# ENGG 5403 Linear System Theory and Design

## Design Problem: An Unmanned Helicopter System

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***You should refer to the problem description on an unmanned helicopter flight control system design in the appendix. You are free to choose any one of the following choices…***

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**Choice 1:**

Using the LQR design, the Kalman filter design and their combination, i.e., the LQG control method to design an appropriate measurement feedback control law that meets all the design specification specified in the problem.

Show all the detailed calculation and simulate your design using MATLAB and/or SIMULINK. Give all the necessary plots that show the evidence of your design.

**Choice 2:**

Using both the $H_2$ and $H_\infty$ control techniques to design appropriate measurement feedback control laws that meet all the design specification specified in the problem. In this problem setting, you might wish to formulate the wind gust as external disturbance to the system.

Show all the detailed calculation and simulate your design using MATLAB and/or SIMULINK. Give all the necessary plots that show the evidence of your design.

**Choice 3:**

Using the loop transfer recovery control technique to design appropriate measurement feedback control laws that meet all the design specification specified in the problem.

Show all the detailed calculation and simulate your design using MATLAB and/or SIMULINK. Give all the necessary plots that show the evidence of your design.

**Choice 4:**

Using whatever control techniques that you like to design appropriate measurement feedback control laws that meet all the design specification specified in the problem.

Show all the detailed calculation and simulate your design using MATLAB and/or SIMULINK. Give all the necessary plots that show the evidence of your design.

# Appendix

## An Unmanned Helicopter Control System Design

adopted from the monograph

Guowei Cai, Ben M. Chen and Tong H. Lee

*Unmanned Rotorcraft Systems*

Springer, New York, 2011

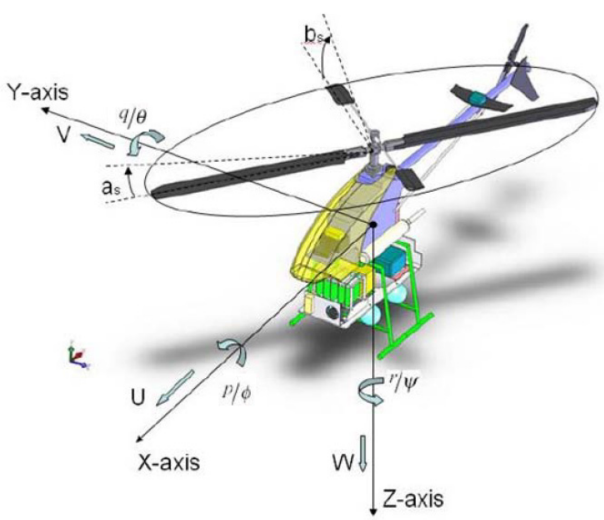
## Flight Control System Design for an Unmanned Helicopter

### 1. The Unmanned Helicopter Platform

The chopper depicted below is HeLion, a fully functional unmanned helicopter constructed by my UAV Research Team. The system consists of a small-scale bare helicopter (Raptor 90) with all necessary accessories onboard and a ground station. The unmanned helicopter system is an integration of advanced technologies developed in communications, computing and control areas. It is an excellent test bed for testing and implementing modern control techniques. We will use this platform for designing flight control laws using the techniques that we have learnt from the class.

### 2. The Linearized Model at Hovering

The dynamic model of the unmanned system can be obtained through an intensive modeling process involving test flights of the actual helicopter in hovering and near hovering flight conditions. In the figure below,




*U*, *V* and *W* are respectively the velocities of the chopper in the *x*, *y* and *z*-axis of the body frame of the helicopter; $\phi$ is the roll angle, $\theta$ is the pitch angle, and $\psi$ is the heading or yaw angle; *p*, *q* and *r* are respectively the angular rates of the roll, pitch and heading motions; and $a_s, b_s$ are respectively the longitudinal and lateral flapping angles of the tip-path-plane. A linearized state space model for the inner-loop of the helicopter system can be described as the following:

$$\dot{x} = Ax + Bu + Ew$$

where

$$x = \begin{pmatrix} \phi \\ \theta \\ p \\ q \\ a_s \\ b_s \\ r \\ \delta_{\text{ped,int}} \\ \psi \end{pmatrix}, \quad u = \begin{pmatrix} \delta_{\text{lat}} \\ \delta_{\text{lon}} \\ \delta_{\text{ped}} \end{pmatrix}, \quad w = \begin{pmatrix} u_{\text{wind}} \\ v_{\text{wind}} \\ w_{\text{wind}} \end{pmatrix},$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\ 0 & 0 & 0 & 0.9992 & 0 & 0 & -0.0389 & 0 & 0 \\ 0 & 0 & -0.0302 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.529 & 0 \\ 0 & 0 & 0 & -0.0707 & 267.7499 & -0.0003 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -3.3607 & 2.2223 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2.4483 & -3.3607 & 0 & 0 & 0 \\ 0 & 0 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.0389 & 0 & 0 & 0.9992 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 43.3635 \\ 0 & 0 & 0 \\ 0.2026 & 2.5878 & 0 \\ 2.5878 & -0.0663 & 0 \\ 0 & 0 & -83.1883 \\ 0 & 0 & -3.8500 \\ 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0001 & 0.1756 & -0.0395 \\ 0 & 0.0003 & 0.0338 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0002 & -0.3396 & 0.6424 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and where $\delta_{\text{ped,int}}$ is associated with a yaw channel built-in controller, and the control input variables $\delta_{\text{lat}}, \delta_{\text{lon}}, \delta_{\text{ped}}$ are respectively corresponding the lateral channel (for roll, left- and right-side tilting motions), the longitudinal channel (for pitch, forward and backward motions), and the pedal channel (for yaw motion) of the helicopter, $w$ represents wind gust disturbance in x-, y- and z-directions. The control inputs are normalized with 1 being equivalent to $\pi/4$ rad. Due to physical limitation, they will have to be kept within the following limits:

$$|\delta_{\text{lat}}| < 0.35, \quad |\delta_{\text{lon}}| < 0.35, \quad |\delta_{\text{ped}}| < 0.4.$$

The ideal measurement output of the system is given by

$$y = \begin{pmatrix} \phi \\ \theta \\ p \\ q \\ r \\ \psi \end{pmatrix} = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x$$

However, the actual measurement output is likely to be noisy. Depended on the control technique used to design a control law for the system, one might wish to add some fictitious noises in the above measurement equation.

## 3. Design Specifications

The primary focus of the flight control system for the hovering condition is to design a control law such that when it is applied to the actual plant

- ✧ it will stabilize the overall system;
- ✧ it will drive the dynamics of the helicopter to the hovering state (all state variables are to be driven to 0) as quickly as possible from a given initial condition.

The following initial condition is to be used for all simulations:

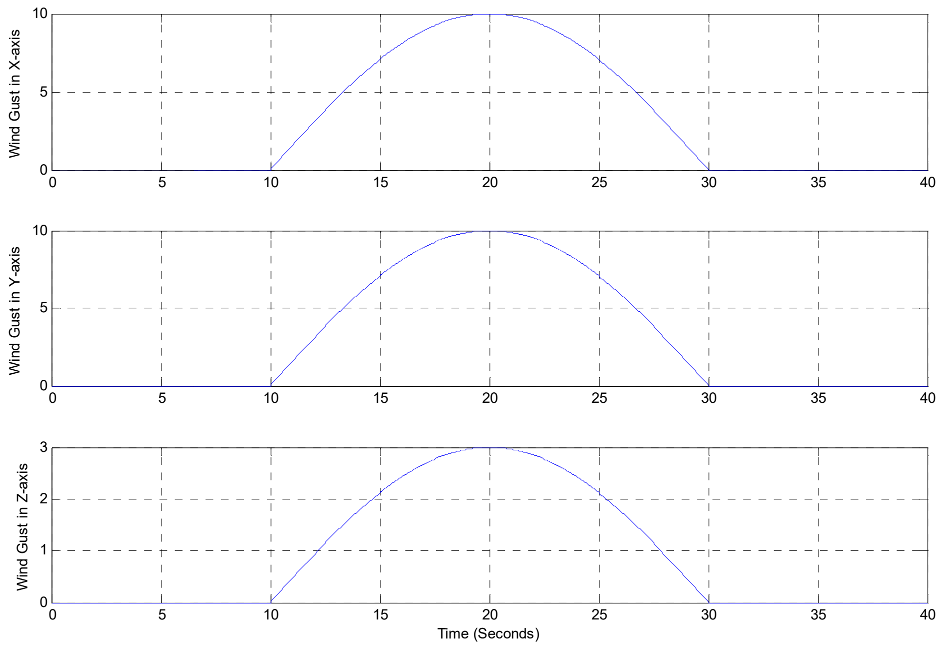
$$x_0 = \begin{pmatrix} 0 \\ -0.1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.1 \end{pmatrix}$$

Physically, it means the helicopter is commanded to hover from a pitch angle of $-0.1$ rad (a nose-down angle of 5.7 degrees) and yaw angle of 0.1 rad (5.7 degrees).

Other design considerations, such as frequency domain requirements on gain and phase margins, are to be ignored. Bonus will be given to those who have shown their design performing well in this category. For simulation purpose, you can take a wind gust

$$w = \begin{pmatrix} u_{\text{wind}} \\ v_{\text{wind}} \\ w_{\text{wind}} \end{pmatrix} = \begin{pmatrix} 10\cos\left(\frac{2\pi}{40}(t-20)\right) \\ 10\cos\left(\frac{2\pi}{40}(t-20)\right) \\ 3\cos\left(\frac{2\pi}{40}(t-20)\right) \end{pmatrix}, \quad 10 \le t \le 30,$$

and $w = 0$ elsewhere. Graphically, it can be depicted as follows:


Wind Gust in X-axis
Wind Gust in Y-axis
Wind Gust in Z-axis
Time (Seconds)
0
5
10
15
20
25
30
35
40