Outline

A Neural Network Based Framework for Financial Models Summary

4.3 Backward Pass (Heston Model)

σ_{imp} | K, τ, S_0, r -> Heston-CaNN -> ρ, k, v_0, $\overline{v}$, γ

How does this relate to our research?

- We hope to be able to use historical asset prices and option prices to be able to calibrate our parameters. This mean, our input depends on the different parameters that make these prices/returns, which would be strike prices, initial price, rate, time to maturity and the implied volatility.
- We then send these input to the CaNN where it would train the model
- After we train the model, we can use our Joint objective function to return the calibrated parameters, where in our case, since we are just caring about the GARCH(1, 1) model would be ω, α, β.

Sampling Training Data

Found in Table 5 of the paper:

	Parameters	**Range**	**Samples**
Market data	Moneyness, $m = \frac{S_0}{K}$ Time to maturity, τ Risk free rate, r European call/put price, $\frac{V}{K}$	$[0.85, 1.15]$ $[0.5, 2.0]$(year) 0.03 $(0.0, 0.6)$	5 7 Fixed, •
Black-Scholes	Implied Volatility	$(0.2, 0.5)$	35

We can use this as a way to sample all our different input parameters, and be able to measure how more accurate the model becomes and its calibration.

During the calibration, they use the total squared error measure $J(\Theta)$:

$$J(\Theta) = \sum \omega\left(\sigma_{\text{imp}}^{\text{ANN}} - \sigma_{\text{imp}}^{*}\right)^2 + \overline{\lambda}|\Theta|$$

Averaged performance of the Backward pass of the CaNN:

- Need to list CPU and GPU spec
  - OS: Linux fedora 6.10.7-200.fc40.x86_64
  - CPU: AMD Ryzen 5 5600G
  - GPU: Radeon 6600

Abosulute deviation from θ^*, Error measure and computational cost.
Error Measure: $J(\Theta)$, MJ and Data Points
Computational Cost: CPU, GPU time and Function evaluations

Summary of Joint Calibrarion Artificial Neural Networks

Goals & Designs

Our goal is to be able to use two forms of input data to be able to calibrate an optimum parameters of the GARCH(1, 1) model. This will be calibrated using joint calibration which will take into account the log likelihood of returns and option prices to be able to consider both physical and risk neutral measures.

The design will make use of a backward pass artificial neural network, where we will do the following:

$$\text{Market Data } (S, K, S_0, r, \tau, \sigma |\ R_t)$$
$$\downarrow$$
$$\text{Joint Calibration Neural Network}$$
$$\downarrow$$
$$\text{GARCH Params } (\omega, \alpha, \beta, \gamma, \lambda)$$

Where we consider the parameters under **P** measure:

- S: Price (a time series of asset prices eg. (10-year daily prices))
- K: Strike Price
- S_0: Initial Price
- r: Risk-free rate (fixed)
- σ: Implied Volatility
- τ: Time to maturity

And we consider the following physical measure:

- R_t: Log return at time t (several GARCH models)

Our goal is to be able to use the Joint Calibration to have the most optimum calibrated GARCH parameters that takes into account both measures. This will come into the calibration phase, where the idea is to utilize the Joint Calibration formula as the objective function for minimization.

Risk Neutralization for One-Component Gaussian Models

- Q: Risk-Neutral Measure
- P: Physical Measure

Using the Radon-Nikodym derivative, we can convert the physical measure to the risk-neutral measure. Let z_t i.i.d $N(0, 1)$, then $\gamma_t = \frac{1}{2}h_t$ since $\exp(\gamma_t) = E_{\text{t-1}}[\exp(\varepsilon_t)]$
The Radon-Nikodym derivative is defined as:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i=1}}^{t}\left(\frac{\mu_i - r_i}{h_i}\varepsilon_i + \frac{1}{2}\left(\frac{\mu_i - r_i}{h_i}\right)^2 h_i\right)\right)$$

NGARCH(1, 1)

For NGARCH(1, 1) using $\varepsilon_t^* = \varepsilon_t + \mu_t - r_t$, the volatility process under Q becomes:

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha(\varepsilon^*_{\text{t-1}} - \mu_{\text{t-1}} + r_{\text{t-1}})^2 \Rightarrow \varepsilon^*_t \mid F_t \sim N(0, h_t)$$
$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t - \frac{1}{2}h_t + \varepsilon^*_t \Rightarrow \varepsilon^*_t \mid F_{\text{t-1}} \sim N(0, h_t)$$
$$E^Q\left[\frac{S_t}{S_{\text{t-1}}} \mid F_{\text{t-1}}\right] = \exp(r_t)$$

Duan

The Physical GARCH Model Duan (1995) comes in the following form:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\varepsilon^2_{\text{t-1}}$$

Assume:
- λ: price of risk (const.)
- $\mu_t = r_t + \lambda\sqrt{h_t}$ or $\lambda = \frac{\mu_t - r_t}{\sqrt{h_t}}$

This corresponds to the following RN-Derivative:

$$\frac{\text{dQ}}{\text{dP}} \mid F_t = \exp\left(-\sum_{\text{i=1}}^{t}\left(\frac{\varepsilon_i}{\sqrt{h_i}}\lambda + \frac{1}{2}\lambda^2\right)\right)$$

With risk-neutral innovations:

$$\begin{aligned} \varepsilon^*_t &= \varepsilon_t + \mu_t - r_t \\ &= \varepsilon_t + \lambda\sqrt{h_t} \end{aligned}$$

The Risk-Neutral GARCH becomes:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r - \frac{1}{2}h_t + \varepsilon^*_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon^*_{\text{t-1}} - \lambda\sqrt{h_{\text{t-1}}}\right)^2$$

HN-GARCH(1, 1)

Starting with the following model of Heston and Nandi (2000):

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r + \lambda h_t + \varepsilon_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon_{\text{t-1}} - c\sqrt{h_{\text{t-1}}}\right)^2$$

Assume $r_t = r, \mu_t = r + \lambda h_t + 0.5h_t$
RN-Derivative:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i}=1}^{t}\left(\left(\lambda + \frac{1}{2}\right)\varepsilon_i + \frac{1}{2}\left(\lambda + \frac{1}{2}\right)^2 h_i\right)\right)$$

$$\varepsilon_t^* = \varepsilon_t + \lambda h_t + 0.5 h_t$$

$$R_t \equiv \ln\left(\frac{S_t}{S_{\mathrm{t-1}}}\right) = r - \frac{1}{2}h_t + \varepsilon_t^*$$

$$h_t = \omega + \beta h_{\mathrm{t-1}} + \alpha\left(\varepsilon_{\mathrm{t-1}}^* - \left(c + \lambda + \frac{1}{2}\right)\sqrt{h_{\mathrm{t-1}}}\right)^2$$

Input Data

So firstly we need to discuss how we get our input parameters?
Risk-Neutral Measure:

- American Option Prices

Physical Measure:

- Historical Asset Prices (change in difference to get log return)
  - Where: $R_t \equiv \ln\left(\frac{S_t}{S_{\mathrm{t-1}}}\right) = \mu_t - \gamma_t + \varepsilon_t$
  - μ_t: Conditional mean of the returns at time t
  - Assume γ_t is defined from $\exp(\gamma_t) = E_{\mathrm{t-1}}[\exp(\varepsilon_t)]$
  - ε_t: the normal innovation at time t where $\varepsilon_t | F_{\mathrm{t-1}} \sim N(0, h_t)$
  - h_t: Conditional variance at time t
    - $h_t = \omega + \alpha\varepsilon_{\mathrm{t-1}}^2 + \beta h_{\mathrm{t-1}}$

Joint Calibration Neural Network

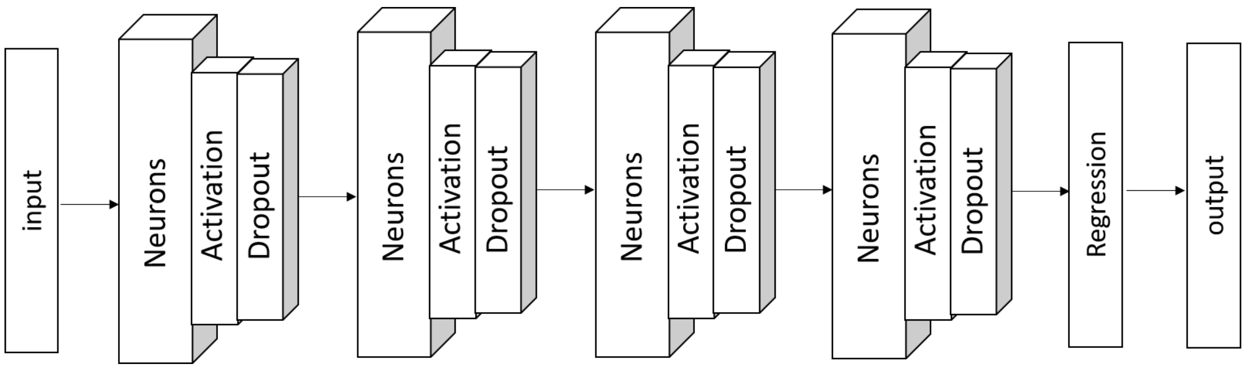
This neural network will be written in Python with a minimum required version of 3.10, and will require the following dependencies:

```
[tool.poetry.dependencies]
python = "^3.10"
numpy = "^1.26.4"
pandas = "^2.2.2"
arch = "^7.0.0"
scikit-learn = "^1.5.0"
matplotlib = "^3.9.0"
scipy = "^1.13.1"
```

The architecture currently will follow simarly to the paper, as the following:

Parameters	Options
Hidden Layers	4
Neurons(each layer except Output) Activation Dropout rate Batch-normalization Initialization Optimizer	200 ReLu 0.0 No Glorot_uniform Adam

Parameters	**Options**
Batch size	1024
Output Layer Activation	SoftPlus (to ensure all GARCH params are positive)
Output Layer Neurons	5 (for each GARCH param)



Where we then hope in the regression phase, to be able to use the Joint Calibration formula in a batching method to be able to calibrate our parameters for the GARCH model.

Essential Equations

These are from GARCH Option Valuation Theory and Evidence:

Return Log Likelihood:

$$\ln L^R \propto -\frac{1}{2} \sum_{\mathrm{t=1}}^{T} \left\{ \ln(h(t)) + \frac{(R(t) - \mu_t - \gamma_t)^2}{h(t)} \right\}$$

where:

- $h(t)$ is the conditional variance
- $R(t)$ represents the return at time t
- μ_t is the conditional mean of the returns at time t
- γ_t is an adjustment term

Options Log Likelihood:

$$\ln L^O \propto -\frac{1}{2} \sum_{\mathrm{i=1}}^{N} \left\{ \ln(\mathrm{IVRMSE}^2) + \left(\frac{e_{\mathrm{i,t}}}{\mathrm{IVRMSE}} \right)^2 \right\}$$

Implied Volatility root mean squared error (IVRMSE) loss function follows:

$$\mathrm{IVRMSE} \approx \sqrt{\frac{1}{N_T} \sum_{\mathrm{i,\ t}}^{N_T} e_{\mathrm{i,t}}^2}$$

where:

- N_T: total number of option prices in the sample

The verga weighted option error follows as:

$$\underbrace{e_{\mathrm{i,t}} = \frac{C_{\mathrm{i,t}} - C_{\mathrm{i,t}}(h_t(\xi^*))}{\mathrm{Vega}_{\ \mathrm{i,t}}}}_{\text{We may also choose to use relative error if Vega is hard or expensive to calculate}}$$

where:

- $\mathrm{Vega}_{\mathrm{i,t}}$ is the Black-Scholes sensitivity of the option prices with respect to volatility
- ξ^* is the vector of risk-neutral parameters to be estimated
- $C_{\mathrm{i,t}} - C_{\mathrm{i,t}}(h_t(\xi^*))$: The corresponding implied volatility from the option price.

