3.1 Questions

Problem 1. Sketch a graph of a continuous function $f(x)$ on the interval $[1, 5]$ with the following properties:

1. $f(x)$ has an absolute maximum value at $x = 5$,
2. $f(x)$ has an absolute minimum value at $x = 2$,
3. $f(x)$ has a local maximum value at $x = 3$,
4. $f(x)$ has a local minimum value at $x = 4$.

Problem 2. Sketch the graph of a function $g(x)$ whose domain is the interval $[3, 8]$ and such that $g(x)$ does not have an absolute maximum nor an absolute minimum.

Problem 3. Sketch the graph of a function $h(x)$ that is continuous on its domain $(3, 8)$ and does not have an absolute maximum nor an absolute minimum.

Problem 4. Why do your examples in problems 2 and 3 not contradict the Extreme Value Theorem?

Problem 5. Write a step-by-step process for finding absolute maximum and absolute minimum values of a continuous function $f(x)$ defined on a closed interval $[a, b]$.

Problem 6. Find all of the critical numbers of the following functions.

(a) $f(x) = \dfrac{7x}{3x^2 + 7}$

(b) $g(x) = (x - 3)(x - 7)^3 + 9$

Problem 7. Consider the function $f(t) = 5\sqrt[3]{t^2} - 2\sqrt[3]{t^5}$

(a) Find the critical numbers of $f(t)$.

(b) Find the absolute minimum and absolute maximum of $f(t)$ on the interval $[-1, 8]$.

3.2 Questions

The **mean value theorem (MVT)** states:

If a function $f(x)$ is (1) continuous on a closed interval $[a, b]$ and (2) differentiable on the open interval (a, b),

then there is some number c satisfying $a < c < b$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Problem 8. Check whether the hypotheses of the MVT apply to the functions given below.

(i) If yes, find a c value so that $f'(c)$ equals the average rate of change of f over the interval.

(ii) If no, explain why not.

(a) $f(x) = x + \dfrac{1}{x}$ on the interval $[1, 3]$.

(b) $f(x) = \tan x$ on the interval $[0, \pi]$.

(c) $f(x) = |x - 1|$ on the interval [-2,2].

(d) $f(x) = \sqrt[3]{x} - x$ on the interval [-1,1].

Problem 9. Give the statements of the following important theorems that we have covered in the course so far.

1. Intermediate Value Theorem:

2. Extreme Value Theorem:

3. Mean Value Theorem: