

1. Compute

$$\frac{20}{2! + 0} + \frac{24}{2! + 4}$$

2. Six people are in a line, including Adam and Ben. Given that Adam is third in line, then what is the probability that Ben is next to him?
3. A triangle has side lengths $2x$, $3x$, and $4x$. If the perimeter of the triangle is equal to 18cm, find the length of the longest side in centimeters.
4. Joseph received 62 pieces of candy from trick-or-treating at Halloween. He gives the largest possible number of candies to his 12 friends, such that each friend gets the same amount, and Joseph still has more candy than any one of them. How many candy pieces does Joseph have now?
5. Harrison's closet contains a red, a blue, and a black shirt; a pair of black pants and a pair of white pants; and a pair of black shoes and a pair of white shoes. If Harrison wears 1 shirt, 1 pair of pants, and 1 pair of shoes, how many outfits can he make if he doesn't want to wear all black?
6. John has a glass of orange juice that is $\frac{5}{8}$ full. He pours all of it into another glass and only ends up filling it halfway. The second glass has a capacity of 40 fluid ounces. What is the total capacity of the first glass in fluid ounces?
7. What is the largest number of sides a regular polygon could have such that it can be inscribed in a square?
8. Lucy has a bag of marbles containing red, blue, and green marbles. The ratio of red to blue marbles is 3 : 2, and the ratio of blue to green marbles is 4 : 5. If Lucy has 45 marbles in total, how many green marbles does she have?
9. Triangle ABC is a right triangle with $AB = 2$ and $\angle BCA = 30^\circ$. Find the product of the side lengths of triangle ABC .
10. Jax has 10 pyramids, 2 cubes, 4 octahedrons, 12 cylinders, and 15 spheres. He places all of these objects in a drawer and draws an object randomly. If a is the minimum number of draws to guarantee that Jax has all of at least 1 type of object, and b is the minimum number of draws to guarantee that he has all 12 cylinders, what is $\frac{a}{b}$?
11. At Mustang High, there are 3 different grade levels (10^{th} to 12^{th}) with an equal number of students in each grade. At the start of every year, around 10% of the students from each grade level receive a trophy that they keep throughout high school. Assume that trophy-winners are chosen uniformly and randomly, and note that a student may have more than one. After 5 years have passed, what percent of the entire school population is expected to have at least one trophy? The school population does not include those that have graduated.
12. Elaine starts with 100,000 strands of hair, each with a mass of 1 mg. In her quest for more massive hair, she purchases a magic brush. Each stroke of the brush through her

hair adds 0.1 mg to each strand but causes 1000 strands to fall. How many times should she run the brush through her hair to achieve the most massive hair?

13. Say that the values of x and y for which the expression $x^2 + y^2 - 6x + 14y - 40$ is at it's minimal value (for real x and y) is for $x = a$ and $y = b$. What is $a + b$?

14. Positive integers x and y satisfy the equation

$$\frac{15}{3+x} = \frac{12+y}{7}$$

Find the sum of all possible values of xy .

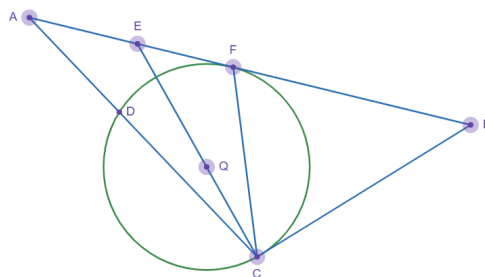
15. Jack and Jill are playing a not-so-fair game. They have a deck of 9 cards, each card numbered 1 through 9. Jack randomly takes 6 cards from the deck and gives the other 3 to Jill. Jack wins if the sum of his cards is greater than the sum of Jill's cards, and loses otherwise. How many different hands of 6 cards are winning for Jack?

16. What are the last two digits of 131^{45} ?

17. Alice and Bob are blowing bubbles together. Alice can blow 2 bubbles per second for 10 seconds, then catches her breath for 2 seconds before continuing. Her bubbles last 15 seconds before popping. Bob can blow 3 bubbles per second for 9 seconds, then catches his breath for 6 seconds. His bubbles last 16 seconds before popping. If Alice and Bob start blowing bubbles at the same time, after 2 minutes, what is the positive difference between the number of Alice's bubbles and the number of Bob's bubbles still in the air?

18. In a standard tic-tac-toe game, 5 moves have been made, i.e. there are 3 X's and 2 O's on the grid. How many board configurations exist where neither player has won yet?

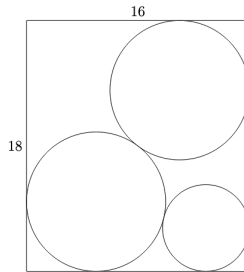
19. We have two circles O and Q tangent at point D , where the radius of O is greater than the radius of Q . Line \overline{AC} is tangent to circles O and Q at points A and E , respectively. Line \overline{BC} is tangent to circles O and Q at points B and F , respectively. Given that $AC = 5\sqrt{3}$ and the radius of O is 5, find the area inside quadrilateral $OACB$ but outside the two circles.



20. Let $f(x) = x^4 - 2x^3 + 8x^2 + x + 2$. Suppose $f(x)$ has roots r_1, r_2, r_3, r_4 . Compute

$$(r_1 + r_2 + r_3)(r_1 + r_2 + r_4)(r_1 + r_3 + r_4)(r_2 + r_3 + r_4).$$

21. Refer to the diagram below. The rectangle has a width of 16 and a height of 18. Two congruent circles, ω_1 and ω_2 , are tangent to each other, and each is tangent to two sides of the rectangle. A third circle, ω_3 , is tangent to ω_1 and to two sides of the rectangle. Find the radius of ω_3 .



22. Max starts at the origin and can only move to the up or the right and has to take 15 or 16 total steps. How many different paths can he take, if he has to take an even number of upward steps?
23. Consider a 13-hour analog clock, where the hour hand points to the numbers 1 through 13, evenly spread around the clock, while the minute hand still points to 60 ticks in 1 full rotation. The clock currently displays 13:00, so the minute and hour hands both point upwards. Let m be the number of minutes it takes for the minute and hour hands to first form a 144° angle, and t be the number of times the minute and hour hands form a 144° angle in the next 13 hours. Compute $m + t$.
24. A four digit number \overline{abcd} is suspicious if $\overline{da} + \overline{bc} = \overline{ab} + \overline{cd}$. How many suspicious numbers are there?
25. We have a circle Q . Lines \overline{AB} and \overline{BC} are tangent to circle Q at points F and C , respectively. Line \overline{CE} is drawn such that it passes through the center of circle Q . The line \overline{AC} intersects circle Q at point D .

Given that $AD = \frac{16}{7}$, $AC = 7$, the area of $\triangle AEC$ is $\frac{24}{5}$, the area of $\triangle EFC$ is $\frac{24}{5}$, and $EC = 6$, find the area of triangle $\triangle BFC$.

