Outline

A Neural Network Based Framework for Financial Models Summary

4.3 Backward Pass (Heston Model)

σ_{imp} | K, τ, S_0, r -> Heston-CaNN -> ρ, k, v_0, $\overline{v}$, γ

How does this relate to our research?

- We hope to be able to use historical asset prices and option prices to be able to calibrate our parameters. This mean, our input depends on the different parameters that make these prices/returns, which would be strike prices, initial price, rate, time to maturity and the implied volatility.
- We then send these input to the CaNN where it would train the model
- After we train the model, we can use our Joint objective function to return the calibrated parameters, where in our case, since we are just caring about the GARCH(1, 1) model would be ω, α, β.

Sampling Training Data

Found in Table 5 of the paper:

	Parameters	**Range**	**Samples**
Market data	Moneyness, $m = \frac{S_0}{K}$ Time to maturity, τ Risk free rate, r European call/put price, $\frac{V}{K}$	$[0.85, 1.15]$ $[0.5, 2.0]$(year) 0.03 $(0.0, 0.6)$	5 7 Fixed, •
Black-Scholes	Implied Volatility	$(0.2, 0.5)$	35

We can use this as a way to sample all our different input parameters, and be able to measure how more accurate the model becomes and its calibration.

During the calibration, they use the total squared error measure $J(\Theta)$:

$$J(\Theta) = \sum \omega\left(\sigma_{\text{imp}}^{\text{ANN}} - \sigma_{\text{imp}}^{*}\right)^2 + \overline{\lambda}|\Theta|$$

Averaged performance of the Backward pass of the CaNN:

- Need to list CPU and GPU spec
  - OS: Linux fedora 6.10.7-200.fc40.x86_64
  - CPU: AMD Ryzen 5 5600G
  - GPU: Radeon 6600

Abosulute deviation from θ^*, Error measure and computational cost.
Error Measure: $J(\Theta)$, MJ and Data Points
Computational Cost: CPU, GPU time and Function evaluations

Summary of Joint Calibrarion Artificial Neural Networks

Goals & Designs

Our goal is to be able to use two forms of input data to be able to calibrate an optimum parameters of the GARCH(1, 1) model. This will be calibrated using joint calibration which will take into account the log likelihood of returns and option prices to be able to consider both physical and risk neutral measures.

The design will make use of a backward pass artificial neural network, where we will do the following:

$$\text{Market Data } (S, K, S_0, r, \tau, \sigma |\ R_t)$$
$$\downarrow$$
$$\text{Joint Calibration Neural Network}$$
$$\downarrow$$
$$\text{GARCH Params } (\omega, \alpha, \beta, \gamma, \lambda)$$

Where we consider the parameters under **P** measure:

- S: Price (a time series of asset prices eg. (10-year daily prices))
- K: Strike Price
- S_0: Initial Price
- r: Risk-free rate (fixed)
- σ: Implied Volatility
- τ: Time to maturity

And we consider the following physical measure:

- R_t: Log return at time t (several GARCH models)

Our goal is to be able to use the Joint Calibration to have the most optimum calibrated GARCH parameters that takes into account both measures. This will come into the calibration phase, where the idea is to utilize the Joint Calibration formula as the objective function for minimization.

Risk Neutralization for One-Component Gaussian Models

- Q: Risk-Neutral Measure
- P: Physical Measure

Using the Radon-Nikodym derivative, we can convert the physical measure to the risk-neutral measure. Let z_t i.i.d $N(0, 1)$, then $\gamma_t = \frac{1}{2} h_t$ since $\exp(\gamma_t) = E_{\text{t-1}}[\exp(\varepsilon_t)]$
The Radon-Nikodym derivative is defined as:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i=1}}^{t} \left(\frac{\mu_i - r_i}{h_i} \varepsilon_i + \frac{1}{2} \left(\frac{\mu_i - r_i}{h_i} \right)^2 h_i \right) \right)$$

NGARCH(1, 1)

For NGARCH(1, 1) using $\varepsilon_t^* = \varepsilon_t + \mu_t - r_t$, the volatility process under Q becomes:

