MAE5403 (Linear System Control Design Problem)

Unmanned Helicopter Controller Design

Lzyue

May 11, 2020

1. **Abstract**

In this Unmanned Helicopter case. We consider 9 state space variable. There are three angle(Roll,Pitch,Yaw) angular rates and three unmeasured variable($a_s, b_s, \delta_{ped,int}$). This system also has three inputs and three dsiturbance wind noise$\delta_{lat}, \delta_{lon}, \delta_{ped}$ and $u_{wind}, u_{wind}, \delta_{wind}$,respectively. I choose LQG technique in this case for Inner-loop Flight control. I also try H_∞,but I have some problem about reduced order Controller design, I can not caculate the right nums. So I decide to use LQG for this case. And I use simulink to simulation my controller,It have good performance that I will show in Part2(Results analysis).

2. **Part1: Mathmactics and Fligh dynamics model analysis**

(a) Here,We Let

$$\dot{x} = Ax + Bu + Ew$$

$$y = C_1 x + D_1 v$$

And the noise **v** is the measurement output noise,In this case i choose the white noise for output measurement noise. The state variable is $x = \begin{Bmatrix} \phi \\ \theta \\ p \\ q \\ a_s \\ b_s \\ r \\ \delta_{ped,int} \\ \psi \end{Bmatrix}$; $u = \begin{Bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{Bmatrix}$; $w = \begin{Bmatrix} u_{wind} \\ v_{wind} \\ w_{wind} \end{Bmatrix}$;

(b) And from dynamics linearization model,we also obtain the A,B and E martix,They showed the fellowing.

$$A=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\ 0 & 0 & 0 & 0.9992 & 0 & 0 & -0.0389 & 0 & 0 \\ 0 & 0 & -0.0302 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.529 & 0 \\ 0 & 0 & 0 & -0.0707 & 267.7499 & -0.0003 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.0000 & -3.3607 & 2.2223 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2.4483 & -3.3607 & 0 & 0 & 0 \\ 0 & 0 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.0389 & 0 & 0 & 0.9992 & 0 & 0 \end{bmatrix};$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 43.3635 \\ 0 & 0 & 0 \\ 0.2026 & 2.5878 & 0 \\ 2.5878 & -0.0663 & 0 \\ 0 & 0 & -83.1883 \\ 0 & 0 & -3.8500 \\ 0 & 0 & 0 \end{bmatrix}; E=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0001 & 0.1756 & -0.0395 \\ 0.0000 & 0.0003 & 0.0338 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0002 & -0.3396 & 0.6424 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$C_1=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

(c) Choose the LQG technique for this case. Before we start the design, we need to know the pair (A,B) and (A,C1)is controllable and observable. It's obvious full rank with the matlab command rank(ctrb(A,B)) and rank(obsv(A,C1)). So we can do the job now. we need to three steps for solving this case, I will show as fellowing:

First:,We need to design a LQR control law $u = -Fx$,and we need to choose the Matirx Q and R($Q \geq 0$,$R \geq 0$). Here is the Q and R matrix in my simulation:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};R = 0.001$$

Then,use Matlab function icare(**Note:**are function can not solve B nonsymmetric,so i choose icare more general function) to solve the Riccati Equtaion $PA + A^TP - PBR^{-1}B^TP + Q = 0,\ P \geq 0$,$F = R^{-1}B^TP$ The matlab code is

```
[X1,K1,L1] = icare(A,[],Q'*Q,[],[],[],-B*B');
F=inv(R)*B'*X1;
Abk=A-B*F;
eigabk=eig(Abk);
```

Then,We obtain the gain F matrix as fellow:

$$F = 1.0e{+}04{*}\begin{bmatrix} 0.0943 & 0.0035 & 0.0652 & 0.0039 & 0.1251 & 1.5944 & 0.0356 & -0.0087 & 0.0332 \\ -0.0046 & 0.0999 & -0.0025 & 0.0724 & 1.0974 & -0.0017 & -0.0012 & 0.0003 & 0.0028 \\ 0.0331 & 0.0042 & 0.0354 & 0.0004 & -0.0045 & -0.0391 & -0.0721 & -0.1079 & -0.0943 \end{bmatrix};$$

And the matrix $A - BF$ is show:

$$Abk = 1.0e + 04 * \begin{bmatrix} 0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 \\ -1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.0883 \\ 0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0139 \\ -0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0858 \\ 2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8429 \\ 0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3630 \\ 0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 \end{bmatrix};$$

And the eigvalues is show:

$$Eigvalues = 1.0e+04 * \begin{bmatrix} -7.9558 + 0.0000i \\ -4.1816 + 0.0000i \\ -2.8032 + 0.0000i \\ -0.0023 + 0.0000i \\ -0.0016 + 0.0000i \\ -0.0005 + 0.0000i \\ -0.0001 + 0.0000i \\ -0.0001 - 0.0000i \\ -0.0001 + 0.0000i \end{bmatrix}; \text{We can see all eigvalue have negative real part.}$$

Here,I also plot bode from the pair(A,B,F,0),The figure shows fellow:

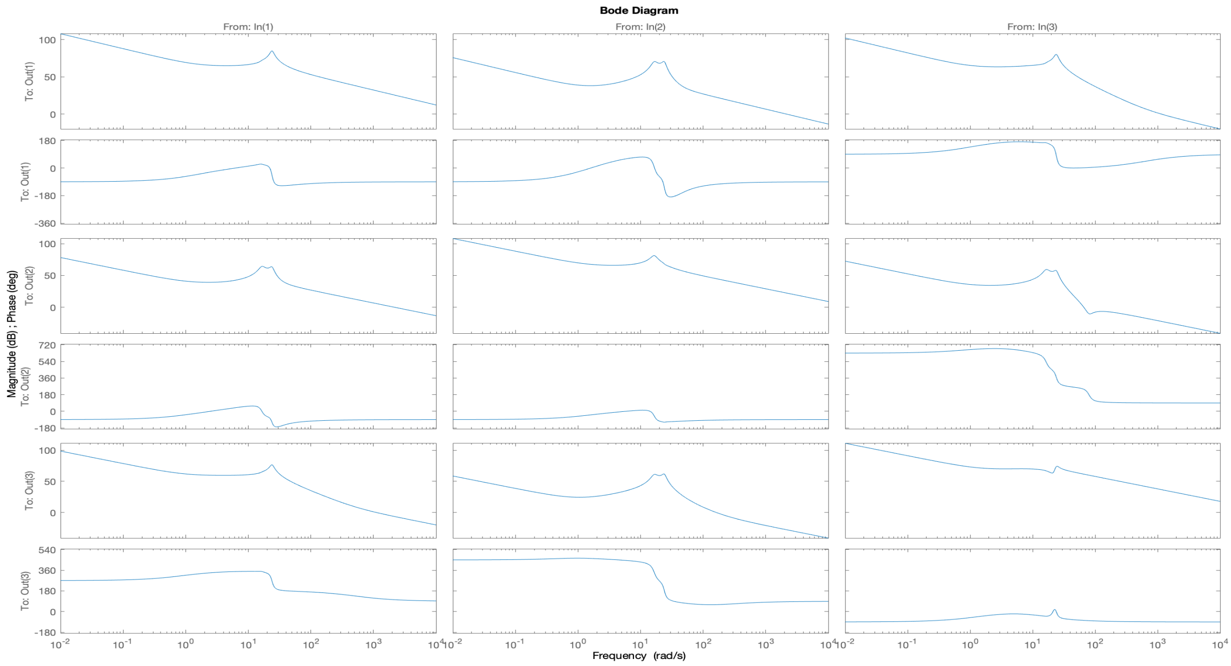


Figure 1: The Plant phase and magnitude

Second:,We need to use Kalman filter for the plant, $\dot{\hat{x}} = A\hat{x} + K_e(y - \hat{y}), \hat{y} = C_1\hat{x}$.

