Sparsity for source localization: application to acoustic source imaging

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***Abstract*— In this document, I summarized a research seminar of our Master ATSI entitled "Sparsity for source localization: application to acoustic source imaging". It was presented by José Picheral and Gilles Chardon , we will explore the concept of sparsity and its applications in source localization, specifically in acoustic source imaging. This work will cover the mathematical foundations of sparsity, its applications in acoustic source imaging, and some challenges of using sparsity in this field.**

I. INTRODUCTION

The field of source localization involves the estimation of the position of a source based on signals received at multiple sensors. The signals received at the sensors can be modeled as a linear function of the source position, as described in a widely cited reference [4]. The relationship between the source position and the sensor signals can be expressed as an equation that includes a measurement matrix, which describes the delays of the signals arriving at the sensors, and a measurement noise term.

Source localization is an important problem in various fields such as wireless communication systems, acoustic source imaging, and radar systems, among others. There are several methods for source localization, including conventional beamforming and the Maximum Likelihood (ML) method. Beamforming is a simple technique that involves weighting the signals collected by each sensor to maximize the output signal in a specific direction. The Maximum Likelihood method is a statistical method that finds the set of parameters that maximizes the likelihood function, which represents the probability of observing the data given the model parameters.

this bibliographic work explores the use of sparsity for source localization, which involves finding the sparse representation of the source position using the measurement data. it provides an overview of the principles and limitations of sparsity-based source localization and discusses its applications to acoustic source imaging.

II. DISCUSSION

A. Model of the Measured Signal by an Array Antenna

The signals received at the sensors can be modeled as a linear function of the source position, as described in [4]. Let $\mathbf{x} \in \mathbb{R}^d$ represent the position of the source and $\mathbf{y} \in \mathbb{R}^m$ represent the signals received at the sensors. The relationship between $\mathbf{x}$ and $\mathbf{y}$ can be expressed as:

$$\mathbf{y} = \mathbf{A}(\mathbf{x}) + \mathbf{n} \tag{1}$$

where $\mathbf{A}(\mathbf{x}) \in \mathbb{R}^m$ is the measurement matrix that describes the delays of the signals arriving at the sensors, and $\mathbf{n} \in \mathbb{R}^m$ is the measurement noise. The measurement matrix $\mathbf{A}(\mathbf{x})$ is a function of the source position $\mathbf{x}$ and the array geometry. In a two-dimensional scenario with n sensors, it can be expressed as:

$$\mathbf{A}(\mathbf{x}) = [a_1(\mathbf{x}), a_2(\mathbf{x}), \ldots, a_n(\mathbf{x})] \tag{2}$$

where $a_i(\mathbf{x}) = e^{-j2\pi \frac{d}{\lambda} \sin\theta_i}$ is the steering vector that describes the delays of the signals arriving at the i-th sensor, d is the inter-sensor distance, λ is the wavelength of the signal, and θ_i is the angle between the direction of the source and the normal to the array at the i-th sensor. The measurement noise $\mathbf{n}$ is typically modeled as zero-mean Gaussian noise with covariance matrix $\mathbf{R}$. The goal of source localization is to estimate $\mathbf{x}$ from the measured signals $\mathbf{y}$.

B. Conventional beamforming

In the field of source localization, Beamforming is a widely used and simple technique for source localization that involves weighting the signals collected by each sensor in an array to maximize the output signal at a specific direction. The basic idea behind beamforming is to apply a set of weights, w_i, to the signals collected by each sensor, $x_i(t)$, to obtain the beamformed signal:

$$y(t) = \sum_{i=1}^{N} w_i x_i(t) \tag{3}$$

where N is the number of sensors in the array. The weights are chosen to maximize the output signal in a specific direction, typically by maximizing the signal-to-noise ratio (SNR) of the beamformed signal [1]. The direction of maximum output is then assumed to be the direction of the source.

One of the commonly used beamforming algorithms is the Delay-and-Sum (DS) beamformer, which applies a simple delay to each sensor signal based on the direction of the source and sums the delayed signals [2]. The DS beamformer has the advantage of being easy to implement and computationally efficient, but it has some limitations. One of the main limitations is that the DS beamformer assumes that the array geometry is known, which may not always be the case in practice [3]. Another limitation is that the DS beamformer assumes that the source is narrowband, meaning that the source signal has a single dominant frequency [4].

These limitations can be overcome by using more sophisticated beamforming algorithms, such as the Minimum Variance Distortionless Response (MVDR) beamformer, which does not require a known array geometry and can handle wideband sources [5]. However, these algorithms can be computationally more demanding and may not always provide better performance than the DS beamformer.

C. Maximum Likelihood Method for Source Localization: Principles and Limitations:

Maximum Likelihood (ML) is a widely used technique for source localization in wireless communication systems. It is a statistical method that provides a way to estimate the unknown parameters of a model that best fits the observed data. The idea behind ML is to find the set of parameters that maximizes the likelihood function, which represents the probability of observing the data given the model parameters[4].

The likelihood function is defined as the product of the probability densities of the observed data given the model parameters. If we assume that the source signals and the measurement noise are independent and Gaussian distributed[4], then the likelihood function is given by:

$$L(\theta) = \prod_{k=1}^{K} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{||\mathbf{y}_k - \mathbf{A}(\theta)\mathbf{s}_k||^2}{2\sigma^2}\right) \tag{4}$$

where $\mathbf{y}_k$ is the measurement vector at time k, $\mathbf{s}_k$ is the source signal vector at time k, $\mathbf{A}(\theta)$ is the steering matrix that depends on the source position θ, σ^2 is the variance of the measurement noise, and K is the total number of measurements. The goal is to find the maximum likelihood estimate $\hat{\theta}$ that maximizes $L(\theta)$.

The ML method requires a statistical model of the source signals and the measurement noise. In other words, it requires the knowledge of the distribution of the source signals and the measurement noise. This can be a limitation of the ML method, as it may not be possible to accurately model the source signals and the measurement noise in real-world scenarios. Another limitation of the ML method is that it may result in a local maximum of the likelihood function, rather than the global maximum, which can lead to suboptimal results.

Overall, the ML method is a powerful tool for source localization in wireless communication systems, but its limitations should be taken into consideration when using it.

D. Reformulation as an Inverse Problem: The Role of Sparsity

The source localization problem can be reformulated as an inverse problem by considering the measurement signals as the inputs and the source positions as the unknown parameters to be estimated. This reformulation allows us to use priors to solve the problem. One such prior is sparsity, which assumes that only a small number of sources are present in the environment. This assumption can be incorporated into the inverse problem formulation by adding a sparsity-promoting term to the objective function. The sparsity prior can be expressed mathematically as:

$$\min_{\mathbf{x}} \left(\frac{1}{2}\left|\mathbf{y} - \mathbf{A}\mathbf{x}\right|_2^2 + \lambda \left|\mathbf{x}\right|_1\right) \tag{5}$$

where $\mathbf{y}$ is the vector of measurement signals, $\mathbf{A}$ is the measurement matrix, $\mathbf{x}$ is the vector of source positions, $|\cdot|_2$ is the Euclidean norm, and $|\cdot|_1$ is the ℓ_1 norm. The term $\lambda\left|\mathbf{x}\right|_1$ promotes sparsity in the solution by encouraging most of the elements of $\mathbf{x}$ to be zero. The value of λ determines the strength of the sparsity promotion.

This reformulation has been extensively studied in the literature and has been shown to be an effective approach for source localization. For example, in [5], it was shown that the sparsity-regularized inverse problem formulation outperforms traditional methods such as the beamforming method in a noisy environment. Similarly, in [6], the authors proposed a sparsity-based algorithm for source localization in wireless sensor networks and showed that it can improve the accuracy of the estimated source positions.

E. The Multiple Signal Classification method

Another widely used method for localizing sources with spatial spread is the MUSIC (Multiple Signal Classification) method. The MUSIC algorithm is based on the concept of subspace projection[7], and it can effectively localize multiple sources even in the presence of noise. Given the measurement matrix $\mathbf{Y}$, the MUSIC algorithm estimates the direction vectors of the sources by performing a singular value decomposition (SVD) of $\mathbf{Y}$. The columns of the resulting orthogonal matrix span the signal subspace, while the orthogonal complement of the signal subspace corresponds to the noise subspace. The MUSIC algorithm then computes the direction of arrival (DOA) of the sources by finding the peaks in the spectral output, which are proportional to the inverse of the singular values of the noise subspace.

$$\mathbf{a}(\theta) = \frac{\mathbf{p}^H(\theta)\mathbf{R}_{yy}}{\mathbf{p}^H(\theta)\mathbf{p}(\theta)} \tag{6}$$

where $\mathbf{a}(\theta)$ is the steering vector, $\mathbf{p}(\theta)$ is the array response vector, and $\mathbf{R}_{yy}$ is the spatial covariance matrix of the received signals. This equation represents the MUSIC-based method for localizing sources with spatial spread [8].

