1. John has a glass of orange juice that is $\frac{5}{8}$ full. He pours all of it into another glass and only ends up filling it halfway. The second glass has a capacity of 40 fluid ounces. What is the total capacity of the first glass in fluid ounces?

2. What is the largest number of sides a regular polygon could have such that it can be inscribed in a square?

3. Lucy has a bag of marbles containing red, blue, and green marbles. The ratio of red to blue marbles is $3 : 2$, and the ratio of blue to green marbles is $4 : 5$. If Lucy has 45 marbles in total, how many green marbles does she have?

4. Triangle ABC is a right triangle with $AB = 2$ and $\angle BCA = 30°$. Find the product of the side lengths of triangle ABC.

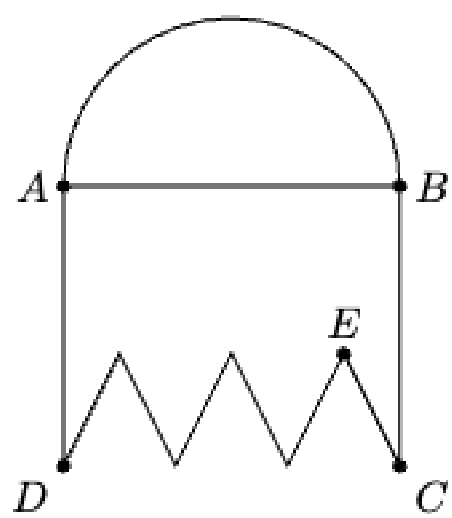
5. Jax has 10 pyramids, 2 cubes, 4 octahedrons, 12 cylinders, and 15 spheres. He places all of these objects in a drawer and draws an object randomly. If a is the minimum number of draws to guarantee that Jax has all of at least 1 type of object, and b is the minimum number of draws to guarantee that he has all 12 cylinders, what is $\frac{a}{b}$?

6. A rectangular garden is 15 feet long and 10 feet wide. If a path of uniform width is built around the garden on all four sides, increasing the total area to 300 square feet, what is the width of the path as a decimal in feet?

7. How many regular polygons have an integer value for their angle measure?

8. Find the units digit of $2^{2024} \times 3^{2024^{2024}} \times 7^{2024^{2024^{2024}}}$.

9. A Pac-Man ghost is made by joining semicircle $\widehat{AB}$ of radius 3 with rectangle $ABCD$ with 3 congruent isosceles triangles cut out, as shown below. Given that $BC = 5$ and $CE = \sqrt{5}$, find the area of the ghost in terms of π.



10. At Mustang High, there are 3 different grade levels (10^{th} to 12^{th}) with an equal number of students in each grade. At the start of every year, around 10% of the students from each grade level receive a trophy that they keep throughout high school. Assume that trophy-winners are chosen uniformly and randomly, and note that a student may have more than one. After 5 years have passed, what percent of the entire school population is expected to have at least one trophy? The school population does not include those that have graduated.

11. Positive integers x and y satisfy the equation

$$\frac{15}{3+x} = \frac{12+y}{7}$$

Find the sum of all possible values of xy.

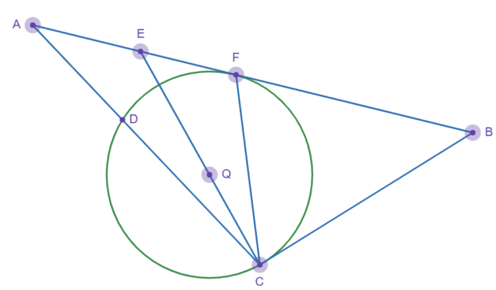
12. Jack and Jill are playing a not-so-fair game. They have a deck of 9 cards, each card numbered 1 through 9. Jack randomly takes 6 cards from the deck and gives the other 3 to Jill. Jack wins if the sum of his cards is greater than the sum of Jill's cards, and loses otherwise. How many different hands of 6 cards are winning for Jack?

13. Alice and Bob are blowing bubbles together. Alice can blow 2 bubbles per second for 10 seconds, then catches her breath for 2 seconds before continuing. Her bubbles last 15 seconds before popping. Bob can blow 3 bubbles per second for 9 seconds, then catches his breath for 6 seconds. His bubbles last 16 seconds before popping. If Alice and Bob start blowing bubbles at the same time, after 2 minutes, what is the positive difference between the number of Alice's bubbles and the number of Bob's bubbles still in the air?

