Question 1

$$X_t = \sin(\omega t + \Theta) + 3\varepsilon_t$$

a) $E[X_t] = E[\sin(\omega t + \Theta) + 3\varepsilon_t]$

$= E[\sin(\omega t + \Theta)] + \underbrace{E[3\varepsilon_t]}_{0}$

$$E[\sin(\omega t + \Theta)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin(\omega t + \Theta)\, d\Theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{\cos(\Theta)\sin(\omega t)}_{0} + \underbrace{\cos(\omega t)\sin(\Theta)}_{0}\, d\Theta$$

$= 0$

$= 0 + 0$

$\therefore m(t) = 0$

b) $\sigma^2(t) = E(X^2)$

$= E[\sin^2(\omega t + \Theta) + 6\sin(\omega t + \Theta) + 9\varepsilon_t^2]$

$= E[\sin^2(\omega t + \Theta)] + \underbrace{E[6\sin(\omega t + \Theta)]}_{0} + E[9\varepsilon_t^2]$

$\because E[\varepsilon_t^2] = 1$, then $E[9\varepsilon^2] = 9$

$$E[\sin^2(\omega t + \Theta)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2(\omega t + \Theta)\, d\Theta$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} 1 - \underbrace{\cos(2\omega t + 2\Theta)}_{0}\, d\Theta$$

$$= \frac{1}{4\pi}\left(\Theta\Big|_{-\pi}^{\pi}\right)$$

$$= \frac{1}{4\pi} \cdot 2\pi$$

$$= \frac{1}{2}$$

$\sigma^2(t) = \frac{1}{2} + 0 + 9$

$= \frac{19}{2}$