Enveloping grasp for multiple objects

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2 authors:

Kensuke Harada
Graduate School of Engineering Science, Osaka University

Makoto Kaneko
Meijo University

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Enveloping Grasp for Multiple Objects

Kensuke Harada Makoto Kaneko

Industrial and Systems Engineering
Hiroshima University
Kagamiyama, Higashi-Hiroshima 739-8527, JAPAN

Abstract

This paper discusses the enveloping grasp of multiple objects under rolling contacts. We first provide a general mathematical formulation on the kinematic relationship for multiple objects enveloped by a multifingered robot hand, and then derive a condition for judging whether the rolling condition can be satisfied at each contact point. We also show a sufficient condition for rolling up general two objects grasped by a multifingered robot hand in contact with them. Finally, an experimental result is shown to confirm how easily two cylindrical objects can be enveloped by a simple grasping motion.

1 Introduction

Multifingered robot hands have a potential advantage to perform various skillful tasks like human hands. While much research has been done on multifingered robot hands, there are two basic grasp patterns. One is the finger tip grasp that emphasizes on dexterity and sensitivity [1, 2, 3], and the other is the enveloping grasp that provides highly stable grasp due to a large number of distributed contacts on the grasped object [4, 5, 6, 7, 8].

So far, most of works have implicitly assumed that a multifingered hand manipulates only one object. Under such a condition, they discussed several grasping issues, such as the stability of grasp, the analysis of contact force, the planning for manipulating an object and so forth. In this paper, we relax the assumption of single object, and discuss how to grasp multiple objects by a multifingered hand.

Suppose that a multifingered hand is grasping two objects by the finger tip grasp. Intuitively, we can imagine that it is difficult for such system to keep a stable grasp and the system will easily fail in grasping for a small disturbance. On the other hand, suppose that a multifingered hand is grasping two objects by enveloping grasp, as shown in Fig.1. It seems that the enveloping grasp can achieve this task even more easily than the finger tip grasp. We can find another advantage for enveloping two objects by a multifingered hand. Suppose that the friction between an object and the link surface is very significant. Under such a condition, since it is hard to lift up the object by slipping over the link surface, we have to provide an alternative scheme based on rolling contact. In such a case, one finger continuously pushes the object so that it may be rolled up over the surface of the other fingers. Generally, this motion planning is too complicated to be easily implemented to the actual system. However, for two objects satisfying the rolling contact each other, we can expect that a multifingered hand can easily achieve an enveloping grasp by simply pushing two links contacting with the objects, as shown in Fig.1. During the lifting phase, links and two objects behave as if they were just connected by mechanical gears. Due to this mechanical properties, achieving an enveloping grasp for two objects seems to be even easier than for a single object under a significant friction.

This work is motivated by these backgrounds. We first provide a general mathematical formulation on the kinematic relationship for multiple objects enveloped by a multifingered hand, and then discuss a condition whether each contact point can satisfy the rolling condition or not. We confirm the condition by a numerical example. We also show a sufficient condition for lifting up two objects by a simple pushing force exerted on the links contacting with them. Finally, an experimental result is shown to see how easily a multifingered robot hand can achieve an enveloping grasp for two cylindrical objects placed on a table.

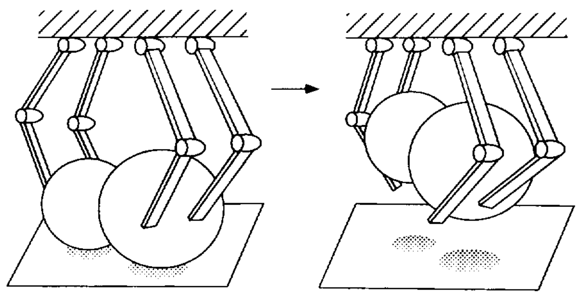


Fig. 1: Enveloping grasp of multiple objects

2 Related Works

Enveloping Grasp:
There have been a number of works concerning the enveloping grasp. Especially, Salisbury et al.[4] has

proposed the Whole-Arm Manipulation(WAM) capable of treating a big and heavy object by using one arm which allows multiple contacts with an object. Bicchi[5], Zhang et al.[6] and Omata et al.[7] analyzed the grasp force of the enveloping grasp. In our previous work, we have proposed the grasping strategies for achieving enveloping grasp work for cylindrical objects[8].

Grasp and Manipulation of Multiple Objects: Dauchez et al.[10] and Kosuge et al.[11] used two manipulators holding two objects independently and tried to apply to an assembly task. However, they have not considered that two manipulators grasp and manipulate two common objects simultaneously. Recently, Aiyama et al.[9] studied a scheme for grasping multiple box type objects stably by using two manipulators. For an assembly task, Mattikalli et al.[12] proposed a stable alignments of multiple objects under the gravitational field. While these works treated multiple objects, they have not considered any manipulation of objects based on rolling contacts.

Grasp by Rolling Contacts: Kerr et al.[1] and Montana[13] formulated the kinematics of manipulation of objects under rolling contacts with the fingertip. Li et al.[14] proposed a motion planning method with nonholonomic constraint. Howard et al.[15] and Maekawa et al.[16] studied the stiffness effect for the object motion with rolling. Cole et al.[2] and Paljug et al.[3] proposed a control scheme for the object motion.

Within our knowledge, this is the first challenge for enveloping multiple objects based on rolling contacts.

