, we have found that the LSM algorithm provides estimates which are very close to what has previously been reported for American option prices in GARCH models. Furthermore, since the simulation method is more flexible than either of the methods above, e.g. these methods cannot accommodate specifications like the FIGARCH (see Duan, 1999), we argue that the algorithm is indeed a useful alternative. A final benefit when using the LSM method is the possibility of gauging the precision of the estimated price, since standard errors of the estimate can be calculated easily using standard techniques or using multiple simulated estimates. As will be discussed in the next section, this will allow us to assess the statistical importance of the American feature.

4. A Monte Carlo study of the GARCH option pricing model

In Duan (1995) it is argued that for European options the GARCH option pricing model can explain some of the regularities found in the empirical option pricing literature like the smiles in implied Black–Scholes volatilities and the underpricing of short-term options and out of the money options. The purpose of our Monte Carlo study is threefold. First of all, since the vast majority of exchange traded options is American style options, it is of interest to see whether the results of Duan (1995) hold when early exercise features are incorporated. Also, from a computationally point of view it is of interest to examine the magnitude of the early exercise premiums since taking account of the American feature complicates the pricing algorithm. We show that these premiums generally are statistically significant and potentially quite large in economic terms. Secondly, we wish to use empirically plausible parameter values such that the results indicates what we might expect to find in real data. The parameter values used in Duan (1995) are far from what we find in our sample for both the individual stocks as well as for the index. We show that this has implications for the pricing performance. Thirdly, we wish to extend the numerical results in Duan (1995) to include asymmetric volatility models and models with long memory. Asymmetries have been shown to be present in many return series, and NGARCH, GJR, or EGARCH models have proven important in modelling this as Section 2 showed. When it comes to option prices, asymmetries are also found, and some authors have argued that for various assets the smiles in implied volatilities are not genuine smiles but rather smirks. Below we show that volatility asymmetries have important implications on the estimated option prices and implied smirks are exactly what would be expected if asymmetries corresponding to leverage effects are incorporated in the volatility process.

4.1. Pricing American options under GARCH

To illustrate the pricing effects we use a set of artificial put options with strike prices ranging from deep out of the money (85% of the stock level) to deep in the money (115% of the stock level) and with short (1 month), middle (3 months) and long maturities (6 months). We take a year to be 252 trading days, such that the options expire in 21, 63, and 126 trading days, respectively. This collection of options covers to a large extent what is actually observed for traded options on individual stocks as well as the Standard and Poor's 100 index. In order to make the American pricing problem interesting, in terms of generating a positive early exercise premium, a nonzero interest rate is needed, as we for now disregard possible dividend payments. We choose a value of 6% for the interest rate.

In Column four of Panel A in Table 5 we show the American prices calculated using a GARCH specification with empirically plausible parameter values. The values we choose are "rounded" averages of the estimates for the individual stock return series in Table 2. Thus, we decide to use $\lambda = 0.05$, $\beta = 0.92$ and $\alpha = 0.06$ in the GARCH model. These values yield a persistence of 0.98, so in order to get an annualized unconditional volatility of 25%, corresponding to what is implied be the average parameter values, we set $\omega = 0.0496$. Column five shows the bias arising, if one were to use the constant volatility (CV) specification in the binomial model (BM) to price options in the situation where the correct specification is a GARCH specification. From this column it is clear that out of the money options in general and out of the money short maturity options in particular are underpriced. Options which are at the money are, on the other hand, overpriced and although this is true for the in the money options also the overpricing is smaller both in absolute terms as well as relative to the actual price. In Panel A of Fig. 3 the implied volatilities from inverting the binomial model for the American prices in Table 5 are plotted as a function of the strike price for the GARCH specification. The plot shows a characteristic smile in the implied volatilities, which is what we would expect from the pricing performance relative to the CV model.

Table 5
American GARCH put option prices and early exercise premiums

Panel A: American price estimates

T	K/S	BM	GARCH		NGARCH		GJR		EGARCH		FIGARCH	
		Price	Price	Bias	Price	Bias	Price	Bias	Price	Bias	Price	Bias
21	85	0.023	0.039 (0.0029)	−0.017	0.065 (0.0037)	−0.042	0.058 (0.0036)	−0.035	0.062 (0.0035)	−0.039	0.039 (0.0028)	−0.017
21	100	2.662	2.614 (0.0247)	0.048	2.648 (0.0268)	0.014	2.630 (0.0259)	0.032	2.661 (0.0274)	0.001	2.614 (0.0254)	0.048
21	115	15.000	15.000 (0.0003)	0.000	15.000 (0.0000)	0.000	15.000 (0.0004)	0.000	15.000 (0.0000)	0.000	15.000 (0.0016)	0.000
63	85	0.407	0.453 (0.0115)	−0.046	0.629 (0.0156)	−0.222	0.570 (0.0143)	−0.163	0.618 (0.0149)	−0.211	0.447 (0.0109)	−0.040
63	100	4.364	4.270 (0.0414)	0.093	4.392 (0.0450)	−0.028	4.338 (0.0449)	0.025	4.424 (0.0461)	−0.061	4.283 (0.0409)	0.081
63	115	15.174	15.149 (0.0391)	0.025	15.074 (0.0334)	0.100	15.098 (0.0371)	0.076	15.095 (0.0358)	0.079	15.149 (0.0408)	0.025
126	85	1.159	1.198 (0.0223)	−0.039	1.548 (0.0276)	−0.388	1.426 (0.0262)	−0.266	1.525 (0.0261)	−0.366	1.195 (0.0216)	−0.035
126	100	5.845	5.727 (0.0524)	0.118	5.977 (0.0580)	−0.132	5.877 (0.0577)	−0.032	6.016 (0.0573)	−0.171	5.742 (0.0556)	0.103
126	115	15.793	15.682 (0.0652)	0.111	15.605 (0.0667)	0.188	15.629 (0.0664)	0.164	15.658 (0.0693)	0.135	15.695 (0.0659)	0.098

This table shows American put option prices and early exercise premiums for a set of artificial options. The interest rate is fixed at 6%, and the parameter values are the ones specified in the text. In the cross-sectional regressions powers of and cross-products between the stock level and the level of the volatility of total order less than or equal to two were used. Exercise is considered once every trading day. Prices reported are averages of 100 calculated prices using 20,000 paths and different seeds in the random number generator, "rann", in Ox. In parentheses standard errors of these price estimates are reported. The column headed Bias reports the difference between the binomial model (BM) price and the average of the simulated prices. The early exercise premiums reported are the averages of the premiums from the 100 price estimates. In parentheses, standard errors of these premiums are reported. The column headed Diff reports the difference between the BM premiums and the average of the simulated premiums.

In columns four and five of Panel B in the same table we report the early exercise premiums, calculated as the difference between the estimated American and European option prices, as well as the difference between this premium and what would be obtained with a CV model. The table shows that the differences in premiums are generally smaller than the differences in the actual price estimates in Panel A. However, the premiums are in most cases not only of economic importance but also statistically significant. In terms of the implied volatilities Panel C of Fig. 3 plots the volatilities implied by inverting the Black–Scholes model for the price of European options with comparable characteristics. Compared to Panel A of the same figure we see that although the pattern is much the same for longer-term options, there are differences for the short-term options. In particular, the slope of the implied volatility curves is generally steeper for the American options than for the European counterparts. Thus, we see that the American feature is important and should not be ignored.