We also use icare to sove Riccati Equtaion:

$P_eA^T + AP_e - P_eC^TR_e^{-1}CP_e + Q_e = 0,\ P_e \geq 0, Ke = P_eC^TR_e^-1$

And here we choose the Q_e and R_e matrix as fellowing:

$$Q_e = 1.0e-03 * \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix};$$

$R_e = 0.004$

Using Matlab code as fellow:

```
[X2,K2,L2] = icare(A,[],Qe'*Qe,[],[],[],-C1*C1');
% Pk=are(A,C1,Qe)
Ke=X2*C1'*inv(Re)
Ack=A-Ke*C1
eigack=eig(Ack)
```

And we obtain Ke and $Ack = A - K_eC_1$ and Ack eigenvalues: Then,We obtain the F matrix as fellow:

$$K_e = \begin{bmatrix} 0.2500 & 0.0000 & 0.0014 & -0.0023 & 0 & 0 & 0.0007 & 0 & 0.0000 \\ 0.0000 & 0.2500 & -0.0009 & 0.0031 & 0 & 0 & -0.0005 & 0 & -0.0000 \\ 0.0014 & -0.0009 & 0.0001 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ -0.0023 & 0.0031 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.0001 \\ 0.0002 & 0.2496 & -0.0012 & 0.0033 & 0 & 0 & -0.0005 & 0 & 0.0097 \\ 0.2500 & 0.0000 & 0.0016 & -0.0021 & 0 & 0 & 0.0011 & 0 & 0.0000 \\ 0.0007 & -0.0005 & 0.0000 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ 0.0002 & -0.0097 & -0.0001 & -0.0000 & 0 & 0 & -0.0000 & 0 & 0.2498 \\ 0.0000 & -0.0000 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.2500 \end{bmatrix};$$

And the matrix $A - K_eC_1$ is show:

$$Ack = 1.0e{+}04* \begin{bmatrix} -0.2500 & -0.0000 & 0.9986 & 0.0023 & 0 & 0 & 0.0002 & 0 & - \\ -0.0000 & -0.2500 & 0.0009 & 0.9961 & 0 & 0 & -0.0384 & 0 & 0 \\ -0.0014 & 0.0009 & -0.0303 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.5290 & 0 \\ 0.0023 & -0.0031 & 0.0000 & -0.0708 & 267.7499 & -0.0003 & 0.0000 & 0 & - \\ -0.0002 & -0.2496 & 0.0012 & -1.0033 & -3.3607 & 2.2223 & 0.0005 & 0 & - \\ -0.2500 & -0.0000 & -1.0016 & 0.0021 & 2.4483 & -3.3607 & -0.0011 & 0 & - \\ -0.0007 & 0.0005 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2000 & 0 \\ -0.0002 & 0.0097 & 0.0001 & 0.0000 & 0 & 0 & -1.0000 & 0 & - \\ -0.0000 & 0.0000 & 0.0000 & 0.0388 & 0 & 0 & 0.9992 & 0 & - \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = \begin{bmatrix} -1.5367 + 23.9237i \\ -1.5367 - 23.9237i \\ -1.6190 + 16.4397i \\ -1.6190 - 16.4397i \\ -13.8925 + 0.0000i \\ -7.7978 + 0.0000i \\ -0.5274 + 0.0000i \\ -0.4993 + 0.0000i \\ -0.4999 + 0.0000i \end{bmatrix}$$; We can also see all eigvalue have negative real part.

At The last: The plant LQG control law is $u = -F\hat{(x)}$

$$\begin{cases} \dot{\hat{x}} = (A - BF - K_eC_1) + K_ey, \\ u = -F\hat{x} \end{cases}$$ And$G = [C_1(A - BF)^{-1}]^{-1}$,Using the Matlab we obtain the G:

$$G = \begin{bmatrix} -942.5693 & -34.5139 & -332.2226 \\ 46.3757 & -998.5361 & -27.8396 \\ -330.7754 & -41.6478 & 942.7901 \end{bmatrix};$$

Clearly, the closed-loop system is characterized by the following state space equation we obtain:

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} A - BF & BF \\ 0 & A - K_eC \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} - \begin{pmatrix} BG \\ 0 \end{pmatrix} r + \widetilde{v}, \widetilde{v} = \begin{pmatrix} v \\ v - K_eW \end{pmatrix} \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + w \end{cases}$$

Then,We let $A_c = \begin{bmatrix} A - BF & BF \\ 0 & A - K_eC \end{bmatrix}$,$B_c = \begin{pmatrix} BG \\ 0 \end{pmatrix}$,$C_c = \begin{bmatrix} C & 0 \end{bmatrix}$,Caculating From Matlab We Obatin the A_c, B_c, C_c as show bellow:

$A_c = 1.0e+04*$

$$\begin{matrix}
0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.0883 & 1.4344 & 0.1806 & 1.5341 & 0.0177 & -0.1934 & -1.6961 & -3.128 \\
0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0139 & 0.0071 & 0.2591 & 0.0067 & 0.1882 & 2.8651 & 0.3186 & 0.0041 \\
-0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0858 & 0.2442 & 0.0023 & 0.1688 & 0.0053 & 0.2509 & 4.1260 & 0.0922 \\
2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8429 & -2.7517 & -0.3465 & -2.9429 & -0.0339 & 0.3711 & 3.2539 & 6.0019 \\
0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3630 & -0.1273 & -0.0160 & -0.1362 & -0.0016 & 0.0172 & 0.1506 & 0.2778 \\
0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0001 & 0.0000 & 0 & 0 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0000 & 0.0001 & 0 & 0 & -0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0585 & 0.0011 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0268 & -0.0000 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & 0.0000 & -0.0001 & -0.0003 & 0.0002 & 0.0000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & -0.0000 & -0.0001 & 0.0000 & 0.0002 & -0.0003 & -0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.002 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0 & 0 & -0.000 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0 & 0 & 0.0001
\end{matrix}$$

The B_c is equal: $B_c = 1.0e+04*\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.4344 & -0.1806 & 4.0883 \\ 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0139 \\ -0.2442 & -0.0023 & -0.0858 \\ 2.7517 & 0.3465 & -7.8429 \\ 0.1273 & 0.0160 & -0.3630 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$;

(d) **Summary**

Here,We use LQG caculating all required martix,those matrix will all use in the Part2.

