Outline

A Neural Network Based Framework for Financial Models Summary

4.3 Backward Pass (Heston Model)

σ_{imp} | K, τ, S_0, r -> Heston-CaNN -> ρ, k, v_0, $\overline{v}$, γ

How does this relate to our research?

- We hope to be able to use historical asset prices and option prices to be able to calibrate our parameters. This mean, our input depends on the different parameters that make these prices/ returns, which would be strike prices, initial price, rate, time to maturity and the implied volatility.
- We then send these input to the CaNN where it would train the model
- After we train the model, we can use our Joint objective function to return the calibrated parameters, where in our case, since we are just caring about the GARCH(1, 1) model would be ω, α, β.

Sampling Training Data

Found in Table 5 of the paper:

	Parameters	**Range**	**Samples**
Market data	Moneyness, $m = \frac{S_0}{K}$ Time to maturity, τ Risk free rate, r European call/put price, $\frac{V}{K}$	$[0.85, 1.15]$ $[0.5, 2.0]$(year) 0.03 $(0.0, 0.6)$	5 7 Fixed, •
Black-Scholes	Implied Volatility	$(0.2, 0.5)$	35

We can use this as a way to sample all our different input parameters, and be able to measure how more accurate the model becomes and its calibration.

During the calibration, they use the total squared error measure $J(\Theta)$:

$$J(\Theta) = \sum \omega\left(\sigma_{\text{imp}}^{\text{ANN}} - \sigma_{\text{imp}}^{*}\right)^2 + \overline{\lambda}|\Theta|$$

Averaged performance of the Backward pass of the CaNN:

- Need to list CPU and GPU spec
  - OS: Linux fedora 6.10.5-200.fc40.x86_64
  - CPU: AMD Ryzen 5 5600G
  - GPU: Radeon 6600

Abosulute deviation from θ^*, Error measure and computational cost.
Error Measure: $J(\Theta)$, MJ and Data Points
Computational Cost: CPU, GPU time and Function evaluations

Summary of Joint Calibrarion Artificial Neural Networks

Goals & Designs

Our goal is to be able to use two forms of input data to be able to calibrate an optimum parameters of the GARCH(1, 1) model. This will be calibrated using joint calibration which will take into account the log likelihood of returns and option prices to be able to consider both physical and risk neutral measures.

The design will make use of a backward pass artificial neural network, where we will do the following:

$$\text{Market Data } (S, K, S_0, r, \tau, \text{ sigma} \mid R_t)$$
$$\downarrow$$
$$\text{Joint Calibration Neural Network}$$
$$\downarrow$$
$$\text{GARCH Params } (\omega, \alpha, \beta, \gamma, \lambda)$$

Where we consider the parameters under **P** measure:
- S: Price (a time series of asset prices eg. (10-year daily prices))
- K: Strike Price
- S_0: Initial Price
- r: Risk-free rate (fixed)
- σ: Implied Volatility
- τ: Time to maturity

And we consider the following physical measure:
- R_t: Log return at time t (several GARCH models)

Our goal is to be able to use the Joint Calibration to have the most optimum calibrated GARCH parameters that takes into account both measures. This will come into the calibration phase, where the idea is to utilize the Joint Calibration formula as the objective function for minimization.

Risk Neutralization for One-Component Gaussian Models

- Q: Risk-Neutral Measure
- P: Physical Measure

Using the Radon-Nikodym derivative, we can convert the physical measure to the risk-neutral measure. Let z_t i.i.d $N(0, 1)$, then $\gamma_t = \frac{1}{2} h_t$ since $\exp(\gamma_t) = E_{\text{t-1}}[\exp(\varepsilon_t)]$
The Radon-Nikodym derivative is defined as:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i=1}}^{t}\left(\frac{\mu_i - r_i}{h_i}\varepsilon_i + \frac{1}{2}\left(\frac{\mu_i - r_i}{h_i}\right)^2 h_i\right)\right)$$

NGARCH(1, 1)

For NGARCH(1, 1) using $\varepsilon_t^* = \varepsilon_t + \mu_t - r_t$, the volatility process under Q becomes:

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha(\varepsilon^*_{\text{t-1}} - \mu_{\text{t-1}} + r_{\text{t-1}})^2 \Rightarrow \varepsilon^*_t \mid F_t \sim N(0, h_t)$$
$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t - \frac{1}{2}h_t + \varepsilon^*_t \Rightarrow \varepsilon^*_t \mid F_{\text{t-1}} \sim N(0, h_t)$$
$$E^Q\left[\frac{S_t}{S_{\text{t-1}}} \mid F_{\text{t-1}}\right] = \exp(r_t)$$