The MUSIC-based method works by estimating t[h]e peaks in the spectrum of the function $\mathbf{a}(\theta)$ which correspond to the directions of arrival of the sources. The method requires the knowledge of the number of sources present in the environment and the array geometry. It is a computationally efficient method and has been widely used for source localization in various applications .

F. Simulation Results for Linear Antenna Array

In this section, we demonstrate the effectiveness of the proposed method, CMF-OLS, by simulating a linear antenna with 19 sensors spaced by a half wavelength[9]. The simulation considers two groups of correlated sources, with 2 sources emitting at 3500 Hz and 3 sources emitting at 3500 Hz, located on a line parallel to the antenna at a distance of 5m. The SNR was set to 0 dB and 500 time samples were collected. The 1D region of interest of the sources was discretized with a 5 mm step, yielding 400 grid points.

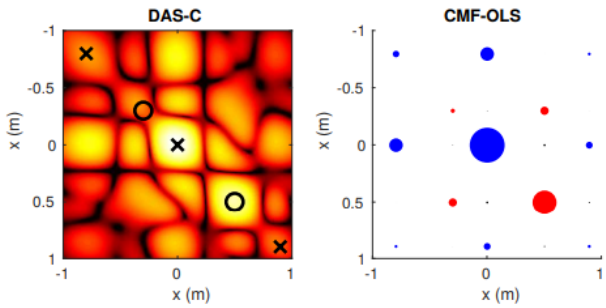

Fig. 1. Simulations with 5 Sources: Visualizing the Source Covariance Matrix with Diagonal Showcasing Source Powers and Off-Diagonal Depicting Correlation between Sources.

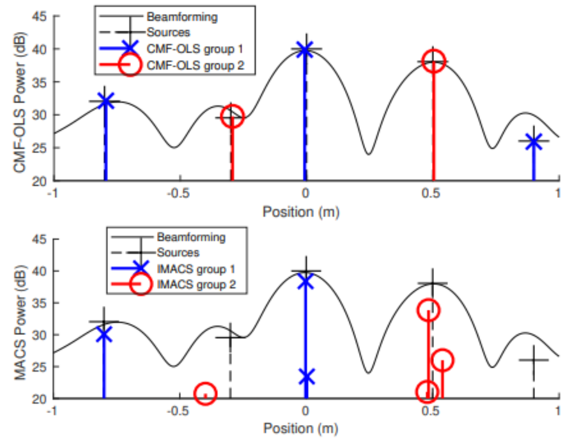

Fig. 2. Simulation Results for 5 Sources - Position and Power Distribution. Upper Panel: CMF-OLS Estimation with K=5.
Lower Panel: IMACS Estimation.

The results of the source covariance matrix estimation provided by the DAS-C beamformer and the CMF-OLS are shown in Fig. 2. The diagonal of the covariance matrix, which represents the output of the standard DAS beamformer, was also plotted. The DAS-C beamformer was not able to identify the two less powerful sources, as they were below the level of the sidelobes of more powerful sources. However, CMF-OLS was able to easily identify the two groups of correlated sources from its estimation. The estimated positions and powers of the sources estimated by CMF-OLS are given in Table I and plotted in Fig. 2. The positions and powers of the five sources were estimated correctly, even for the sources that could not be identified by beamforming.

The source powers estimated by IMACS were also plotted. The regularization parameter had to be set manually, and the rank of the covariance matrix was assumed to be $Lh = 2$. 50 iterations were used in the estimation. IMACS did not accurately estimate the weaker sources, and the power was spread over contiguous grid points, making the estimation of the power difficult. The energy of the residual before each step of CMF-OLS was plotted, and the number of sources was estimated by the location of the discontinuity in the decay of the energy, which was at $K = 5$ iterations. The singular values of the estimated covariance matrix with 5 iterations were also plotted, showing that the sources could be separated into two correlated groups.

The computation time for CMF-OLS was 0.003s, which was about 10 000 times faster than the IMACS algorithm, which is known as one of the fastest approaches for source covariance matrix estimation.

In conclusion, the simulation results demonstrate the effectiveness and efficiency of the proposed method, CMF-OLS, in estimating the positions and powers of correlated sources.

III. Conclusion

In conclusion, the field of source localization involves the estimation of the position of a source based on signals received by multiple sensors. There are various methods for source localization, including conventional beamforming and the Maximum Likelihood method. This bibliographic work explores the use of sparsity for source localization and provides an overview of the principles and limitations of sparsity-based source localization, with a focus on its applications to acoustic source imaging.

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