3. **Part2: Results** First,I will give the simulink block diagram show figure 2,It has 6 main block,the detail can be found in figure 2:

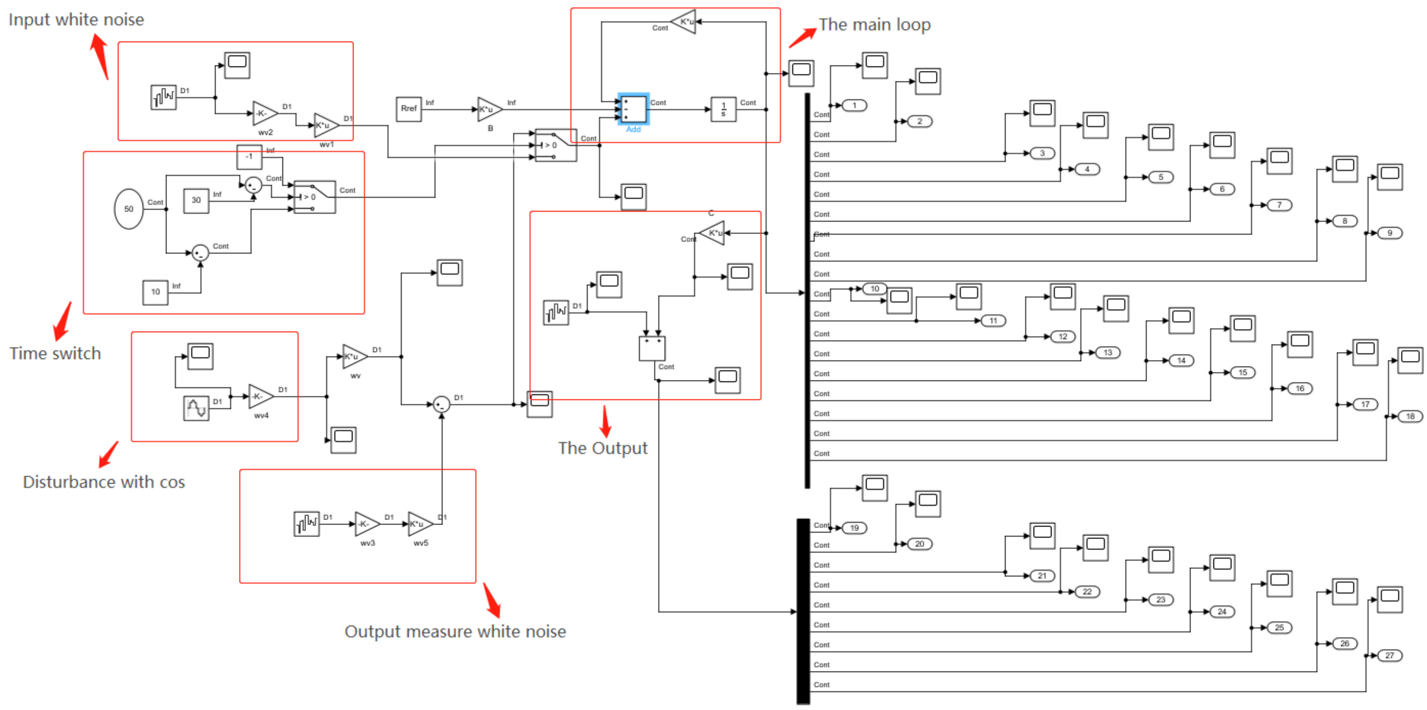


Figure 2: The Unmanned Helicopter LQG Controller diagram

Then,I will show the state and error,and y output from figure 3 and figure 4:

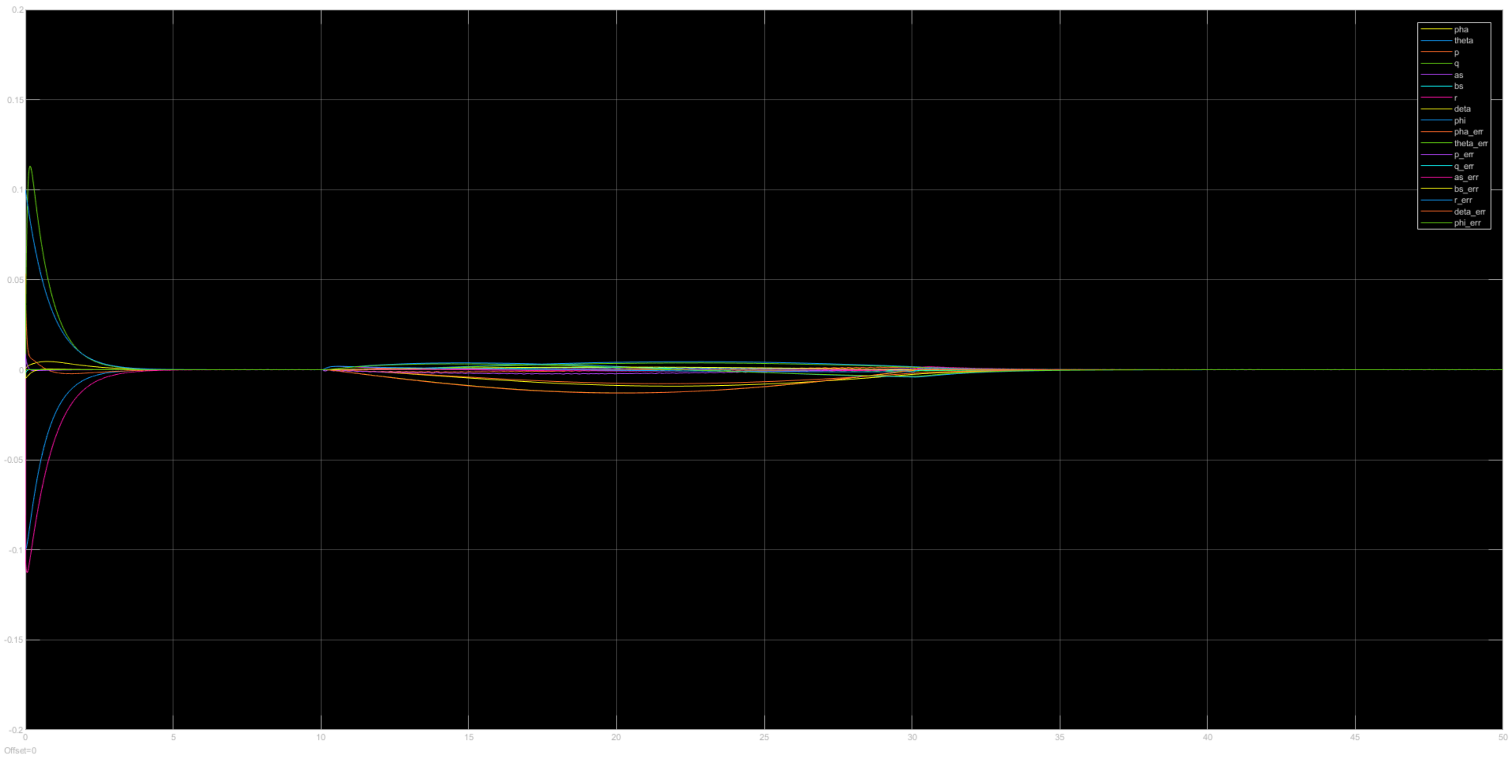


Figure 3: The Plant state x and error

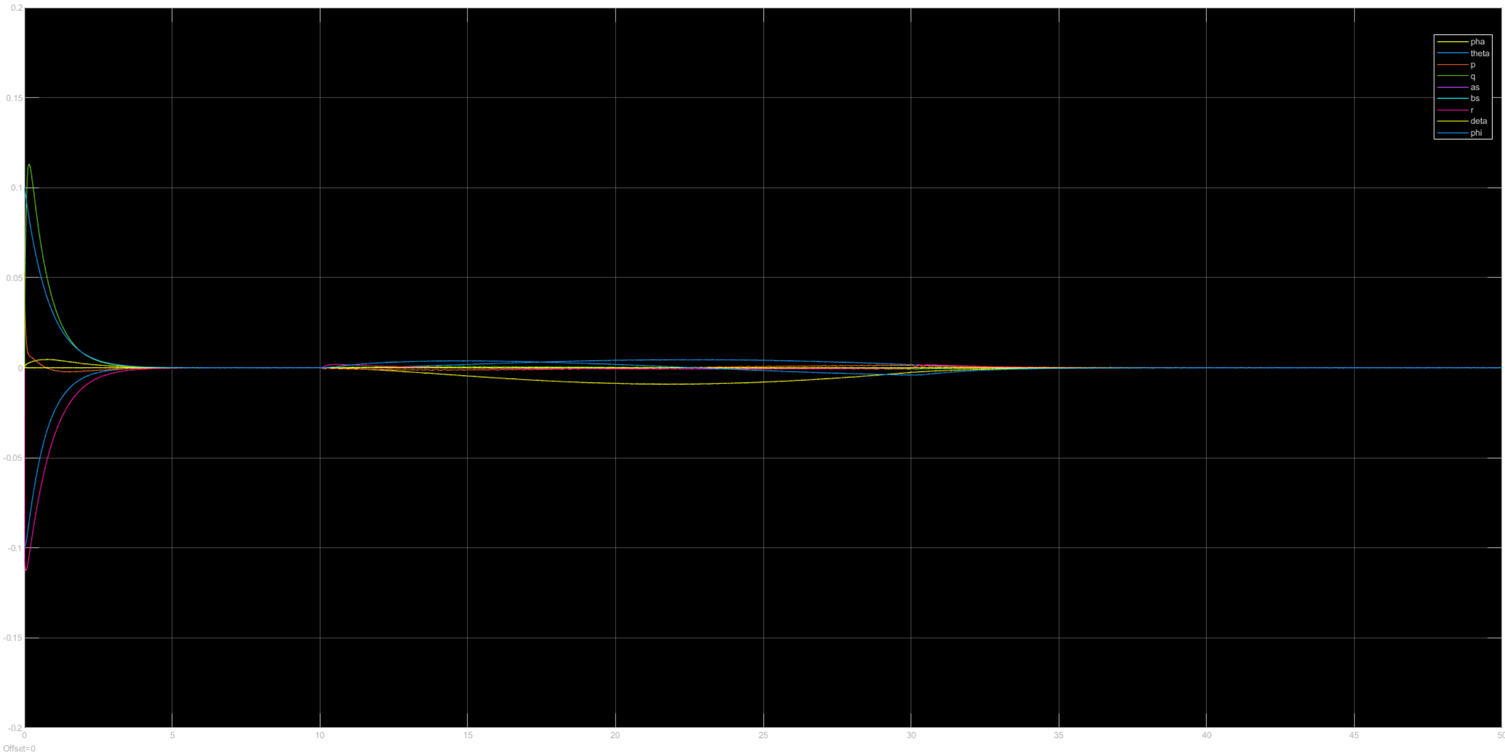
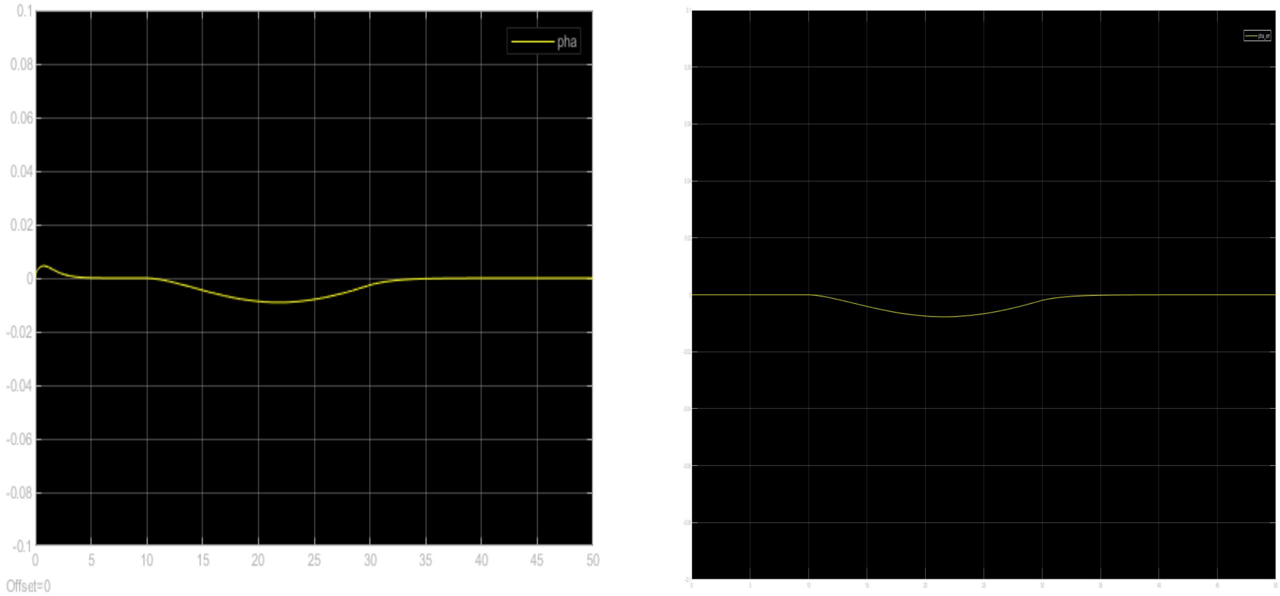


Figure 4: The Plant y output

At last,I will show all state and output respectively.