Except for the use of trading days instead of calendar days the artificial options above are comparable to the ones used in Duan (1995) by the put-call parity when incorporating the nonzero interest rate in the moneyness. Thus, the results in Panel C of Fig. 3 are comparable

Panel B: Early exercise premiums

T	K/S	BM	GARCH		NGARCH		GJR		EGARCH		FIGARCH	
		Prem.	Prem.	Diff	Prem.	Diff	Prem.	Diff	Prem.	Diff	Prem.	Diff
21	85	0.000	0.002 (0.0013)	−0.001	0.003 (0.0014)	−0.002	0.002 (0.0014)	−0.002	0.002 (0.0017)	−0.002	0.002 (0.0011)	−0.001
21	100	0.034	0.029 (0.0139)	0.005	0.027 (0.0150)	0.007	0.027 (0.0142)	0.007	0.027 (0.0140)	0.007	0.028 (0.0138)	0.006
21	115	0.455	0.469 (0.0497)	−0.014	0.509 (0.0507)	−0.054	0.493 (0.0504)	−0.039	0.502 (0.0510)	−0.047	0.469 (0.0495)	−0.015
63	85	0.006	0.012 (0.0065)	−0.006	0.016 (0.0081)	−0.009	0.014 (0.0073)	−0.008	0.016 (0.0075)	−0.009	0.013 (0.0064)	−0.006
63	100	0.127	0.111 (0.0258)	0.016	0.104 (0.0289)	0.023	0.104 (0.0275)	0.022	0.106 (0.0283)	0.021	0.111 (0.0260)	0.016
63	115	0.775	0.770 (0.0656)	0.005	0.831 (0.0712)	−0.056	0.803 (0.0697)	−0.028	0.807 (0.0692)	−0.031	0.767 (0.0656)	0.008
126	85	0.036	0.042 (0.0130)	−0.006	0.051 (0.0181)	−0.014	0.047 (0.0163)	−0.011	0.053 (0.0161)	−0.017	0.042 (0.0135)	−0.006
126	100	0.285	0.253 (0.0329)	0.032	0.245 (0.0391)	0.039	0.245 (0.0373)	0.040	0.254 (0.0390)	0.030	0.254 (0.0351)	0.031
126	115	1.154	1.120 (0.0737)	0.034	1.121 (0.0771)	0.033	1.117 (0.0767)	0.038	1.108 (0.0757)	0.046	1.122 (0.0724)	0.032

to the corresponding figures of Duan (1995). However, the panel shows that by using empirically relevant values for the parameters, in particular the α parameter, the curvature of the smile becomes less pronounced. We note that it is well known in the GARCH model that α determines the kurtosis (see Engle and Bollerslev, 1986). In terms of option prices this is seen to correspond to the curvature in the implied smiles, and for higher values of α the implied smile is more curved (more detailed results on this matter are available upon request).

4.2. Increasing the complexity of the volatility specification

When comparing the various volatility specifications within the GARCH framework it seems reasonable that we consider models with the same characteristics at least in terms of the level of persistence and the level of implied volatility. We do this by first choosing the persistence parameters ($\beta+\alpha$ in the GARCH model, $\beta+\alpha\ (1+\gamma^2)$ in the NGARCH model, $\beta+\alpha(1+\gamma/2)$ in the GJR model, and β in the EGARCH model) and then adjusting ω in order to yield the same unconditional variance, $E\ [h]$, across the models. Thus, in the

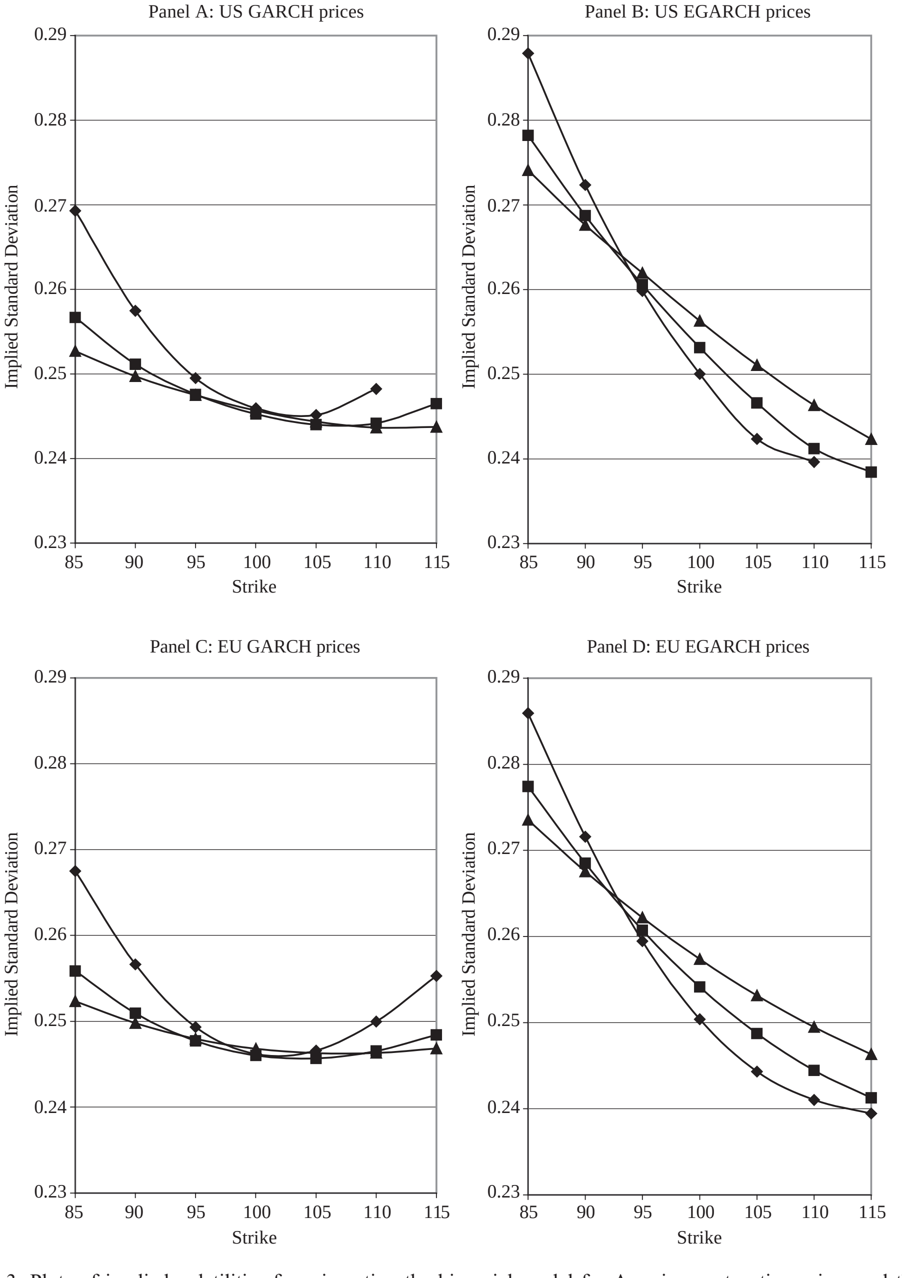

Fig. 3. Plots of implied volatilities from inverting the binomial model for American put option prices and the Black–Scholes Model for European prices. In all plots a ♦ indicates short-term options (1 month or 21 trading days), a ■ indicates middle-term options (3 months or 63 trading days), whereas a ▲ is for long-term options (6 months or 126 trading days).

GARCH model we set ω GARCH$=(1-\beta-\alpha)*E[h]$, in the NGARCH model we set $\omega_{\mathrm{NGARCH}}=(1-\beta-\alpha(1+\gamma^2))*E[h]$, whereas in the GJR model we set $\omega_{\mathrm{GJR}}=(1-\beta-\alpha(1+\gamma/2))*E[h]$. To get an expression for ω in the EGARCH model were write it in its infinite order ARCH form as

$$\ln h_t = \frac{\omega}{(1-\beta)} + \sum_{i=0}^{\infty} \alpha\beta^i g(z_{t-1-i}).$$