Duan

The Physical GARCH Model Duan (1995) comes in the following form:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r_t + \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\varepsilon^2_{\text{t-1}}$$

Assume:

- λ: price of risk (const.)
- $\mu_t = r_t + \lambda\sqrt{h_t}$ or $\lambda = \frac{\mu_t - r_t}{\sqrt{h_t}}$

This corresponds to the following RN-Derivative:

$$\frac{\text{dQ}}{\text{dP}} \mid F_t = \exp\left(-\sum_{\text{i=1}}^{t}\left(\frac{\varepsilon_i}{\sqrt{h_i}}\lambda + \frac{1}{2}\lambda^2\right)\right)$$

With risk-neutral innovations:

$$\begin{aligned}\varepsilon^*_t &= \varepsilon_t + \mu_t - r_t \\ &= \varepsilon_t + \lambda\sqrt{h_t}\end{aligned}$$

The Risk-Neutral GARCH becomes:

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r - \frac{1}{2}h_t + \varepsilon^*_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon^*_{\text{t-1}} - \lambda\sqrt{h_{\text{t-1}}}\right)^2$$

HN-GARCH(1, 1)

Starting with the following model of Heston and Nandi (2000):

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r + \lambda h_t + \varepsilon_t$$
$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon_{\text{t-1}} - c\sqrt{h_{\text{t-1}}}\right)^2$$

Assume $r_t = r, \mu_t = r + \lambda h_t + 0.5h_t$
RN-Derivative:

$$\frac{\mathrm{dQ}}{\mathrm{dP}} \mid F_t = \exp\left(-\sum_{\mathrm{i=1}}^{t}\left(\left(\lambda + \frac{1}{2}\right)\varepsilon_i + \frac{1}{2}\left(\lambda + \frac{1}{2}\right)^2 h_i\right)\right)$$

$$\varepsilon_t^* = \varepsilon_t + \lambda h_t + 0.5 h_t$$

$$R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = r - \frac{1}{2} h_t + \varepsilon_t^*$$

$$h_t = \omega + \beta h_{\text{t-1}} + \alpha\left(\varepsilon_{\text{t-1}}^* - \left(c + \lambda + \frac{1}{2}\right)\sqrt{h_{\text{t-1}}}\right)^2$$

Input Data

So firstly we need to discuss how we get our input parameters?
Risk-Neutral Measure:

- American Option Prices

Physical Measure:

- Historical Asset Prices (change in difference to get log return)
  - Where: $R_t \equiv \ln\left(\frac{S_t}{S_{\text{t-1}}}\right) = \mu_t - \gamma_t + \varepsilon_t$
  - μ_t: Conditional mean of the returns at time t
  - Assume γ_t is defined from $\exp(\gamma_t) = E_{\text{t-1}}[\exp(\varepsilon_t)]$
  - ε_t: the normal innovation at time t where $\varepsilon_t | F_{\text{t-1}} \sim N(0, h_t)$
  - h_t: Conditional variance at time t
    - $h_t = \omega + \alpha \varepsilon_{\text{t-1}}^2 + \beta h_{\text{t-1}}$

Joint Calibration Neural Network

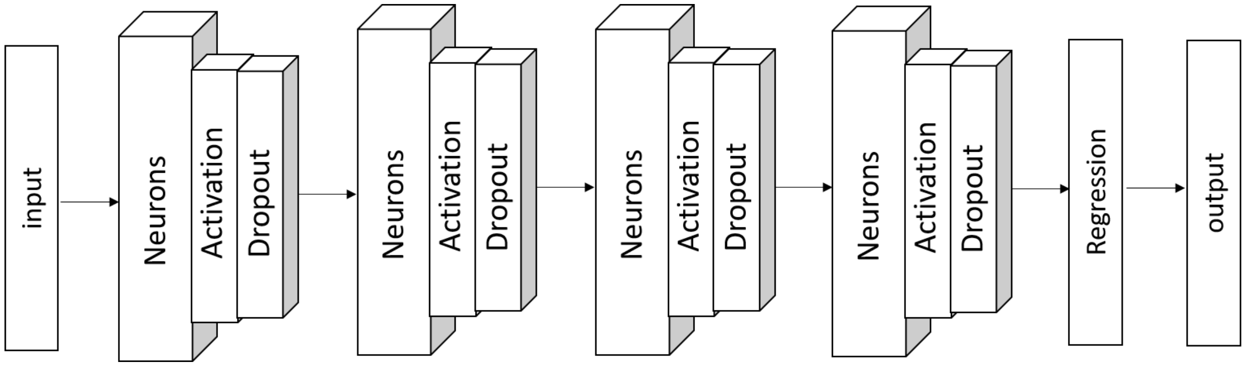
This neural network will be written in Python with a minimum required version of 3.10, and will require the following dependencies:

```toml
[tool.poetry.dependencies]
python = "^3.10"
numpy = "^1.26.4"
pandas = "^2.2.2"
arch = "^7.0.0"
scikit-learn = "^1.5.0"
matplotlib = "^3.9.0"
scipy = "^1.13.1"
```

