

MAE5403 (Linear System Control Design Problem)

Unmanned Helicopter Controller Design

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1. Abstract

In this Unmanned Helicopter case. We consider 9 state space variable. There are three angle(Roll,Pitch,Yaw) angular rates and three unmeasured variable($a_s, b_s, \delta_{ped,int}$). This system also has three inputs and three disturbance wind noise $\delta_{lat}, \delta_{lon}, \delta_{ped}$ and $u_{wind}, v_{wind}, w_{wind}$, respectively. I choose LQG technique in this case for Inner-loop Flight control. I also try H_∞ , but I have some problem about reduced order Controller design, I can not calculate the right nums. So I decide to use LQG for this case. And I use simulink to simulation my controller, It have good performance that I will show in Part2(Results analysis).

2. Part1: Mathematics and Flight dynamics model analysis

(a) Here, We Let

$$\dot{x} = Ax + Bu + Ew$$

$$y = C_1x + D_1v$$

And the noise v is the measurement output noise, In this case i choose the white noise for output

measurement noise. The state variable is $x = \begin{Bmatrix} \phi \\ \theta \\ p \\ q \\ a_s \\ b_s \\ r \\ \delta_{ped,int} \\ \psi \end{Bmatrix}$; $u = \begin{Bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{Bmatrix}$; $w = \begin{Bmatrix} u_{wind} \\ v_{wind} \\ w_{wind} \end{Bmatrix}$;

(b) And from dynamics linearization model, we also obtain the A, B and E matrix. They showed the following.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0.0009 & 0 & 0 \\ 0 & 0 & 0 & 0.9992 & 0 & 0 & -0.0389 & 0 & 0 \\ 0 & 0 & -0.0302 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.529 & 0 \\ 0 & 0 & 0 & -0.0707 & 267.7499 & -0.0003 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.0000 & -3.3607 & 2.2223 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 2.4483 & -3.3607 & 0 & 0 & 0 \\ 0 & 0 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0.0389 & 0 & 0 & 0.9992 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 43.3635 \\ 0 & 0 & 0 \\ 0.2026 & 2.5878 & 0 \\ 2.5878 & -0.0663 & 0 \\ 0 & 0 & -83.1883 \\ 0 & 0 & -3.8500 \\ 0 & 0 & 0 \end{bmatrix}; E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0001 & 0.1756 & -0.0395 \\ 0.0000 & 0.0003 & 0.0338 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0002 & -0.3396 & 0.6424 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};$$

(c) Choose the LQG technique for this case. Before we start the design, we need to know the pair (A,B) and (A,C1) is controllable and observable. It's obvious full rank with the matlab command rank(ctrb(A,B)) and rank(observ(A,C1)). So we can do the job now. we need to three steps for solving this case, I will show as following:

First:, We need to design a LQR control law $u = -Fx$, and we need to choose the Matrix Q and R ($Q \geq 0, R \geq 0$). Here is the Q and R matrix in my simulation:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R = 0.001$$

Then, use Matlab function `icare` (**Note:** are function can not solve B nonsymmetric, so i choose `icare` more general function) to solve the Riccati Equation $PA + A^T P - PBR^{-1}B^T P + Q = 0$, $P \geq 0$, $F = R^{-1}B^T P$. The matlab code is

```
[X1,K1,L1] = icare(A,[],Q'*Q,[],[],[],-B*B');
F=inv(R)*B'*X1;
Abk=A-B*F;
eigabk=eig(Abk);
```

Then, We obtain the gain F matrix as follow:

$$F = 1.0e+04 * \begin{bmatrix} 0.0943 & 0.0035 & 0.0652 & 0.0039 & 0.1251 & 1.5944 & 0.0356 & -0.0087 & 0.0332 \\ -0.0046 & 0.0999 & -0.0025 & 0.0724 & 1.0974 & -0.0017 & -0.0012 & 0.0003 & 0.0028 \\ 0.0331 & 0.0042 & 0.0354 & 0.0004 & -0.0045 & -0.0391 & -0.0721 & -0.1079 & -0.0943 \end{bmatrix};$$

And the matrix $A - BF$ is show:

$$Abk = 1.0e + 04 * \begin{bmatrix} 0 & 0 & 0.0001 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0 \\ -1.4344 & -0.1806 & -1.5341 & -0.0177 & 0.1934 & 1.7547 & 3.1298 & 4.6733 & 4.0883 \\ 0 & 0 & 0 & -0.0000 & 0.0268 & -0.0000 & 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0067 & -0.1883 & -2.8654 & -0.3184 & -0.0041 & 0.0010 & -0.0139 \\ -0.2442 & -0.0023 & -0.1689 & -0.0053 & -0.2506 & -4.1263 & -0.0922 & 0.0225 & -0.0858 \\ 2.7517 & 0.3465 & 2.9430 & 0.0339 & -0.3711 & -3.2539 & -6.0041 & -8.9653 & -7.8429 \\ 0.1273 & 0.0160 & 0.1362 & 0.0016 & -0.0172 & -0.1506 & -0.2779 & -0.4154 & -0.3630 \\ 0 & 0 & 0 & 0.0000 & 0 & 0 & 0.0001 & 0 & 0 \end{bmatrix};$$

And the eigvalues is show:

$$Eigvalues = 1.0e + 04 * \begin{bmatrix} -7.9558 + 0.0000i \\ -4.1816 + 0.0000i \\ -2.8032 + 0.0000i \\ -0.0023 + 0.0000i \\ -0.0016 + 0.0000i \\ -0.0005 + 0.0000i \\ -0.0001 + 0.0000i \\ -0.0001 - 0.0000i \\ -0.0001 + 0.0000i \end{bmatrix} ; \text{ We can see all eigvalue have negative real part.}$$

Here, I also plot bode from the pair(A,B,F,0), The figure shows follow:

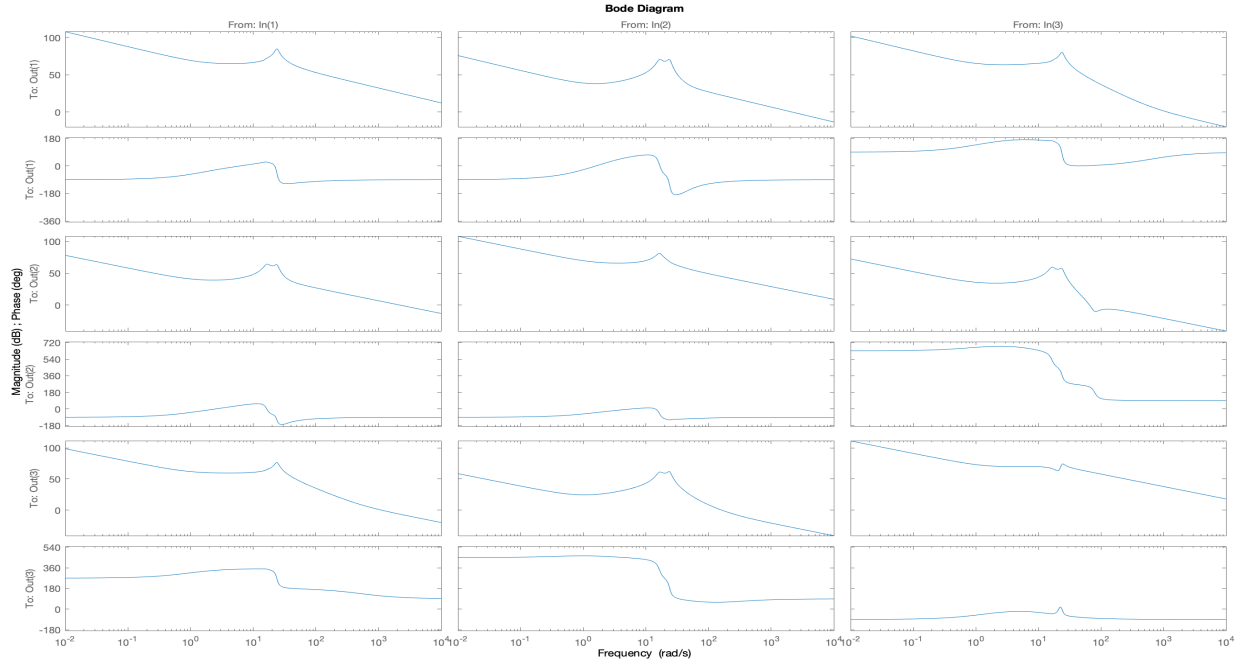


Figure 1: The Plant phase and magnitude

Second:, We need to use Kalman filter for the plant, $\dot{\hat{x}} = A\hat{x} + K_e(y - \hat{y}), \hat{y} = C_1\hat{x}$.

