MAE5403 (Linear System Control Design Problem)

Unmanned Helicopter Controller Design

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1. **Abstract**

In this Unmanned Helicopter case. We consider 9 state space variable. There are three angle(Roll,Pitch,Yaw) angular rates and three unmeasured variable($a_s, b_s, \delta_{ped,int}$). This system also has three input and three dsiturbance $\delta_{lat}, \delta_{lon}, \delta_{ped}$ and $u_{wind}, u_{wind}, \delta_{wind}$,respectively. I choose LQG technique in this case for Inner-loop Flight control. I also try H_∞,but I have some problem about reduced order Controller design, I can not caculate the right nums. So I decide to use LQG for this case. And I use simulink to simulation my controller,It have good performance that I will show in Part2(Results analysis).

2. **Part1: Mathmactics and Fligh dynamics model analysis**

   (a) Here,We Let

$$\dot{x} = Ax + Bu + Ew$$

$$y = C_1 x + D_1 v$$

And the noise **v** is the measurement output noise,In this case i choose the white noise for output measurement noise. The state variable is $x = \begin{Bmatrix} \phi \\ \theta \\ p \\ q \\ a_s \\ b_s \\ r \\ \delta_{ped,int} \\ \psi \end{Bmatrix}$; $u = \begin{Bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{Bmatrix}$; $w = \begin{Bmatrix} u_{wind} \\ v_{wind} \\ w_{wind} \end{Bmatrix}$;

(b) And from dynamics linearization model,we also obtain the A,B and E martix,They showed the fellowing.

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\ 0 & 0 & 0 & 0.9992 & 0 & 0 & -0.0389 & 0 & 0 \\ 0 & 0 & -0.0302 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.529 & 0 \\ 0 & 0 & 0 & -0.0707 & 267.7499 & -0.0003 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.0000 & -3.3607 & 2.2223 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2.4483 & -3.3607 & 0 & 0 & 0 \\ 0 & 0 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.0389 & 0 & 0 & 0.9992 & 0 & 0 \end{bmatrix};$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 43.3635 \\ 0 & 0 & 0 \\ 0.2026 & 2.5878 & 0 \\ 2.5878 & -0.0663 & 0 \\ 0 & 0 & -83.1883 \\ 0 & 0 & -3.8500 \\ 0 & 0 & 0 \end{bmatrix}; E=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0001 & 0.1756 & -0.0395 \\ 0.0000 & 0.0003 & 0.0338 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0002 & -0.3396 & 0.6424 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; C_1=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Choose the LQG technique for this. Before we start the design we need to know the pair (A,B) and (A,C1)is controllable and observable. It's obvious full rank with the matlab command rank(ctrb(A,B)) and rank(obsv(A,C1)). So we can do the job now. we need to three steps, they show as fellowing: **First:**,We need to design a LQR control law $u = -Fx$,and we need to choose the Matirx Q and R($Q \geq 0$,$R \geq 0$). Here In my simulation I choose the Q and R matrix as fellow.

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R = 0.001$$

Then,use Matlab function icare to solve the Riccati Equtaion $PA + A^T P - PBR^{-1}B^T P + Q = 0$,

$P \geq 0$,$F = R^{-1}B^TP$ The matlab function is

```
[X1,K1,L1] = icare(A,[],Q'*Q,[],[],[],-B*B');
F=inv(R)*B'*X1;
Abk=A-B*F;
eigabk=eig(Abk);
```

Then,We obtain the F matrix as fellow:

$$F = 1.0e{+}04 * \begin{bmatrix} 0.0943 & 0.0035 & 0.0652 & 0.0039 & 0.1251 & 1.5944 & 0.0356 & -0.0087 & 0.0332 \\ -0.0046 & 0.0999 & -0.0025 & 0.0724 & 1.0974 & -0.0017 & -0.0012 & 0.0003 & 0.0028 \\ 0.0331 & 0.0042 & 0.0354 & 0.0004 & -0.0045 & -0.0391 & -0.0721 & -0.1079 & -0.0943 \end{bmatrix};$$

And the matrix $A - BF$ is show:

$$Abk = 1.0e{+}04 * \begin{bmatrix} 0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 \\ -1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.08 \\ 0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0 \\ -0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0 \\ 2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8 \\ 0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3 \\ 0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = 1.0e+04 * \begin{bmatrix} -7.9558 + 0.0000i \\ -4.1816 + 0.0000i \\ -2.8032 + 0.0000i \\ -0.0023 + 0.0000i \\ -0.0016 + 0.0000i \\ -0.0005 + 0.0000i \\ -0.0001 + 0.0000i \\ -0.0001 - 0.0000i \\ -0.0001 + 0.0000i \end{bmatrix}$$; We can see all eigvalue have negative real part.

