

Training Data Generation

Current Idea

1. Given a set of parameters for GARCH (in Physical measure)
2. Given the initial asset price S_0 , use Monte Carlo method to simulate a path of asset prices, S_1, S_2, \dots, S_N , with say $N = 500$ (Under **P** measure)
3. Select last 30-50 days on the path, for each day, use the selected asset price (under **Q**) as the initial price to generate American option prices with various strike prices (11-17) and maturities (7 days to 1 year). **Pay attention to the transformation from the physical measure to the risk-neutral measure.**

Pseudo Code

The Steps that we are following:

1. Initialize Option and Monte Carlo Parameters
 - r : The risk-free rate
 - S_0 : The initial asset price
 - h_0 : Initial volatility
 - N : Number of time steps for simulation
 - M : Number of Monte Carlo paths
2. Initialize HN-GARCH parameters under P measure
 - $\theta = (\alpha, \beta, \omega, \gamma, \lambda)$
3. Simulate paths using Monte Carlo Simulation
4. Risk Neutralize HN-GARCH parameters
 - $\theta^* = (\alpha_Q, \rho, \omega_Q, \gamma_Q, \lambda_Q)$
5. Initialize Willow Tree parameters
6. Generate data for the days up to the maturity

Project Structure:

MATLAB Files:

American.m	gen_PoWiner.m	nodes_Winer.m
Prob_Xt.m	zq.m	datagen.m
impVol_HN.m	impvol.m	probcali.m
main.m	Prob_ht.m	sign.m
genhDelta.m	Prob.m	TreeNodes_ht_HN.m
Treenodes_JC_h.m	Treenodes_JC_X.m	TreeNodes_logSt_HN.m

Dependencies: f_hhh.mexa64

Output Files: annual.csv, half.csv, quarter.csv, week.csv

The datagen() Function

```
function [A_sig, A_prices] = datagen(maturity, r, S0, N, M, h0, alpha, beta, omega,
gamma, lambda, path_days)
% Parameters:
% maturity - Time to maturity of the option.
% r - Risk-free rate.
% S0 - Initial asset price.
% N - Number of time steps for simulation.
% M - Number of Monte Carlo paths.
% h0 - Initial volatility.
% alpha - Alpha parameter for HN-GARCH model.
% beta - Beta parameter for HN-GARCH model.
% omega - Omega parameter for HN-GARCH model.
% gamma - Gamma parameter for HN-GARCH model.
% lambda - Lambda parameter for HN-GARCH model.
% path_days - Number of days in the path to consider.
% Output:
% A_sig - Implied volatilities for the American options.
% A_prices - Prices of the American options.

% 3. Simulate paths using Monte Carlo simulation under P measure
numPoint = N + 1;
Z = randn(numPoint + 1, M);
ht = nan(numPoint + 1, M); Xt = nan(numPoint + 1, M);
ht(1,:) = h0 * ones(1, M);
Xt(1,:) = log(S0) * ones(1, M);
for i = 2:numPoint
    ht(i,:) = omega+alpha*(Z(i-1,:)-gamma*sqrt(ht(i-1,:))).^2 + beta*ht(i-1,:);
    Xt(i,:) = Xt(i-1,:) + (r - 0.5 * ht(i,:)) + sqrt(ht(i,:)) .* Z(i,:);
end
S = exp(Xt);
% Get the last 'path_days' days on the path
S = S(end - (path_days - 1):end, :);
% 4. Risk-neutralize GARCH parameters (Q measure)
eta = 0;
omega_Q = omega / (1 - 2 * alpha * eta);
gamma_Q = gamma * (1 - 2 * alpha * eta);
alpha_Q = alpha / (1 - 2 * alpha * eta)^2;
lambda_Q = lambda * (1 - 2 * alpha * eta);
rho = lambda_Q + gamma_Q + 1/2;
% 5. Initialize Willow Tree parameters
m_h = 6; m_x = 30;
% 6. Generate Data for the days up to the maturity
% Generate strike prices based on moneyness through the maturity
strike_prices = linspace(0.8 * S0, 1.2 * S0, maturity);
A_sig = zeros(maturity, 1); A_prices = zeros(maturity, 1);
% Setting the initial price to use the days from S
% and wrapping around if necessary.
S0 = S(mod(0:(maturity - 1), path_days) + 1, :);
parfor j = 1:maturity % iterate through the maturities in parallel
    [A_sig(j), A_prices(j), ~] = impVol_HN(r, lambda_Q, omega_Q, rho, alpha_Q,
gamma_Q, h0, S0(j), strike_prices(j), T, N, m_h, m_x, -1);
end
end
```