Exponentiating and taking expectations we get the following expression for ω_{EGARCH} after some rearranging

$$\omega_{\mathrm{EGARCH}} = \ln\left(\frac{E[h]}{\Pi_{i=0}^{\infty} E\left[\exp\left\{\alpha\beta^i g(z)\right\}\right]}\right) * (1-\beta),$$

where E $[\exp\{bg(z)\}]$ is given by Eq. (A1.3) in Nelson (1991) with $\theta=1$. In practice we truncated the infinite sum at 1000. For the FIGARCH model, denote by δ the cumulative effects of $[1-[1-\beta L]^{-1}\phi(L)(1-L)^d]$ which allows us to define $\omega_{\mathrm{FIGARCH}}=(1-\delta)*(1-\beta)*E[h]$. As it was the case for the GARCH specification above we request that the parameter values for λ, β, α, γ, θ, d, and $E[h]$ are empirically plausible and again we choose values close to the actual averages of the estimated parameter values from the individual return series from 1976 through 1995 reported in Table 2. After rounding we decide to use $\beta=0.92$, $\alpha=0.048$, and $\gamma=0.5$ in the NGARCH model and $\beta=0.92$, $\alpha=0.08$, and $\gamma=-0.5$ in the GJR model. These values yield a persistence of 0.98 as in the GARCH model, so in order to get an annualized unconditional volatility of 25% we again set $\omega=0.0496$. In the EGARCH model we set $\beta=0.98$, $\alpha=0.11$, and $\theta=-0.35$, and to get the correct volatility $\omega=0.0166$. In the FIGARCH specification we set $\beta=0.65$, $\alpha=0.40$, and $d=0.35$, which implies that $\omega=0.0958$. Furthermore, we set $\lambda=0.05$ in all the models. In order to start up the simulations we need either values for h_0 and ε_0 for the GARCH, NGARCH, GJR, and EGARCH models or we can specify h_1. We choose the latter as does Duan (1995) and set h_1 equal to the unconditional expectation, that is $E[h]$. For the FIGARCH model the algorithm is initialized by equating all the unobserved values of ε_t^2, $t=0, -1, -2, \ldots$, with their unconditional expectation.

4.3. Pricing American options with time-varying volatility specifications

Column six in Table 5 shows the NGARCH prices with the parameter values above and column seven the bias arising, if one were to use the CV model to price options in the situation where the correct specification is a NGARCH specification. From these columns it is clear that the underpricing of out of the money options by the CV model is even more pronounced, compared to the GARCH results in columns four and five. Furthermore, once asymmetries are introduced in the volatility model mispricing is present for long maturities, too, e.g. the underpricing by the CV model relative to the NGARCH for long-term out of the money options is 34% compared to 3% relative to the GARCH model.

Columns eight and nine show that the same conclusions will be reached if the GJR model is the correct one although the degree of mispricing by the CV model, in terms of the absolute size of the bias, is generally smaller than when the correct model has an NGARCH specification. For the EGARCH model in columns ten and eleven the results are in all respects close to those from the NGARCH model. Panel B of Fig. 3 plots the implied volatilities for the EGARCH model and shows that for this asymmetric model the smile in volatilities is turned into a smirk. We note that the plots from the NGARCH and the GJR specifications resemble this plot, and we conclude that the possibility of having asymmetry parameters in the volatility process allows extra flexibility in the pricing model when it comes to generating smiles or smirks.

The added flexibility from allowing the returns to have long memory is, on the other hand, very limited for the set of options with short-term to expiration. This is in agreement with what has been found in e.g. Bollerslev and Mikkelsen (1996) and we see that the effects of the long memory parameter becomes pronounced only as the maturity of the options increases. However, for the parameter values observed in the data the absolute size of the bias is generally smaller than for the simple GARCH specification.

5. Option data and pricing results

The option data we consider have been extracted from the Berkeley Options DataBase (BODB) database. This database contains, among other observations, all quotes and trades during the day. The amount of data is substantial, and at the present we limit the period under consideration to January, 1991 through December, 1995. After this the Chicago Board Options Exchange (CBOE) discontinued the supply of data to BODB and more recent data of equivalent quality has not been made available to us since. We choose a sample of major US stocks from the Dow Jones Industrial Index for which options are actively traded. In particular, we use General Motors (GM), International Business Machines (IBM), and Merck and Company Inc. (MRK). The reason for choosing these three stocks is that for the period under consideration options on these three stocks were the most traded in terms of actual trades as well as in terms of total volume. Apart form the individual stock options we include options on the Standard and Poor's 100 index (OEX). This index is the broadest index for which options are traded on the CBOE, and for this reason it has been the focus of much research. For the American style options the Standard and Poor's 100 index constitutes the natural counterpart to the European style options on the Standard and Poor's 500 index.

In our sample of options we take data on a weekly basis, selecting each Wednesday. If the Wednesday is a no trade day or no options for the particular asset were traded we pick the Thursday immediately after. At any particular day we sample an end of day option price for all contracts, that is combinations of strike price and maturity, for which the traded volume during the day was at least five contracts. This is not a guarantee against thin trade effects, but it should go quite a way in terms of minimizing the problem. Furthermore, the choice of five contracts is more than what has been used in the previous literature. For each contract the observed end of day price is the bid-ask midpoint immediately before 3PM. Valid quotes are the ones for which the bid-ask midpoint conforms to the simple arbitrage bounds in Hull (1997). We end up with 1835 options for GM, 4745 for IBM, 1844 for MRK, and 8291 for the index. On average 65% of the stock options are calls whereas these constitute 42% of the index options.

5.1. Option characteristics and the "empirical regularities"

To get an idea about the characteristics of the options we split the options into three categories of maturity. Short-term options are options with maturities of less than $T=42$ trading days corresponding roughly to 2 months. The middle term is maturities of more than 42 but less than 126 trading days. Long-term maturity options are the rest, that is options with more than 126 trading days, the equivalence of 6 months, to maturity. Note that there will be no index options in this category in our data set, simply because none are traded. In terms of maturity the individual stock option data is much alike across the stocks although slightly fewer of the options on IBM fall within the short-term category. The major difference in maturity is between the put and call options. For the put options around 62% are short term, but this proportion is around 10 percentage points smaller for the call options. For the index options the vast majority of the options are short-term options and the proportion is 89% for the calls and 84% for the calls.

We also split the options into three categories of moneyness which we define as the ratio between the asset price and the discounted strike price, $\text{Mon}=S/(K*\exp(-r*T))$. Put options that are in the money will have low values and we choose 0.95 as the cut of point. Out of the money options have a moneyness of 1.05 or more, and at the money options are defined broadly as the ones in between. For the call options these categories are defined the opposite way with the in the money options having $\text{Mon}>1.05$ and so on. Generally most of the options fall within the at the money category although the proportion is much larger for the index options than for the individual stock options (e.g. only 8% of the call options on the index are either in the money or out of the money). For call options on the individual stocks the proportion of in the money options is 23% whereas the out of the money options constitute 30% of our sample. For the individual put options the proportion of in the money options is 10 percentage points smaller, 13%, whereas the out of the money options constitute 38% of our sample. For the index put options, the distribution is approximately 65% to 35% between the at the money and out of the money with only 0.2% of the options being in the money.

Previous research has documented the relationship between the volatility implied from inverting constant volatility models and moneyness, the so-called smile effects, as well as the maturity effects, which refers to the relationship between time to maturity and the implied volatilities. These findings have become almost empirical regularities. In order to examine these effects for the sample of options considered in this paper, Table 6 reports the result from the following regression

$$\sigma_{\text{imp}} = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \beta_1 T^{\text{P}} + \beta_2 \left(T^{\text{P}}\right)^2 + \alpha_3 \text{Mon} + \alpha_4 \text{Mon}^2 + \beta_3 \text{Mon}^{\text{P}} + \beta_4 \left(\text{Mon}^{\text{P}}\right)^2 + \varepsilon, \tag{22}$$

where σ_{imp} is the implied volatility from a simple binomial model with no dividend corrections and where a superscript "P" denotes values from the put options only. The table clearly indicates that a significant proportion of the variation in the implied volatilities can be explained by these factors. Indeed, for the index options 60% of this variation can be explained as functions of the time to maturity and the moneyness as indicated by the value

