

Question 1

$$X_t = \sin(\omega t + \Theta) + 3\varepsilon_t$$

$$\begin{aligned} a) E[X_t] &= E[\sin(\omega t + \Theta) + 3\varepsilon_t] \\ &= E[\sin(\omega t + \Theta)] + \underbrace{E[3\varepsilon_t]}_0 \end{aligned}$$

$$E[\sin(\omega t + \Theta)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(\omega t + \Theta) d\Theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\cos(\Theta)\sin(\omega t)}_0 + \underbrace{\cos(\omega t)\sin(\Theta)}_0 d\Theta$$

$$= 0$$

$$= 0 + 0$$

$$\therefore m(t) = 0$$

$$\begin{aligned} b) \sigma^2(t) &= E(X^2) \\ &= E[\sin^2(\omega t + \Theta) + 6\sin(\omega t + \Theta) + 9\varepsilon_t^2] \\ &= E[\sin^2(\omega t + \Theta)] + \underbrace{E[6\sin(\omega t + \Theta)]}_0 + E[9\varepsilon_t^2] \end{aligned}$$

$$\because E[\varepsilon_t^2] = 1, \text{ then } E[9\varepsilon_t^2] = 9$$

$$\begin{aligned} E[\sin^2(\omega t + \Theta)] &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin^2(\omega t + \Theta) d\Theta \\ &= \frac{1}{4\pi} \int_{-\pi}^{\pi} 1 - \underbrace{\cos(2\omega t + 2\Theta)}_0 d\Theta \end{aligned}$$

$$= \frac{1}{4\pi} (\Theta \Big|_{-\pi}^{\pi})$$

$$= \frac{1}{4\pi} \cdot 2\pi$$

$$= \frac{1}{2}$$

$$\sigma^2(t) = \frac{1}{2} + 0 + 9$$

$$= \frac{19}{2}$$