(a) State:pha

(b) Error:pha

Figure 5: Pha state and error

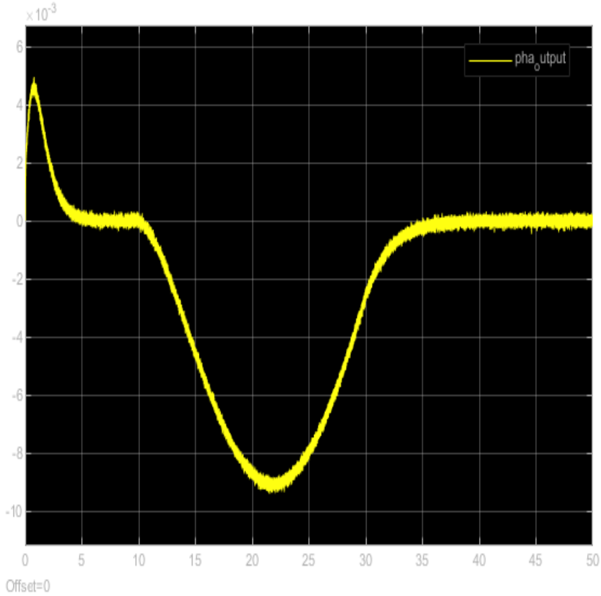
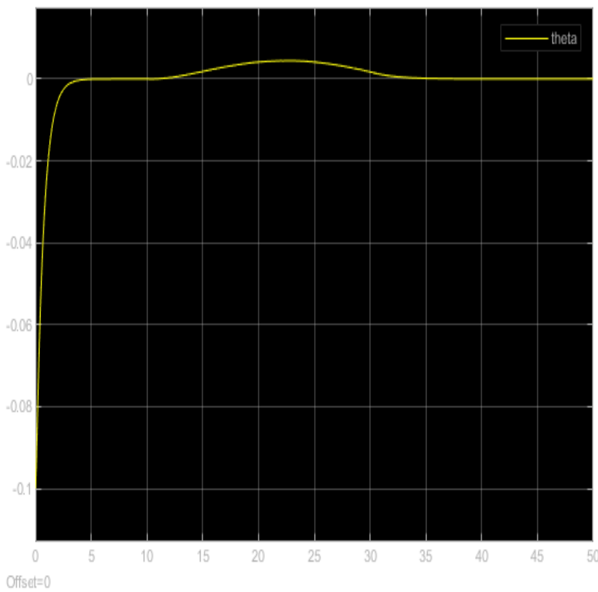
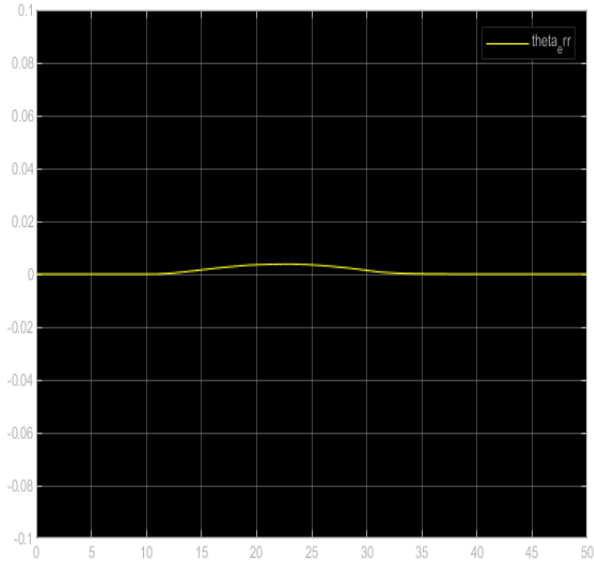


Figure 6: The Plant Pha output



(a) State:Theta



(b) Error:Theta

Figure 7: Pha state and error

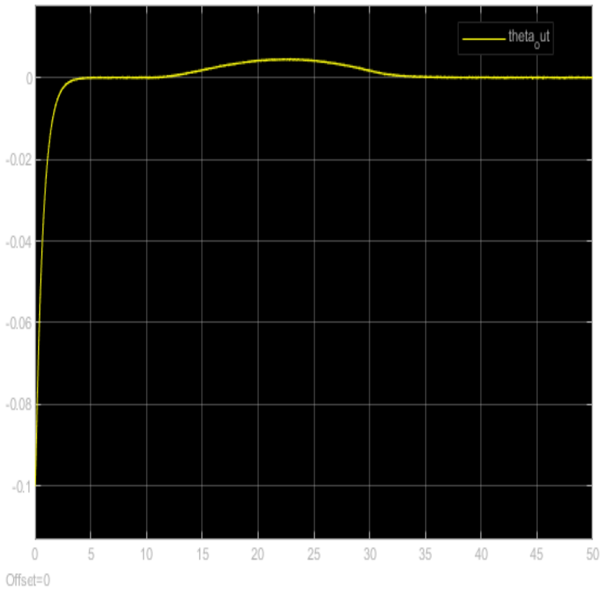
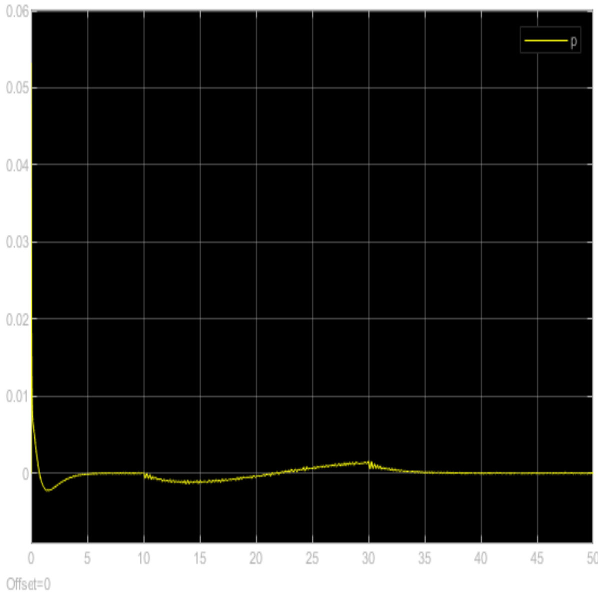
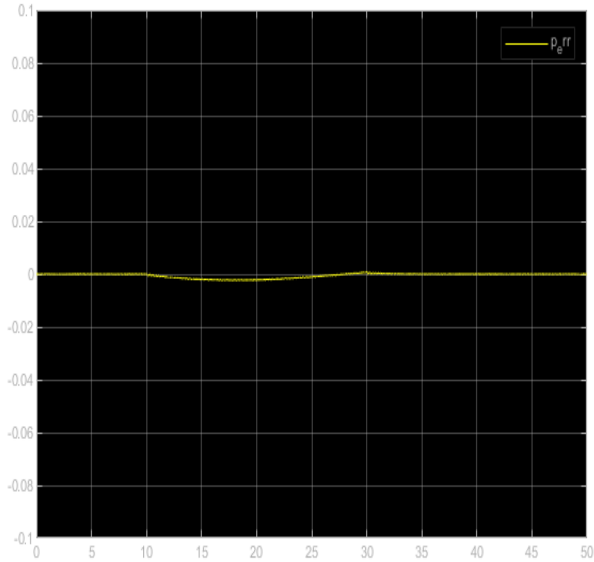


Figure 8: The Plant Theta output



(a) State:p



(b) Error:p

Figure 9: p state and error

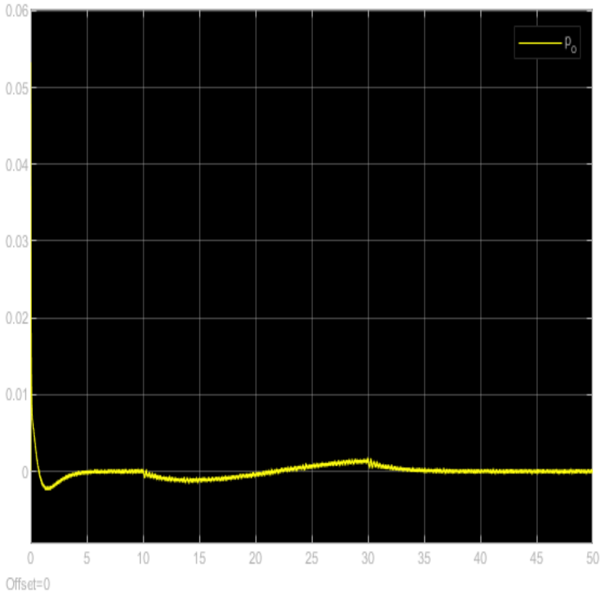
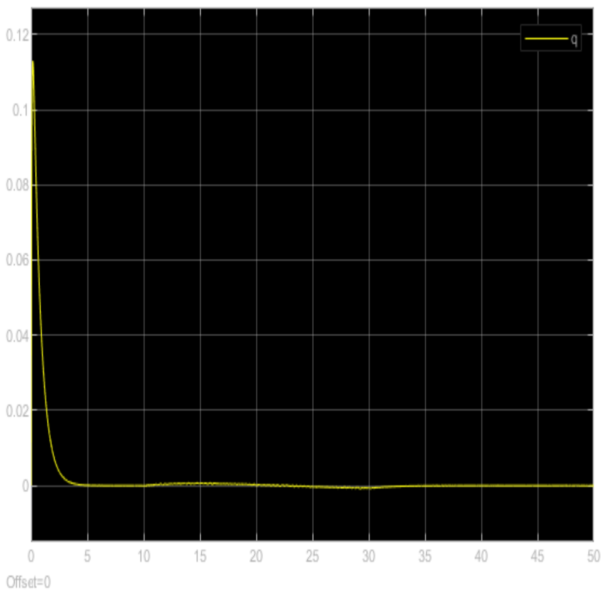
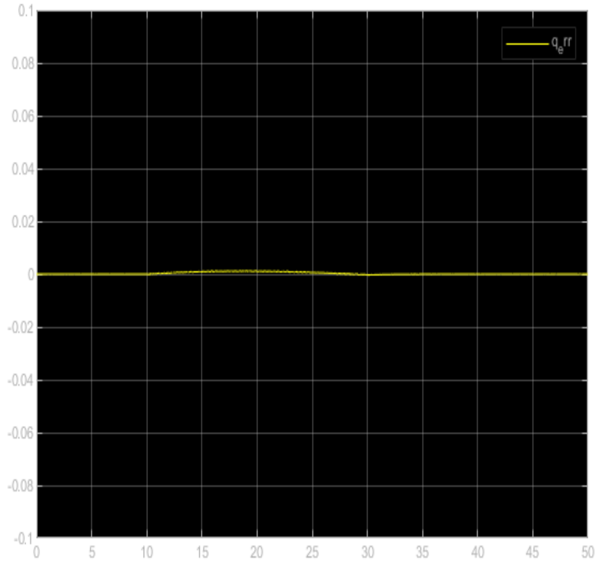