Table 6
Implied volatility regression results

Parameter	GM	IBM	MRK	OEX
Cons	2.1307 (0.1449)	1.7921 (0.1078)	4.0557 (0.4394)	6.0486 (0.5658)
	[0.1059]	[0.0552]	[0.0444]	[0.0136]
T	−0.2168 (0.0498)	−0.1756 (0.0239)	−0.1444 (0.0408)	−0.3391 (0.0336)
	[0.0103]	[0.0113]	[0.0068]	[0.0122]
T^2	0.2474 (0.0751)	0.1409 (0.0363)	0.0579 (0.0634)	0.9626 (0.1419)
	[0.0059]	[0.0032]	[0.0005]	[0.0055]
T^{P}	0.3045 (0.0927)	0.1150 (0.0425)	0.0958 (0.1057)	0.0838 (0.0443)
	[0.0059]	[0.0015]	[0.0004]	[0.0004]
$(T^{\mathrm{P}})^2$	−0.4428 (0.1553)	−0.0744 (0.0690)	−0.0393 (0.1748)	−0.0827 (0.1784)
	[0.0044]	[0.0002]	[0.0000]	[0.0000]
Mon	−3.6801 (0.2703)	−2.9931 (0.2134)	−7.6371 (0.8722)	−12.0338 (1.1540)
	[0.1028]	[0.0399]	[0.0401]	[0.0130]
Mon^2	1.8511 (0.1280)	1.4754 (0.1073)	3.8309 (0.4327)	6.1190 (0.5887)
	[0.0922]	[0.0384]	[0.0410]	[0.0129]
$\mathrm{Mon}^{\mathrm{P}}$	0.0202 (0.0596)	−0.0168 (0.0541)	0.2333 (0.1173)	−0.0429 (0.0782)
	[0.0001]	[0.0000]	[0.0022]	[0.0000]
$(\mathrm{Mon}^{\mathrm{P}})^2$	0.0202 (0.0586)	0.0291 (0.0519)	−0.2091 (0.1094)	0.0672 (0.0781)
	[0.0001]	[0.0001]	[0.0020]	[0.0001]
R^2	0.48	0.32	0.51	0.60

This table reports the results from regressing the implied volatilities from the binomial model on the time to maturity in years and time squared as well as moneyness and moneyness squared. Parameters with superscript “P” allow for special effects for the put options. In parentheses, we report heteroscedasticity and autocorrelation consistent standard errors (HACSE) of the estimates as they are reported by the PcGive software. In brackets the partial R^2 are reported.

of the R^2. For the individual options 40% of the variation can be explained by these variables on average. The partial R^2 values in brackets indicate that the moneyness alone can explain a substantial part of the variation in the implied volatilities, and the parameter estimates are consistent with those of a smile shape.

Thus, to conclude, we have documented findings supporting a strong relationship between maturity and moneyness on one side and the implied volatility on the other side. In particular the connection between moneyness and the implied volatility, the implied smile or smirk, is strong in the sample we consider. We note that this was exactly the effects one would find if the CV model was used on option prices from a GARCH framework. Furthermore, our sample consists of a majority of short-term options which again are the ones for which the GARCH framework produces the largest differences relative to the CV model. Thus, we have reason to believe that the suggested extensions can explain some of the mispricing of the CV model.

5.2. Empirical option pricing with GARCH

In our empirical application we use a total of $M=20{,}000$ paths and we assume that the options can be exercised only once a day. This not only fits nicely into the discrete framework of the GARCH models, but it probably also serves as an adequate approximation to the possibility of continuous trade. In estimating the conditional expectations, apart from a constant term, we use powers of and cross products between the asset price and the level of the volatility. For computational convenience we use the fact that the option price is homogenous of degree one in the level of the underlying asset. Therefore, it is possible to use the same simulated paths to price all the options on a single day by scaling the simulated pathwise stock prices with the contemporaneously recorded level of the underlying asset.

Although term structure effects could be included by interpolating the interest rate with the term matching the maturity of the option, the results, available on request, indicate that this has little effects on the pricing performance for the option sample considered here. Thus, when simulating the future stock price we assume that the interest rate is constant throughout the life of the option and we set it equal to the present short rate. This we take to be a one month LIBOR rate which was extracted from Datastream. Furthermore, one might argue that the mean specification used is restrictive. However, allowing other mean specifications generally do not change the dynamics under the equivalent martingale measure as noted in Footnote 3. First of all, the estimates of the parameters in the volatility specification generally do not change. Secondly, although, for example, a mean parameter would enter the variance process in (20), the change is typically negligible due to the size of the estimate.

In terms of the dividend payments we make the following three assumptions: First, we assume that only cash dividend payments are important for our purpose. This assumption is validated by the fact that exchange traded options, in general, are protected against other forms of dividends like, say stock splits. Secondly, we make the assumption that both the ex-dividend day and the size of the dividends are known in advance. Of course, this is not strictly correct, although dividends seem to be paid regularly with fairly stable amounts through the period we consider. Finally, we assume that the effect of a cash dividend payment fully spills over on the stock price. Thus, if day t is an ex-dividend day and the simulated risk-neutral return is R_t, the end of day stock price is calculated from the price at the previous day as

$$S_t = S_{t-1} * \exp(R_t) - d_{t-1}.$$

Treating cash dividend payments as known both in size and timing and letting the payment spill over on the stock level seems to be the standard procedure (see Hull, 1997). For the index we do not have the actual dividend data. Instead we use data on the dividend yield (DY) supplied by Datastream to calculate actual dividend amounts.

The benchmark, with which we compare the pricing errors, is the model with constant volatility. However, due to the inclusion of dividends and the possibility of multiple payments until expiration the binomial model is not easy to apply (see Hull, 1997). Instead we use the CV formulation for $g(\cdot)$ and price options using simulation. We use all the historical observations available at any time t, $\mathcal{F}_{t-1}$, in the estimation part of the algorithm. Thus, the models are reestimated on a weekly basis and the most resent parameter estimates are used in the simulation. Furthermore, at any time t we set r in the mean equation used for simulation equal to the present short rate as does Duan (1995).

5.2.1. Overall model performance

In the literature a number of different metrics have been used to gauge the performance of alternate option pricing models (see Bollerslev and Mikkelsen, 1999). Denoting the kth price estimate by $\tilde{P}_k$ and the kth observed price by P_k some of these are the mean bias, $\mathrm{BIAS} \equiv K^{-1}\sum_{k=1}^{K}\left(\tilde{P}_k - P_k\right)$, the mean absolute error, $\mathrm{AE} \equiv K^{-1}\sum_{k=1}^{K}|\tilde{P}_k - P_k|$, and the mean squared error, $\mathrm{SE} \equiv K^{-1}\sum_{k=1}^{K}\left(\tilde{P}_k - P_k\right)^2$. We also use the corresponding relative metrics: the relative mean bias, $\mathrm{RBIAS} \equiv K^{-1}\sum_{k=1}^{K}\frac{\left(\tilde{P}_k - P_k\right)}{P_k}$, the relative mean absolute error, $\mathrm{RAE} \equiv K^{-1}\sum_{k=1}^{K}\frac{|\tilde{P}_k - P_k|}{P_k}$, and the relative mean squared error, $\mathrm{RSE} \equiv K^{-1}\sum_{k=1}^{K}\frac{\left(\tilde{P}_k - P_k\right)^2}{P_k^2}$. Table 7 provides summary statistics for each of these metrics for the individual stock options, whereas Tables 8 and 9 report the results from applying these metrics to all the stock options and the index options respectively, with Panel A reporting results for put options, Panel B for call options and Panel C for all options.