Joint Log Likelihood

$$L_{\mathrm{joint}} = \frac{T + N_T}{2} \frac{L^R}{T} + \frac{T + N_T}{2} \frac{L^O}{N}$$

where:

- T is the number of days in the return sample
- N_T is the total number of option contracts

Training Data Generation

Current Idea

1. Given a set of parameters for GARCH (in Physical measure)
2. Given the initial asset price S_0, use Monte Carlo method to simulate a path of asset prices, $S_1, S_2, ...S_N$, with say $N = 500$ (Under **P** measure)
3. Select last 30-50 days on the path, for each day, use the selected asset price (under **Q**) as the initial price to generate American option prices with various strike prices (11-17) and maturities (7 days to 1 year). **Pay attention to the transformation from the physical measure to the risk-neutral measure**.

Pseudo Code

The Steps that we are following:

1. Initialize Option and Monte Carlo Parameters
   - `r`: The risk-free rate
   - `S0`: The initial asset price
   - `h0`: Initial volatility
   - `N`: Number of time steps for simulation
   - `M`: Number of Monte Carlo paths
2. Initialize HN-GARCH parameters under P measure
   - $\theta = (\alpha, \beta, \omega, \gamma, \lambda)$
3. Simulate paths using Monte Carlo Simulation
4. Risk Neutralize HN-GARCH parameters
   - $\theta^* = \left(\alpha_Q, \rho, \omega_Q, \gamma_Q, \lambda_Q\right)$
5. Initialize Willow Tree parameters
6. Generate data for the days up to the maturity

Project Structure:

MATLAB Files:

`American.m`	`gen_PoWiner.m`	`nodes_Winer.m`
`Prob_Xt.m`	`zq.m`	`datagen.m`
`impVol_HN.m`	`impvol.m`	`probcali.m`
`main.m`	`Prob_ht.m`	`sign.m`
`genhDelta.m`	`Prob.m`	`TreeNodes_ht_HN.m`
`Treenodes_JC_h.m`	`Treenodes_JC_X.m`	`TreeNodes_logSt_HN.m`

Dependencies: `f_hhh.mexa64`
Output Files: `annual.csv`, `half.csv`, `quarter.csv`, `week.csv`

The `datagen()` Function

```
function [A_sig, A_prices, S0] = datagen(maturity, r, S0, N, M, h0, alpha, beta, omega,
gamma, lambda, path_days)
    % Parameters:
    %   maturity - Time to maturity of the option.
    %   r - Risk-free rate.
    %   S0 - Initial asset price.
    %   N - Number of time steps for simulation.
    %   M - Number of Monte Carlo paths.
    %   h0 - Initial volatility.
    %   alpha - Alpha parameter for HN-GARCH model.
    %   beta - Beta parameter for HN-GARCH model.
    %   omega - Omega parameter for HN-GARCH model.
    %   gamma - Gamma parameter for HN-GARCH model.
    %   lambda - Lambda parameter for HN-GARCH model.
    %   path_days - Number of days in the path to consider.
    % Output:
    %   A_sig - Implied volatilities for the American options.
    %   A_prices - Prices of the American options.
    %   S0 - Initial prices used for each maturity.

    % 3. Simulate paths using Monte Carlo simulation under P measure
    numPoint = N + 1;
    Z = randn(numPoint, M);
    ht = nan(numPoint, M);
    Xt = nan(numPoint, M);
    ht(1,:) = h0 * ones(1, M);
    Xt(1,:) = log(S0) * ones(1, M);
    for i = 2:numPoint
          ht(i,:) = omega + alpha * (Z(i-1,:) - gamma * sqrt(ht(i-1,:))).^2 + beta *
ht(i-1,:);
        Xt(i,:) = Xt(i-1,:) + (r - 0.5 * ht(i,:)) + sqrt(ht(i,:)) .* Z(i,:);
    end
    S = exp(Xt);

    % Get the last 'path_days' days on the path
    S = S(end - path_days + 1:end, :);

    % 4. Risk-neutralize GARCH parameters (Q measure)
    eta = 0;
    omega_Q = omega / (1 - 2 * alpha * eta);
    gamma_Q = gamma * (1 - 2 * alpha * eta);
    alpha_Q = alpha / (1 - 2 * alpha * eta)^2;
    lambda_Q = lambda * (1 - 2 * alpha * eta);
    rho = lambda_Q + gamma_Q + 1/2;

    % 5. Initialize Willow Tree parameters
    m_h = 6;
    m_x = 30;

    % 6. Generate Data for the days up to the maturity
    % Generate strike prices based on moneyness through the maturity
    strike_prices = linspace(0.8 * S0, 1.2 * S0, maturity);
    A_sig = zeros(maturity, 1);
    A_prices = zeros(maturity, 1);

    % Prepare initial prices for each maturity
    S0 = S(mod(0:(maturity - 1), path_days) + 1);

    parfor j = 1:maturity % iterate through the maturities in parallel
          [A_sig(j), A_prices(j), ~] = impVol_HN(r, lambda_Q, omega_Q, rho, alpha_Q,
gamma_Q, h0, S0(j), strike_prices(j), j, N, m_h, m_x, -1);
    end
```