Figure 10: The Plant p output



(a) State:q



(b) Error:q

Figure 11: q state and error

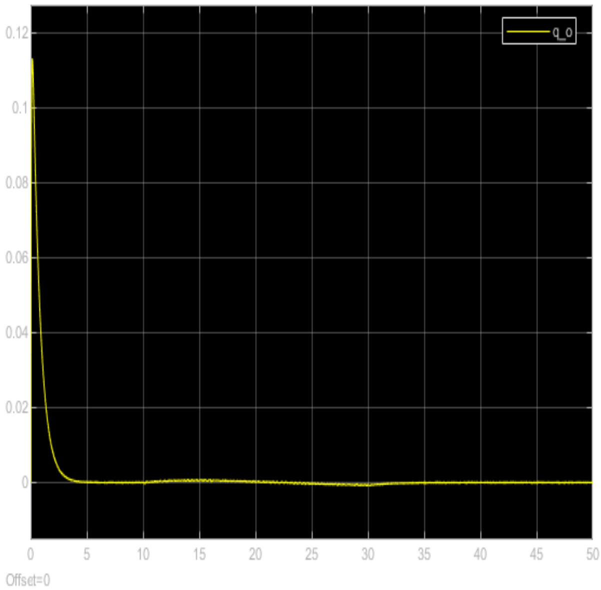
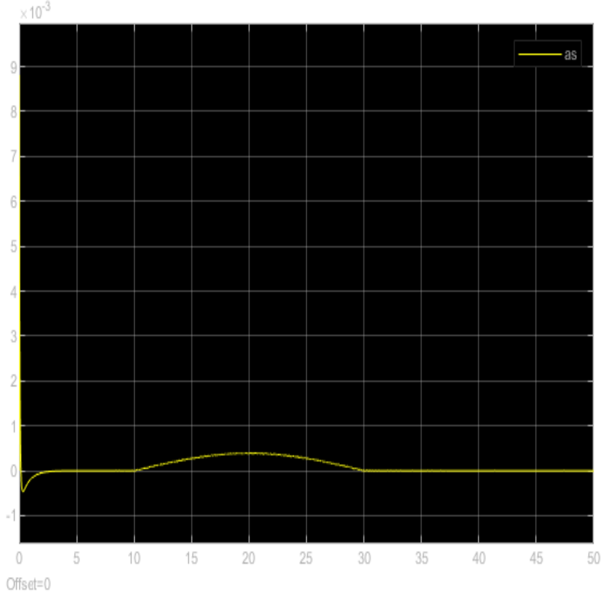
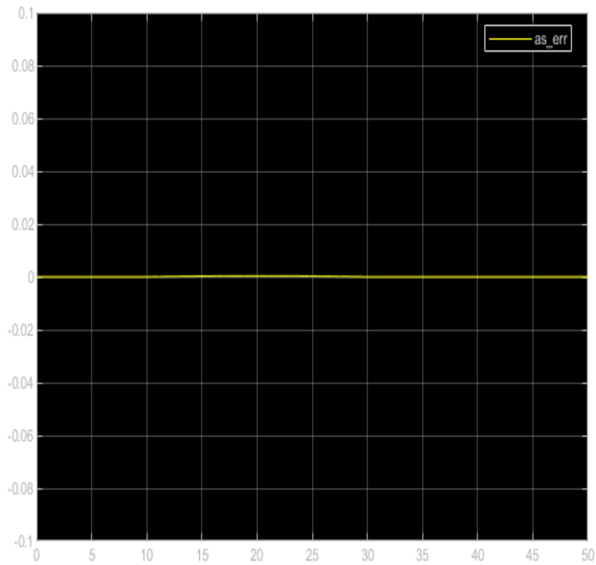


Figure 12: The Plant q output



(a) State:as



(b) Error:as

Figure 13: as state and error

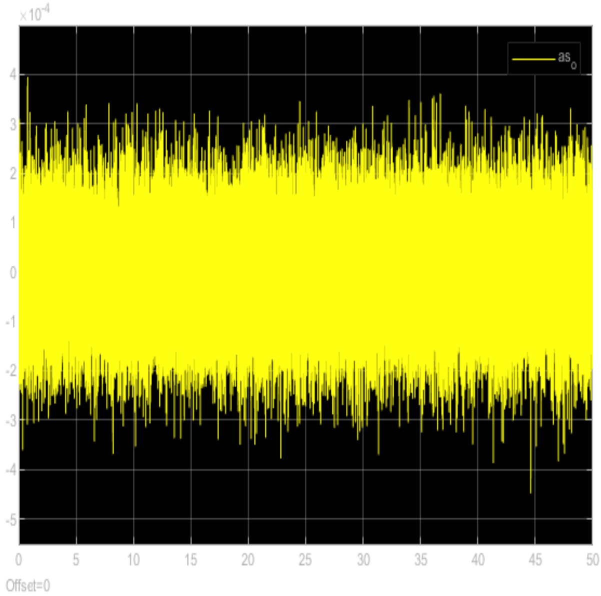
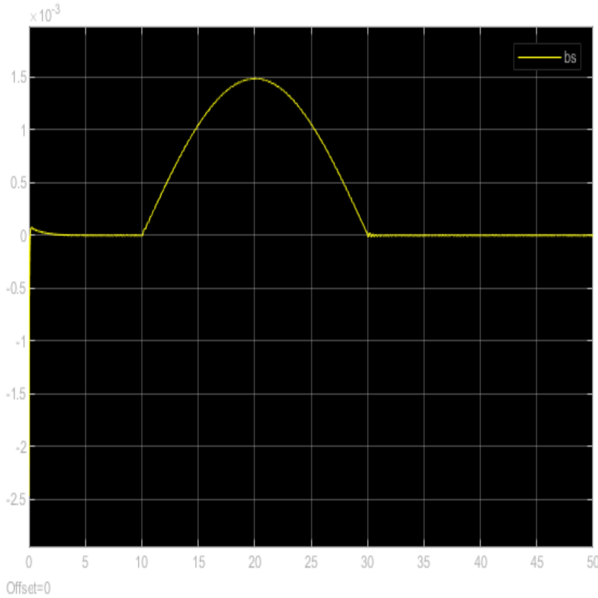
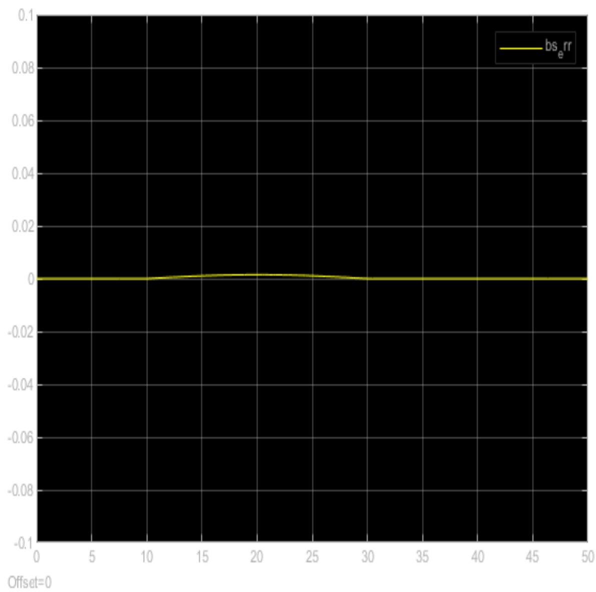