Second:,We need to use Kalman filter for the plant, $\dot{\hat{x}} = A\hat{x} + K_e(y - \hat{y}), \hat{y} = C_1\hat{x}$

We also use icare to sove Riccati Equtaion:$P_eA^T + AP_e - P_eC^TR_e^{-1}CP_e + Q_e = 0$, $P_e \geq 0$,$Ke = P_eC^TR_e^-1$

And here we choose the Q_e and R_e matrix is:

$$Q_e = 1.0e-03 * \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix} ;R_e =$$

0.004

Using Matlab code as fellow:

```
[X2,K2,L2] = icare(A,[],Qe'*Qe,[],[],[],-C1*C1');
% Pk=are(A,C1,Qe)
Ke=X2*C1'*inv(Re)
Ack=A-Ke*C1
eigack=eig(Ack)
```

And we obtain Ke and $Ack = A - K_eC_1$ and Ack eigenvalues: Then,We obtain the F matrix as fellow:

$$K_e = \begin{bmatrix} 0.2500 & 0.0000 & 0.0014 & -0.0023 & 0 & 0 & 0.0007 & 0 & 0.0000 \\ 0.0000 & 0.2500 & -0.0009 & 0.0031 & 0 & 0 & -0.0005 & 0 & -0.0000 \\ 0.0014 & -0.0009 & 0.0001 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ -0.0023 & 0.0031 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.0001 \\ 0.0002 & 0.2496 & -0.0012 & 0.0033 & 0 & 0 & -0.0005 & 0 & 0.0097 \\ 0.2500 & 0.0000 & 0.0016 & -0.0021 & 0 & 0 & 0.0011 & 0 & 0.0000 \\ 0.0007 & -0.0005 & 0.0000 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ 0.0002 & -0.0097 & -0.0001 & -0.0000 & 0 & 0 & -0.0000 & 0 & 0.2498 \\ 0.0000 & -0.0000 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.2500 \end{bmatrix};$$

And the matrix $A - K_eC_1$ is show:

$$Ack = 1.0e{+}04* \begin{bmatrix} -0.2500 & -0.0000 & 0.9986 & 0.0023 & 0 & 0 & 0.0002 & 0 \\ -0.0000 & -0.2500 & 0.0009 & 0.9961 & 0 & 0 & -0.0384 & 0 \\ -0.0014 & 0.0009 & -0.0303 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.5290 \\ 0.0023 & -0.0031 & 0.0000 & -0.0708 & 267.7499 & -0.0003 & 0.0000 & 0 \\ -0.0002 & -0.2496 & 0.0012 & -1.0033 & -3.3607 & 2.2223 & 0.0005 & 0 \\ -0.2500 & -0.0000 & -1.0016 & 0.0021 & 2.4483 & -3.3607 & -0.0011 & 0 \\ -0.0007 & 0.0005 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2000 \\ -0.0002 & 0.0097 & 0.0001 & 0.0000 & 0 & 0 & -1.0000 & 0 \\ -0.0000 & 0.0000 & 0.0000 & 0.0388 & 0 & 0 & 0.9992 & 0 \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = \begin{bmatrix} -1.5367 + 23.9237i \\ -1.5367 - 23.9237i \\ -1.6190 + 16.4397i \\ -1.6190 - 16.4397i \\ -13.8925 + 0.0000i \\ -7.7978 + 0.0000i \\ -0.5274 + 0.0000i \\ -0.4993 + 0.0000i \\ -0.4999 + 0.0000i \end{bmatrix}$$; We can also see all eigvalue have negative real part.

The last:,The plant LQG control law is $u = -F\hat{(x)}$

$$\begin{cases} \dot{\hat{x}} = (A - BF - K_e C_1) + K_e y, \\ u = -F\hat{x} \end{cases}$$ And,$G = [C_1(A - BF)^{-1}]^{-1}$,Using the Matlab we obtain the G:

$$G = \begin{bmatrix} -942.5693 & -34.5139 & -332.2226 \\ 46.3757 & -998.5361 & -27.8396 \\ -330.7754 & -41.6478 & 942.7901 \end{bmatrix};$$

Clearly, the closed-loop system is characterized by the following state space equation we can obtain:

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} - \begin{pmatrix} BG \\ 0 \end{pmatrix} r + \widetilde{v}, \widetilde{v} = \begin{pmatrix} v \\ v - K_e W \end{pmatrix} \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + w \end{cases}$$ Then,We let

$A_c = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix}$,$B_c = \begin{pmatrix} BG \\ 0 \end{pmatrix}$,$C_c = \begin{bmatrix} C & 0 \end{bmatrix}$,Caculating From Matlab We Obatin the A_c, B_c, C_c as show bellow:

$A_c = 1.0e + 04 *$

$$
\begin{matrix}
0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.0883 & 1.4344 & 0.1806 & 1.5341 & 0.0177 & -0.1934 & -1.6961 & -3.128 \\
0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0139 & 0.0071 & 0.2591 & 0.0067 & 0.1882 & 2.8651 & 0.3186 & 0.0041 \\
-0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0858 & 0.2442 & 0.0023 & 0.1688 & 0.0053 & 0.2509 & 4.1260 & 0.0922 \\
2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8429 & -2.7517 & -0.3465 & -2.9429 & -0.0339 & 0.3711 & 3.2539 & 6.0019 \\
0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3630 & -0.1273 & -0.0160 & -0.1362 & -0.0016 & 0.0172 & 0.1506 & 0.2778 \\
0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0001 & 0.0000 & 0 & 0 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0000 & 0.0001 & 0 & 0 & -0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0585 & 0.0011 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0268 & -0.0000 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0000 & -0.0001 & -0.0003 & 0.0002 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & -0.0001 & 0.0000 & 0.0002 & -0.0003 & -0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0022 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0 & 0 & -0.0001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0 & 0 & 0.0001
\end{matrix}
$$