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha(\varepsilon^*_{\text{t-1}} - \mu_{\text{t-1}} + r_{\text{t-1}})^2 \Rightarrow \varepsilon^*_t \mid F_t \sim N(0, h_t)$$

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t - \frac{1}{2}h_t + \varepsilon^*_t \Rightarrow \varepsilon^*_t \mid F_{\text{t-1}} \sim N(0, h_t)$$

$$E^Q\left[\frac{S_t}{S_{\text{t-1}}} \mid F_{\text{t-1}}\right] = \exp(r_t)$$

Duan

The Physical GARCH Model Duan (1995) comes in the following form:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t$$

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\varepsilon^2_{\text{t-1}}$$

Assume:

- λ: price of risk (const.)
- $\mu_t = r_t + \lambda\sqrt{h_t}$ or $\lambda = \frac{\mu_t - r_t}{\sqrt{h_t}}$

This corresponds to the following RN-Derivative:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\text{i=1}}^{t}\left(\frac{\varepsilon_i}{\sqrt{h_i}}\lambda + \frac{1}{2}\lambda^2\right)\right)$$

With risk-neutral innovations:

$$\begin{aligned}\varepsilon^*_t &= \varepsilon_t + \mu_t - r_t \\ &= \varepsilon_t + \lambda\sqrt{h_t}\end{aligned}$$

The Risk-Neutral GARCH becomes:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r - \frac{1}{2}h_t + \varepsilon^*_t$$

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon^*_{\text{t-1}} - \lambda\sqrt{h_{\text{t-1}}}\right)^2$$

HN-GARCH(1, 1)

Starting with the following model of Heston and Nandi (2000):

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r + \lambda h_t + \varepsilon_t$$

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon_{\text{t-1}} - c\sqrt{h_{\text{t-1}}}\right)^2$$

Assume $r_t = r, \mu_t = r + \lambda h_t + 0.5h_t$
RN-Derivative:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i=1}}^{t}\left(\left(\lambda + \frac{1}{2}\right)\varepsilon_i + \frac{1}{2}\left(\lambda + \frac{1}{2}\right)^2 h_i\right)\right)$$

$$\varepsilon_t^* = \varepsilon_t + \lambda h_t + 0.5 h_t$$

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r - \frac{1}{2}h_t + \varepsilon_t^*$$

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon_{\text{t-1}}^* - \left(c + \lambda + \frac{1}{2}\right)\sqrt{h_{\text{t-1}}}\right)^2$$

Input Data

So firstly we need to discuss how we get our input parameters?
Risk-Neutral Measure:

- American Option Prices

Physical Measure:

- Historical Asset Prices (change in difference to get log return)
  - Where: $R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = \mu_t - \gamma_t + \varepsilon_t$
  - μ_t: Conditional mean of the returns at time t
  - Assume γ_t is defined from $\exp(\gamma_t) = E_{\text{t-1}}[\exp(\varepsilon_t)]$
  - ε_t: the normal innovation at time t where $\varepsilon_t | F_{\text{t-1}} \sim N(0, h_t)$
  - h_t: Conditional variance at time t
    - $h_t = \omega + \alpha\varepsilon_{\text{t-1}}^2 + \beta h_{\text{t-1}}$