We also use icare to solve Riccati Equation:

$$P_e A^T + A P_e - P_e C^T R_e^{-1} C P_e + Q_e = 0, P_e \geq 0, K_e = P_e C^T R_e^{-1}$$

And here we choose the Q_e and R_e matrix as following:

$$Q_e = 1.0e - 03 * \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix};$$

$$R_e = 0.004$$

Using Matlab code as follow:

```
[X2,K2,L2] = icare(A,[],Qe'*Qe,[],[],[],-C1*C1');
% Pk=are(A,C1,Qe)
Ke=X2*C1'*inv(Re)
Ack=A-Ke*C1
eigack=eig(Ack)
```

And we obtain K_e and $Ack = A - K_e C_1$ and Ack eigenvalues: Then, We obtain the F matrix as follow:

$$K_e = \begin{bmatrix} 0.2500 & 0.0000 & 0.0014 & -0.0023 & 0 & 0 & 0.0007 & 0 & 0.0000 \\ 0.0000 & 0.2500 & -0.0009 & 0.0031 & 0 & 0 & -0.0005 & 0 & -0.0000 \\ 0.0014 & -0.0009 & 0.0001 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ -0.0023 & 0.0031 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.0001 \\ 0.0002 & 0.2496 & -0.0012 & 0.0033 & 0 & 0 & -0.0005 & 0 & 0.0097 \\ 0.2500 & 0.0000 & 0.0016 & -0.0021 & 0 & 0 & 0.0011 & 0 & 0.0000 \\ 0.0007 & -0.0005 & 0.0000 & -0.0000 & 0 & 0 & 0.0000 & 0 & -0.0000 \\ 0.0002 & -0.0097 & -0.0001 & -0.0000 & 0 & 0 & -0.0000 & 0 & 0.2498 \\ 0.0000 & -0.0000 & -0.0000 & 0.0001 & 0 & 0 & -0.0000 & 0 & 0.2500 \end{bmatrix};$$

And the matrix $A - K_e C_1$ is show:

$$Ack = 1.0e+04* \begin{bmatrix} -0.2500 & -0.0000 & 0.9986 & 0.0023 & 0 & 0 & 0.0002 & 0 & - \\ -0.0000 & -0.2500 & 0.0009 & 0.9961 & 0 & 0 & -0.0384 & 0 & 0 \\ -0.0014 & 0.0009 & -0.0303 & -0.0056 & -0.0003 & 585.1165 & 11.4448 & -59.5290 & 0 \\ 0.0023 & -0.0031 & 0.0000 & -0.0708 & 267.7499 & -0.0003 & 0.0000 & 0 & - \\ -0.0002 & -0.2496 & 0.0012 & -1.0033 & -3.3607 & 2.2223 & 0.0005 & 0 & - \\ -0.2500 & -0.0000 & -1.0016 & 0.0021 & 2.4483 & -3.3607 & -0.0011 & 0 & - \\ -0.0007 & 0.0005 & 0.0579 & 0.0108 & 0.0049 & 0.0037 & -21.9557 & 114.2000 & 0 \\ -0.0002 & 0.0097 & 0.0001 & 0.0000 & 0 & 0 & -1.0000 & 0 & - \\ -0.0000 & 0.0000 & 0.0000 & 0.0388 & 0 & 0 & 0.9992 & 0 & - \end{bmatrix}$$

And the eigvalues is show

$$Eigvalues = \begin{bmatrix} -1.5367 + 23.9237i \\ -1.5367 - 23.9237i \\ -1.6190 + 16.4397i \\ -1.6190 - 16.4397i \\ -13.8925 + 0.0000i \\ -7.7978 + 0.0000i \\ -0.5274 + 0.0000i \\ -0.4993 + 0.0000i \\ -0.4999 + 0.0000i \end{bmatrix}; \text{ We can also see all eigvalue have negative real part.}$$

At The last:The plant LQG control law is $u = -F\hat{x}$

$$\begin{cases} \dot{\hat{x}} = (A - BF - K_e C_1) \hat{x} + K_e y, \\ u = -F\hat{x} \end{cases} \quad \text{And } G = [C_1(A - BF)^{-1}]^{-1}, \text{ Using the Matlab we obtain the } G:$$

$$G = \begin{bmatrix} -942.5693 & -34.5139 & -332.2226 \\ 46.3757 & -998.5361 & -27.8396 \\ -330.7754 & -41.6478 & 942.7901 \end{bmatrix};$$

Clearly, the closed-loop system is characterized by the following state space equation we obtain:

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} - \begin{pmatrix} BG \\ 0 \end{pmatrix} r + \tilde{v}, \tilde{v} = \begin{pmatrix} v \\ v - K_e W \end{pmatrix} \\ y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + w \end{cases}$$

Then, We let $A_c = \begin{bmatrix} A - BF & BF \\ 0 & A - K_e C \end{bmatrix}$, $B_c = \begin{pmatrix} BG \\ 0 \end{pmatrix}$, $C_c = \begin{bmatrix} C & 0 \end{bmatrix}$, Caculating From Matlab We Obatin the A_c, B_c, C_c as show bellow:

$A_c = 1.0e + 04 *$

0	0	0.0001	0	0	0	0.0000	0	0	0	0	0	0	0	0	0
0	0	0	0.0001	0	0	-0.0000	0	0	0	0	0	0	0	0	0
-1.4344	-0.1806	-1.5341	-0.0177	0.1934	1.7547	3.1298	4.6733	4.0883	1.4344	0.1806	1.5341	0.0177	-0.1934	-1.6961	-3.1280
0	0	0	-0.0000	0.0268	-0.0000	0	0	0	0	0	0	0	0	0	0
-0.0071	-0.2591	-0.0067	-0.1883	-2.8654	-0.3184	-0.0041	0.0010	-0.0139	0.0071	0.2591	0.0067	0.1882	2.8651	0.3186	0.0041
-0.2442	-0.0023	-0.1689	-0.0053	-0.2506	-4.1263	-0.0922	0.0225	-0.0858	0.2442	0.0023	0.1688	0.0053	0.2509	4.1260	0.0922
2.7517	0.3465	2.9430	0.0339	-0.3711	-3.2539	-6.0041	-8.9653	-7.8429	-2.7517	-0.3465	-2.9429	-0.0339	0.3711	3.2539	6.0019
0.1273	0.0160	0.1362	0.0016	-0.0172	-0.1506	-0.2779	-0.4154	-0.3630	-0.1273	-0.0160	-0.1362	-0.0016	0.0172	0.1506	0.2778
0	0	0	0.0000	0	0	0.0001	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0001	0.0000	0	0	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0000	0.0001	0	0	-0.0000
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	-0.0000	-0.0000	-0.0000	0.0585	0.0011
0	0	0	0	0	0	0	0	0	0.0000	-0.0000	0.0000	-0.0000	0.0268	-0.0000	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	0.0000	-0.0001	-0.0003	0.0002	0.0000
0	0	0	0	0	0	0	0	0	-0.0000	-0.0000	-0.0001	0.0000	0.0002	-0.0003	-0.0000
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0023
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0	0	-0.0000
0	0	0	0	0	0	0	0	0	-0.0000	0.0000	0.0000	0.0000	0	0	0.0001