The main Code

This will contain generating data for different maturities and configuring the parameters for both the Option and HN-GARCH.

```
maturities = [5, 63, 126, 252]; % week, 3 months, 6 months and one year of trading
days
filenames = {'week.csv', 'quarter.csv', 'half.csv', 'annual.csv'};
% 1. Initialize Option Parameters
r = 0.05/252; % Risk-free rate
S0 = 100; % Initial asset price
N = 100; % Number of time steps for simulation (ex. 500)
M = 10000; % Number of Monte Carlo paths
h0 = (0.2^2)/252; % Initial volatility

% 2. Initialize HN-GARCH parameters under P Measure
alpha = 1.33e-6;
beta = 0.586;
omega = 4.96e-6;
gamma = 484.69;
lambda = 0.5;

path_days = 50;

parfor i = 1:length(maturities)
    [sig, V] = datagen(maturities(i), r, S0, N, M, h0, alpha, beta, omega, gamma,
    lambda, path_days);
    data = table(V, sig, 'VariableNames', {'Option Price', 'Implied Volatility'});
    writetable(data, filenames{i});
end
```

Downloading the Code: Available under [Github](#)

Generated Data

Week

Option Price	Implied Volatility
36.8178263574187	0.147327423095703
46.7785582767416	0.157432317733765
57.0272091662167	0.166537761688232
67.0332631013716	0.173696279525757
76.9315818148281	0.179845809936523

3 Months

Option Price	Implied Volatility
36.5600267989308	0.146834373474121
37.838229760055	0.148760318756104
38.6630645436978	0.149825572967529
38.94097134216	0.149765491485596
39.7499309766126	0.150805950164795
40.5043481737101	0.15174674987793
41.2146551135058	0.152600765228271
41.8122031897004	0.153214931488037
42.7477864587875	0.15459156036377
43.5456488717973	0.15551233291626
44.5654108907963	0.156750202178955
45.3579463181082	0.157594680786133
46.0338671807266	0.158233165740967
46.4469090549655	0.158373832702637
47.7572902244408	0.160250663757324
48.505145933108	0.161055088043213
49.1723390839423	0.161704063415527
49.9543809623368	0.162595272064209
49.9817566421013	0.161921501159668
50.3784032899062	0.162011623382568
51.0501814249374	0.162662744522095
51.8403674635294	0.163570404052734
52.788237955443	0.164823770523071
53.6385998998422	0.165787220001221
53.1713746420422	0.164102554321289
54.1428497351705	0.16540002822876
54.7691635371043	0.165846824645996
55.1556469421606	0.165880441665649
55.9788163924708	0.166629314422607