Joint Calibration Neural Network

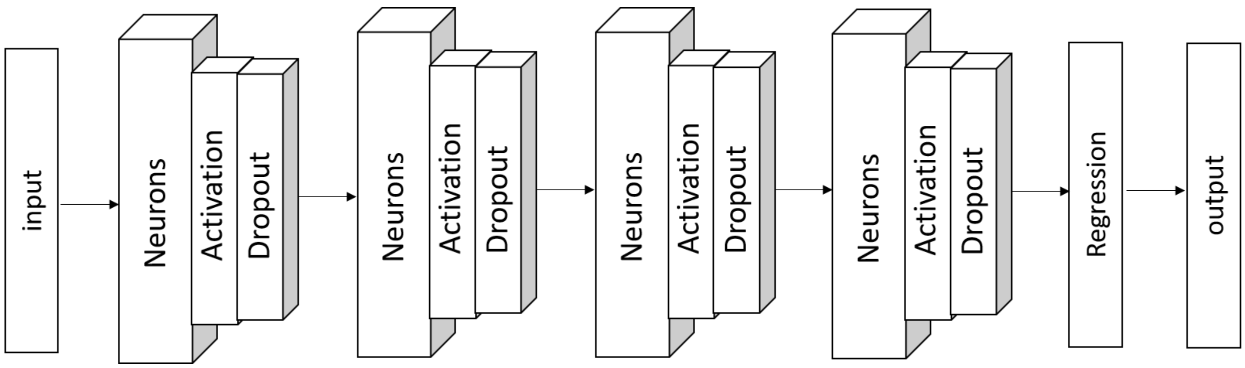
This neural network will be written in Python with a minimum required version of 3.10, and will require the following dependencies:

```
[tool.poetry.dependencies]
python = "^3.10"
numpy = "^1.26.4"
pandas = "^2.2.2"
arch = "^7.0.0"
scikit-learn = "^1.5.0"
matplotlib = "^3.9.0"
scipy = "^1.13.1"
```

The architecture currently will follow simarly to the paper, as the following:

Parameters	**Options**
Hidden Layers	4
Neurons(each layer except Output) Activation Dropout rate Batch-normalization Initialization Optimizer	200 ReLu 0.0 No Glorot_uniform Adam

Parameters	**Options**
Batch size Output Layer Activation Output Layer Neurons	1024 SoftPlus (to ensure all GARCH params are positive) 5 (for each GARCH param)



Where we then hope in the regression phase, to be able to use the Joint Calibration formula in a batching method to be able to calibrate our parameters for the GARCH model.

Essential Equations

These are from GARCH Option Valuation Theory and Evidence:

Return Log Likelihood:

$$\ln L^R \propto -\frac{1}{2}\sum_{\mathrm{t=1}}^{T}\left\{\ln(h(t)) + \frac{(R(t)-\mu_t-\gamma_t)^2}{h(t)}\right\}$$

where:

- $h(t)$ is the conditional variance
- $R(t)$ represents the return at time t
- μ_t is the conditional mean of the returns at time t
- γ_t is an adjustment term

Options Log Likelihood:

$$\ln L^O \propto -\frac{1}{2}\sum_{\mathrm{i=1}}^{N}\left\{\ln(\mathrm{IVRMSE}^2) + \left(\frac{e_{\mathrm{i,t}}}{\mathrm{IVRMSE}}\right)^2\right\}$$

Implied Volatility root mean squared error (IVRMSE) loss function follows:

$$\mathrm{IVRMSE} \approx \sqrt{\frac{1}{N_T}\sum_{\mathrm{i,\,t}}^{N_T} e_{\mathrm{i,t}}^2}$$

where:

- N_T: total number of option prices in the sample

The verga weighted option error follows as:

$$\underbrace{e_{\mathrm{i,t}} = \frac{C_{\mathrm{i,t}} - C_{\mathrm{i,t}}(h_t(\xi^*))}{\mathrm{Vega}_{\ \mathrm{i,t}}}}_{\text{We may also choose to use relative error if Vega is hard or expensive to calculate}}$$

where:

- $\mathrm{Vega}_{\mathrm{i,t}}$ is the Black-Scholes sensitivity of the option prices with respect to volatility
- ξ^* is the vector of risk-neutral parameters to be estimated
- $C_{\mathrm{i,t}} - C_{\mathrm{i,t}}(h_t(\xi^*))$: The corresponding implied volatility from the option price.

Joint Log Likelihood

$$L_{\mathrm{joint}} = \frac{T+N_T}{2}\frac{L^R}{T} + \frac{T+N_T}{2}\frac{L^O}{N}$$

where:

- T is the number of days in the return sample
- N_T is the total number of option contracts

Training Data Generation

Current Idea

1. Given a set of parameters for GARCH (in Physical measure)
2. Given the initial asset price S_0, use Monte Carlo method to simulate a path of asset prices,
$$S_1, S_2, ...S_N, \text{ with say } N = 500 \text{ (Under } \mathbf{P} \text{ measure)}$$
3. Select last 30-50 days on the path, for each day, use the selected asset price (under **Q**) as the initial price to generate American option prices with various strike prices (11-17) and maturities (7 days to 1 year). **Pay attention to the transformation from the physical measure to the risk-neutral measure**.