The first thing to note from Table 7 when comparing the results for the CV model and the GARCH model is that the latter is the preferred one irrespective of which metric is used for all the stock options. In fact, for the individual options the CV specification is generally the worst performing model of all the volatility specifications, although the GJR

Table 7
Overall performance for the individual stocks

	BIAS	AE	SE	RBIAS	RAE	RSE	Overall
Panel A: General Motors (a total of 1835 options)							
CV	−0.2248	0.2516	0.1044	−0.1617	0.1870	0.0701	36
GARCH	−0.0506	0.1722	0.0520	−0.0228	0.1281	0.0360	27
NGARCH	−0.0500	0.1625	0.0504	−0.0294	0.1236	0.0354	22
GJR	−0.0437	0.1641	0.0493	−0.0240	0.1248	0.0368	23
EGARCH	−0.0075	0.1378	0.0331	−0.0076	0.1094	0.0290	8
FIGARCH	−0.0433	0.1433	0.0374	−0.0211	0.1074	0.0268	10
Best	EGARCH	EGARCH	EGARCH	EGARCH	FIGARCH	FIGARCH	EGARCH
Worst	CV	CV	CV	CV	CV	CV	CV
Panel B: International Business Machines (a total of 4745 options)							
CV	−0.3784	0.4361	0.3466	−0.2094	0.2417	0.1183	34
GARCH	−0.1241	0.3499	0.2809	−0.0745	0.1935	0.1033	17
NGARCH	−0.1697	0.3701	0.2991	−0.1057	0.2077	0.1067	27
GJR	−0.1430	0.3682	0.3475	−0.0945	0.2084	0.1345	29
EGARCH	−0.0991	0.2941	0.1803	−0.0869	0.1725	0.0720	8
FIGARCH	−0.1021	0.3140	0.2117	−0.0624	0.1735	0.0807	11
Best	EGARCH	EGARCH	EGARCH	FIGARCH	EGARCH	EGACH	EGARCH
Worst	CV	CV	GJR	CV	CV	GJR	CV
Panel C: Merck and Company Inc. (a total of 1844 options)							
CV	−0.2028	0.2926	0.1466	−0.1249	0.1975	0.0800	36
GARCH	−0.1777	0.2476	0.1147	−0.1052	0.1600	0.0551	30
NGARCH	−0.1652	0.2325	0.1003	−0.1042	0.1540	0.0526	18
GJR	−0.1665	0.2380	0.1052	−0.1042	0.1566	0.0538	24
EGARCH	−0.0981	0.2029	0.0756	−0.0715	0.1425	0.0485	9
FIGARCH	−0.1341	0.2065	0.0890	−0.0702	0.1312	0.0410	9
Best	EGARCH	EGARCH	EGARCH	FIGARCH	FIGARCH	FIGARCH	EGARCH/ FIGARCH
Worst	CV	CV	CV	CV	CV	CV	CV

This table shows the performance of the different volatility specifications. We report results for all the metrics described in Section 5.2.1. The column headed Overall sums the ranking across all the metrics. Thus, if a specification performs best for all metrics it gets the number 6. If it performs worst, it gets the number 36.

Table 8
Overall performance for the stocks

	BIAS	AE	SE	RBIAS	RAE	RSE	Overall
Panel A: Put options (number of options is 3009)							
CV	−0.2882	0.3444	0.2179	−0.1954	0.2322	0.1071	35
GARCH	−0.0962	0.2775	0.1857	−0.0797	0.1808	0.0772	26
NGARCH	−0.0821	0.2717	0.1882	−0.0550	0.1698	0.0731	19
GJR	−0.0769	0.2780	0.2260	−0.0597	0.1746	0.0884	25
EGARCH	−0.0403	0.2297	0.1212	−0.0402	0.1497	0.0529	6
FIGARCH	−0.0797	0.2398	0.1338	−0.0655	0.1585	0.0616	15
Best	EGARCH	EGARCH	EGARCH	EGARCH	EGARCH	EGARCH	EGARCH
Worst	CV	CV	GJR	CV	CV	CV	CV
Panel B: Call options (number of options is 5415)							
CV	−0.3167	0.3757	0.2679	−0.1723	0.2134	0.0952	35
GARCH	−0.1330	0.2950	0.1996	−0.0646	0.1670	0.0785	17
NGARCH	−0.1763	0.3075	0.2088	−0.1075	0.1820	0.0828	28
GJR	−0.1541	0.3048	0.2315	−0.0932	0.1813	0.0995	27
EGARCH	−0.1004	0.2459	0.1275	−0.0808	0.1536	0.0600	10
FIGARCH	−0.1055	0.2608	0.1541	−0.0493	0.1450	0.0595	9
Best	EGARCH	EGARCH	EGARCH	FIGARCH	FIGARCH	FIGARCH	FIGARCH
Worst	CV	CV	CV	CV	CV	GJR	CV
Panel C: All options (total number of options is 8424)							
CV	−0.3065	0.3645	0.2501	−0.1805	0.2201	0.0994	36
GARCH	−0.1198	0.2888	0.1947	−0.0700	0.1719	0.0781	18
NGARCH	−0.1426	0.2947	0.2014	−0.0888	0.1776	0.0793	26
GJR	−0.1265	0.2952	0.2295	−0.0812	0.1789	0.0955	28
EGARCH	−0.0790	0.2401	0.1253	−0.0663	0.1522	0.0575	8
FIGARCH	−0.0963	0.2533	0.1468	−0.0551	0.1498	0.0603	10
Best	EGARCH	EGARCH	EGARCH	FIGARCH	FIGARCH	EGARCH	EGARCH
Worst	CV	CV	CV	CV	CV	CV	CV

See the notes to Table 7.

specification has slightly larger pricing errors using the squared error, SE, and the relative squared error, RSE, for IBM. Table 8 shows that this continues to be the case when all the individual options are considered together as well as when put and call options are considered separately.

Tables 7 and 8 also show that although the GARCH and NGARCH specifications consistently out-perform the CV models both on an individual stock basis as well as when considering different types of options these specifications always have larger pricing errors than the exponential version, and seen across the metrics the EGARCH specification is the best performing one. The FIGARCH model also performs better than the GARCH specification. However, overall the results are less favorable than for the EGARCH version because of the relatively bad performance of the FIGARCH for the put options. We note that the preferred models in terms of option pricing is thus to a large extent the same as when using the returns series.

Panel C of Table 9 shows that models with GARCH volatility on average provide much better estimates for the index options than the CV model which severely overestimates the

Table 9
Table of overall performance for individual stocks

	BIAS	AE	SE	RBIAS	RAE	RSE	Overall
Panel A: Put options (number of options is 4804)							
CV	0.7197	1.1548	2.4709	0.0685	0.3716	0.2168	29
GARCH	−0.4220	0.6875	0.8469	−0.2919	0.3356	0.2085	25
NGARCH	−0.3222	0.6306	0.7366	−0.2466	0.3016	0.1750	12
GJR	−0.3070	0.6492	0.7811	−0.2461	0.3064	0.1787	14
EGARCH	−0.3085	0.6217	0.7204	−0.2486	0.3034	0.1793	13
FIGARCH	−0.6582	0.7844	1.0567	−0.3539	0.3743	0.2448	33
Best	GJR	EGARCH	EGARCH	CV	NGARCH	NGARCH	NGARCH
Worst	CV	CV	CV	FIGARCH	FIGARCH	FIGARCH	CV
Panel B: Call options (number of options is 3487)							
CV	1.7385	1.8506	5.0782	1.0900	1.1106	3.6561	36
GARCH	0.3931	0.7257	0.9177	0.3205	0.3872	0.4822	27
NGARCH	0.3428	0.6828	0.8359	0.2563	0.3413	0.3695	16
GJR	0.4008	0.7295	0.9407	0.2977	0.3749	0.4451	27
EGARCH	0.3167	0.6944	0.8600	0.2274	0.3313	0.3418	14
FIGARCH	0.1413	0.6069	0.6212	0.1938	0.2867	0.2456	6
Best	FIGARCH	FIGARCH	FIGARCH	FIGARCH	FIGARCH	FIGARCH	FIGARCH
Worst	CV	CV	CV	CV	CV	CV	CV
Panel C: All options (number of options is 8291)							
CV	1.1482	1.4474	3.5675	0.4981	0.6824	1.6633	36
GARCH	−0.0792	0.7035	0.8767	−0.0343	0.3573	0.3236	25
NGARCH	−0.0425	0.6525	0.7784	−0.0351	0.3183	0.2568	13
GJR	−0.0093	0.6830	0.8482	−0.0174	0.3352	0.2907	15
EGARCH	−0.0456	0.6522	0.7791	−0.0484	0.3151	0.2476	13
FIGARCH	−0.3219	0.7097	0.8735	−0.1235	0.3375	0.2451	24
Best	GJR	EGARCH	NGARCH	GJR	EGARCH	FIGARCH	NGARCH/ EGARCH
Worst	CV	CV	CV	CV	CV	CV	CV

See the notes to Table 7.

prices. This indicates that the specifications with time-varying volatility generally provide better predictions of the future volatility. In particular, the CV model overestimates the subsequent level of volatility, and one might argue that this could be attributed to the inclusion of the stock market crash in October 1987 in the sample. However, our numerical results show that excluding the crash and using only return data from 1988 has only a relatively small impact on this, and even if the period for estimation is limited to 1990 and onwards the BIAS is still 0.5183 and the RBIAS equals 0.2161 for the CV model. Thus, it remains unclear what period should be used for estimation if the CV model is to produce “unbiased” results. However, since the procedure of using different periods for estimation purposes for the CV model and for the models with time-varying volatility is inconsistent with the presented framework, we only report the results from using the full period for estimation.