The B_c is equal: $B_c = 1.0e+04 * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.4344 & -0.1806 & 4.0883 \\ 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0139 \\ -0.2442 & -0.0023 & -0.0858 \\ 2.7517 & 0.3465 & -7.8429 \\ 0.1273 & 0.0160 & -0.3630 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$;

(d) Summary Here,We use LQG caculating all required martix,those matrix will all use in the Part2.

3. **Part2: Results** First,I will give the simulink block diagram show figure 1:

References

[1] G. O. Young, "Synthetic structure of industrial plastics," in *Plastics,* 2$^{\text{nd}}$ ed., vol. 3, J. Peters, Ed. New York, NY, USA: McGraw-Hill, 1964, pp. 15–64.

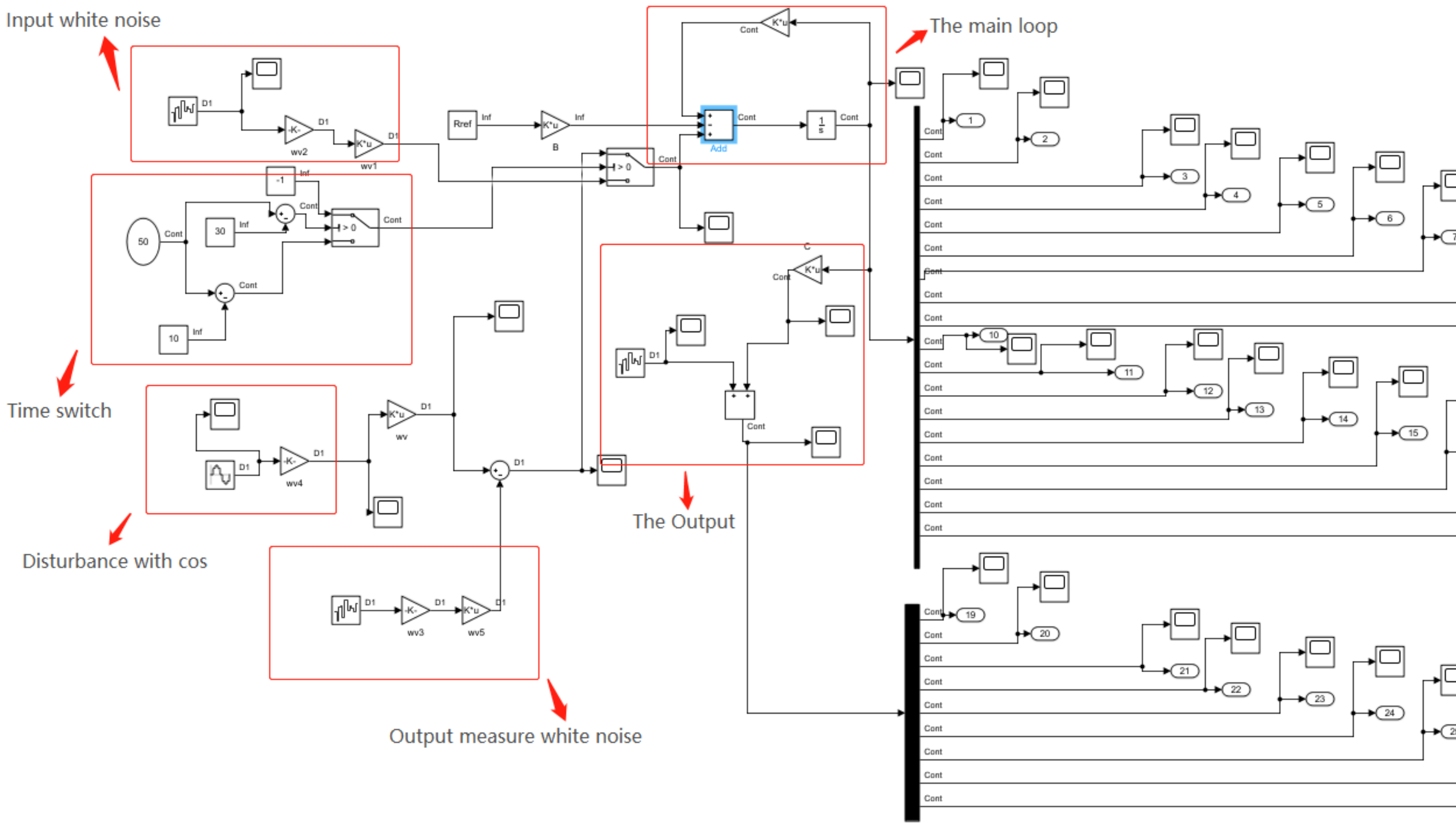

Figure 1: The Unmanned Helicopter LQG Controller diagram