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.4344 & -0.1806 & 4.0883 \\ 0 & 0 & 0 \\ -0.0071 & -0.2591 & -0.0139 \\ -0.2442 & -0.0023 & -0.0858 \\ 2.7517 & 0.3465 & -7.8429 \\ 0.1273 & 0.0160 & -0.3630 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

The B_c is equal: $B_c = 1.0e + 04 *$

(d) Summary

Here, We use LQG caculating all required martix, those matrix will all use in the Part2.

- Part2: Results** First, I will give the simulink block diagram show figure 2, It has 6 main block, the detail can be found in figure 2:

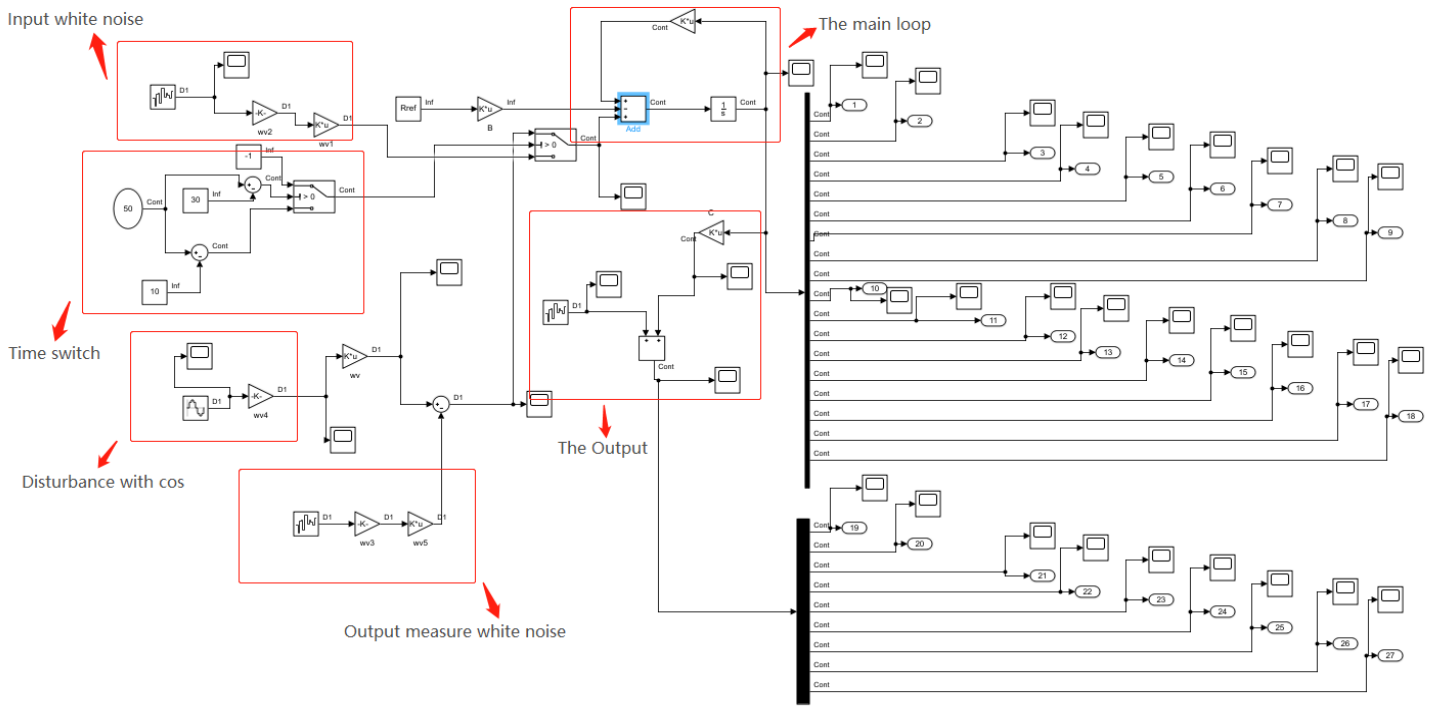


Figure 2: The Unmanned Helicopter LQG Controller diagram

Then,I will show the state and error,and y output from figure 3 and figure 4:

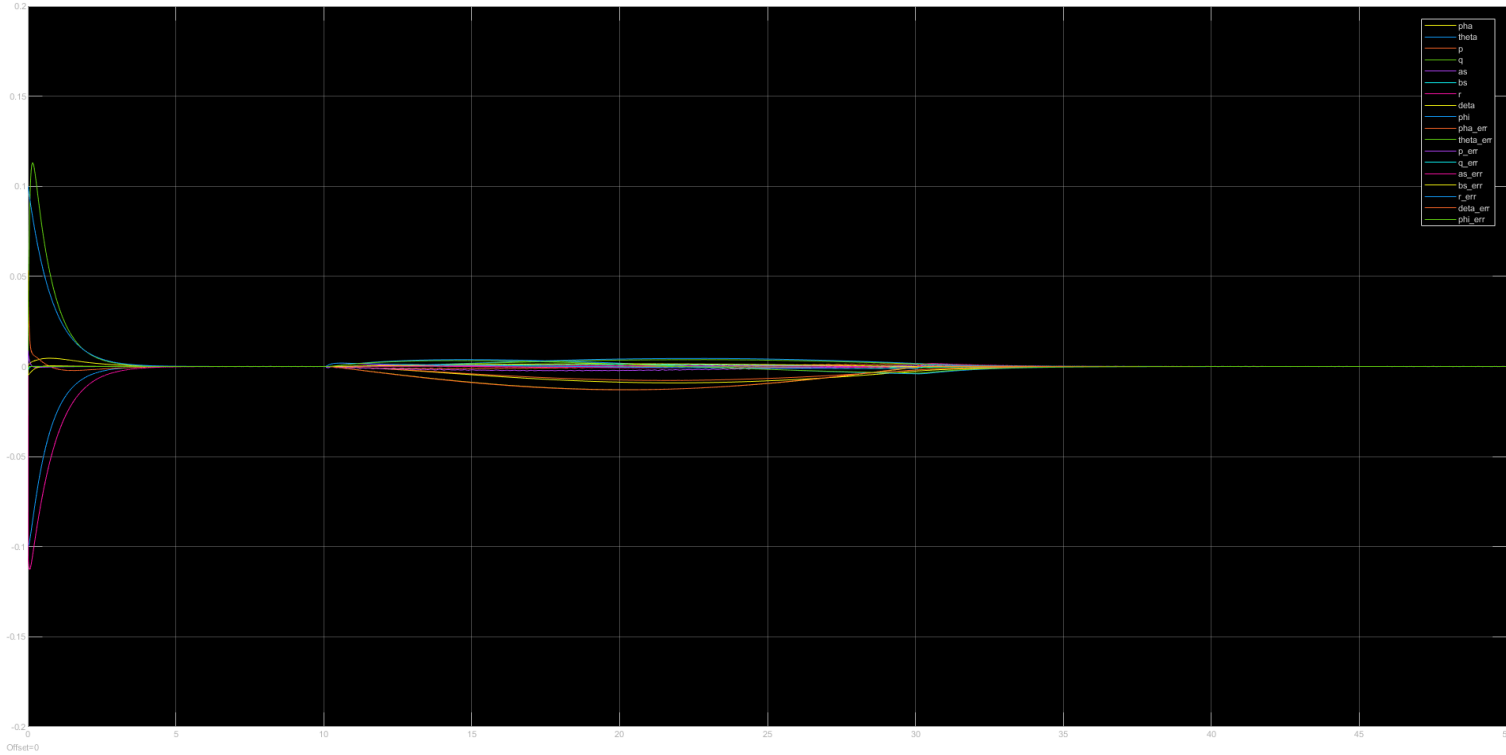


Figure 3: The Plant state x and error

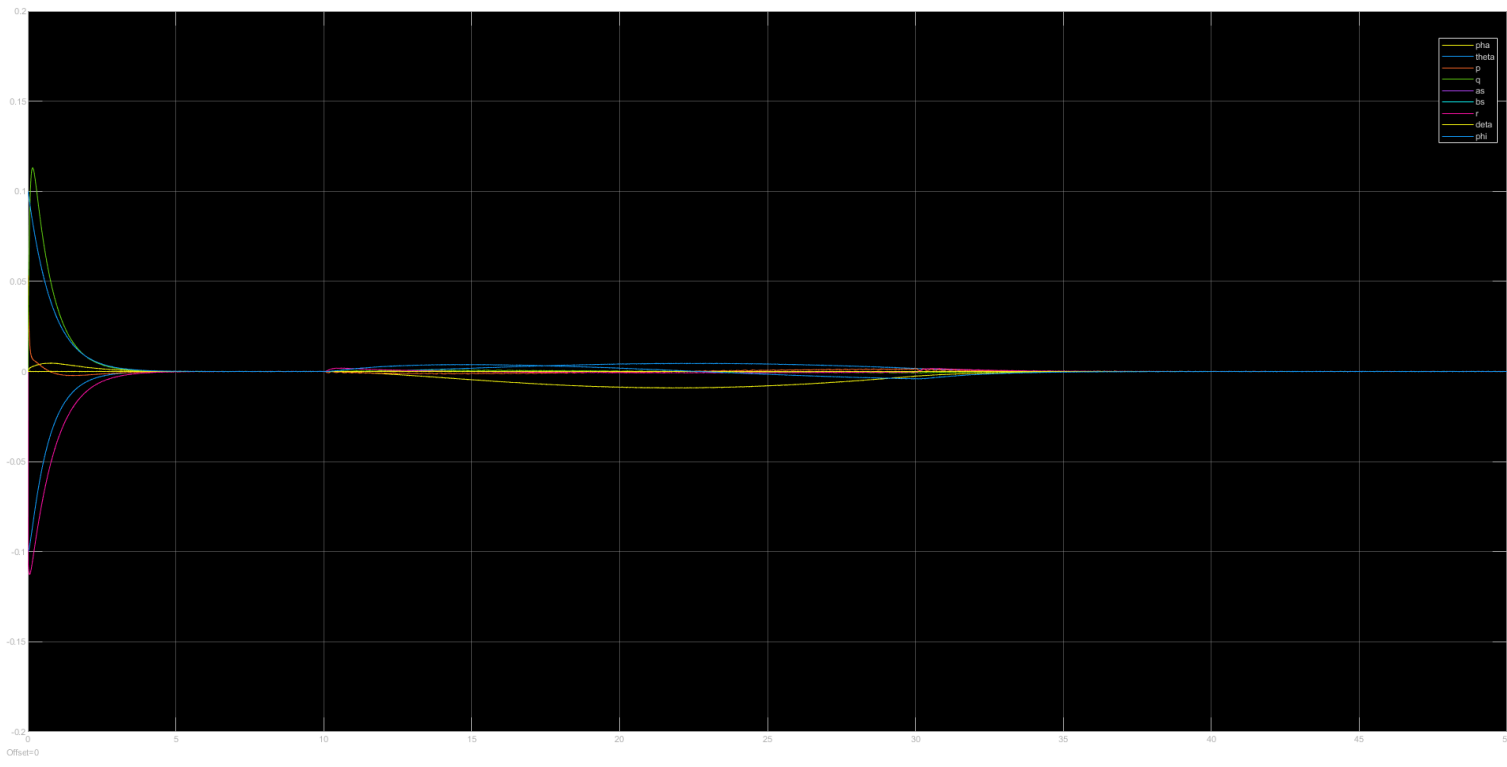
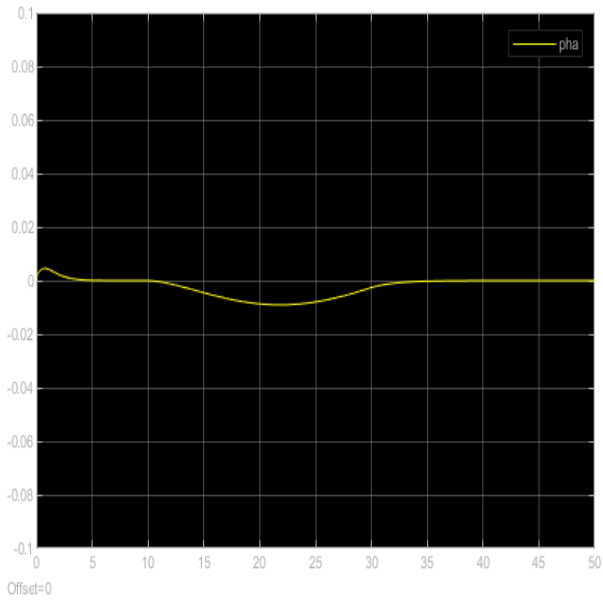
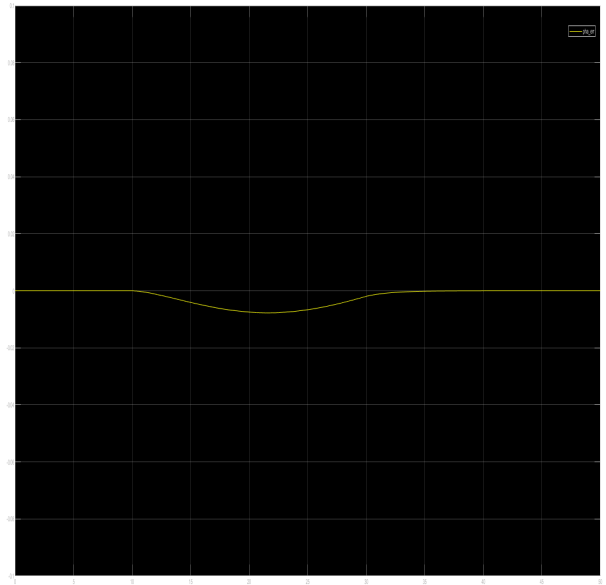


Figure 4: The Plant y output

At last,I will show all state and output respectively.