For the different types of index options clear results obtain for the call options only where the CV model consistently performs the worst and where the FIGARCH model consistently performs the best. For the put options the CV model performs the worst for the first three metrics whereas the FIGARCH model has the largest pricing errors using the relative metrics. Seen across all the metrics the NGARCH specification is the preferred

Table 10
Maturity and moneyness effects for options on individual stocks

		Short term		Middle term		Long term	
		BIAS	RBIAS	BIAS	RBIAS	BIAS	RBIAS
Panel A: Put options							
CV	ITM	−0.1326	−0.0282	−0.3954	−0.0615	−0.4043	−0.0379
	ATM	−0.2038	−0.1004	−0.3484	−0.0975	−0.5177	−0.0997
	OTM	−0.2868	−0.4855	−0.4169	−0.2827	−0.5697	−0.2483
	ITM	−0.0153 (*)	−0.0045 (*)	0.03042	−0.0024	−0.1077 (*)	−0.0082
GARCH	ATM	−0.0792	−0.0315	−0.0766	−0.0245	−0.1952	−0.0460
	OTM	−0.1505	−0.2625	−0.1308	−0.0889	−0.2473	−0.1104
	ITM	−0.0367	−0.0087	−0.0435	−0.0127	−0.2261	−0.0189
NGARCH	ATM	−0.0854	−0.0298	−0.0966	−0.0274	−0.2283	−0.0495
	OTM	−0.1077	−0.1884 (*)	−0.0492	−0.0260	−0.1549	−0.0641
	ITM	−0.0200	−0.0067	−0.0013 (*)	−0.0093	−0.1598	−0.0133
GJR	ATM	−0.0806	−0.0329	−0.0798	−0.0265	−0.1701	−0.0392
	OTM	−0.1098	−0.1939	−0.0610	−0.0430	−0.1565	−0.0683
	ITM	−0.0474	−0.0102	−0.0658	−0.0135	−0.1948	−0.0144
EGARCH	ATM	−0.0659 (*)	−0.0231	0.0206 (*)	0.0027 (*)	−0.0231 (*)	−0.0026 (*)
	OTM	−0.1008 (*)	−0.1919	0.0249 (*)	0.0128 (*)	0.0449 (*)	0.0160 (*)
	ITM	−0.0194	−0.0050	0.0085	−0.0015 (*)	−0.1319	−0.0078 (*)
FIGARCH	ATM	−0.0667	−0.0213 (*)	−0.0326	−0.0066	−0.0697	−0.0098
	OTM	−0.1472	−0.2455	−0.1115	−0.0686	−0.1521	−0.0524
Panel B: Call options							
CV	ITM	−0.1948	−0.0363	−0.4560	−0.0719	−0.4663	−0.0615
	ATM	−0.2068	−0.0949	−0.4014	−0.1048	−0.5761	−0.1197
	OTM	−0.2160	−0.4662	−0.4047	−0.3236	−0.5811	−0.2950
	ITM	−0.1007	−0.0153	−0.2090	−0.0329	−0.2490	−0.0339
GARCH	ATM	−0.0952	−0.0298	−0.1700	−0.0382	−0.2701	−0.0561
	OTM	−0.0792	−0.2073	−0.1325	−0.0959	−0.1899	−0.1096
	ITM	−0.0787	−0.0109 (*)	−0.1693	−0.0253	−0.1864	−0.0260
NGARCH	ATM	−0.1129	−0.0466	−0.2232	−0.0548	−0.3808	−0.0791
	OTM	−0.1322	−0.3252	−0.2791	−0.2264	−0.3899	−0.2078
	ITM	−0.0841	−0.0128	−0.1763	−0.0281	−0.1666	−0.0248
GJR	ATM	−0.1048	−0.0454	−0.1906	−0.0472	−0.2864	−0.0604
	OTM	−0.1055	−0.2737	−0.2323	−0.1908	−0.3090	−0.1738
	ITM	−0.0766 (*)	−0.0114	−0.0821 (*)	−0.0140 (*)	0.0334 (*)	−0.0003 (*)
EGARCH	ATM	−0.0866	−0.0364	−0.0672 (*)	−0.0141 (*)	−0.0840 (*)	−0.0127 (*)
	OTM	−0.1218	−0.3114	−0.1778	−0.1608	−0.2034	−0.1187
	ITM	−0.0982	−0.0148	−0.1734	−0.0257	−0.1665	−0.0218
FIGARCH	ATM	−0.0801 (*)	−0.0172 (*)	−0.1185	−0.0190	−0.1593	−0.0259
	OTM	−0.0750 (*)	−0.1851 (*)	−0.1081 (*)	−0.0769 (*)	−0.1101 (*)	−0.0669 (*)

This table shows results for BIAS and RBIAS across maturity and moneyness for options on the stocks. An asterisk indicates the model with the smallest pricing error for a particular combination of maturity and moneyness.

one for the put options since this specification never performs really badly. However, the EGARCH and the GJR models performs nearly as well as the NGARCH model.

Relative to the overall performance, Panels A and B of Table 9 indicate that all the models underprice the put options and overprice the call options. Thus, although care has to be taken when using the GARCH option pricing model on broad indices (see Duan, 1995, footnote 7), we conjecture that mispricing of the put and call options respectively is at least in part caused by using too high an interest rate or to low dividend payments (see Hull, 1997). Because of these problems we exclude the index options from the rest of the analysis in Sections 5.2.2 and 5.2.3.

5.2.2. Maturity and moneyness effects for individual stock options

In Table 10 we report the results for the BIAS and RBIAS metrics for put and call options on stocks in terms of the time to maturity, T, and the moneyness, Mon (results for the other metrics are available upon request). Examining the result for the relative bias for the CV specification first, the table show that the degree of mispricing is largest for out of the money options with short term to maturity, which is in line with what has been found previously in the literature and consistent with the findings in Table 6 with regards to implied volatilities.

Comparing the results from the CV and the GARCH specifications we see that the GARCH specification has smaller pricing errors for virtually all combinations of maturity and moneyness, and in terms of relative bias the pricing errors from the GARCH model are less than half of those from the CV model in almost all situations. Thus, allowing for time-varying volatility of the GARCH type explains a large fraction of the mispricing often found for models with constant volatility.

However, although the GARCH model performs better than the other time-varying volatility specifications for in the money put options with short and long maturity, this is a small proportion of the options since this partition constitutes less than 9% of the put options and around 3% of the total number of options. We therefore conclude that asymmetries, especially of the exponential type, or long memory are important features when pricing put options. This is also so for the call options and Panel B of Table 10 shows that the GARCH specifications are dominated by specifications with asymmetries, again of the exponential type, or long memory for all combinations of maturity and moneyness. Note that while the NGARCH model has the smallest RBIAS for in the money short-term options the errors for the EGARCH model are less than 5% larger in absolute value. We also note that the long memory FIGARCH specification has the lowest pricing errors for the out of money call options irrespective of the term to maturity.