The `main` Code

This will contain generating data for different maturities and configuring the parameters for both the Option and HN-GARCH.

```
maturities = [5, 63, 126, 252]; % week, 3 months, 6 months and one year of trading days
filenames = {'week.csv', 'quarter.csv', 'half.csv', 'annual.csv'};
% 1. Initialize Option Parameters
r = 0.05/252; % Risk-free rate
S0 = 100; % Initial asset price
N = 100; % Number of time steps for simulation (ex. 500)
M = 10000; % Number of Monte Carlo paths
h0 = (0.2^2)/252; % Initial volatility

% 2. Initialize HN-GARCH parameters under P Measure
alpha = 1.33e-6;
beta = 0.586;
omega = 4.96e-6;
gamma = 484.69;
lambda = 0.5;

path_days = 50;

parfor i = 1:length(maturities)
     [sig, V, S] = datagen(maturities(i), r, S0, N, M, h0, alpha, beta, omega, gamma,
lambda, path_days);
    data = table(V, sig, S', 'VariableNames', {'V', 'Sigma', 'S'});
    writetable(data, filenames{i});
end
```

Downloading the Code: Available under **Github**

Generated Data

Week

V	Sigma	S
37.5787797250986	1	104.668228980692
47.4151914954578	1	105.041996110373
57.0334461885849	0.93803596496582	105.956732031607
66.9976083581371	0.845191955566406	106.017525133789
76.7922428142381	0.782032012939453	106.49857049938

3 Months

V	Sigma	S
38.3623197306296	1	102.730300344903
39.250003248286	1	102.128307895461
40.4348673655296	0.866586685180664	100.791319950664
41.9810713149062	0.763522148132324	98.5608029880303
43.0293581250349	0.688857078552246	97.561712468532
43.7317063137896	0.631765365600586	97.4181493120252
44.5435395129996	0.588425636291504	97.0038268303109
44.7129016709972	0.549392700195312	98.1783069553716
45.1313343320254	0.51860237121582	98.7368407159232
45.7123486344422	0.49354362487793	98.8933095149815
46.3192597995617	0.472209930419922	98.9857412909193
47.1669944855991	0.45503044128418	98.4826686628901
47.4594943637135	0.436985969543457	99.3526061211913
48.2395436128517	0.423463821411133	99.0168965764348
48.742874175257	0.409994125366211	99.3654688921808
49.9647609557609	0.401521682739258	97.9371940855519
50.9880463670644	0.39309024810791	97.0000586182414
50.9826281485278	0.380248069763184	98.6066376564767
52.2177890914371	0.374224662780762	97.1455718020798
52.8164016264526	0.365696907043457	97.2585564679354
52.5946764487619	0.354772567749023	99.39996871208
52.67478208421	0.345417976379395	100.795059379417
53.2589998265968	0.338886260986328	100.943594834625
53.2696836015643	0.330329895019531	102.510346245811
53.6987529581726	0.32402515411377	103.042521192881