The architecture currently will follow simarly to the paper, as the following:

Parameters	**Options**
Hidden Layers	4
Neurons(each layer)	200
Activation	ReLu
Dropout rate	0.0
Batch-normalization	No
Initialization	Glorot_uniform

Parameters	Options
Optimizer Batch size	Adam 1024



Where we then hope in the regression phase, to be able to use the Joint Calibration formula in a batching method to be able to calibrate our parameters for the GARCH model.

Essential Equations

These are from GARCH Option Valuation Theory and Evidence:

Return Log Likelihood:

$$\ln L^R \propto -\frac{1}{2}\sum_{\text{t=1}}^{T}\left\{\ln(h(t)) + \frac{(R(t)-\mu_t-\gamma_t)^2}{h(t)}\right\}$$

where:

- $h(t)$ is the conditional variance
- $R(t)$ represents the return at time t
- μ_t is the conditional mean of the returns at time t
- γ_t is an adjustment term

Options Log Likelihood:

$$\ln L^O \propto -\frac{1}{2}\sum_{\text{i=1}}^{N}\left\{\ln(\text{IVRMSE}^2) + \left(\frac{e_{\text{i,t}}}{\text{IVRMSE}}\right)^2\right\}$$

Implied Volatility root mean squared error (IVRMSE) loss function follows:

$$\text{IVRMSE} \approx \sqrt{\frac{1}{N_T}\sum_{\text{i, t}}^{N_T} e_{\text{i,t}}^2}$$

where:

- N_T: total number of option prices in the sample

The verga weighted option error follows as:

$$\underbrace{e_{\text{i,t}} = \frac{C_{\text{i,t}} - C_{\text{i,t}}(h_t(\xi^*))}{\text{Vega}_{\text{ i,t}}}}_{\text{We may also choose to use relative error if Vega is hard or expensive to calculate}}$$

where:

- $\text{Vega}_{\text{i,t}}$ is the Black-Scholes sensitivity of the option prices with respect to volatility
- ξ^* is the vector of risk-neutral parameters to be estimated
- $C_{\text{i,t}} - C_{\text{i,t}}(h_t(\xi^*))$: The corresponding implied volatility from the option price.

Joint Log Likelihood

$$L_{\text{joint}} = \frac{T+N_T}{2}\frac{L^R}{T} + \frac{T+N_T}{2}\frac{L^O}{N}$$

where:

- T is the number of days in the return sample
- N_T is the total number of option contracts

Training Data Generation

Current Idea

1. Given a set of parameters for GARCH (in Physical measure)
2. Given the initial asset price S_0, use Monte Carlo method to simulate a path of asset prices, $S_1, S_2, ...S_N$, with say $N = 500$ (Under **P** measure)
3. Select last 30-50 days on the path, for each day, use the selected asset price (under **Q**) as the initial price to generate American option prices with various strike prices (11-17) and maturities (7 days to 1 year). **Pay attention to the transformation from the physical measure to the risk-neutral measure**.