14. What are the last two digits of 131^{45}?

15. In a standard tic-tac-toe game, 5 moves have been made, i.e. there are 3 X's and 2 O's on the grid. How many board configurations exist where neither player has won yet?

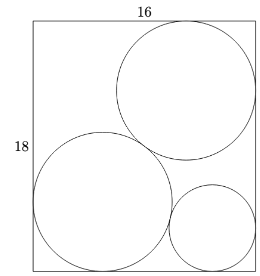
16. We have two circles O and Q tangent at point D, where the radius of O is greater than the radius of Q. Line $\overline{AC}$ is tangent to circles O and Q at points A and E, respectively. Line $\overline{BC}$ is tangent to circles O and Q at points B and F, respectively. Given that $AC = 5\sqrt{3}$ and the radius of O is 5, find the area inside quadrilateral $OACB$ but outside the two circles.



17. Let $f(x) = x^4 - 2x^3 + 8x^2 + x + 2$. Suppose $f(x)$ has roots r_1, r_2, r_3, r_4. Compute

$$(r_1+r_2+r_3)(r_1+r_2+r_4)(r_1+r_3+r_4)(r_2+r_3+r_4).$$

18. Refer to the diagram below. The rectangle has a width of 16 and a height of 18. Two congruent circles, ω_1 and ω_2, are tangent to each other, and each is tangent to two sides of the rectangle. A third circle, ω_3, is tangent to ω_1 and to two sides of the rectangle. Find the radius of ω_3.



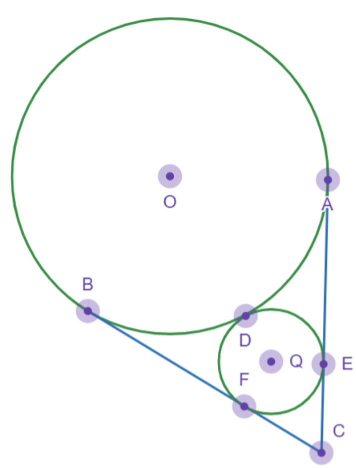
19. Consider a 13-hour analog clock, where the hour hand points to the numbers 1 through 13, evenly spread around the clock, while the minute hand still points to 60 ticks in 1 full rotation. The clock currently displays 13:00, so the minute and hour hands both point upwards. Let m be the number of minutes it takes for the minute and hour hands to first form a $144°$ angle, and t be the number of times the minute and hour hands form a $144°$ angle in the next 13 hours. Compute $m + t$.

20. Max starts at the origin and can only move to the up or the right and has to take 15 or 16 total steps. How many different paths can he take, if he has to take an even number of upward steps?

21. A four digit number $\overline{abcd}$ is suspicious if $\overline{da}+\overline{bc} = \overline{ab}+\overline{cd}$. How many suspicious numbers are there?

22. We have a circle Q. Lines $\overline{AB}$ and $\overline{BC}$ are tangent to circle Q at points F and C, respectively. Line $\overline{CE}$ is drawn such that it passes through the center of circle Q. The line $\overline{AC}$ intersects circle Q at point D.

    Given that $AD = \frac{16}{7}$, $AC = 7$, the area of $\triangle AEC$ is $\frac{24}{5}$, the area of $\triangle EFC$ is $\frac{24}{5}$, and $EC = 6$, find the area of triangle $\triangle BFC$.



23. Let a_1, a_2, a_3, a_4, and a_5 be the (not necessarily real) solutions to $x^5 + 2x^4 + 3x^3 + 4x^2 + 5x + 6 = 0$. Find the value of $(a_1)^7 + (a_2)^7 + (a_3)^7 + (a_4)^7 + (a_5)^7$.

24. How many 6 digit numbers can be created using the digits 1, 3, 5, 6 if:

    1. a prime digit cannot go after a composite digit

    2. the first digit must be 1

    3. every 6 must have a 1 before it.

25. Let $S = \{1, 3, 7, \ldots, 993, 997, 999\}$ be the set of positive integers less than 1000 that are divisible by neither 2 nor 5. The product of the elements of S is computed. What is the remainder when this product is divided by 1000?