56.6767728348353	0.167176246643066
57.5725602516113	0.168067455291748
58.3345531034212	0.168738603591919
59.3414198795941	0.169857263565063
59.5193486337769	0.169496059417725
60.3590631447097	0.1703200340271
61.4437808568462	0.171615123748779
61.6144444959818	0.171213388442993
62.6298385475495	0.172378540039062
63.7126349359526	0.173737525939941
64.5062983687353	0.174536228179932
65.3765925252094	0.175503969192505
65.8441576393859	0.175646781921387
66.5730700635252	0.176327705383301
67.0772617669549	0.17654275894165
67.7198470424805	0.176971435546875
68.8375584301847	0.178085327148438
69.4019170889499	0.178322792053223
69.9305915443399	0.17851734161377
71.0538046158352	0.179696321487427
0	0.75
68.7738916874684	0.17437481880188
70.0525184463837	0.176072597503662
70.8775866896062	0.176845550537109
71.1554539703295	0.176662445068359
71.9646255586904	0.17725682258606
72.7192182546412	0.177772521972656
73.4296809340978	0.178240776062012
74.0273446796613	0.178529500961304
74.9631981166391	0.179345369338989
75.7612543151339	0.179957389831543
76.7813159735817	0.180923461914062
77.5740468533512	0.181561231613159
78.2501038592641	0.181993007659912

6 Months

Option Price	Implied Volatility
39.7745442303503	0.153676748275757
40.2417013782174	0.15440845489502
40.1482326211424	0.153807640075684

40.4860977651485	0.154224395751953
40.6874916257496	0.154326438903809
41.1651861563163	0.155060291290283
41.6339230055312	0.155632019042969
41.4956430715265	0.155074596405029
41.5527029940939	0.154852867126465
41.8779830839429	0.155181407928467
42.0054982969109	0.1551194190979
42.3122894147101	0.155387878417969
42.755639476573	0.155889987945557
43.3505975507406	0.156688213348389
44.2538416335556	0.158097267150879
44.5071965777931	0.158265829086304
44.9690366698257	0.1588454246521
45.5736406025845	0.159725189208984
45.9000501363021	0.160038948059082
46.2258600605157	0.16035795211792
46.0637715262076	0.159676551818848
46.6039491353158	0.160428524017334
46.9374773947644	0.16076135635376
46.88857714499	0.160307645797729
47.3737492712995	0.160919189453125
47.656047069316	0.161176204681396
47.7123761105056	0.160942077636719
48.1659588696408	0.161521911621094
48.9221557948837	0.162744522094727
49.308729503316	0.163201332092285
50.3489629792158	0.165133476257324
51.2368276917631	0.166460037231445
51.4248872031987	0.166472196578979
51.3994837476182	0.166104078292847
51.3856327694319	0.165763378143311
51.8501843468134	0.166257381439209
51.5993969353885	0.165509462356567
51.7927400582371	0.165531873703003
51.9457975597075	0.165483474731445
52.5336730266891	0.16618013381958
52.9402608821029	0.166569709777832
53.1723587554392	0.166656732559204
53.7187091598829	0.167266845703125

54.401137214151	0.168196678161621
54.9325395432338	0.168832778930664
55.4248293644186	0.169407844543457
56.3468177107097	0.17083215713501
56.1491649568899	0.170073747634888
56.1628206671635	0.169708251953125
0	0.75
55.7525620634895	0.168288946151733
56.2197861726215	0.168798446655273
56.1262587533167	0.16829252243042
56.4641597830321	0.168571472167969
56.6655596149801	0.168590784072876
57.1433206817112	0.169115543365479
57.6121302828543	0.169630289077759
57.4737736847784	0.169036388397217
57.5308079408447	0.168803691864014
57.8561211577723	0.169049263000488
57.9836261479652	0.168943643569946
58.2904463400797	0.169156074523926
58.7338632798458	0.169615983963013
59.3289377539508	0.170358896255493
60.2323523853851	0.171704292297363
60.4857240028273	0.17182445526123
60.9476280010851	0.172340393066406
61.5523504084749	0.173161745071411
61.8787987384351	0.173426866531372
62.2046413420147	0.173691272735596
62.0424649434966	0.172978401184082
62.5827322409927	0.173664093017578
62.9162949725892	0.173946619033813
62.8673403661098	0.173456192016602
63.3525821877537	0.174038171768188
63.6349025583263	0.174211502075195
63.6912016850191	0.173933267593384
64.144846802046	0.174452781677246
64.9011775712944	0.175602912902832
65.2878092106579	0.176000595092773
66.3282433533727	0.177513122558594
67.2162860495653	0.178647041320801
67.4043441397029	0.178622722625732

