

## Training Data Generation

### Current Idea

1. Given a set of parameters for GARCH (in Physical measure)
2. Given the initial asset price  $S_0$ , use Monte Carlo method to simulate a path of asset prices,  $S_1, S_2, \dots, S_N$ , with say  $N = 500$  (Under **P** measure)
3. Select last 30-50 days on the path, for each day, use the selected asset price (under **Q**) as the initial price to generate American option prices with various strike prices (11-17) and maturities (7 days to 1 year). **Pay attention to the transformation from the physical measure to the risk-neutral measure.**

### Pseudo Code

#### The Steps that we are following:

1. Initialize Option and Monte Carlo Parameters
  - $r$ : The risk-free rate
  - $S_0$ : The initial asset price
  - $h_0$ : Initial volatility
  - $N$ : Number of time steps for simulation
  - $M$ : Number of Monte Carlo paths
2. Initialize HN-GARCH parameters under P measure
  - $\theta = (\alpha, \beta, \omega, \gamma, \lambda)$
3. Simulate paths using Monte Carlo Simulation
4. Risk Neutralize HN-GARCH parameters
  - $\theta^* = (\alpha_Q, \rho, \omega_Q, \gamma_Q, \lambda_Q)$
5. Initialize Willow Tree parameters
6. Generate data for the days up to the maturity

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#### Project Structure:

#### MATLAB Files:

American.m	gen_PoWiner.m	nodes_Winer.m
Prob_Xt.m	zq.m	datagen.m
impVol_HN.m	impvol.m	probcali.m
main.m	Prob_ht.m	sign.m
genhDelta.m	Prob.m	TreeNodes_ht_HN.m
Treenodes_JC_h.m	Treenodes_JC_X.m	TreeNodes_logSt_HN.m

**Dependencies:** f\_hhh.mexa64

**Output Files:** annual.csv, half.csv, quarter.csv, week.csv

## The datagen() Function

```
function [A_sig, A_prices, S0] = datagen(maturity, r, S0, N, M, h0, alpha, beta,
omega, gamma, lambda, path_days)
% Parameters:
% maturity - Time to maturity of the option.
% r - Risk-free rate.
% S0 - Initial asset price.
% N - Number of time steps for simulation.
% M - Number of Monte Carlo paths.
% h0 - Initial volatility.
% alpha - Alpha parameter for HN-GARCH model.
% beta - Beta parameter for HN-GARCH model.
% omega - Omega parameter for HN-GARCH model.
% gamma - Gamma parameter for HN-GARCH model.
% lambda - Lambda parameter for HN-GARCH model.
% path_days - Number of days in the path to consider.
% Output:
% A_sig - Implied volatilities for the American options.
% A_prices - Prices of the American options.

% 3. Simulate paths using Monte Carlo simulation under P measure
numPoint = N + 1;
Z = randn(numPoint + 1, M);
ht = nan(numPoint + 1, M); Xt = nan(numPoint + 1, M);
ht(1,:) = h0 * ones(1, M);
Xt(1,:) = log(S0) * ones(1, M);
for i = 2:numPoint
    ht(i,:) = omega+alpha*(Z(i-1,:)-gamma*sqrt(ht(i-1,:))).^2 + beta*ht(i-1,:);
    Xt(i,:) = Xt(i-1,:) + (r - 0.5 * ht(i,:)) + sqrt(ht(i,:)) .* Z(i,:);
end
S = exp(Xt);
% Get the last 'path_days' days on the path
S = S(end - (path_days - 1):end, :);
% 4. Risk-neutralize GARCH parameters (Q measure)
eta = 0;
omega_Q = omega / (1 - 2 * alpha * eta);
gamma_Q = gamma * (1 - 2 * alpha * eta);
alpha_Q = alpha / (1 - 2 * alpha * eta)^2;
lambda_Q = lambda * (1 - 2 * alpha * eta);
rho = lambda_Q + gamma_Q + 1/2;
% 5. Initialize Willow Tree parameters
m_h = 6; m_x = 30;
% 6. Generate Data for the days up to the maturity
% Generate strike prices based on moneyness through the maturity
strike_prices = linspace(0.8 * S0, 1.2 * S0, maturity);
A_sig = zeros(maturity, 1); A_prices = zeros(maturity, 1);
% Setting the initial price to use the days from S
% and wrapping around if necessary.
S0 = S(mod(0:(maturity - 1), path_days) + 1);
parfor j = 1:maturity % iterate through the maturities in parallel
    [A_sig(j), A_prices(j), ~] = impVol_HN(r, lambda_Q, omega_Q, rho, alpha_Q,
gamma_Q, h0, S0(j), strike_prices(j), maturity, N, m_h, m_x, -1);
end
end
```

