

Q3

$$f(x, y) = 4e^{-2y} \quad 0 < 4x < y$$

$$\begin{aligned} f_X(x) &= \int_{4x}^{\infty} 4e^{-2y} dy \\ &= [-2e^{-2y}]_{4x}^{\infty} \\ &= 0 - (-2e^{-2 \cdot 4x}) \\ &= 2e^{-2 \cdot 4x} \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \int_0^{y/4} 4e^{-2y} dx \\ &= 4e^{-2y} [x]_0^{y/4} \\ &= \frac{4}{4} ye^{-2y} \end{aligned}$$

$$f_{Y/X}(y/x) = \frac{f(x, y)}{f_X(x)} = \frac{4e^{-2y}}{2e^{-2 \cdot 4x}}$$

$$f_{Y/X}(x, y) = 2e^{2 \cdot 4x - 2y}$$

$$E[Y/X] = \int_{4x}^{\infty} y 2e^{2 \cdot 4x - 2y} dy$$

Use IBP with

$$u = y \quad du = dy$$

$$dv = 2e^{2 \cdot 4x - 2y} \quad v = -e^{2 \cdot 4x - 2y}$$

$$\begin{aligned} &= uv - \int v du \\ &= -ye^{2 \cdot 4x - 2y} + \int_{4x}^{\infty} e^{2 \cdot 4x - 2y} dy \\ &= -ye^{2 \cdot 4x - 2y} - \frac{1}{2} [e^{2 \cdot 4x - 2y}]_{4x}^{\infty} \\ &= -ye^{2 \cdot 4x - 2y} - \frac{1}{2} [0 - (-1)] \\ &= \frac{1}{2} - ye^{2 \cdot 4x - 2y} \end{aligned}$$