67.3788771409684	0.178238868713379
67.3649780721592	0.177883863449097
67.8295962033576	0.178315162658691
67.5787029459563	0.177577495574951
67.7720467928272	0.177565574645996
67.9250955270829	0.177487373352051
68.5130646693643	0.178109407424927
68.9197067521691	0.178443431854248
69.1518133606396	0.178491353988647
69.6982470089083	0.17905855178833
70.3807905707591	0.17986798286438
70.9122725995061	0.180431842803955
71.4046360069924	0.180936813354492
72.3267967643812	0.182228565216064
72.1290490336779	0.181487798690796
72.1426602308671	0.181141376495361
0	0.75
71.7322117765369	0.179729461669922
72.1995003687965	0.180166959762573
72.1059044794745	0.179665088653564
72.4438393300558	0.179886341094971
72.6452406186895	0.17988109588623
73.1230686668412	0.180339813232422
73.5919349458114	0.180781602859497
73.4535074300211	0.180200099945068
73.5105089139635	0.179953098297119
73.8358529672101	0.180151700973511
73.9633418937591	0.180022954940796
74.2701885375306	0.180190086364746
74.7136564439123	0.180583477020264
75.3088168221735	0.181237936019897
76.212399429009	0.18244743347168
76.4657837813122	0.182524681091309
76.9277503152576	0.182974815368652
77.5325520670569	0.183690547943115
77.8590299988534	0.183902263641357
78.1849020570751	0.184112787246704
78.0226406498355	0.183427810668945
78.5629853806867	0.184032440185547
78.8965794429512	0.184253215789795

78.8475632137016	0.183776378631592
79.3328732247164	0.184275388717651
79.6152124738322	0.184403419494629

One Year

Option Price	Implied Volatility
39.4235565325853	0.152856349945068
39.8357873363572	0.153633594512939
39.5570009334124	0.152816772460938
40.1606260889611	0.154027462005615
40.2167579519335	0.153972148895264
40.7770830302166	0.155091285705566
40.8766497195349	0.155127048492432
40.4338283513495	0.153922080993652
40.5331029613919	0.153968334197998
41.063791673604	0.155028104782104
40.5182361994944	0.153563737869263
41.0582384480557	0.154642581939697
40.9770832784384	0.154264450073242
41.3562172107	0.154968738555908
42.2396154252162	0.156478881835938
42.7669453607337	0.157320022583008
42.358661894635	0.156378984451294
42.4298828222129	0.156359434127808
42.5562585555524	0.156443119049072
42.4840358898559	0.156156539916992
42.5143517986366	0.156059503555298
43.0423463322926	0.156897783279419
42.7781256820852	0.156246423721313
43.1109040107781	0.156715869903564
43.0749900398822	0.156494617462158
43.2503754205889	0.156673431396484
43.0922772360794	0.15622353553772
43.2306755634296	0.156329154968262
42.8416651422596	0.155472755432129
42.8334414154253	0.155315160751343
43.1885100730408	0.155804634094238
43.4875555924425	0.156198501586914
43.7354460019012	0.156505107879639
43.8211368924788	0.156511545181274