(a) State:pha



(b) Error:pha

Figure 5: Pha state and error

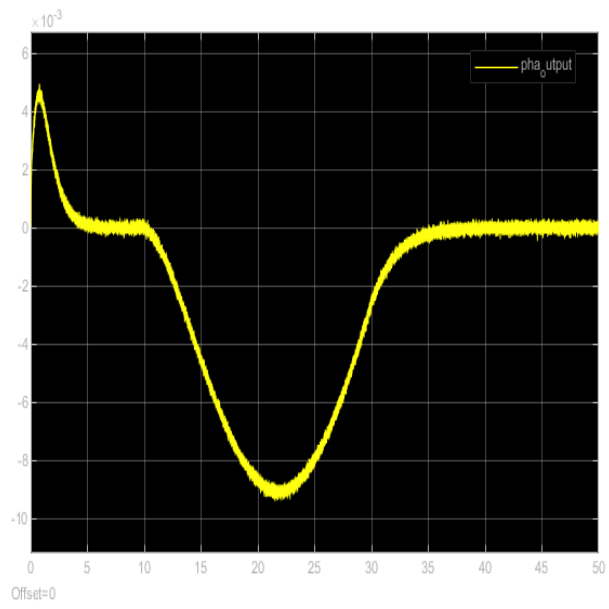
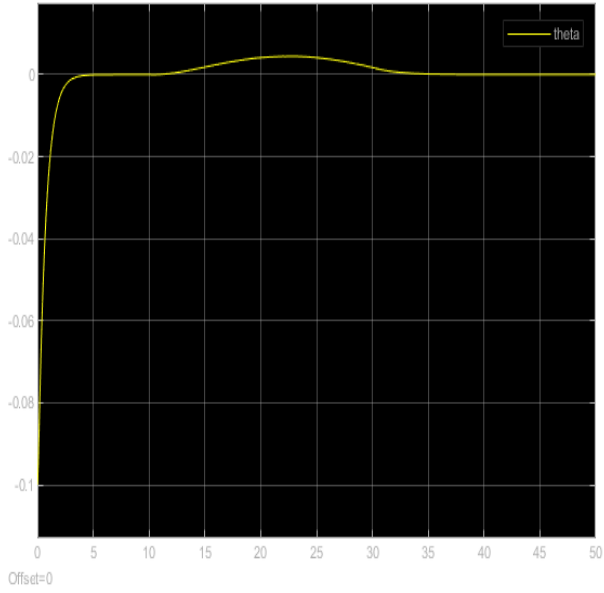
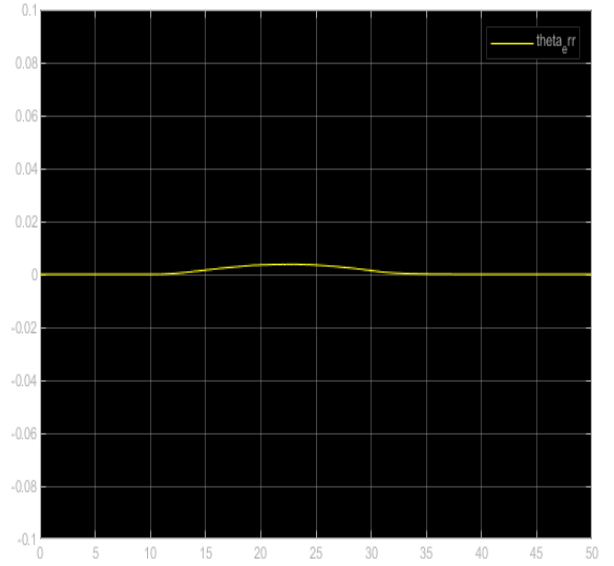


Figure 6: The Plant Pha output



(a) State:Theta



(b) Error:Theta

Figure 7: Pha state and error

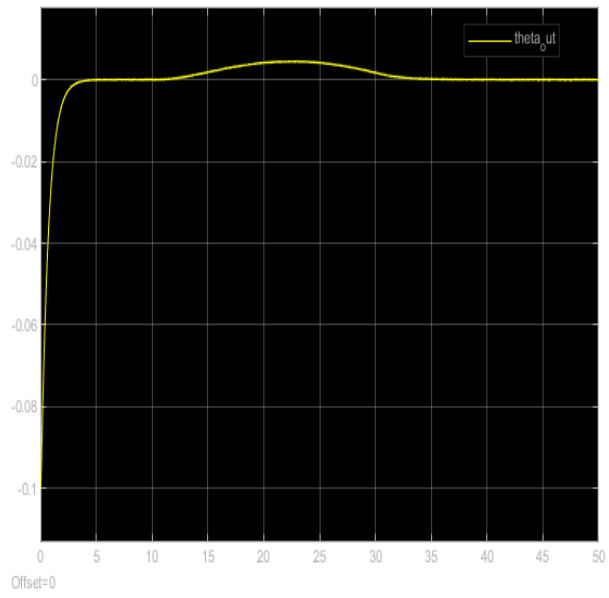
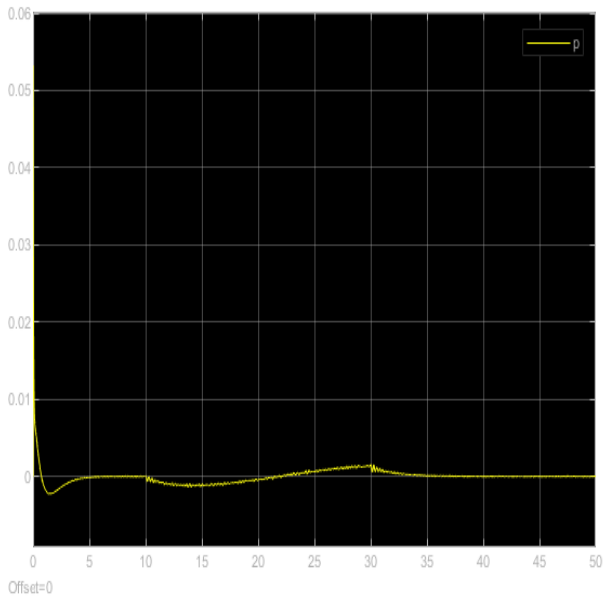
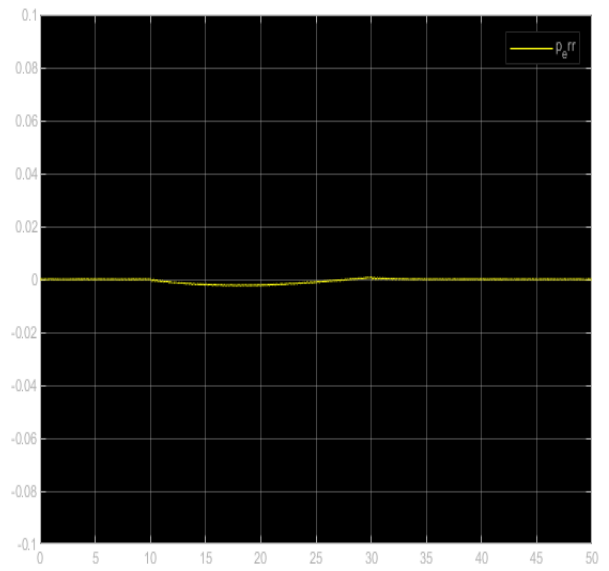