Pseudo Code

The Steps that we are following:

1. Initialize Option and Monte Carlo Parameters
   - `r`: The risk-free rate
   - `S0`: The initial asset price
   - `h0`: Initial volatility
   - `N`: Number of time steps for simulation
   - `M`: Number of Monte Carlo paths
2. Initialize HN-GARCH parameters under P measure
   - $\theta = (\alpha, \beta, \omega, \gamma, \lambda)$
3. Simulate paths using Monte Carlo Simulation
4. Risk Neutralize HN-GARCH parameters
   - $\theta^* = \left(\alpha_Q, \rho, \omega_Q, \gamma_Q, \lambda_Q\right)$
5. Initialize Willow Tree parameters
6. Generate data for the days up to the maturity

Project Structure:

MATLAB Files:

`American.m`	`gen_PoWiner.m`	`nodes_Winer.m`
`Prob_Xt.m`	`zq.m`	`datagen.m`
`impVol_HN.m`	`impvol.m`	`probcali.m`
`main.m`	`Prob_ht.m`	`sign.m`
`genhDelta.m`	`Prob.m`	`TreeNodes_ht_HN.m`
`Treenodes_JC_h.m`	`Treenodes_JC_X.m`	`TreeNodes_logSt_HN.m`

Dependencies: `f_hhh.mexa64`
Output Files: `annual.csv`, `half.csv`, `quarter.csv`, `week.csv`

The `datagen()` Function

```
function [A_sig, A_prices] = datagen(maturity, r, S0, N, M, h0, alpha, beta, omega,
gamma, lambda, path_days)
% Parameters:
%   maturity - Time to maturity of the option.
%   r - Risk-free rate.
%   S0 - Initial asset price.
%   N - Number of time steps for simulation.
%   M - Number of Monte Carlo paths.
%   h0 - Initial volatility.
%   alpha - Alpha parameter for HN-GARCH model.
%   beta - Beta parameter for HN-GARCH model.
%   omega - Omega parameter for HN-GARCH model.
%   gamma - Gamma parameter for HN-GARCH model.
%   lambda - Lambda parameter for HN-GARCH model.
%   path_days - Number of days in the path to consider.
% Output:
%   A_sig - Implied volatilities for the American options.
%   A_prices - Prices of the American options.

% 3. Simulate paths using Monte Carlo simulation under P measure
    numPoint = N + 1;
    Z = randn(numPoint + 1, M);
    ht = nan(numPoint + 1, M); Xt = nan(numPoint + 1, M);
    ht(1,:) = h0 * ones(1, M);
    Xt(1,:) = log(S0) * ones(1, M);
    for i = 2:numPoint
        ht(i,:) = omega+alpha*(Z(i-1,:)-gamma*sqrt(ht(i-1,:))).^2 + beta*ht(i-1,:);
        Xt(i,:) = Xt(i-1,:) + (r - 0.5 * ht(i,:)) + sqrt(ht(i,:)) .* Z(i,:);
    end
    S = exp(Xt);
    % Get the last 'path_days' days on the path
    S = S(end - (path_days - 1):end, :);
    % 4. Risk-neutralize GARCH parameters (Q measure)
    eta = 0;
    omega_Q = omega / (1 - 2 * alpha * eta);
    gamma_Q = gamma * (1 - 2 * alpha * eta);
    alpha_Q = alpha / (1 - 2 * alpha * eta)^2;
    lambda_Q = lambda * (1 - 2 * alpha * eta);
    rho = lambda_Q + gamma_Q + 1/2;
    % 5. Initialize Willow Tree parameters
    m_h = 6; m_x = 30;
    % 6. Generate Data for the days up to the maturity
    % Generate strike prices based on moneyness through the maturity
    strike_prices = linspace(0.8 * S0, 1.2 * S0, maturity);
    A_sig = zeros(maturity, 1); A_prices = zeros(maturity, 1);
    % Setting the initial price to use the days from S
    % and wrapping around if necessary.
    S0 = S(mod(0:(maturity - 1), path_days) + 1, :);
    parfor j = 1:maturity % iterate through the maturities in parallel
         [A_sig(j), A_prices(j), ~] = impVol_HN(r, lambda_Q, omega_Q, rho, alpha_Q,
gamma_Q,h0, S0(j), strike_prices(j), T, N, m_h, m_x, -1);
    end
end
```

The `main` Code

This will contain generating data for different maturities and configuring the parameters for both the Option and HN-GARCH.

```
maturities = [5, 63, 126, 252]; % week, 3 months, 6 months and one year of trading days
filenames = {'week.csv', 'quarter.csv', 'half.csv', 'annual.csv'};
% 1. Initialize Option Parameters
r = 0.05/252; % Risk-free rate
S0 = 100; % Initial asset price
N = 100; % Number of time steps for simulation (ex. 500)
M = 10000; % Number of Monte Carlo paths
h0 = (0.2^2)/252; % Initial volatility

% 2. Initialize HN-GARCH parameters under P Measure
alpha = 1.33e-6;
beta = 0.586;
omega = 4.96e-6;
gamma = 484.69;
lambda = 0.5;

path_days = 50;

parfor i = 1:length(maturities)
   [sig, V] = datagen(maturities(i), r, S0, N, M, h0, alpha, beta, omega, gamma, lambda,
path_days);
    data = table(V, sig, 'VariableNames', {'Option Price', 'Implied Volatility'});
    writetable(data, filenames{i});
end
```

