

MAE5403 (Linear System Control Design Problem)

Unmanned Helicopter Controller Design

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1. Abstract

In this Unmanned Helicopter case. We consider 9 state space variable. There are three angle(Roll,Pitch,Yaw) angular rates and three unmeasured variable($a_s, b_s, \delta_{ped,int}$). This system also has three input and three disturbance $\delta_{lat}, \delta_{lon}, \delta_{ped}$ and $u_{wind}, v_{wind}, w_{wind}$, respectively. I choose LQG technique in this case for Inner-loop Flight control. I also try H_∞ , but I have some problem about reduced order Controller design, I can not calculate the right nums. So I decide to use LQG for this case. And I use simulink to simulation my controller, It have good performance that I will show in Part2(Results analysis).

2. Part1: Mathmactics and Fligh dynamics model analysis

(a) Here, We Let

$$\dot{x} = Ax + Bu + Ew$$

$$y = C_1x + D_1v$$

And the noise v is the measurement output noise, In this case i choose the white noise for output

measurement noise. The state variable is $x = \begin{Bmatrix} \phi \\ \theta \\ p \\ q \\ a_s \\ b_s \\ r \\ \delta_{ped,int} \\ \psi \end{Bmatrix}$; $u = \begin{Bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{Bmatrix}$; $w = \begin{Bmatrix} u_{wind} \\ v_{wind} \\ w_{wind} \end{Bmatrix}$;

(b) And from dynamics linearization model, we also obtain the A, B and E matrix. They showed the following.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\ 0 & 0 & 0 & 0.9992 & 0 & 0 & -0.0389 & 0 & 0 \\ 0 & 0 & -0.0302 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.529 & 0 \\ 0 & 0 & 0 & -0.0707 & 267.7499 & -0.0003 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.0000 & -3.3607 & 2.2223 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2.4483 & -3.3607 & 0 & 0 & 0 \\ 0 & 0 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.0389 & 0 & 0 & 0.9992 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 43.3635 \\ 0 & 0 & 0 \\ 0.2026 & 2.5878 & 0 \\ 2.5878 & -0.0663 & 0 \\ 0 & 0 & -83.1883 \\ 0 & 0 & -3.8500 \\ 0 & 0 & 0 \end{bmatrix}; E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0001 & 0.1756 & -0.0395 \\ 0.0000 & 0.0003 & 0.0338 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0002 & -0.3396 & 0.6424 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) Choose the LQG technique for this. Before we start the design we need to know the pair (A,B) and (A,C1) is controllable and observable. It's obvious full rank with the matlab command rank(ctrb(A,B)) and rank(observ(A,C1)). So we can do the job now. we need to three steps, they show as following:

First: We need to design a LQR control law $u = -Fx$, and we need to choose the Matrix Q and R ($Q \geq 0, R \geq 0$). Here In my simulation I choose the Q and R matrix as follow.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R = 0.001$$

Then, use Matlab function icare to solve the Riccati Equation $PA + A^T P - PBR^{-1}B^T P + Q = 0$,

$P \geq 0, F = R^{-1}B^T P$ The matlab function is

```
[X1,K1,L1] = icare(A,[],Q'*Q,[],[],[],-B*B');
F=inv(R)*B'*X1;
Abk=A-B*F;
eigabk=eig(Abk);
```

Then, We obtain the F matrix as follow:

$$F = 1.0e+04 * \begin{bmatrix} 0.0943 & 0.0035 & 0.0652 & 0.0039 & 0.1251 & 1.5944 & 0.0356 & -0.0087 & 0.0332 \\ -0.0046 & 0.0999 & -0.0025 & 0.0724 & 1.0974 & -0.0017 & -0.0012 & 0.0003 & 0.0028 \\ 0.0331 & 0.0042 & 0.0354 & 0.0004 & -0.0045 & -0.0391 & -0.0721 & -0.1079 & -0.0943 \end{bmatrix};$$