Figure 14: The Plant as output



(a) State:bs



(b) Error:bs

Figure 15: bs state and error

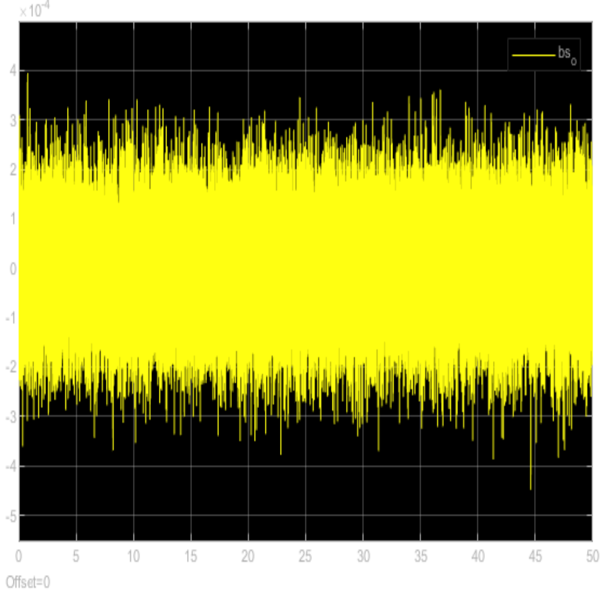
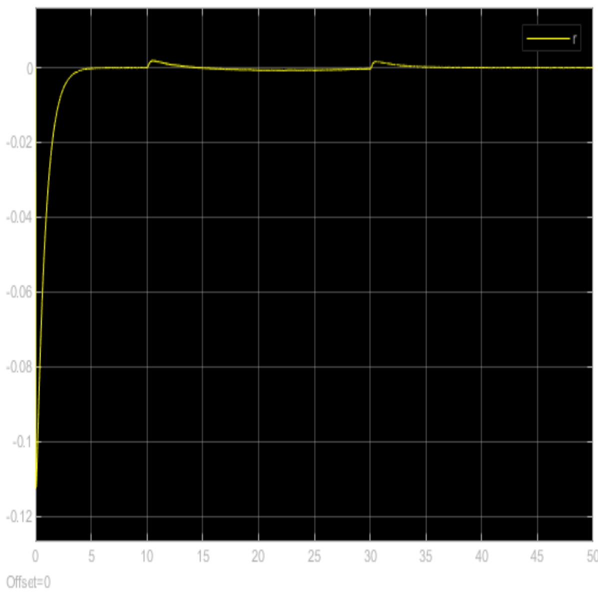
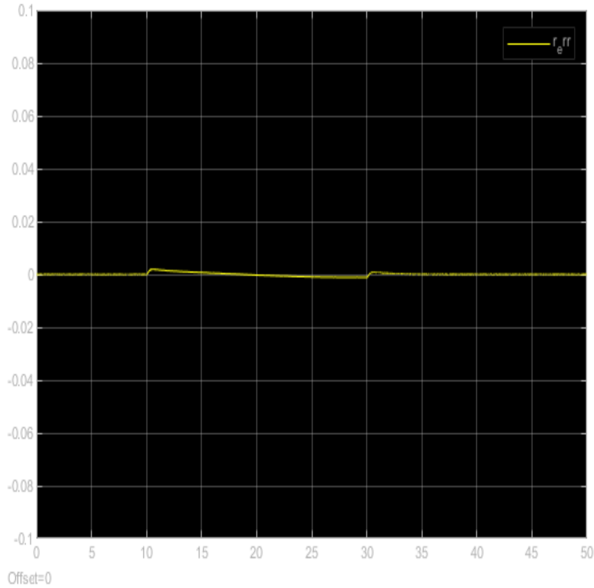


Figure 16: The Plant bs output



(a) State:r



(b) Error:r

Figure 17: r state and error

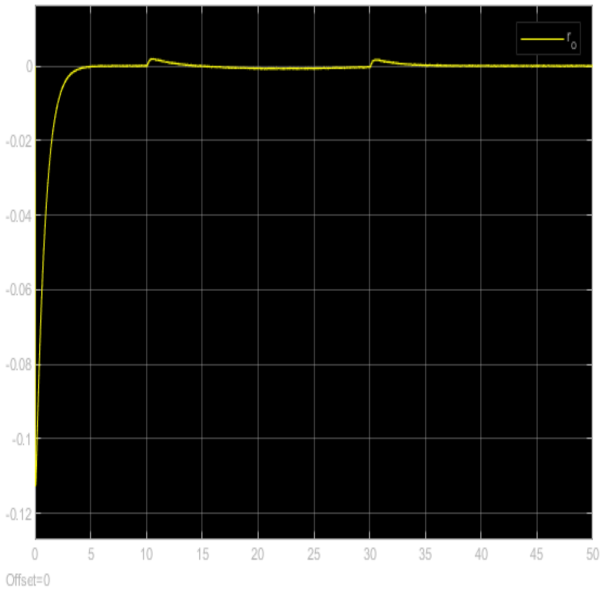
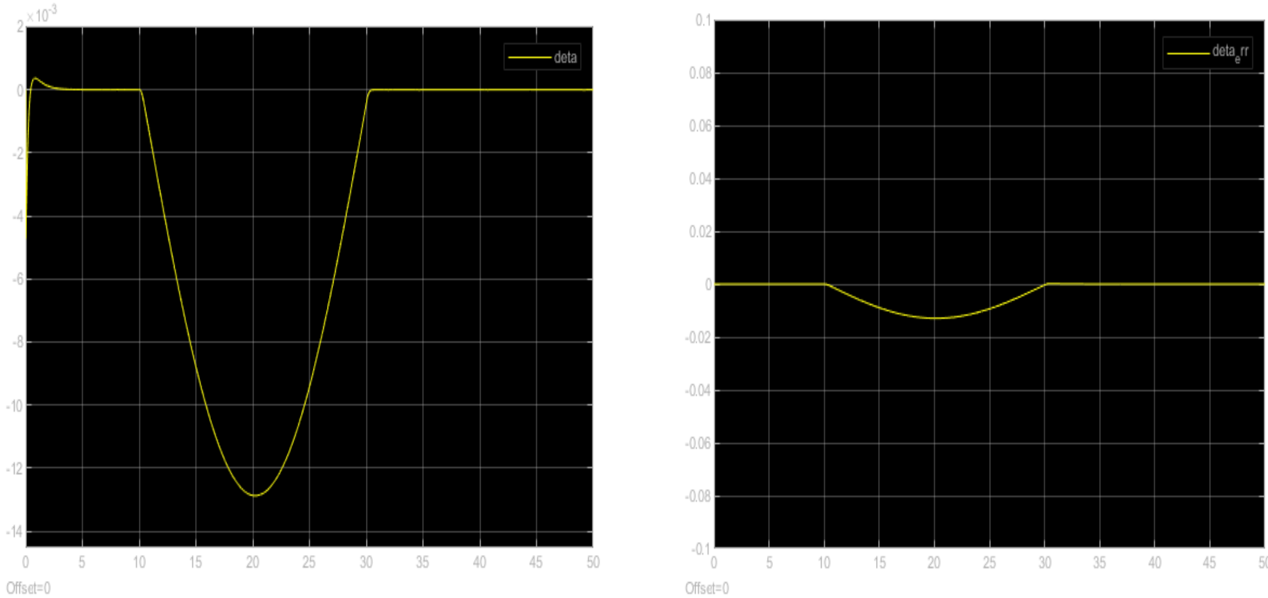