Downloading the Code: Available under **Github**

Generated Data

Week

Option Price	Implied Volatility
36.8178263574187	0.147327423095703
46.7785582767416	0.157432317733765
57.0272091662167	0.166537761688232
67.0332631013716	0.173696279525757
76.9315818148281	0.179845809936523

3 Months

Option Price	Implied Volatility
36.5600267989308	0.146834373474121
37.838229760055	0.148760318756104
38.6630645436978	0.149825572967529
38.94097134216	0.149765491485596
39.7499309766126	0.150805950164795
40.5043481737101	0.15174674987793
41.2146551135058	0.152600765228271
41.8122031897004	0.153214931488037
42.7477864587875	0.15459156036377
43.5456488717973	0.15551233291626
44.5654108907963	0.156750202178955
45.3579463181082	0.157594680786133
46.0338671807266	0.158233165740967
46.4469090549655	0.158373832702637
47.7572902244408	0.160250663757324
48.505145933108	0.161055088043213
49.1723390839423	0.161704063415527
49.9543809623368	0.162595272064209
49.9817566421013	0.161921501159668
50.3784032899062	0.162011623382568
51.0501814249374	0.162662744522095
51.8403674635294	0.163570404052734
52.788237955443	0.164823770523071
53.6385998998422	0.165787220001221
53.1713746420422	0.164102554321289

54.1428497351705	0.16540002822876
54.7691635371043	0.165846824645996
55.1556469421606	0.165880441665649
55.9788163924708	0.166629314422607
56.6767728348353	0.167176246643066
57.5725602516113	0.168067455291748
58.3345531034212	0.168738603591919
59.3414198795941	0.169857263565063
59.5193486337769	0.169496059417725
60.3590631447097	0.1703200340271
61.4437808568462	0.171615123748779
61.6144444959818	0.171213388442993
62.6298385475495	0.172378540039062
63.7126349359526	0.173737525939941
64.5062983687353	0.174536228179932
65.3765925252094	0.175503969192505
65.8441576393859	0.175646781921387
66.5730700635252	0.176327705383301
67.0772617669549	0.17654275894165
67.7198470424805	0.176971435546875
68.8375584301847	0.178085327148438
69.4019170889499	0.178322792053223
69.9305915443399	0.17851734161377
71.0538046158352	0.179696321487427
0	0.75
68.7738916874684	0.17437481880188
70.0525184463837	0.176072597503662
70.8775866896062	0.176845550537109
71.1554539703295	0.176662445068359
71.9646255586904	0.17725682258606
72.7192182546412	0.177772521972656
73.4296809340978	0.178240776062012
74.0273446796613	0.178529500961304
74.9631981166391	0.179345369338989
75.7612543151339	0.179957389831543
76.7813159735817	0.180923461914062

77.5740468533512	0.181561231613159
78.2501038592641	0.181993007659912

6 Months

Option Price	Implied Volatility
39.7745442303503	0.153676748275757
40.2417013782174	0.15440845489502
40.1482326211424	0.153807640075684
40.4860977651485	0.154224395751953
40.6874916257496	0.154326438903809
41.1651861563163	0.155060291290283
41.6339230055312	0.155632019042969
41.4956430715265	0.155074596405029
41.5527029940939	0.154852867126465
41.8779830839429	0.155181407928467
42.0054982969109	0.1551194190979
42.3122894147101	0.155387878417969
42.755639476573	0.155889987945557
43.3505975507406	0.156688213348389
44.2538416335556	0.158097267150879
44.5071965777931	0.158265829086304
44.9690366698257	0.1588454246521
45.5736406025845	0.159725189208984
45.9000501363021	0.160038948059082
46.2258600605157	0.16035795211792
46.0637715262076	0.159676551818848
46.6039491353158	0.160428524017334
46.9374773947644	0.16076135635376
46.88857714499	0.160307645797729
47.3737492712995	0.160919189453125
47.656047069316	0.161176204681396
47.7123761105056	0.160942077636719
48.1659588696408	0.161521911621094
48.9221557948837	0.162744522094727
49.308729503316	0.163201332092285
50.3489629792158	0.165133476257324

51.2368276917631	0.166460037231445
51.4248872031987	0.166472196578979
51.3994837476182	0.166104078292847
51.3856327694319	0.165763378143311
51.8501843468134	0.166257381439209
51.5993969353885	0.165509462356567
51.7927400582371	0.165531873703003
51.9457975597075	0.165483474731445
52.5336730266891	0.16618013381958
52.9402608821029	0.166569709777832
53.1723587554392	0.166656732559204
53.7187091598829	0.167266845703125
54.401137214151	0.168196678161621
54.9325395432338	0.168832778930664
55.4248293644186	0.169407844543457
56.3468177107097	0.17083215713501
56.1491649568899	0.170073747634888
56.1628206671635	0.169708251953125
0	0.75
55.7525620634895	0.168288946151733
56.2197861726215	0.168798446655273
56.1262587533167	0.16829252243042
56.4641597830321	0.168571472167969
56.6655596149801	0.168590784072876
57.1433206817112	0.169115543365479
57.6121302828543	0.169630289077759
57.4737736847784	0.169036388397217
57.5308079408447	0.168803691864014
57.8561211577723	0.169049263000488
57.9836261479652	0.168943643569946
58.2904463400797	0.169156074523926
58.7338632798458	0.169615983963013
59.3289377539508	0.170358896255493
60.2323523853851	0.171704292297363
60.4857240028273	0.17182445526123
60.9476280010851	0.172340393066406

61.5523504084749	0.173161745071411
61.8787987384351	0.173426866531372
62.2046413420147	0.173691272735596
62.0424649434966	0.172978401184082
62.5827322409927	0.173664093017578
62.9162949725892	0.173946619033813
62.8673403661098	0.173456192016602
63.3525821877537	0.174038171768188
63.6349025583263	0.174211502075195
63.6912016850191	0.173933267593384
64.144846802046	0.174452781677246
64.9011775712944	0.175602912902832
65.2878092106579	0.176000595092773
66.3282433533727	0.177513122558594
67.2162860495653	0.178647041320801
67.4043441397029	0.178622722625732
67.3788771409684	0.178238868713379
67.3649780721592	0.177883863449097
67.8295962033576	0.178315162658691
67.5787029459563	0.177577495574951
67.7720467928272	0.177565574645996
67.9250955270829	0.177487373352051
68.5130646693643	0.178109407424927
68.9197067521691	0.178443431854248
69.1518133606396	0.178491353988647
69.6982470089083	0.17905855178833
70.3807905707591	0.17986798286438
70.9122725995061	0.180431842803955
71.4046360069924	0.180936813354492
72.3267967643812	0.182228565216064
72.1290490336779	0.181487798690796
72.1426602308671	0.181141376495361
0	0.75
71.7322117765369	0.179729461669922
72.1995003687965	0.180166959762573
72.1059044794745	0.179665088653564

72.4438393300558	0.179886341094971
72.6452406186895	0.17988109588623
73.1230686668412	0.180339813232422
73.5919349458114	0.180781602859497
73.4535074300211	0.180200099945068
73.5105089139635	0.179953098297119
73.8358529672101	0.180151700973511
73.9633418937591	0.180022954940796
74.2701885375306	0.180190086364746
74.7136564439123	0.180583477020264
75.3088168221735	0.181237936019897
76.212399429009	0.18244743347168
76.4657837813122	0.182524681091309
76.9277503152576	0.182974815368652
77.5325520670569	0.183690547943115
77.8590299988534	0.183902263641357
78.1849020570751	0.184112787246704
78.0226406498355	0.183427810668945
78.5629853806867	0.184032440185547
78.8965794429512	0.184253215789795
78.8475632137016	0.183776378631592
79.3328732247164	0.184275388717651
79.6152124738322	0.184403419494629

One Year

Option Price	Implied Volatility
39.4235565325853	0.152856349945068
39.8357873363572	0.153633594512939
39.5570009334124	0.152816772460938
40.1606260889611	0.154027462005615
40.2167579519335	0.153972148895264
40.7770830302166	0.155091285705566
40.8766497195349	0.155127048492432
40.4338283513495	0.153922080993652
40.5331029613919	0.153968334197998
41.063791673604	0.155028104782104