### The main Code

This will contain generating data for different maturities and configuring the parameters for both the Option and HN-GARCH.

```
maturities = [5, 63, 126, 252]; % week, 3 months, 6 months and one year of trading
days
filenames = {'week.csv', 'quarter.csv', 'half.csv', 'annual.csv'};
% 1. Initialize Option Parameters
r = 0.05/252; % Risk-free rate
S0 = 100; % Initial asset price
N = 100; % Number of time steps for simulation (ex. 500)
M = 10000; % Number of Monte Carlo paths
h0 = (0.2^2)/252; % Initial volatility

% 2. Initialize HN-GARCH parameters under P Measure
alpha = 1.33e-6;
beta = 0.586;
omega = 4.96e-6;
gamma = 484.69;
lambda = 0.5;

path_days = 50;

parfor i = 1:length(maturities)
    [sig, V, S] = datagen(maturities(i), r, S0, N, M, h0, alpha, beta, omega,
gamma, lambda, path_days);
    data = table(V, sig, S', 'VariableNames', {'V', 'Sigma', 'S'});
    writetable(data, filenames{i});
end
```

**Downloading the Code:** Available under [Github](#)

## Generated Data

### Week

V	Sigma	S
38.814441784109	0.662147521972656	101.61135506747
48.3309515483963	0.700183868408203	102.776692148378
58.0781375279794	0.734198570251465	103.373131087257
68.2612206916722	0.76637077331543	102.893271027282
78.6384931858621	0.794336318969727	101.934887847694

### 3 Months

V	Sigma	S
37.9692448096908	0.18695592880249	103.690932422543
38.437539487218	0.18737268447876	104.126197855748
38.7870527530073	0.187482833862305	104.855283365265
39.3006038540439	0.188010692596436	105.178617004234
39.9649541048298	0.188933372497559	105.128960460577
40.022412978275	0.188273906707764	106.580445804649
40.6209415960836	0.189016342163086	106.693598643484
41.5079795980806	0.190505981445312	106.09312539076
42.7124764014314	0.192942142486572	104.707459369881
44.1852424726793	0.195672035217285	102.658337440818
44.8344616259078	0.196388721466064	102.646170847224
45.6869575505859	0.197580814361572	102.131271250952
45.465325063497	0.196302890777588	104.272947148018
46.2233036453953	0.197258472442627	103.991811460948
46.7455288502909	0.197679996490479	104.293750047832
47.1660372736841	0.197866916656494	104.84725847128
47.3233344081321	0.197455883026123	106.051751127266
47.9467126610625	0.198097705841064	106.103514124769
48.5266134301996	0.198644638061523	106.262807298999
48.9912046268538	0.19892406463623	106.707288520639
49.6911332125652	0.199744939804077	106.569725800539
50.2984428218594	0.200354814529419	106.661265035587
51.3294086033997	0.201982975006104	105.705057580781
51.3566258674071	0.201207160949707	107.231226769061
52.2291659679604	0.202455520629883	106.666822842013
52.8365125910846	0.20307731628418	106.758270537919
53.543627829414	0.20394229888916	106.60299140258
54.2955477760111	0.204930782318115	106.336948368317
55.0851722221636	0.206027030944824	105.977663294242