Figure 18: The Plant r output



(a) State:deta

(b) Error:deta

Figure 19: deta state and error

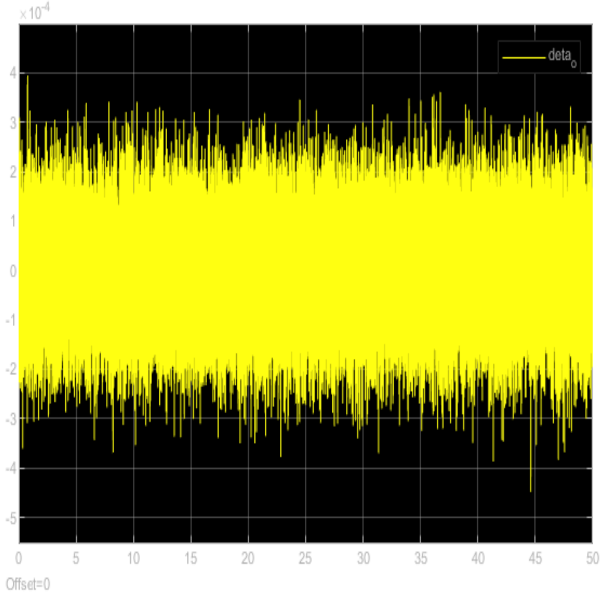
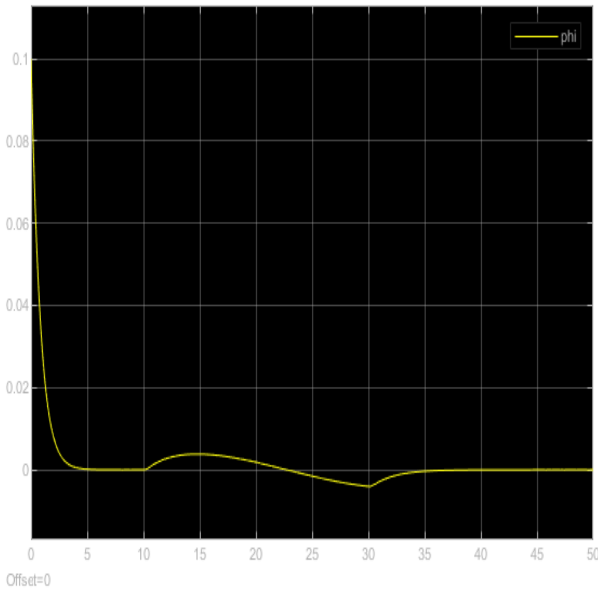
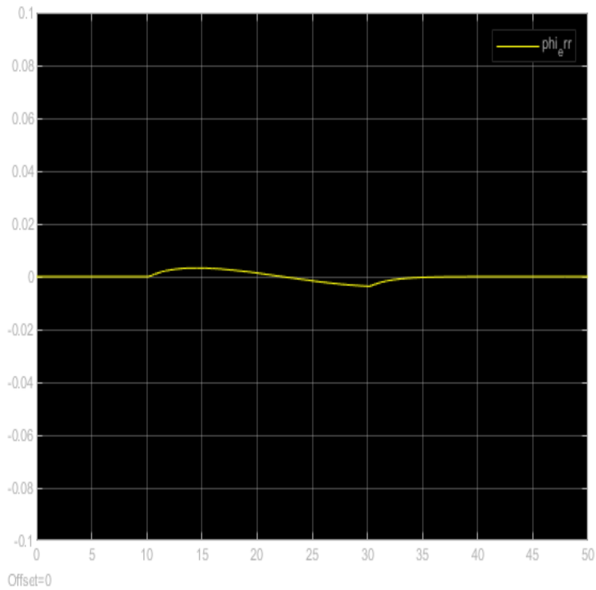


Figure 20: The Plant deta output



(a) State:phi



(b) Error:phi

Figure 21: phi state and error

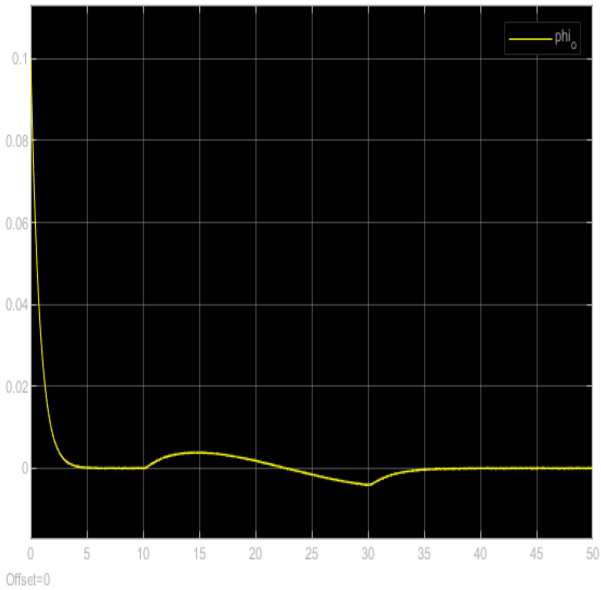


Figure 22: The Plant phi output

References

[1] Liu X , Chen B M , Lin Z . Linear systems toolkit in Matlab: structural decompositions and their applications[J]. Journal of Control Theory and Applications, 2005, 3(3):p.287-294.

[2] C. T. Chen, Linear System Theory and Design, Holt, Rinehart Winston, 1984

[3] B. M. Chen, Robust and $H\infty$ Control, Springer, 2000

[4] G. Cai, B. M. Chen, T. H. Lee, Unmanned Rotorcraft Systems, Springer, 2011