40.5182361994944	0.153563737869263
41.0582384480557	0.154642581939697
40.9770832784384	0.154264450073242
41.3562172107	0.154968738555908
42.2396154252162	0.156478881835938
42.7669453607337	0.157320022583008
42.358661894635	0.156378984451294
42.4298828222129	0.156359434127808
42.5562585555524	0.156443119049072
42.4840358898559	0.156156539916992
42.5143517986366	0.156059503555298
43.0423463322926	0.156897783279419
42.7781256820852	0.156246423721313
43.1109040107781	0.156715869903564
43.0749900398822	0.156494617462158
43.2503754205889	0.156673431396484
43.0922772360794	0.15622353553772
43.2306755634296	0.156329154968262
42.8416651422596	0.155472755432129
42.8334414154253	0.155315160751343
43.1885100730408	0.155804634094238
43.4875555924425	0.156198501586914
43.7354460019012	0.156505107879639
43.8211368924788	0.156511545181274
43.6888244638003	0.156121730804443
44.0383785376669	0.156610012054443
44.4277353371777	0.157177925109863
44.1018757314978	0.156425952911377
44.2319267463757	0.156513690948486
44.2969363400745	0.156483173370361
44.3907707301024	0.15650486946106
44.8589094232881	0.157212734222412
45.4392375042146	0.158146858215332
46.3179637835003	0.159702777862549
46.1526299232392	0.159205913543701
46.4164993276284	0.159560680389404

46.2889053901982	0.159137725830078
46.4174087115951	0.159228324890137
46.6017514987736	0.1593017578125
0	0.75
47.3804535160907	0.160644054412842
47.7927359907155	0.161316633224487
47.5138930263583	0.160565376281738
48.1175949790902	0.161633014678955
48.1737239269849	0.161568641662598
48.7340992294736	0.162572383880615
48.8336675815138	0.162598371505737
48.3907907112336	0.16148042678833
48.4900669532144	0.161510467529297
49.0208027661168	0.162441253662109
48.475175681807	0.161123275756836
49.0152312931328	0.162056922912598
48.9340587391146	0.161708354949951
49.3132237989988	0.16231632232666
50.1967270370235	0.164032936096191
50.7241176347352	0.165019989013672
50.315765217396	0.163905620574951
50.3869838845488	0.163867473602295
50.5133662410159	0.16395092010498
50.4411182110382	0.163599491119385
50.4714252709947	0.163474321365356
50.9994911148289	0.164433479309082
50.7352141760327	0.163662672042847
51.0680323722508	0.164193391799927
51.0320988851625	0.163922071456909
51.2074987827744	0.164111137390137
51.0493614023753	0.163579940795898
51.1877682892816	0.163686752319336
50.7986958227063	0.162680149078369
50.7904623980572	0.162482976913452
51.1455596469433	0.16303825378418
51.4446347661595	0.163480281829834

51.6925513706034	0.1639404296875
51.778242331372	0.163813591003418
51.6458948600071	0.163347005844116
51.9954915033977	0.163898229598999
52.3848902833132	0.164539337158203
52.0589714610941	0.163654327392578
52.1890296910616	0.16374397277832
52.254036023902	0.16375732421875
52.3478717957602	0.16370964050293
52.8160641433289	0.164508819580078
53.3964460775834	0.165553092956543
54.2752727580464	0.166883707046509
54.1098974131517	0.166444540023804
54.3737953164329	0.166739940643311
54.2461688414039	0.166368007659912
54.3746767284744	0.166433334350586
54.5590342710101	0.166590690612793
0	0.75
55.3378187063321	0.167618036270142
55.7501379166567	0.168184757232666
55.4712530432618	0.167531967163086
56.0750135137518	0.168435573577881
56.1311386698255	0.168369770050049
56.6915675598343	0.169219017028809
56.7911370631221	0.169230461120605
56.3481996180124	0.168262004852295
56.4474769780011	0.168243408203125
56.9782630056785	0.169063568115234
56.4325636543584	0.167915344238281
56.9726705409802	0.168714046478271
56.8914785712904	0.16840386390686
57.2706765983684	0.168899536132812
58.1542811716363	0.170369148254395
58.6817228857515	0.171197414398193
58.2733116772897	0.17024040222168
58.3445278812546	0.170196056365967