Pseudo Code

The Steps that we are following:

1. Initialize Option and Monte Carlo Parameters
   - `r`: The risk-free rate
   - `S0`: The initial asset price
   - `h0`: Initial volatility
   - `N`: Number of time steps for simulation
   - `M`: Number of Monte Carlo paths
2. Initialize HN-GARCH parameters under P measure
   - $\theta = (\alpha, \beta, \omega, \gamma, \lambda)$
3. Simulate paths using Monte Carlo Simulation
4. Risk Neutralize HN-GARCH parameters
   - $\theta^* = \left(\alpha_Q, \rho, \omega_Q, \gamma_Q, \lambda_Q\right)$
5. Initialize Willow Tree parameters
6. Generate data for the days up to the maturity

Project Structure:

MATLAB Files:

`American.m`	`gen_PoWiner.m`	`nodes_Winer.m`
`Prob_Xt.m`	`zq.m`	`datagen.m`
`impVol_HN.m`	`impvol.m`	`probcali.m`
`main.m`	`Prob_ht.m`	`sign.m`
`genhDelta.m`	`Prob.m`	`TreeNodes_ht_HN.m`
`Treenodes_JC_h.m`	`Treenodes_JC_X.m`	`TreeNodes_logSt_HN.m`

Dependencies: `f_hhh.mexa64`
Output Files: `annual.csv`, `half.csv`, `quarter.csv`, `week.csv`

The `datagen()` Function

```
function [A_sig, A_prices, S0] = datagen(maturity, r, S0, N, M, h0, alpha, beta, omega,
gamma, lambda, path_days)
    % Parameters:
    %   maturity - Time to maturity of the option.
    %   r - Risk-free rate.
    %   S0 - Initial asset price.
    %   N - Number of time steps for simulation.
    %   M - Number of Monte Carlo paths.
    %   h0 - Initial volatility.
    %   alpha - Alpha parameter for HN-GARCH model.
    %   beta - Beta parameter for HN-GARCH model.
    %   omega - Omega parameter for HN-GARCH model.
    %   gamma - Gamma parameter for HN-GARCH model.
    %   lambda - Lambda parameter for HN-GARCH model.
    %   path_days - Number of days in the path to consider.
    % Output:
    %   A_sig - Implied volatilities for the American options.
    %   A_prices - Prices of the American options.
    %   S0 - Initial prices used for each maturity.

    % 3. Simulate paths using Monte Carlo simulation under P measure
    numPoint = N + 1;
    Z = randn(numPoint, M);
    ht = nan(numPoint, M);
    Xt = nan(numPoint, M);
    ht(1,:) = h0 * ones(1, M);
    Xt(1,:) = log(S0) * ones(1, M);
    for i = 2:numPoint
          ht(i,:) = omega + alpha * (Z(i-1,:) - gamma * sqrt(ht(i-1,:))).^2 + beta *
ht(i-1,:);
        Xt(i,:) = Xt(i-1,:) + (r - 0.5 * ht(i,:)) + sqrt(ht(i,:)) .* Z(i,:);
    end
    S = exp(Xt);

    % Get the last 'path_days' days on the path
    S = S(end - path_days + 1:end, :);

    % 4. Risk-neutralize GARCH parameters (Q measure)
    eta = 0;
    omega_Q = omega / (1 - 2 * alpha * eta);
    gamma_Q = gamma * (1 - 2 * alpha * eta);
    alpha_Q = alpha / (1 - 2 * alpha * eta)^2;
    lambda_Q = lambda * (1 - 2 * alpha * eta);
    rho = lambda_Q + gamma_Q + 1/2;

    % 5. Initialize Willow Tree parameters
    m_h = 6;
    m_x = 30;

    % 6. Generate Data for the days up to the maturity
    % Generate strike prices based on moneyness through the maturity
    strike_prices = linspace(0.8 * S0, 1.2 * S0, maturity);
    A_sig = zeros(maturity, 1);
    A_prices = zeros(maturity, 1);

    % Prepare initial prices for each maturity
    S0 = zeros(maturity, 1);
    for j = 1:maturity
        index = mod(j - 1, path_days) + 1;
        S0(j) = mean(S(index, :));  % Use the mean across all paths
    end
```