55.7108423075485	0.206705093383789	106.023832190243
56.4933637127819	0.207576513290405	105.68211271263
57.098583439006	0.208047389984131	105.778870296068
57.5877496641619	0.208288192749023	106.162626400449
58.0821289672916	0.208534717559814	106.533491476468
58.0378140371639	0.207725048065186	108.236460947629
58.8819216173523	0.208654403686523	107.742501326228
58.6671846778805	0.20752477645874	109.866909797231
59.6676116691669	0.208740949630737	108.986384430305
60.7259974345308	0.210102558135986	107.962560354411
60.9083543550394	0.209717750549316	109.105002107885
61.72119174529	0.210597038269043	108.68837613693
61.842056719207	0.210086345672607	109.982876699275
62.3815574392225	0.210412502288818	110.242164417829
62.9727528609195	0.210841178894043	110.373620197435
63.7214556253922	0.211586952209473	110.115602419736
64.6971888215003	0.212812900543213	109.296251247348
65.6182847117955	0.213950157165527	108.612000543118
66.0382337753782	0.214021921157837	109.166924609619
66.6196698154324	0.214435577392578	109.32254337002
0	0.75	NaN
70.1861407620832	0.220852375030518	103.690932422543
70.6544909333134	0.220971584320068	104.126197855748
71.0040004077953	0.220871925354004	104.855283365265
71.517623361547	0.221071720123291	105.178617004234
72.1821087537665	0.221544742584229	105.128960460577
72.2394309116172	0.220917224884033	106.580445804649
72.838065186055	0.221264839172363	106.693598643484
73.7253380824768	0.22213888168335	106.09312539076
74.930235620504	0.223626136779785	104.707459369881
76.4035100899881	0.225705146789551	102.658337440818
77.0528605861453	0.226174354553223	102.646170847224
77.9055748231668	0.227073669433594	102.131271250952
77.6836607240223	0.225760698318481	104.272947148018

## 6 Months

V	Sigma	S
42.4198629821335	0.142680644989014	92.6696740085822
42.8197760161736	0.143128395080566	92.4708590163621
43.2685603791703	0.143677711486816	92.1511785388827

43.594944238755	0.143986701965332	92.1342619650625
43.4008122040984	0.143263339996338	93.4048050509039
43.9962588226267	0.144100427627563	92.7223768542464
44.5407677315844	0.144859313964844	92.1659464418753
44.7213857685711	0.144865751266479	92.5095692866383
45.4046179114877	0.145919322967529	91.6100187023159
45.5736490463057	0.145889282226562	91.9823007683958
45.6900182361661	0.145780324935913	92.484837695867
46.0655805215364	0.14619779586792	92.3462824503503
46.2059689582806	0.146120071411133	92.7894098046201
46.7408243609534	0.146879196166992	92.2568916547356
47.4415492969097	0.148006916046143	91.3141462664972
48.0449683345171	0.148760795593262	90.6120653811929
48.5581434876918	0.149327278137207	90.1331904745747
48.3983505238751	0.148743629455566	91.3187369726133
48.5104779904173	0.148628950119019	91.8317565463546
49.0570556839533	0.149245023727417	91.2702609656598
49.6380635884706	0.14993143081665	90.6236663416917
49.9071539268836	0.150080442428589	90.7484856230034
50.3668649450818	0.150565624237061	90.4018744399922
51.0900204629486	0.151536464691162	89.4037297353052
51.6312003278908	0.152194023132324	88.8556601247041
52.0072638979404	0.152547359466553	88.7159479001527
51.4766270960426	0.151219367980957	90.8185469212974
51.5257569792665	0.150975704193115	91.4873578599741
51.9382813301614	0.15138053894043	91.2574456232753
52.0323354520314	0.1512131690979	91.8151529046378
52.2775060392412	0.151315689086914	91.9991289334219
52.5045284592273	0.151388645172119	92.2279878097034
53.0309741781607	0.151997089385986	91.7163426408562
53.3354893089797	0.152209281921387	91.7535687891988
53.5683272326152	0.152288913726807	91.9680517519377
53.9834912277531	0.152702808380127	91.7316446774415
54.4367534306012	0.153191089630127	91.4010212237438
54.8139537699744	0.153537750244141	91.2584977832298
55.6416882958169	0.154764652252197	90.0018185265509
55.732893558783	0.15456485748291	90.56654779256
56.1094734450331	0.154923439025879	90.4255801427464
56.2393833909477	0.154800176620483	90.8945934519102
56.6453980434499	0.155215501785278	90.6808437394549

