Robot Dynamics Ex 2a:
Theory Recap

$$M(q) = \sum_{i=1}^{n_b} \left({}_A J_{Si}^T \, m_i \, {}_A J_{Si} + {}_B J_{Ri} \, {}_B \Theta_{Si} \, {}_B J_{Ri} \right)$$

$$b(\dot q, q) = \sum_{i=1}^{n_b} \left({}_A J_{Si} \, m_i \, {}_A \dot J_{Si} \, \dot q + {}_B J_{Ri}^T \left({}_B \Theta_{Si} \, {}_B \dot J_{Ri} \, \dot q + {}_B \Omega_{Si} \times {}_B \Theta_{Si} \, {}_B \Omega_{Si} \right) \right)$$

$$g(q) = \sum_{i=1}^{n_B} - {}_A J_{Si}^T \, {}_A F_{g_i}$$

We need for each body i:

$$ {}_I J_{S_i}(q) = {}_I J_{IS_i} $$

$$ {}_I \dot{J}_{S_i}(q) $$

$$ {}_I J_{R_i}(q) $$

$$ {}_I \dot{J}_{R_i}(q) $$

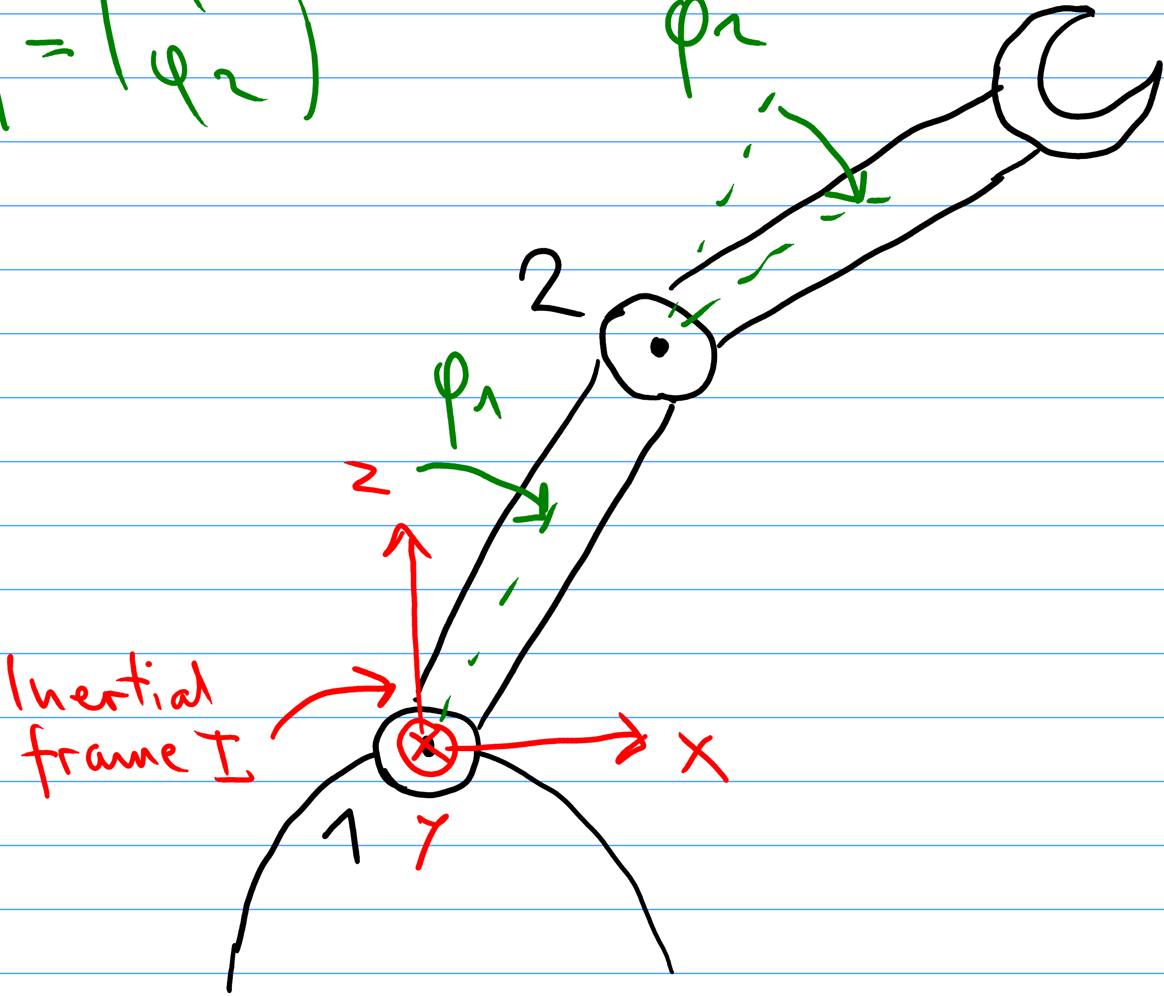
${}_I \Theta_{S_i}$ Inertia Matrix
Inertial Tensor