43.6888244638003	0.156121730804443
44.0383785376669	0.156610012054443
44.4277353371777	0.157177925109863
44.1018757314978	0.156425952911377
44.2319267463757	0.156513690948486
44.2969363400745	0.156483173370361
44.3907707301024	0.15650486946106
44.8589094232881	0.157212734222412
45.4392375042146	0.158146858215332
46.3179637835003	0.159702777862549
46.1526299232392	0.159205913543701
46.4164993276284	0.159560680389404
46.2889053901982	0.159137725830078
46.4174087115951	0.159228324890137
46.6017514987736	0.1593017578125
0	0.75
47.3804535160907	0.160644054412842
47.7927359907155	0.161316633224487
47.5138930263583	0.160565376281738
48.1175949790902	0.161633014678955
48.1737239269849	0.161568641662598
48.7340992294736	0.162572383880615
48.8336675815138	0.162598371505737
48.3907907112336	0.16148042678833
48.4900669532144	0.161510467529297
49.0208027661168	0.162441253662109
48.475175681807	0.161123275756836
49.0152312931328	0.162056922912598
48.9340587391146	0.161708354949951
49.3132237989988	0.16231632232666
50.1967270370235	0.164032936096191
50.7241176347352	0.165019989013672
50.315765217396	0.163905620574951
50.3869838845488	0.163867473602295
50.5133662410159	0.16395092010498
50.4411182110382	0.163599491119385
50.4714252709947	0.163474321365356
50.9994911148289	0.164433479309082
50.7352141760327	0.163662672042847
51.0680323722508	0.164193391799927

51.0320988851625	0.163922071456909
51.2074987827744	0.164111137390137
51.0493614023753	0.163579940795898
51.1877682892816	0.163686752319336
50.7986958227063	0.162680149078369
50.7904623980572	0.162482976913452
51.1455596469433	0.16303825378418
51.4446347661595	0.163480281829834
51.6925513706034	0.1639404296875
51.778242331372	0.163813591003418
51.6458948600071	0.163347005844116
51.9954915033977	0.163898229598999
52.3848902833132	0.164539337158203
52.0589714610941	0.163654327392578
52.1890296910616	0.16374397277832
52.254036023902	0.16375732421875
52.3478717957602	0.16370964050293
52.8160641433289	0.164508819580078
53.3964460775834	0.165553092956543
54.2752727580464	0.166883707046509
54.1098974131517	0.166444540023804
54.3737953164329	0.166739940643311
54.2461688414039	0.166368007659912
54.3746767284744	0.166433334350586
54.5590342710101	0.166590690612793
0	0.75
55.3378187063321	0.167618036270142
55.7501379166567	0.168184757232666
55.4712530432618	0.167531967163086
56.0750135137518	0.168435573577881
56.1311386698255	0.168369770050049
56.6915675598343	0.169219017028809
56.7911370631221	0.169230461120605
56.3481996180124	0.168262004852295
56.4474769780011	0.168243408203125
56.9782630056785	0.169063568115234
56.4325636543584	0.167915344238281
56.9726705409802	0.168714046478271
56.8914785712904	0.16840386390686
57.2706765983684	0.168899536132812

58.1542811716363	0.170369148254395
58.6817228857515	0.171197414398193
58.2733116772897	0.17024040222168
58.3445278812546	0.170196056365967
58.4709142537749	0.170254707336426
58.3986469097545	0.16994571685791
58.4289467012576	0.169829845428467
58.9570637408062	0.170635938644409
58.6927449332466	0.169968128204346
59.0255913924021	0.170166015625
58.9896428565891	0.170168399810791
59.1650525276007	0.170303344726562
59.0068857450407	0.169856071472168
59.1452980603393	0.169936656951904
58.7561649743928	0.169070482254028
58.7479204339819	0.168892621994019
59.1030479113307	0.16935396194458
59.4021513947685	0.169721364974976
59.6500862900487	0.169998645782471
59.7357764874701	0.169983386993408
59.6034026430934	0.169577836990356
59.9530295201308	0.170032501220703
60.3424632086459	0.170572757720947
60.0164952775271	0.169812440872192
60.146557955297	0.169877529144287
60.2115610952809	0.169825077056885
60.3053970604391	0.169825077056885
60.7736335712915	0.170497417449951
61.3540728460373	0.17139196395874
62.2329922926536	0.172891139984131
62.0675863235758	0.172384262084961
62.3315040560815	0.172710418701172
62.2038521012369	0.172273635864258
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