Figure 8: The Plant Theta output



(a) State: p



(b) Error: p

Figure 9: p state and error

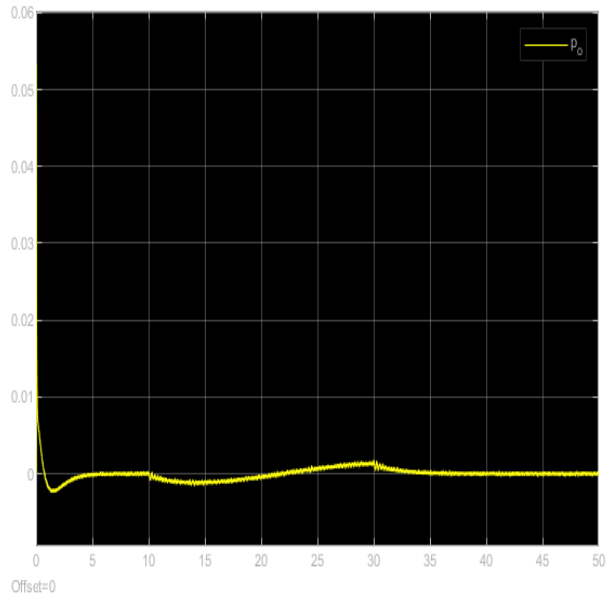
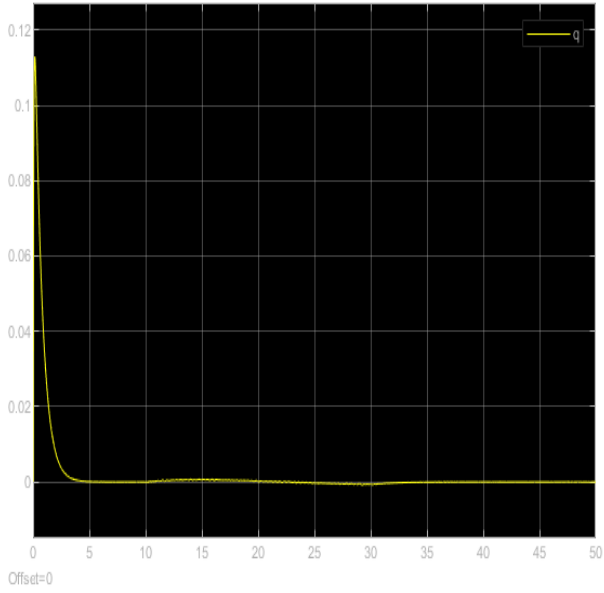
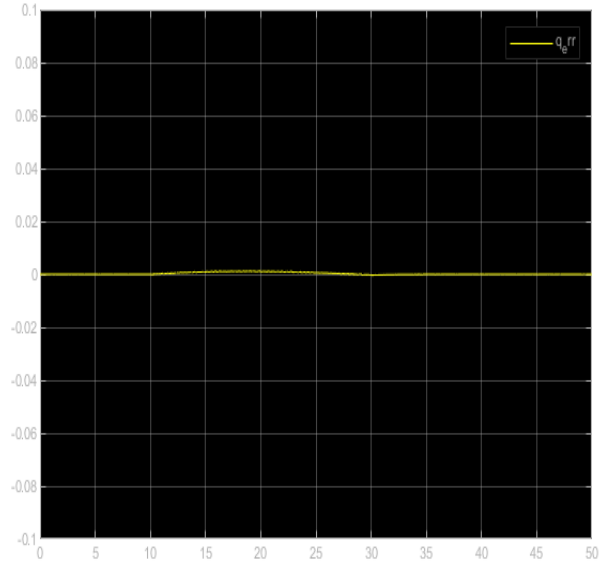


Figure 10: The Plant p output



(a) State: q



(b) Error: q

Figure 11: q state and error

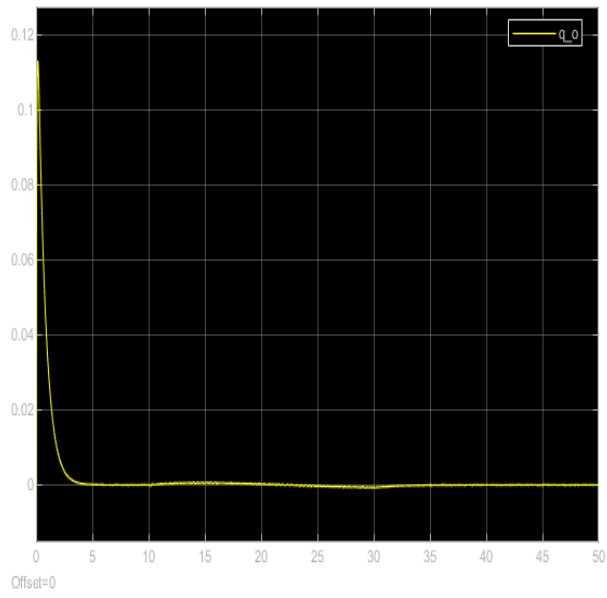
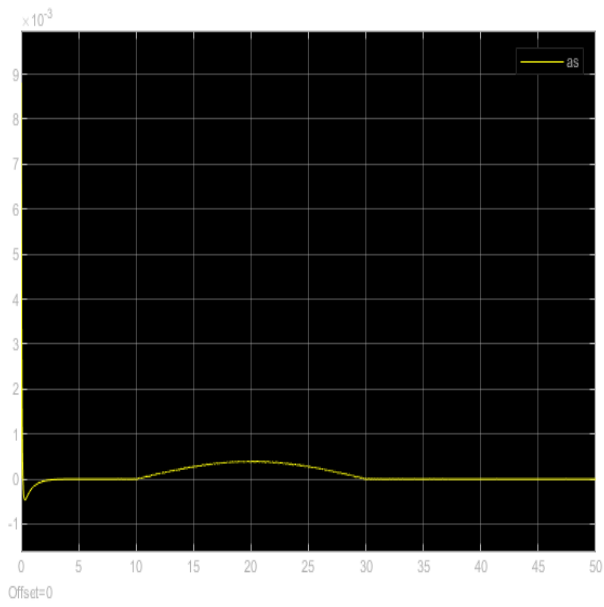
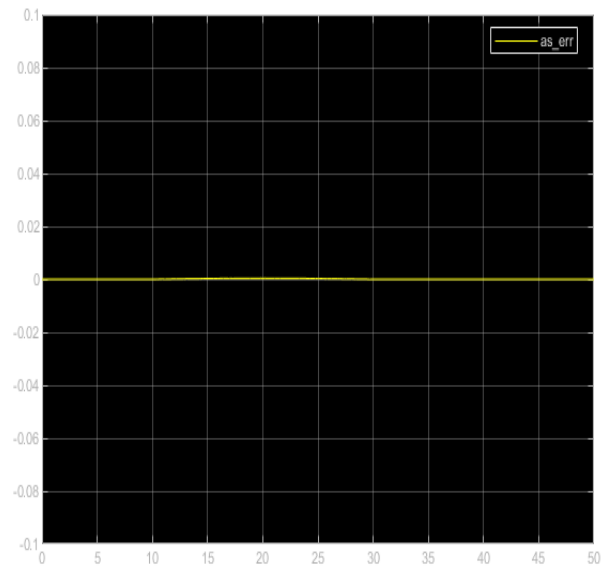