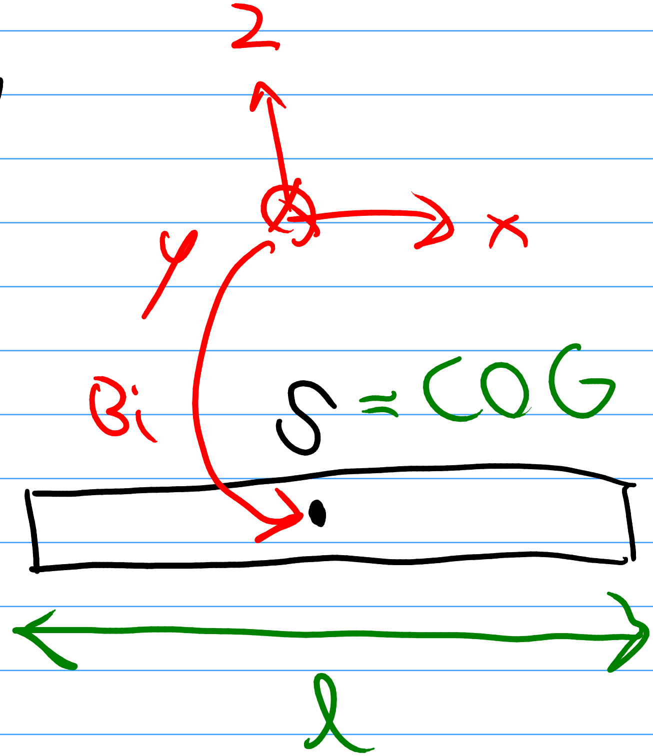
${}_I \Omega_{S_i}$ angular velocity

m_i mass of body i

Example: Two link robot arm:

$$q = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$





Body 1:

$$ {}_I J_{S1} = \left({}_I n_1 \times {}_I r_{IS1} \quad 0 \right) = \begin{pmatrix} \frac{l}{2} c_1 & 0 \\ 0 & 0 \\ -\frac{l}{2} s_1 & 0 \end{pmatrix} $$

$$ {}_I \dot{J}_{S1} = \frac{\partial J_{Si}}{\partial q} \dot{q} = \begin{pmatrix} -\frac{l}{2} s_1 & 0 \\ 0 & 0 \\ -\frac{l}{2} c_1 & 0 \end{pmatrix} \dot{q} $$

$$ {}_I J_{R1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad {}_I \dot{J}_{Ri} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} $$

Body 2:

$$
{}_I J_{S2} = \begin{pmatrix} l(c_1 + \frac{1}{2} c_{12}) & \frac{1}{2} l c_{12} \\ 0 & 0 \\ -l(s_1 + \frac{1}{2} s_{12} & -\frac{1}{2} l s_{12} \end{pmatrix}
$$

$$
{}_I \dot{J}_{S2} = \begin{pmatrix} -l(s_1 + \frac{1}{2} s_{12} & -\frac{1}{2} l s_{12} \\ 0 & 0 \\ -l(c_1 + \frac{1}{2} c_{12} & -\frac{1}{2} l c_{12} \end{pmatrix}
$$

$$
{}_I J_{R2} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix} \qquad {}_I \dot{J}_{R2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}
$$

Inertial Matrix:

In general <ins></ins>variant around COG

$$ {}_{K}\Theta_{Si} = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{zy} & I_{zz} \end{pmatrix} $$

$$ {}_{Bi}\Theta_{Si} = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12} m l^2 & 0 \\ 0 & 0 & \frac{1}{12} m l^2 \end{pmatrix} $$

uniform slender rod

$$ {}_{I}\Theta_{Si} = C_{IBi} \, {}_{Bi}\Theta_{Si} \cdot C_{BiI} $$

Angular Velocity

$${}_I\Omega_{S_i} = {}_I J_{R_i} \dot{q}$$

start put everything together:

$$M = \sum M_i$$

$$M_1 = \begin{pmatrix} \frac{l}{2}c_1 & 0 & -\frac{l}{2}s_1 \\ 0 & 0 & 0 \end{pmatrix} m \begin{pmatrix} \frac{l}{2}c_1 & 0 \\ 0 & 0 \\ -\frac{l}{2}s_1 & 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}ml^2 & 0 \\ 0 & 0 & \frac{1}{12}ml^2 \end{pmatrix}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

converting ${}_{B_i}\Theta_{S_i}$ to ${}_I\Theta_{S_i}$ is omitted for clarity, the result is the same. (only rot. around y)

$$= \begin{pmatrix} \frac{13}{12} m l^2 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{same for } M_2$$

$\Rightarrow$ similar $b(\dot{q}, q)$

$$g(q) = - {}_I J_{S1}^T \begin{pmatrix} 0 \\ 0 \\ -m \cdot g \end{pmatrix} - {}_I J_{S2}^T \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}$$

$$M(q)\ddot{q} + b(\dot{q}, q) + g(q) = \tau + J_c^T(q) F_c$$

set of n_B differential equations describing the system dynamics in joint space.