54.3019638405791	0.318791389465332	103.144077034553
54.9178838762248	0.313860893249512	103.214199865825
55.1853859176626	0.307965278625488	104.14588354514
55.6103063260215	0.302887439727783	104.688300196615
56.4772679450567	0.299284934997559	104.137661713078
57.3329170108701	0.29587459564209	103.615027224817
57.3250554658182	0.290232181549072	105.227563435954
58.1175485594782	0.287031173706055	104.861080748191
58.3346254441609	0.282428741455078	105.917411869
59.5611470823184	0.280749320983887	104.477699538756
59.9417326710549	0.276906967163086	105.129714731833
60.9420473830306	0.274658203125	104.249374466263
60.9458302381919	0.270322322845459	105.833049315234
61.8241391036273	0.268222808837891	105.254397339924
62.1518525708728	0.26478385925293	106.03712402
62.6315809757038	0.261858940124512	106.443979888075
63.3030558707737	0.259538173675537	106.376723461078
64.0387103524465	0.257478713989258	106.150771285746
64.4151274722036	0.254582405090332	106.813056754132
65.0460026092302	0.25242805480957	106.846168239013
65.9934818568403	0.251173496246338	106.09647369712
66.9191795338371	0.249950885772705	105.400671635043
67.9743978443574	0.24916410446167	104.384626129004
68.2625181395983	0.246386051177979	105.265191861077
69.5546809917664	0.24638557434082	103.663372454842
70.5763656684374	0.245467185974121	102.730300344903
71.4641355513093	0.244152545928955	102.128307895461
72.6492328233409	0.243541240692139	100.791319950664
74.1957814020219	0.243837833404541	98.5608029880303
75.2442282353149	0.243122577667236	97.561712468532
75.9465912582397	0.241663932800293	97.4181493120252
76.7584896350636	0.240513324737549	97.0038268303109
76.9276612105059	0.23790168762207	98.1783069553716
77.3460047452393	0.235932350158691	98.7368407159232
77.9269928912391	0.234380722045898	98.8933095149815
78.5338909185604	0.232925891876221	98.9857412909193

79.3816935635992	0.232062816619873	98.4826686628901
79.6740525843912	0.229976654052734	99.3526061211913

6 Months

V	Sigma	S
39.3490194504374	1	100.290005197638
39.3860448284208	1	100.988671467819
39.7275804646038	0.861743927001953	100.934222886362
40.2924860662102	0.75092887878418	100.327331465366
40.3989567905426	0.671308517456055	100.854239711924
40.6529653815528	0.613828659057617	101.016256875327
41.2067866321221	0.571760177612305	100.436816607069
41.9413701890653	0.539224624633789	99.4103282006533
41.8479943806366	0.506922721862793	100.431460401524
42.2315634012528	0.482342720031738	100.273058051303
42.5386711452556	0.460762977600098	100.303749209413
43.2239684404695	0.443915367126465	99.399139350677
43.3623692746639	0.426472663879395	99.8470518498227
43.1355838762604	0.409220695495605	101.19811275876
43.5356300124335	0.396506309509277	100.998944607801
43.7860807180175	0.38438606262207	101.169742401078
44.2717565836049	0.374382972717285	100.75883003314
44.7863996787771	0.365413665771484	100.276280315249
45.1651003096965	0.356642723083496	100.129914300273
45.8931284191789	0.35004711151123	99.1196481887807
46.0412541424224	0.341604709625244	99.5434921911984
46.5259116171794	0.335119724273682	99.1350988937563
47.062551589052	0.329325675964355	98.5981745815367
47.8715725124413	0.325091361999512	97.3876701172062
47.9508693874595	0.318210601806641	97.9817066910342
47.4500230309003	0.309480667114258	100.010392961257
47.4907097274316	0.303286552429199	100.699911074413
47.7075377437783	0.298060417175293	100.953842854625
47.7429120605967	0.292466163635254	101.656496496797
47.7334100219518	0.287002086639404	102.470128103867
47.6943785312767	0.281692504882812	103.356810798578

47.8672874153291	0.277348518371582	103.719347070604
48.121131681861	0.273451805114746	103.881721074723
48.5266729675602	0.270212650299072	103.668944128902
48.5834628176367	0.266035079956055	104.318635026674
49.0787269264214	0.263382911682129	103.883985538872
49.091815330453	0.259379386901855	104.641743645791
48.984866423902	0.255176544189453	105.696368929512
49.4564912372094	0.25283145904541	105.320155129369
49.0172614167669	0.247969150543213	107.196549054498
49.1836516392938	0.244979381561279	107.575192694269
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One Year

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