Figure 12: The Plant q output



(a) State:as



(b) Error:as

Figure 13: as state and error

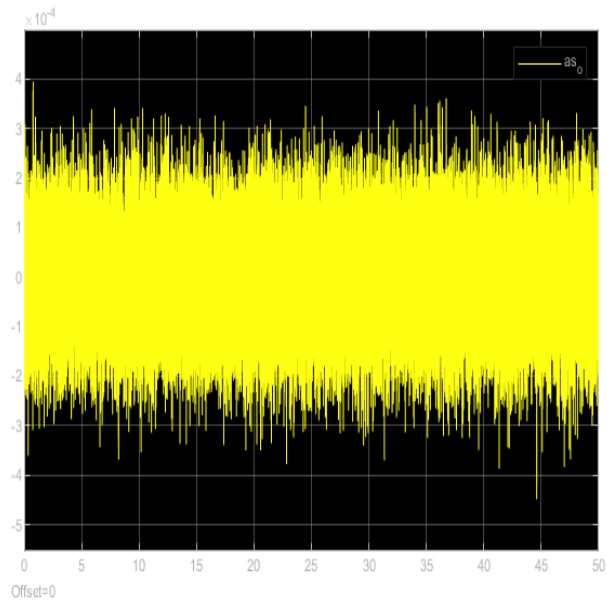
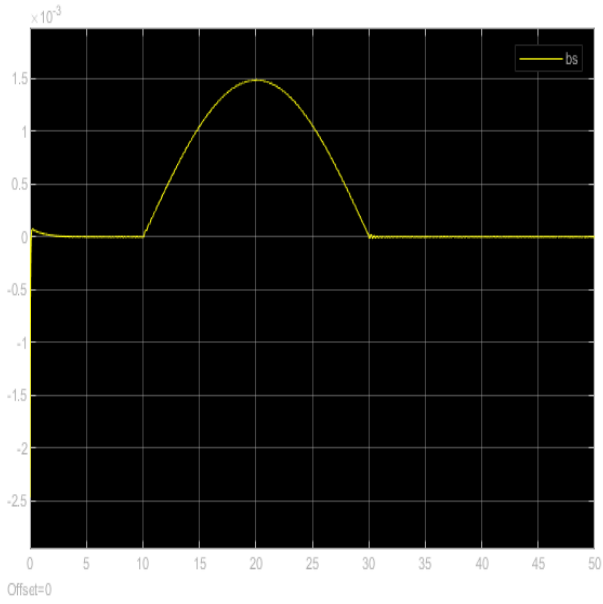
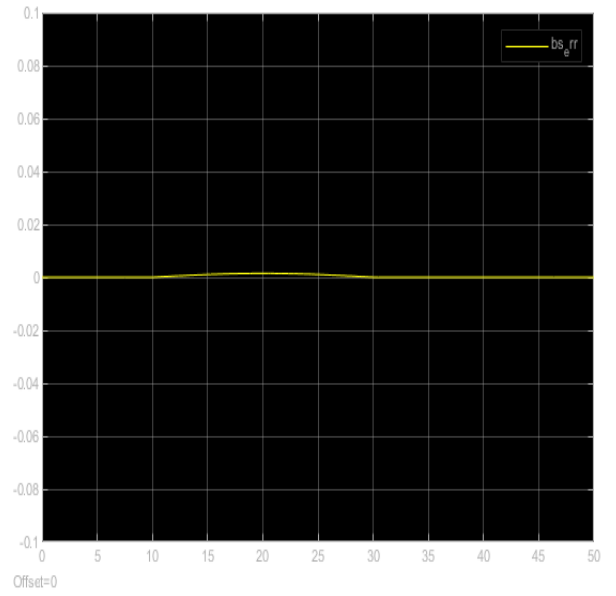


Figure 14: The Plant as output



(a) State:bs



(b) Error:bs

Figure 15: bs state and error

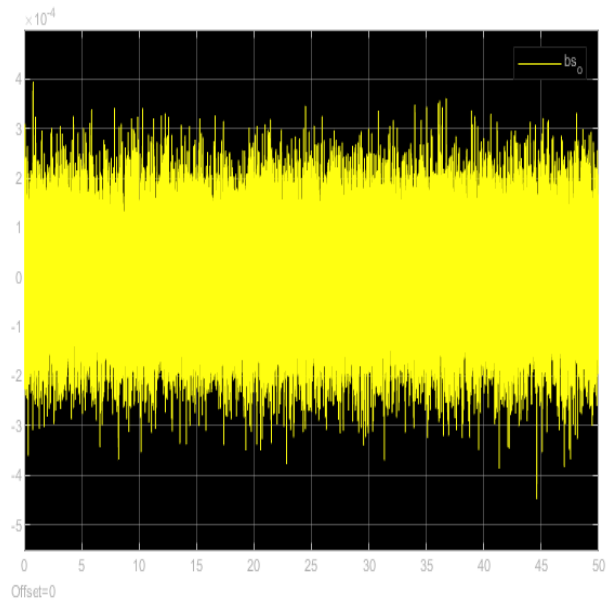
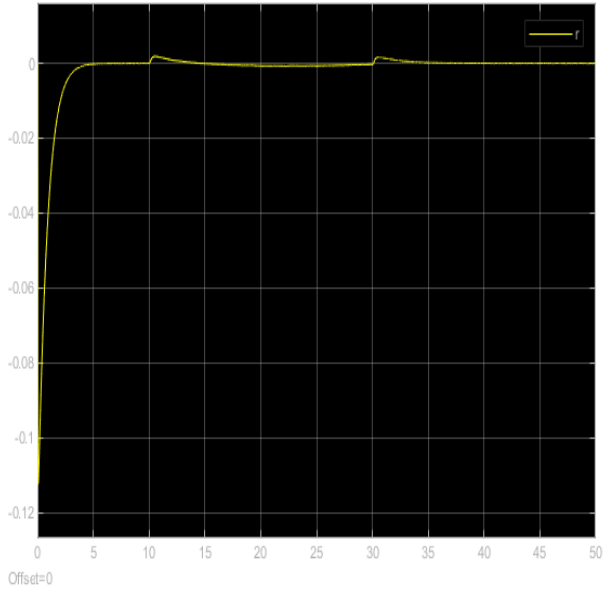
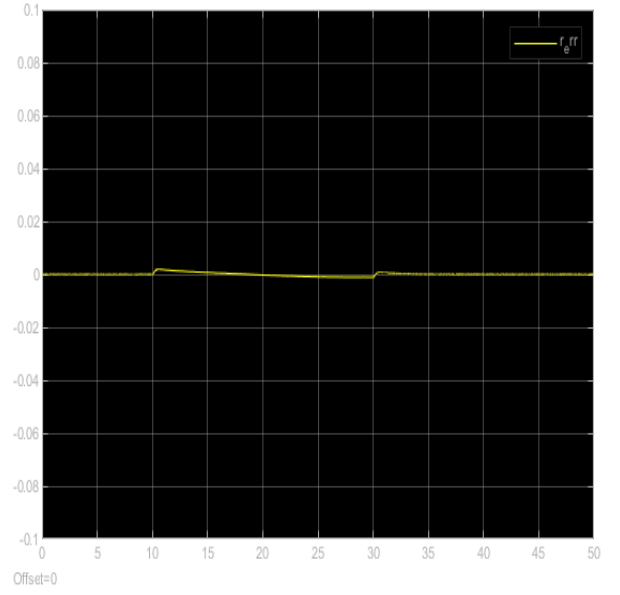