56.7816683570442	0.155102252960205	91.1341288710506
57.6041115722746	0.156355857849121	89.8906175609244
57.3976896365845	0.155549049377441	91.1913261156786
58.0442129338482	0.156452417373657	90.3828367805177
58.1459802557379	0.156262874603271	90.9214423224784
57.9836424321114	0.15555477142334	92.1131378802047
0	0.75	NaN
58.3977344746885	0.155602931976318	92.6696740085822
58.7977096282453	0.156003475189209	92.4708590163621
59.2465619592242	0.15650463104248	92.1511785388827
59.5729786337043	0.156763792037964	92.1342619650625
59.3787554077876	0.155992984771729	93.4048050509039
59.9743004409117	0.156779527664185	92.7223768542464
60.5188928299151	0.157477855682373	92.1659464418753
60.6995098239376	0.157442569732666	92.5095692866383
61.3828848772623	0.158381700515747	91.6100187023159
61.551910580687	0.158335208892822	91.9823007683958
61.6682589999417	0.158201932907104	92.484837695867
62.0438760116892	0.158478498458862	92.3462824503503
62.1842506740814	0.158389091491699	92.7894098046201
62.7191929935054	0.158902168273926	92.2568916547356
63.4200381942537	0.159687995910645	91.3141462664972
64.0235711031336	0.160336256027222	90.6120653811929
64.5368377169927	0.160844802856445	90.1331904745747
64.3769438363809	0.160250663757324	91.3187369726133
64.4890475133785	0.160103321075439	91.8317565463546
65.035729327053	0.160652160644531	91.2702609656598
65.6168286950122	0.161283016204834	90.6236663416917
65.8859365417923	0.161389827728271	90.7484856230034
66.3457102641663	0.161818027496338	90.4018744399922
67.0690083312213	0.162709951400757	89.4037297353052
67.6102805490942	0.163284301757812	88.8556601247041
67.9863861572456	0.163599491119385	88.7159479001527
67.4555534827899	0.16230297088623	90.8185469212974
67.5046456072385	0.162034749984741	91.4873578599741
67.9172275284603	0.162368774414062	91.2574456232753
68.0112516039137	0.162191867828369	91.8151529046378
68.2564355458058	0.162256717681885	91.9991289334219
68.4834667371686	0.162290811538696	92.2279878097034
69.0100065129176	0.162833690643311	91.7163426408562