The `main` Code

This will contain generating data for different maturities and configuring the parameters for both the Option and HN-GARCH.

```
maturities = [5, 63, 126, 252]; % week, 3 months, 6 months and one year of trading days
filenames = {'week.csv', 'quarter.csv', 'half.csv', 'annual.csv'};
% 1. Initialize Option Parameters
r = 0.05/252; % Risk-free rate
S0 = 100; % Initial asset price
N = 100; % Number of time steps for simulation (ex. 500)
M = 10000; % Number of Monte Carlo paths
h0 = (0.2^2)/252; % Initial volatility

% 2. Initialize HN-GARCH parameters under P Measure
alpha = 1.33e-6;
beta = 0.586;
omega = 4.96e-6;
gamma = 484.69;
lambda = 0.5;

path_days = 50;

parfor i = 1:length(maturities)
     [sig, V, S] = datagen(maturities(i), r, S0, N, M, h0, alpha, beta, omega, gamma,
lambda, path_days);
    data = table(V, sig, S, 'VariableNames', {'V', 'Sigma', 'S'});
    writetable(data, filenames{i});
end
```

Downloading the Code: Available under **Github**

Generated Data

Week

V	Sigma	S
38.9854834477284	1	101.189098534199
48.9683598206385	1	101.201215992988
58.9487091702125	0.955499649047852	101.221221613324
68.9309834778906	0.86296272277832	101.237887537136
78.9112359050815	0.796268463134766	101.260721512983

3 Months

V	Sigma	S
39.0510743469898	1	101.026879826771
39.6854068520908	1	101.051470243331
40.3209269701966	0.865146636962891	101.073117230628
40.9521771639181	0.752840042114258	101.105337746153
41.5916104733991	0.67668342590332	101.117334459905
42.2283085006425	0.620756149291992	101.136089489197
42.8661808501481	0.576913833618164	101.151934062621
43.5080363797581	0.541690826416016	101.157921585204
44.1444585656682	0.512613296508789	101.177358129599
44.7858405430586	0.488150596618652	101.184543826779
45.4239025671957	0.467179298400879	101.19993341769
46.0567445740421	0.448959350585938	101.228225786776
46.6890495005708	0.432924270629883	101.25784014088
47.3225096135259	0.418725967407227	101.284598528532
47.9571879497999	0.406038284301758	101.308369290254
48.5929596126335	0.394607543945312	101.32943016291
49.2288493577897	0.384280204772949	101.350192792799
49.866621324473	0.37486743927002	101.366294664155
50.5034031640065	0.366250038146973	101.384838752337
51.1380488953958	0.358321189880371	101.408682188367
51.7788152548515	0.351031303405762	101.417399621107
52.4152363051278	0.34426212310791	101.436855704402
53.0517023215322	0.337976455688477	101.456194351266
53.6837120360436	0.33210563659668	101.486546193722
54.3145820204676	0.326408863067627	101.519710138108