And the matrix $A - BF$ is show:

$$Abk = 1.0e+04 * \begin{bmatrix} 0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 \\ -1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.08 \\ 0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0 \\ -0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0 \\ 2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8 \\ 0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3 \\ 0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = 1.0e + 04 * \begin{bmatrix} -7.9558 + 0.0000i \\ -4.1816 + 0.0000i \\ -2.8032 + 0.0000i \\ -0.0023 + 0.0000i \\ -0.0016 + 0.0000i \\ -0.0005 + 0.0000i \\ -0.0001 + 0.0000i \\ -0.0001 - 0.0000i \\ -0.0001 + 0.0000i \end{bmatrix}; \text{ We can see all eigvalue have negative real part.}$$

Second: We need to use Kalman filter for the plant, $\dot{\hat{x}} = A\hat{x} + K_e(y - \hat{y}), \hat{y} = C_1\hat{x}$

We also use icare to solve Riccati Equation: $P_e A^T + A P_e - P_e C^T R_e^{-1} C P_e + Q_e = 0, P_e \geq 0, K_e = P_e C^T R_e^{-1}$

And here we choose the Q_e and R_e matrix is:

$$Q_e = 1.0e-03 * \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}; R_e = 0.004$$

Using Matlab code as follow:

```
[X2,K2,L2] = icare(A,[],Qe'*Qe,[],[],[],-C1*C1');
% Pk=are(A,C1,Qe)
Ke=X2*C1'*inv(Re)
Ack=A-Ke*C1
eigack=eig(Ack)
```

And we obtain K_e and $Ack = A - K_e C_1$ and Ack eigenvalues: Then, We obtain the F matrix as follow:

$$K_e = \begin{bmatrix} 0.2500 & 0.0000 & 0.0014 & -0.0023 & 0 & 0 & 0.0007 & 0 & 0.0000 \\ 0.0000 & 0.2500 & -0.0009 & 0.0031 & 0 & 0 & -0.0005 & 0 & -0.0000 \\ 0.0014 & -0.0009 & 0.0001 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ -0.0023 & 0.0031 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.0001 \\ 0.0002 & 0.2496 & -0.0012 & 0.0033 & 0 & 0 & -0.0005 & 0 & 0.0097 \\ 0.2500 & 0.0000 & 0.0016 & -0.0021 & 0 & 0 & 0.0011 & 0 & 0.0000 \\ 0.0007 & -0.0005 & 0.0000 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ 0.0002 & -0.0097 & -0.0001 & -0.0000 & 0 & 0 & -0.0000 & 0 & 0.2498 \\ 0.0000 & -0.0000 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.2500 \end{bmatrix};$$

And the matrix $A - K_e C_1$ is show:

$$Ack = 1.0e+04 * \begin{bmatrix} -0.2500 & -0.0000 & 0.9986 & 0.0023 & 0 & 0 & 0.0002 & 0 & - \\ -0.0000 & -0.2500 & 0.0009 & 0.9961 & 0 & 0 & -0.0384 & 0 & 0 \\ -0.0014 & 0.0009 & -0.0303 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.5290 & 0 \\ 0.0023 & -0.0031 & 0.0000 & -0.0708 & 267.7499 & -0.0003 & 0.0000 & 0 & - \\ -0.0002 & -0.2496 & 0.0012 & -1.0033 & -3.3607 & 2.2223 & 0.0005 & 0 & - \\ -0.2500 & -0.0000 & -1.0016 & 0.0021 & 2.4483 & -3.3607 & -0.0011 & 0 & - \\ -0.0007 & 0.0005 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2000 & 0 \\ -0.0002 & 0.0097 & 0.0001 & 0.0000 & 0 & 0 & -1.0000 & 0 & - \\ -0.0000 & 0.0000 & 0.0000 & 0.0388 & 0 & 0 & 0.9992 & 0 & - \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = \begin{bmatrix} -1.5367 + 23.9237i \\ -1.5367 - 23.9237i \\ -1.6190 + 16.4397i \\ -1.6190 - 16.4397i \\ -13.8925 + 0.0000i \\ -7.7978 + 0.0000i \\ -0.5274 + 0.0000i \\ -0.4993 + 0.0000i \\ -0.4999 + 0.0000i \end{bmatrix}; \text{ We can also see all eigvalue have negative real part.}$$