69.314546594941	0.162998676300049	91.7535687891988
69.5473923521315	0.163041591644287	91.9680517519377
69.9626077231704	0.163398027420044	91.7316446774415
70.4159304003734	0.163824558258057	91.4010212237438
70.7931730493269	0.164120197296143	91.2584977832298
71.6210720506583	0.165236711502075	90.0018185265509
71.712247555559	0.165015459060669	90.56654779256
72.0888714257456	0.165319442749023	90.4255801427464
72.2187632163481	0.165167331695557	90.8945934519102
72.624826808227	0.165521621704102	90.6808437394549
72.7610805121072	0.165383577346802	91.1341288710506
73.5836738607329	0.166520595550537	89.8906175609244
73.3771520395669	0.165734767913818	91.1913261156786
74.0237827208925	0.166543245315552	90.3828367805177
74.1255250486856	0.16633415222168	90.9214423224784
73.963098052221	0.16563892364502	92.1131378802047
0	0.75	NaN
74.3771912730022	0.165619373321533	92.6696740085822
74.7772138983103	0.165960073471069	92.4708590163621
75.2261255784149	0.166392803192139	92.1511785388827
75.5525718496041	0.16660213470459	92.1342619650625
75.3582517025529	0.165854692459106	93.4048050509039
75.9538917317908	0.166551828384399	92.7223768542464
76.4985667351858	0.167167186737061	92.1659464418753
76.6791778959375	0.16710090637207	92.5095692866383
77.3626622918019	0.167988777160645	91.6100187023159
77.5316779624163	0.167899370193481	91.9823007683958
77.6480031454353	0.167710781097412	92.484837695867
78.0236619059512	0.168014049530029	92.3462824503503
78.1640193542236	0.167870283126831	92.7894098046201
78.6990433568934	0.168477058410645	92.2568916547356
79.4000118240169	0.169328689575195	91.3141462664972
80.0036270724371	0.169889688491821	90.6120653811929
80.5169654372401	0.170324802398682	90.1331904745747
80.3569866434726	0.169766187667847	91.3187369726133
80.4690714032321	0.169611692428589	91.8317565463546
81.0158199890388	0.170086860656738	91.2702609656598
81.597015791	0.1706223487854	90.6236663416917
81.866138248329	0.170695781707764	90.7484856230034
82.3259765856246	0.171055316925049	90.4018744399922



83.0493909925959	0.171827793121338	89.4037297353052
83.5907287750926	0.172331809997559	88.8556601247041
83.9668725917493	0.172574043273926	88.7159479001527

### One Year

V	Sigma	S
40.5607576048387	0.101242065429688	97.2432712016125
40.3308817278241	0.100824356079102	98.2056823589616
40.5218519404505	0.1009681224823	98.1267963513526
40.9807110888467	0.10145902633667	97.3850742233566
41.4120197342115	0.101923942565918	96.7115201534895
41.2645962120908	0.101618528366089	97.4699199026055
40.3199714560763	0.100295543670654	100.200830280153
40.5595019217407	0.100509881973267	100.001791905539
40.1656689451548	0.0998046398162842	101.369917306049
40.3356963305513	0.0999464988708496	101.342838522682
41.0206913090129	0.100787162780762	100.041565158627
41.0054446708335	0.100667715072632	100.472920370783
41.647844439333	0.101376533508301	99.2770646959442
42.064344968271	0.101811408996582	98.6401502671016
42.6208398971414	0.102442979812622	97.6568679877472
42.6285093299023	0.102341651916504	98.031512494082
42.6926273781177	0.102321624755859	98.2664937605601
42.8144227638406	0.102370738983154	98.3587717249854
42.5688137394547	0.101937770843506	99.3600986569465
42.5125486944736	0.101758480072021	99.8929454874426
42.4513235931564	0.101574420928955	100.438064958061
42.358853708918	0.101353645324707	101.060493223209
42.6452661015993	0.101614713668823	100.745455154429
42.6999124755337	0.10158109664917	101.003874471118
42.6733014742827	0.101442337036133	101.463348464312
42.9144834484486	0.101650238037109	101.260223771298
43.5106724107085	0.102309942245483	100.178714184586
43.8563726032296	0.102655410766602	99.7170145096427
44.1349179582915	0.102917671203613	99.4214670440353
44.4687045460295	0.103257656097412	98.98924360388
45.2283737951138	0.104196548461914	97.5033158027007
45.578496140522	0.104578256607056	97.0306979699765
46.0299964217684	0.105113506317139	96.3072797334017
46.1458771948238	0.105156421661377	96.4141954872519

46.0385196907156	0.104873180389404	97.0734102730992
46.1152019034639	0.104859828948975	97.2773045880952
45.7742224543911	0.104248762130737	98.5145153919741
45.9898543017533	0.104433298110962	98.3746361799451
46.1289074614783	0.104509592056274	98.4242226671062
46.3160424920711	0.104654550552368	98.3548529829771
46.6535852276128	0.105016946792603	97.9133695605463
46.5063312544707	0.104681491851807	98.6712895572838
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