5.2.3. Empirical regularities

In Table 11 we report the results of the following regression

$$\mathrm{RBIAS}_i = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \beta_1 T^{\mathrm{P}} + \beta_2 \left(T^{\mathrm{P}}\right)^2 + \alpha_3 \mathrm{Mon} + \alpha_4 \mathrm{Mon}^2 + \beta_3 \mathrm{Mon}^{\mathrm{P}} + \beta_4 \left(\mathrm{Mon}^{\mathrm{P}}\right)^2 + \gamma \sigma_i^2 + \varepsilon, \tag{23}$$

where i denotes one of the models and σ_i^2 is the one-step ahead prediction of the variance from that model. To save space we only report the results for a selection of

Table 11
RBIAS regression results for selected volatility specifications

	GM				IBM				MRK			
	CV	GARCH	NGARCH	EGARCH	CV	GARCH	NGARCH	EGARCH	CV	GARCH	NGARCH	EGARCH
Cons	−4.680	−0.874	−1.892	−1.472	−6.804	−1.332	−2.677	−2.820	−6.323	−4.557	−4.684	−4.466
	(0.195)	(0.186)	(0.183)	(0.179)	(0.163)	(0.193)	(0.185)	(0.146)	(0.448)	(0.317)	(0.303)	(0.321)
	[0.240]	[0.012]	[0.056]	[0.036]	[0.270]	[0.010]	[0.042]	[0.073]	[0.098]	[0.102]	[0.115]	[0.096]
T	0.208	0.076	0.090	0.241	0.365	0.432	0.312	0.585	−0.341	−0.315	−0.276	−0.136
	(0.109)	(0.117)	(0.114)	(0.112)	(0.085)	(0.130)	(0.124)	(0.098)	(0.141)	(0.118)	(0.113)	(0.119)
	[0.002]	[0.000]	[0.000]	[0.003]	[0.004]	[0.002]	[0.001]	[0.008]	[0.003]	[0.004]	[0.003]	[0.001]
T^2	−0.306	−0.299	−0.262	−0.337	−0.321	−0.501	−0.336	−0.531	0.673	0.597	0.574	0.489
	(0.194)	(0.208)	(0.203)	(0.199)	(0.141)	(0.217)	(0.207)	(0.163)	(0.242)	(0.202)	(0.194)	(0.205)
	[0.001]	[0.001]	[0.001]	[0.002]	[0.001]	[0.001]	[0.001]	[0.002]	[0.004]	[0.005]	[0.005]	[0.003]
T^{P}	0.049	−0.189	0.008	−0.022	0.351	0.180	0.286	0.301	0.957	0.929	1.039	1.081
	(0.191)	(0.205)	(0.200)	(0.196)	(0.145)	(0.222)	(0.212)	(0.167)	(0.276)	(0.231)	(0.221)	(0.234)
	[0.000]	[0.001]	[0.000]	[0.000]	[0.001]	[0.000]	[0.000]	[0.001]	[0.007]	[0.009]	[0.012]	[0.012]
$(T^{\mathrm{P}})^2$	0.412	0.555	0.235	0.213	−0.015	0.141	−0.120	−0.148	−1.275	−1.251	−1.461	−1.589
	(0.357)	(0.382)	(0.374)	(0.366)	(0.259)	(0.397)	(0.380)	(0.299)	(0.503)	(0.420)	(0.402)	(0.426)
	[0.001]	[0.001]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.004]	[0.005]	[0.007]	[0.008]
Mon	4.507	1.126	2.622	2.123	5.596	1.816	3.727	4.043	10.185	7.816	7.867	7.780
	(0.309)	(0.332)	(0.325)	(0.318)	(0.236)	(0.360)	(0.345)	(0.271)	(0.739)	(0.616)	(0.590)	(0.625)
	[0.104]	[0.006]	[0.035]	[0.024]	[0.106]	[0.005]	[0.024]	[0.045]	[0.094]	[0.081]	[0.088]	[0.078]
Mon^2	−1.622	−0.476	−0.980	−0.822	−1.779	−0.615	−1.219	−1.390	−4.338	−3.317	−3.250	−3.326
	(0.140)	(0.150)	(0.146)	(0.143)	(0.113)	(0.172)	(0.164)	(0.129)	(0.364)	(0.303)	(0.290)	(0.307)
	[0.069]	[0.006]	[0.024]	[0.018]	[0.050]	[0.003]	[0.012]	[0.024]	[0.072]	[0.061]	[0.064]	[0.060]
$\mathrm{Mon}^{\mathrm{P}}$	2.882	0.684	0.617	0.478	4.349	1.568	1.854	1.834	3.360	2.706	2.650	2.248
	(0.093)	(0.100)	(0.097)	(0.095)	(0.078)	(0.115)	(0.109)	(0.086)	(0.161)	(0.135)	(0.129)	(0.136)
	[0.345]	[0.025]	[0.022]	[0.014]	[0.394]	[0.038]	[0.057]	[0.087]	[0.192]	[0.181]	[0.187]	[0.129]
$(\mathrm{Mon}^{\mathrm{P}})^2$	−2.871	−0.680	−0.584	−0.445	−4.121	−1.572	−1.804	−1.797	−3.428	−2.780	−2.703	−2.308
	(0.087)	(0.093)	(0.091)	(0.089)	(0.071)	(0.106)	(0.102)	(0.080)	(0.154)	(0.129)	(0.123)	(0.131)
	[0.375]	[0.028]	[0.022]	[0.014]	[0.415]	[0.044]	[0.062]	[0.096]	[0.213]	[0.203]	[0.208]	[0.146]
σ_t^2	0.618	0.056	0.056	0.035	1.323	0.005	0.005	−0.002	0.196	0.000	−0.003	−0.020
	(0.037)	(0.003)	(0.003)	(0.003)	(0.052)	(0.002)	(0.002)	(0.002)	(0.100)	(0.004)	(0.004)	(0.004)
	[0.132]	[0.193]	[0.180]	[0.072]	[0.121]	[0.001]	[0.001]	[0.000]	[0.002]	[0.000]	[0.000]	[0.011]
R^2	0.45	0.21	0.23	0.12	0.47	0.06	0.12	0.19	0.27	0.25	0.28	0.22

This table shows the results from regressing RBIAS on a number of variables in the time $t-1$ information set according to (23). In parenthesis, we report standard errors of the estimates; in brackets the partial R^2 are reported. NGAR and EGAR is short for NGARCH and EGARCH, respectively.

the models used above. Since all the right hand variables are in the time $t-1$ information set, regressions of this type indicate systematic pricing errors, and in this respect the first line in each panel corresponds to the previous regression of implied volatilities from the CV model in (22). In particular, the R^2 indicates how large a proportion of the variation in the relative pricing error that can be attributed to these variables and could thus potentially be incorporated in the pricing formula. Obviously, other variables could be included in the regression. However, our focus has been on the moneyness and maturity effects and for this reason we limit attention to the variables included in (23).

Comparing the results for the CV and GARCH models for GM and IBM in the table, the first thing to note is that the R^2 is much smaller for the model with time-varying volatility than for the model with constant volatility. Although the parameter estimates are still significant decrease in the R^2 can be attributed to a decrease in the partial R^2 for the moneyness variables shown in the square brackets. Also, the parameter estimates are generally smaller in numerical terms for the specifications with time-varying volatility than for the CV model. The results for MRK shows that although the R^2 is smaller for the GARCH specification than for the CV specifications there is still a large fraction of the variation in the pricing errors which can be explained by the moneyness variables. However, when more complicated volatility specifications are allowed for, e.g. the EGARCH specification, the R^2 drops from 28% to 22%.

6. Conclusion

In this paper we consider the problem of pricing options when the underlying asset exhibits time-varying volatility. We model time-varying volatility in the context of GARCH processes, since these processes have been used extensively in the literature to model asset returns, and in many cases evidence has been found in favor of them. When the volatility is allowed to vary through time in a way that depends on lagged innovations to the return process and lagged volatilities, it is generally not possible to derive the future distribution of the underlying asset. Thus, no analytical option pricing formula exists, and alternative numerical methods must be used to price derivatives. Simulation techniques have been applied, but the empirical application of the GARCH option pricing model has been hampered by the lack of simulation techniques able to incorporate early exercise features.

In the present paper we show how a new simulation technique, using simple least squares regressions (the LSM method of Longstaff and Schwartz (2001)), can be used to price American options, with the possibility of early exercise. The method is simple to implement and GARCH processes can be readily accommodated. We show that the method compares favorably to the limited results previously obtained in the literature. Furthermore, since theoretical results comparing the performance of option pricing models with GARCH type volatility processes with the constant volatility cases are limited for the European options and non-existing for American style options, we present a Monte Carlo study of the characteristics of the model. This study shows that the GARCH model can accommodate the smile in implied volatilities found empirically, and that the asymmetric models like the NGARCH, GJR, and EGARCH models have the further ability to generate implied volatility smirks. On the other hand, allowing for long memory seems to have less effects for options with the characteristics considered.

Empirically, we use our proposed algorithm to price traded options on three individual stocks and the Standard and Poor's 100 Index. These options are American, and to our knowledge this is the first paper to price options of this type, when the volatility process is allowed to be of the GARCH type. The empirical analysis shows that GARCH effects are important when pricing options on individual stocks. Furthermore, the asymmetric effects, in particular of the exponential type, are important for both put and call options. For the index options the overall performance suggests that the models with time-varying volatility generally provide good predictions of future volatility, whereas the constant volatility model overshoots the future level. However, the put and call results respectively indicate pricing errors which are consistent with other misspecifications. In particular, it appears as though the interest rate used is too high or the dividend payments too low.

Although our results show that allowing for GARCH type volatility specifications can explain a large fraction of the mispricing often found for models with constant volatility, systematic mispricing still occurs in the time-varying volatility models examined in the present paper. In particular, we find a significant relationship between the relative bias of the estimates and moneyness. Explaining these persisting "empirical regularities" constitutes an obvious area for future research. In particular, the effects of non-normal errors should be explored. Other extensions include increasing the number of individual stocks, as well as examining the results for index options in more detail, in particular focussing on extensions to the GARCH option pricing models which are more suitable for this type of asset.

Acknowledgements

The author wishes to thank an anonymous referee, Asger Lunde, Ton C.F. Vorst, Peter Løchte Jørgensen, and participants at the Financial Econometrics Lunch Group at Duke University for valuable comments and suggestions. All remaining errors are the author's responsibility. Financial support from SSF (Grant 95015328) is gratefully appreciated.

References

Amin, K.I., Ng, V.K., 1993a. ARCH processes and option valuation. Working Paper. Department of Finance, University of Michigan.

Amin, K.I., Ng, V.K., 1993b. Option valuation with systematic stochastic volatility. Journal of Finance 48 (3), 881–910.

Andersen, T.G., Benzoni, L., Lund, J., 2002. An empirical investigation of continuous-time equity return models. Journal of Finance 57 (3), 1239–1284.

Baillie, R.T., Bollerslev, T., Mikkelsen, H.O., 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 74, 3–30.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52 (5), 2003–2049.

Bates, D.S., 2000. Post-'87 crash fears in the SP 500 futures option market. Journal of Econometrics 94, 181–238.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637–654.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307–327.

Bollerslev, T., Mikkelsen, H.O., 1996. Modelling and pricing long memory in stock market volatility. Journal of Econometrics 73, 151–184.

Bollerslev, T., Mikkelsen, H.O., 1999. Long-term equity anticipation securities and stock market volatility dynamics. Journal of Econometrics 92, 75–99.

Bollerslev, T., Wooldridge, J.M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews 11 (2), 143–172.

Bollerslev, T., Chou, R.Y., Kroner, K.F., 1992. ARCH modelling in finance. Journal of Econometrics 52, 5–59.

Bollerslev, T., Engle, R.F., Nelson, D.B., 1994. ARCH models. In: Engle, R.F., McFadden, D.L. (Eds.), Handbook of Econometrics, vol. 4. Elsevier Science B.V., pp. 2959–3038.

Boyle, P.P., 1977. Options: a Monte Carlo approach. Journal of Financial Economics 4, 323–338.

Brennan, M.J., 1979. The pricing of contingent claims in discrete time models. Journal of Finance 24 (1), 53–68.

Carriere, J.F., 1996. Valuation of the early-exercise price for options using simulations and non-parametric regression. Insurance. Mathematics & Economics 19, 19–30.

Christoffersen, P., Jacobs, K., 2002. Which volatility model for option valuation. Working Paper. CIRANO.

CRSP, 1998. CRSP Stock File Guide. Center for Research in Security Prices, Graduate School of Business, University of Chicago.

Doornik, J.A., 2001. Ox: An Object-Oriented Matrix Language, 4th edn. Timberlake Consultants Press, London.

Duan, J.-C., 1995. The GARCH option pricing model. Mathematical Finance 5 (1), 13–32.

Duan, J.-C., 1997. Augmented GARCH(p,q) process and its diffusion limit. Journal of Econometrics 79, 97–127.

Duan, J.-C., 1999. Conditionally fat-tailed distributions and the volatility smile in options, Mimeo, Hong Kong University of Science and Technology.

Duan, J.-C., Simonato, J.-G., 2001. American option pricing under GARCH by a Markov chain approximation. Journal of Economic Dynamics and Control 25, 1689–1718.

Duan, J.-C., Zhang, H., 2001. Pricing Hang Seng index options around the Asian financial crisis — a GARCH approach. Journal of Banking and Finance 25, 1989–2014.

Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50 (4), 987–1007.

Engle, R.E., Bollerslev, T., 1986. Modelling the persistence of conditional variances. Econometric Reviews 5 (1), 1–50.

Engle, R.E., Ng, V.K., 1993. Measuring and testing the impact of news on volatility. Journal of Finance 48 (5), 1749–1778.

Gallant, R., Tauchen, G., 1998. Reprojecting partially observed systems with application to interest rate diffusions. Journal of the American Statistical Association 93 (441), 10–24.

Glosten, L.R., Jagannathan, R., Runkle, D.E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48 (5), 1779–1801.

Harrison, J.M., Kreps, D.M., 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20, 381–408.

Heston, S.L., Nandi, S., 2000. A closed-form GARCH option valuation model. Review of Financial Studies 13 (3), 585–625.

Hsieh, K.C., Ritchken, P., 2000. An empirical comparison of GARCH option pricing models. Working Paper. Case Western Reserve University.

Hull, J.C., 1997. Options, Futures, and Other Derivatives. Prentice-Hall, Inc., Upper Saddle River, NJ.

Hull, J., White, A., 1987. The pricing of options on assets with stochastic volatilities. Journal of Finance 42 (2), 281–300.

Laurent, S., Peters, J.-P., 2002. A tutorial for G@RCH 2.3, a complete Ox package for estimating and forecasting ARCH models, Mimeo, Université de Liège, Belgium and Maastricht University, Netherlands.

Longstaff, F.A., Schwartz, E.S., 2001. Valuing American options by simulation: a simple least-squares approach. Review of Financial Studies 14, 113–147.

Moreno, M., Navas, J.F., 2003. On the robustness of least-squares Monte Carlo (LSM) for pricing American options. Review of Derivatives Research 6, 107–128.

Myers, R.J., Hanson, S.D., 1993. Pricing commodity options when the underlying futures price exhibits time-varying volatility. American Journal of Agricultural Economics 75, 121–130.

Nelson, D.B., 1990. ARCH models as diffusion approximations. Journal of Econometrics 45, 7–38.

Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: a new approach. Econometrica 59 (2), 347–370.

Poon, S.-H., Granger, C.W.J., 2003. Forecasting volatility in financial markets: a review. Journal of Economic Literature XLI, 478–539.

Ritchken, P., Trevor, R., 1999. Pricing options under generalized GARCH and stochastic volatility processes. Journal of Finance 59 (1), 377–402.

Stentoft, L., 2004a. Assessing the least squares Monte-Carlo approach to American option valuation. Review of Derivatives Research 7 (3), 129–168.

Stentoft, L., 2004b. Convergence of the Least Squares Monte Carlo approach to American option valuation. Management Science 50 (9), 1193–1203.

Tsitsiklis, J.N., Van Roy, B., 2001. Regression methods for pricing complex American-style options. IEEE Transactions on Neural Networks 12 (4), 694–703.

Wiggins, J.B., 1987. Option values under stochastic volatility: theory and empirical estimates. Journal of Financial Economics 19, 351–372.