Q3

$$f(x,y) = 4e^{-2y} \qquad 0 < 4x < y$$

$$f_X(x) = \int_{4x}^{\infty} 4e^{-2y}\, dy$$

$$= [-2e^{-2y}]_{4x}^{\infty}$$

$$= 0 - (-2e^{-8x})$$

$$= 2e^{-8x}$$

$$f_Y(y) = \int_0^{y/4} 4e^{-2y}\, dx$$

$$= 4e^{-2y}\,[x]_0^{y/4}$$

$$= \frac{4}{4} y e^{-2y}$$

$$f_{Y/X}(Y/X) = \frac{f(x,y)}{f_X(x)} = \frac{4e^{-2y}}{2e^{-8x}}$$

$$f_{Y/X}(x,y) = 2e^{8x-2y}$$

$$E[Y/X] = \int_{4x}^{\infty} y\, 2e^{8x-2y}\, dy$$

Use IBP with

$u = y \qquad du = dy$

$dv = 2e^{8x-2y} \qquad v = -e^{8x-2y}$

$$= uv - \int v\, du$$

$$= -ye^{8x-2y} + \int_{4x}^{\infty} e^{8x-2y}\, dy$$

$$= -ye^{8x-2y} - \frac{1}{2}[e^{8x-2y}]_{4x}^{\infty}$$

$$= -ye^{8x-2y} - \frac{1}{2}[0-(-1)]$$

$$= \frac{1}{2} - ye^{8x-2y}$$