The last: The plant LQG control law is $u = -F\hat{x}$

$$\begin{cases} \dot{\hat{x}} = (A - BF - K_e C_1) \hat{x} + K_e y, \\ u = -F\hat{x} \end{cases} \quad \text{And, } G = [C_1(A - BF)^{-1}]^{-1}, \text{ Using the Matlab we obtain the } G:$$

$$G = \begin{bmatrix} -942.5693 & -34.5139 & -332.2226 \\ 46.3757 & -998.5361 & -27.8396 \\ -330.7754 & -41.6478 & 942.7901 \end{bmatrix};$$

Clearly, the closed-loop system is characterized by the following state space equation we can

$$\text{obtain: } \begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} - \begin{pmatrix} BG \\ 0 \end{pmatrix} r + \tilde{v}, \tilde{v} = \begin{pmatrix} v \\ v - K_e W \end{pmatrix} \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + w \end{cases} \quad \text{Then, We let}$$

$$A_c = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix}, B_c = \begin{pmatrix} BG \\ 0 \end{pmatrix}, C_c = \begin{bmatrix} C & 0 \end{bmatrix}, \text{ Caculating From Matlab We Obatin the}$$

A_c, B_c, C_c as show bellow:

$$A_c = 1.0e + 04 *$$

0	0	0.0001	0	0	0	0.0000	0	0	0	0	0	0	0	0	0
0	0	0	0.0001	0	0	-0.0000	0	0	0	0	0	0	0	0	0
-1.4344	-0.1806	-1.5341	-0.0177	0.1934	1.7547	3.1298	4.6733	4.0883	1.4344	0.1806	1.5341	0.0177	-0.1934	-1.6961	-3.1288
0	0	0	-0.0000	0.0268	-0.0000	0	0	0	0	0	0	0	0	0	0
-0.0071	-0.2591	-0.0067	-0.1883	-2.8654	-0.3184	-0.0041	0.0010	-0.0139	0.0071	0.2591	0.0067	0.1882	2.8651	0.3186	0.0041
-0.2442	-0.0023	-0.1689	-0.0053	-0.2506	-4.1263	-0.0922	0.0225	-0.0858	0.2442	0.0023	0.1688	0.0053	0.2509	4.1260	0.0922
2.7517	0.3465	2.9430	0.0339	-0.3711	-3.2539	-6.0041	-8.9653	-7.8429	-2.7517	-0.3465	-2.9429	-0.0339	0.3711	3.2539	6.0019
0.1273	0.0160	0.1362	0.0016	-0.0172	-0.1506	-0.2779	-0.4154	-0.3630	-0.1273	-0.0160	-0.1362	-0.0016	0.0172	0.1506	0.2778
0	0	0	0.0000	0	0	0.0001	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0001	0.0000	0	0	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0000	0.0001	0	0	-0.0000
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0585	0.0011
0	0	0	0	0	0	0	0	0	0.0000	-0.0000	0.0000	-0.0000	0.0268	-0.0000	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0000	-0.0001	-0.0003	0.0002	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	-0.0001	0.0000	0.0002	-0.0003	-0.0000
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0023
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0	0	-0.0003
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0	0	0.0001

The B_c is equal: $B_c = 1.0e + 04 *$

0	0	0
0	0	0
-1.4344	-0.1806	4.0883
0	0	0
-0.0071	-0.2591	-0.0139
-0.2442	-0.0023	-0.0858
2.7517	0.3465	-7.8429
0.1273	0.0160	-0.3630
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0

;

(d) Summary Here, We use LQG calculating all required matrix, those matrix will all use in the Part2.

3. **Part2: Results** First, I will give the simulink block diagram show figure 1, It has 6 main block, the detail can be found in figure 1:

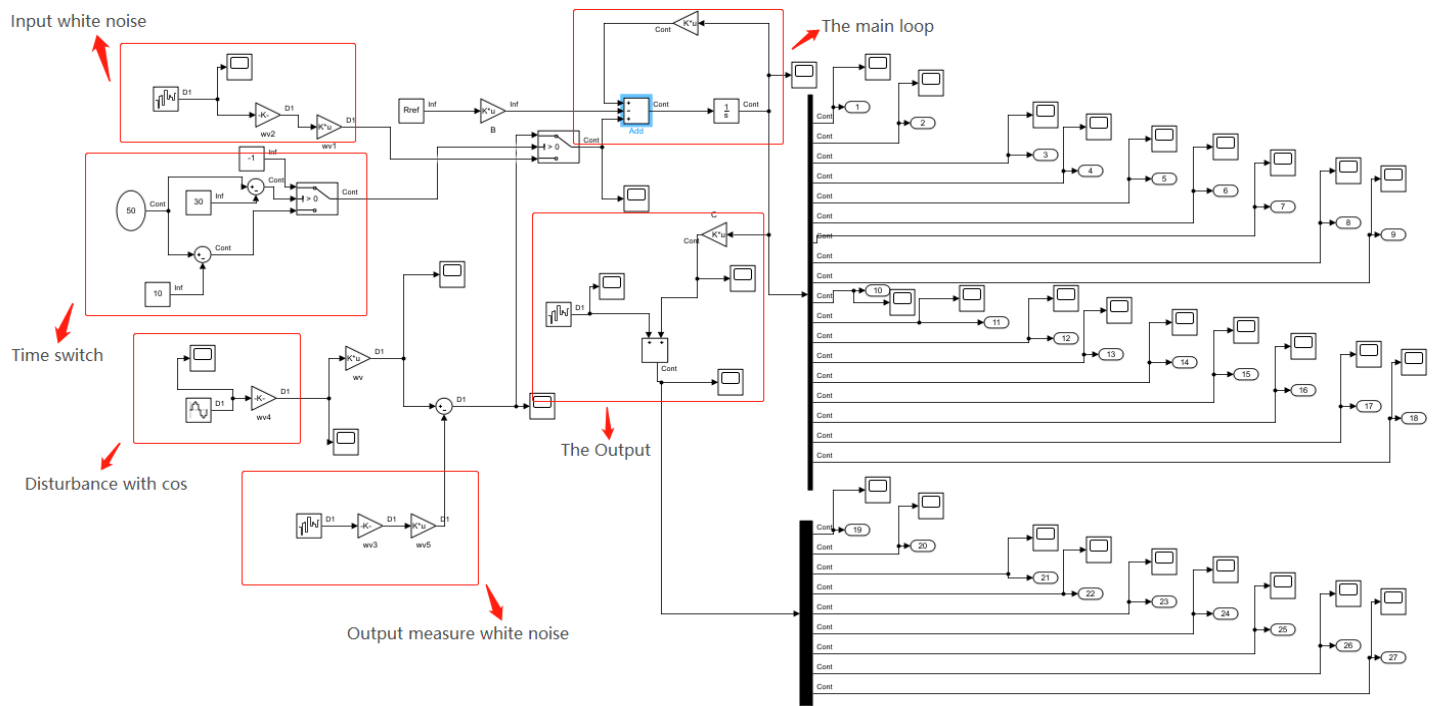


Figure 1: The Unmanned Helicopter LQG Controller diagram

Then,I will show the state and error ,and y output from figure 2 and figure 3:

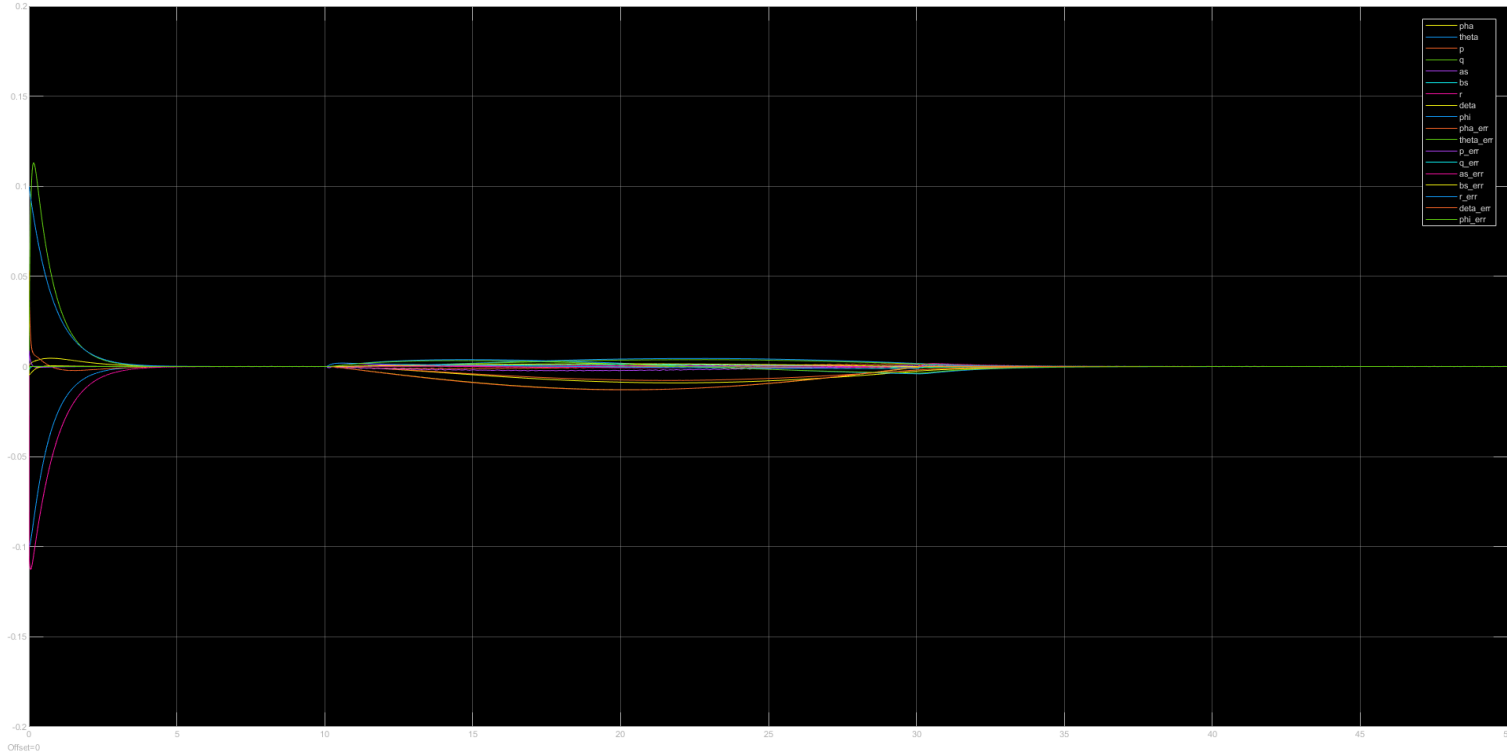


Figure 2: The Plant state x and error

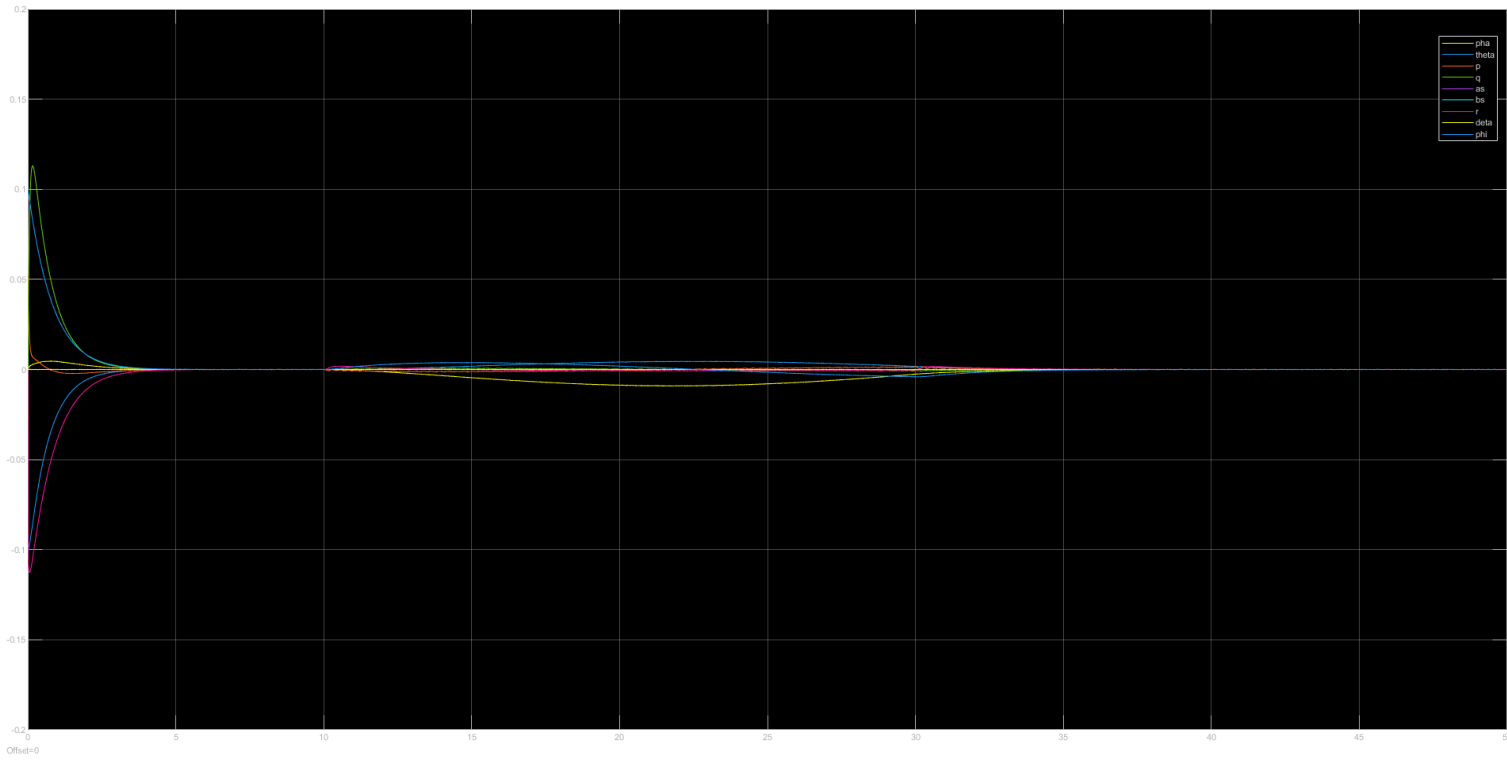
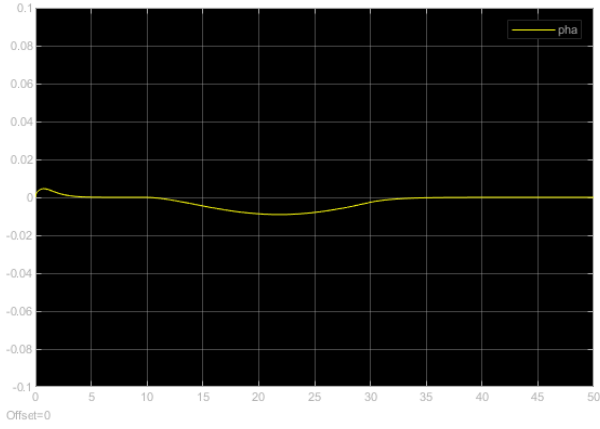
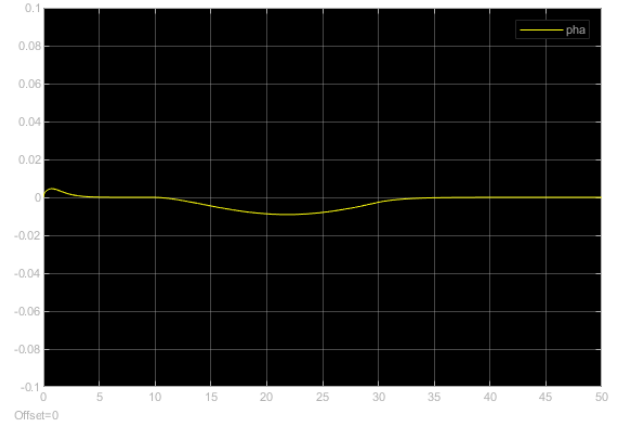


Figure 3: The Plant y output

At last,I will show all state and output respectively.



(a) State:pha



(b) Error:pha

Figure 4: Pha state and error

References

- [1] G. O. Young, "Synthetic structure of industrial plastics," in *Plastics*, 2nd ed., vol. 3, J. Peters, Ed. New York, NY, USA: McGraw-Hill, 1964, pp. 15–64.