54.9512876390125	0.320987701416016	101.538461072396
55.5883891465299	0.315900325775146	101.556242079809
56.2250714743149	0.311102390289307	101.575053321427
56.8566608718758	0.306559562683105	101.60645138977
57.4974148435234	0.302290916442871	101.615182358277
58.133482868599	0.298233985900879	101.635493828118
58.7720294969009	0.294391632080078	101.649680402613
59.4084496192628	0.290756225585938	101.669146571356
60.0456363111905	0.287269115447998	101.686711124509
60.6809824050582	0.283955574035645	101.708820378855
61.319575152924	0.280799865722656	101.722895898701
61.9570258546426	0.277798175811768	101.739788919065
62.5968216099354	0.274930477142334	101.750877409419
63.2353928315312	0.272183895111084	101.765016026158
63.8706516635216	0.269543647766113	101.787345811337
64.5112563917238	0.267031669616699	101.796452041561
65.1442944902842	0.264599323272705	101.824259939322
65.7868227302905	0.262293815612793	101.828597879695
66.4216287541902	0.260063171386719	101.852022220932
67.0582041747677	0.25791072845459	101.871073116049
67.6923001135659	0.255839824676514	101.896274966377
68.3211948628662	0.253830432891846	101.93432953503
68.9563515529101	0.251914024353027	101.956896459157
69.5934581879314	0.249927043914795	101.974636268468
70.2294234816612	0.247963428497314	101.995191590045
71.265376100797	0.246913433074951	101.026879826771
71.8997169513184	0.245074272155762	101.051470243331
72.5352459006719	0.243296146392822	101.073117230628
73.1664953470102	0.241567611694336	101.105337746153
73.8059227779318	0.239906787872314	101.117334459905
74.4426139506429	0.238293647766113	101.136089489197
75.0804820319868	0.236732959747314	101.151934062621
75.7223462906378	0.235228538513184	101.157921585204
76.3587676089485	0.233756065368652	101.177358129599
77.0001420995031	0.232337474822998	101.184543826779
77.6381954219531	0.230955123901367	101.19993341769

78.2710268165037	0.229591369628906	101.228225786776
78.9033209349059	0.228280067443848	101.25784014088

6 Months

V	Sigma	S
39.0415480030629	1	101.050440278229
39.3455605356018	1	101.088796984574
39.6560316052704	0.860857009887695	101.111177458841
39.9690137689538	0.747398376464844	101.127344437964
40.2845441784764	0.670185089111328	101.13720600075
40.5953035724897	0.613310813903809	101.158864207775
40.902660321726	0.569192886352539	101.188948955425
41.2117133839103	0.533738136291504	101.21484019609
41.5217180309283	0.504456520080566	101.238375037161
41.8356466394233	0.479774475097656	101.252202459938
42.1458663620486	0.458572387695312	101.275199290242
42.4637834660193	0.440072059631348	101.279156554502
42.775476259858	0.42362117767334	101.298504173231
43.0908123432294	0.409012794494629	101.308838417228
43.4016047383173	0.395902633666992	101.330406703654
43.7139982773759	0.384067535400391	101.348012092461
44.0282953957837	0.373331069946289	101.360912605289
44.3381319829693	0.363508224487305	101.384854150874
44.6505614414879	0.354496955871582	101.402380426635
44.9597328415356	0.346179485321045	101.427960903354
45.2672061595353	0.338473320007324	101.457737757087
45.5758379069063	0.331318378448486	101.484646754161
45.8798675952712	0.32463264465332	101.522933807146
46.1962383566495	0.31843376159668	101.530697823745
46.510850216563	0.312618255615234	101.542808589575
46.8201707255023	0.307129859924316	101.568002078469
47.1356793819328	0.301987648010254	101.577888861286
47.4471238722174	0.297121047973633	101.597829282031
47.763762121667	0.292540550231934	101.604935560733
48.0716200003148	0.288173675537109	101.633752099089
48.3832519186909	0.284037590026855	101.65323260757

48.6981576128969	0.2801194190979	101.664614160251
49.0078856600541	0.276378631591797	101.688796730823
49.3131082443205	0.27278995513916	101.724118108382
49.6299739020689	0.26934814453125	101.730643533091
49.9435977718698	0.266170501708984	101.745182704065
50.2570146630581	0.263067245483398	101.760230650512
50.5650959636464	0.260082244873047	101.788470348245
50.8764423143185	0.257228851318359	101.808632591188
51.186923411337	0.2544846534729	101.830944900279
51.5001074650027	0.251846313476562	101.846575189629
51.8147904416777	0.249351501464844	101.858495770282
52.128671806607	0.246920108795166	101.872395544385
52.4419901979603	0.244584083557129	101.887684401318
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One Year

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