Figure 16: The Plant bs output



(a) State:r



(b) Error:r

Figure 17: r state and error

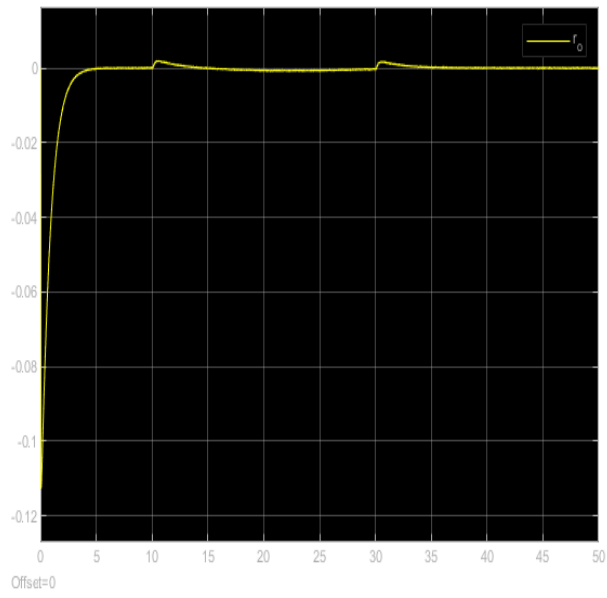
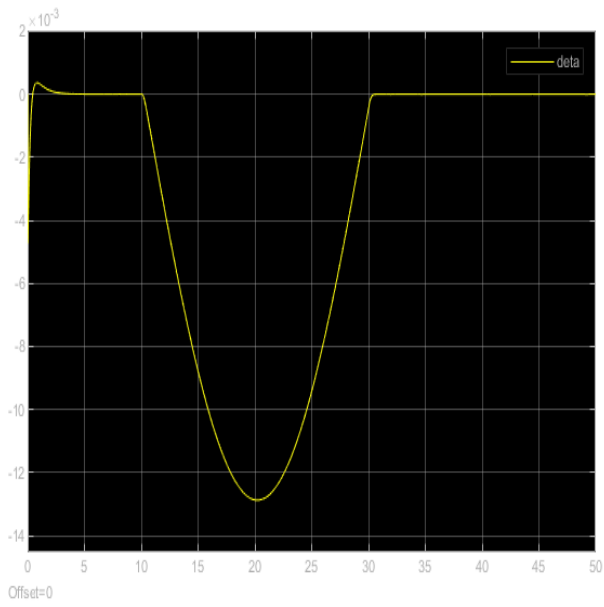
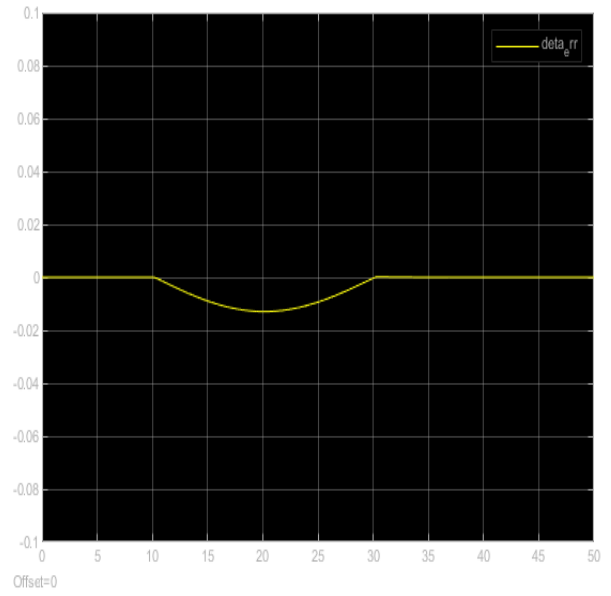


Figure 18: The Plant r output



(a) State:delta



(b) Error:delta

Figure 19: δ state and error

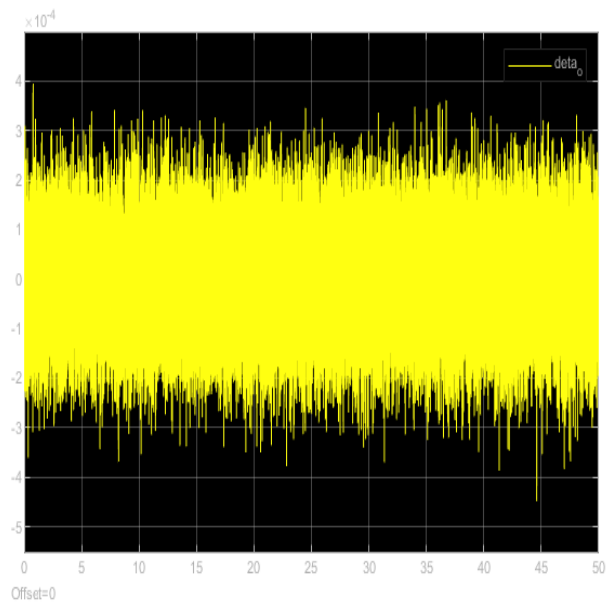
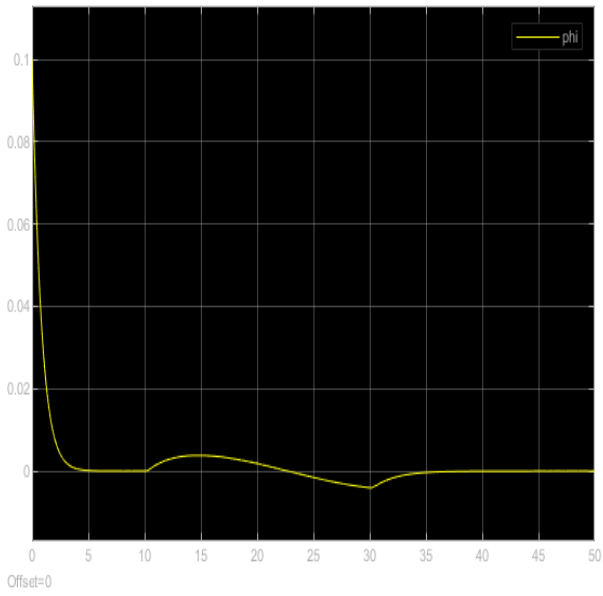
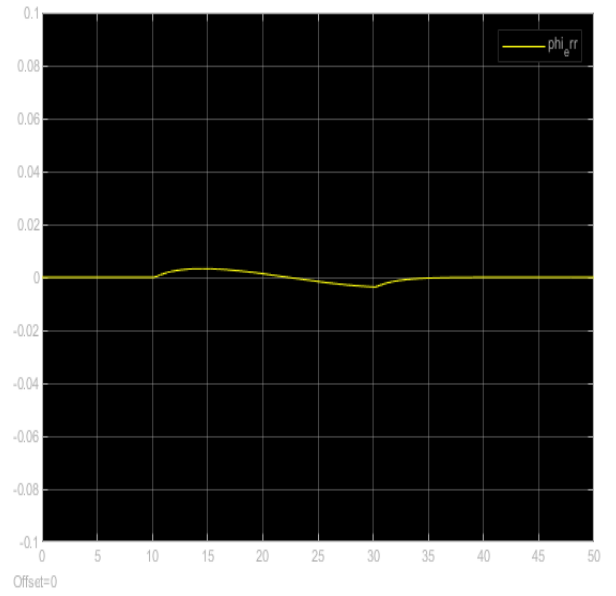


Figure 20: The Plant deta output



(a) State:phi



(b) Error:phi

Figure 21: phi state and error

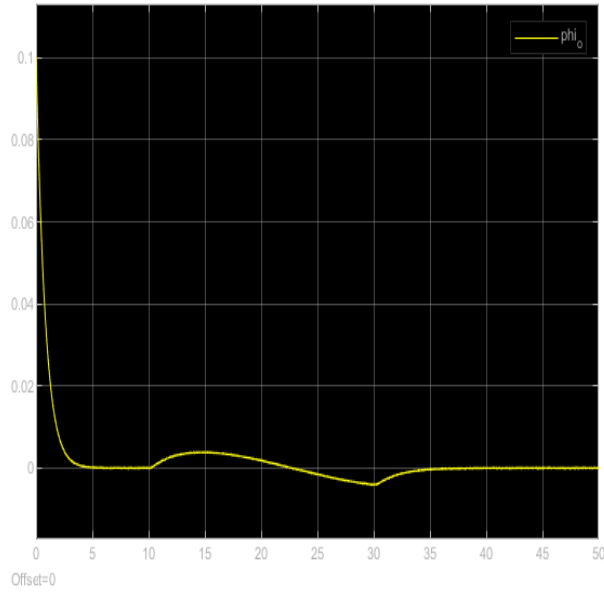


Figure 22: The Plant phi output

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