58.4709142537749	0.170254707336426
58.3986469097545	0.16994571685791
58.4289467012576	0.169829845428467
58.9570637408062	0.170635938644409
58.6927449332466	0.169968128204346
59.0255913924021	0.170166015625
58.9896428565891	0.170168399810791
59.1650525276007	0.170303344726562
59.0068857450407	0.169856071472168
59.1452980603393	0.169936656951904
58.7561649743928	0.169070482254028
58.7479204339819	0.168892621994019
59.1030479113307	0.16935396194458
59.4021513947685	0.169721364974976
59.6500862900487	0.169998645782471
59.7357764874701	0.169983386993408
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59.9530295201308	0.170032501220703
60.3424632086459	0.170572757720947
60.0164952775271	0.169812440872192
60.146557955297	0.169877529144287
60.2115610952809	0.169825077056885
60.3053970604391	0.169825077056885
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61.3540728460373	0.17139196395874
62.2329922926536	0.172891139984131
62.0675863235758	0.172384262084961
62.3315040560815	0.172710418701172
62.2038521012369	0.172273635864258
62.3323642540145	0.172337055206299
62.5167312555267	0.17250657081604
0	0.75
63.2955854864805	0.173650503158569
63.7079423107807	0.174283981323242
63.4290131644482	0.173526763916016
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64.3903477942801	0.173893451690674
64.9305074338772	0.174778461456299
64.8492946119182	0.174417972564697
65.2285263291069	0.174984931945801
66.1122223862696	0.17661714553833
66.6397149121024	0.177303791046143
66.2312439631172	0.176449298858643
66.3024570849118	0.176391124725342
66.42884689251	0.176448345184326
66.3565595166176	0.176092147827148
66.3868513924975	0.175948619842529
66.9150193235398	0.176794052124023
66.6506577970821	0.176084995269775
66.9835320804312	0.176572799682617
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67.1229867002929	0.176448583602905
66.9647897385662	0.175917863845825
67.1032069095665	0.175999402999878
66.7140221889561	0.175013780593872
66.7057654383543	0.174803018569946
67.060923754177	0.175314664840698
67.3600446063199	0.175717353820801
67.6079972951966	0.176021099090576
67.6936861202595	0.175994396209717
67.5612861119113	0.175528049468994
67.9109418271348	0.176033496856689
68.3004100244889	0.17662525177002
67.9743920918654	0.175763368606567
68.1044586389948	0.17582631111145
68.1694579628595	0.175758838653564

68.2632935179182	0.1757493019104
68.7315738465817	0.176493167877197
69.3120701936433	0.177263259887695
70.1910762061191	0.178493976593018
70.0256449686084	0.178067207336426
70.2895760136227	0.178328275680542
70.1619038534651	0.177966117858887
70.2904196924388	0.178009748458862
70.474794245622	0.178142070770264
0	0.75
71.2537126956791	0.179065704345703
71.6661066109226	0.179581165313721
71.3871325592436	0.178956508636475
71.9910131250193	0.179784774780273
72.047128533064	0.179704666137695
72.6076672445212	0.180478096008301
72.707237300354	0.180477142333984
72.2641717804006	0.179559230804443
72.3634496236675	0.179554462432861
72.8943384445259	0.180268526077271
72.3484866537052	0.179197788238525
72.8886985434663	0.179924011230469
72.8074642681123	0.179614543914795
73.1867291471382	0.18007230758667
74.0705092251598	0.181402921676636
74.5980531087141	0.182151556015015
74.1895205732303	0.181250095367432
74.2607299927757	0.18119478225708
74.3871227563844	0.18123459815979
74.3148143618067	0.180933475494385
74.3450977541536	0.180810213088989
74.8733169676709	0.181538820266724
74.6089112446307	0.1810302734375
74.9418134008045	0.181299686431885
74.9058324879394	0.181062459945679
75.0812602575773	0.181185007095337

74.9230345034231	0.18074369430542
75.0614539180695	0.180803060531616
74.6722239017068	0.179991722106934
74.6639543513315	0.179811716079712
75.0191429735457	0.180222988128662
75.3182828914394	0.180543899536133
75.5662443548035	0.18084716796875
75.6519322796878	0.180758476257324
75.5195173997054	0.180368423461914
75.8691889215956	0.18077564239502
76.258690411179	0.181254625320435
75.9326309510095	0.18060302734375
76.0626992438953	0.180586338043213
76.1276964340104	0.180525779724121
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76.6898447842485	0.181111335754395
77.2703988850881	0.1819167137146
78.1494900657636	0.183272838592529
77.9840351769683	0.182796478271484
78.2479833140265	0.183074474334717
78.1202859131017	0.182665109634399
78.2488049646374	0.182707548141479
78.4331894709436	0.182847499847412
0	0.75
79.2121584524149	0.183861017227173
79.6245907170499	0.184419631958008