3 Modeling

Fig.2 shows the hand system enveloping m objects and n fingers, where finger j contacts with object i, and additionally object i has a common contact point with object l. Σ_R, Σ_{Bi} $(i = 1, \cdots, m)$ and Σ_{Fjk} $(j = 1, \cdots, n, \quad k = 1, \cdots, c_j)$ denote the coordinate systems fixed at the base, at the center of gravity of the object i and at the finger link including the kth contact of finger j, respectively. Let $\boldsymbol{p}_{Bi}$ and $\boldsymbol{R}_{Bi}$ be the position vector and the rotation matrix of Σ_{Bi}, and $\boldsymbol{p}_{Fjk}$ and $\boldsymbol{R}_{Fjk}$ be those of Σ_{Fjk}, with respect to Σ_R, respectively. ${}^{Bi}\boldsymbol{p}_{Cjk}$ and ${}^{Fjk}\boldsymbol{p}_{Cjk}$ are the position vectors of the kth contact point of finger j with respect to Σ_{Bi} and Σ_{Fjk}, respectively. ${}^{Bi}\boldsymbol{p}_{COt}(t = 1, \cdots, r)$ is the position vector of the common contact point between object i and object l with respect to Σ_{Bi}.

3.1 Constraint Condition

In this subsection, we derive the constraint condition. The contact point between object i and the kth contact of finger j can be expressed by Σ_{Bi} and Σ_{Fjk}. Similarly, the contact point between object i and object l can be expressed by Σ_{Bi} and Σ_{Bl}. As a result, we have the following relationships:

$$\boldsymbol{p}_{Bi} + \boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{Cjk} = \boldsymbol{p}_{Fjk} + \boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{p}_{Cjk}, \qquad (1)$$

$$\boldsymbol{p}_{Bi} + \boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{COt} = \boldsymbol{p}_{Bl} + \boldsymbol{R}_{Bl}{}^{Bl}\boldsymbol{p}_{COt}. \qquad (2)$$

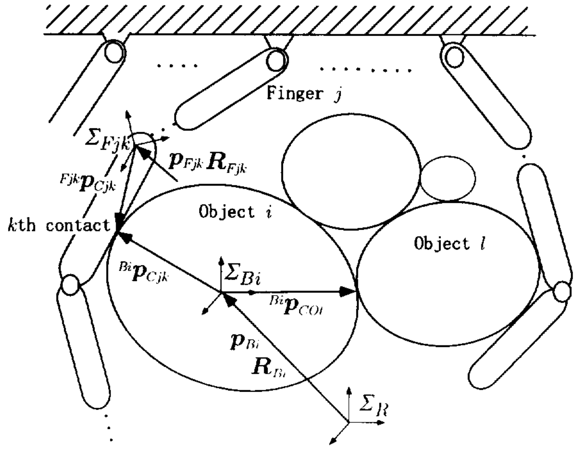


Fig. 2: Model of the system

Suppose that there is no slipping at each contact point. This means implicitly that we consider a sufficiently large coefficient of friction at each contact point. The small displacement of object i and the finger j should be same at the contact point. Also, the small displacement of two objects should be same at the common contact point between them. These discussions lead to the following equations[1]:

$$\boldsymbol{R}_{Bi}{}^{Bi}\Delta\boldsymbol{p}_{Cjk} = \boldsymbol{R}_{Fjk}{}^{Fjk}\Delta\boldsymbol{p}_{Cjk}, \qquad (3)$$

$$\boldsymbol{R}_{Bi}{}^{Bi}\Delta\boldsymbol{p}_{COt} = \boldsymbol{R}_{Bl}{}^{Bl}\Delta\boldsymbol{p}_{COt}, \qquad (4)$$

where $\Delta *$ denotes the finite differentiation of $*$. Substituting eq.(3) and eq.(4) into the differentiation of eq.(1) and eq.(2), respectively, we can derive

$$\boldsymbol{D}_{Bjk}\begin{bmatrix} \Delta\boldsymbol{p}_{Bi} \\ \Delta\boldsymbol{\phi}_{Bi} \end{bmatrix} = \boldsymbol{D}_{Fjk}\begin{bmatrix} \Delta\boldsymbol{p}_{Fjk} \\ \Delta\boldsymbol{\phi}_{Fjk} \end{bmatrix}, \qquad (5)$$

$$\boldsymbol{D}_{Oit}\begin{bmatrix} \Delta\boldsymbol{p}_{Bi} \\ \Delta\boldsymbol{\phi}_{Bi} \end{bmatrix} = \boldsymbol{D}_{Olt}\begin{bmatrix} \Delta\boldsymbol{p}_{Bl} \\ \Delta\boldsymbol{\phi}_{Bl} \end{bmatrix}, \qquad (6)$$

$$\boldsymbol{D}_{Bjk} = [I \quad -(\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{Cjk}\times)], \qquad (7)$$

$$\boldsymbol{D}_{Fjk} = [I \quad -(\boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{p}_{Cjk}\times)], \qquad (8)$$

$$\boldsymbol{D}_{Oit} = [I \quad -(\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{COt}\times)], \qquad (9)$$

where I, $(\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{Cjk}\times)$, $(\boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{p}_{Cjk}\times)$ and $(\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{p}_{COt}\times)$ denote the identity and the skew-symmetric matrices, respectively, and $\Delta\boldsymbol{\phi}_{Bi}$ and $\Delta\boldsymbol{\phi}_{Fjk}$ denote the small angular displacement vectors of Σ_{Bi} and Σ_{Fjk} with respect to Σ_R, respectively. Since the displacement of the finger link including the kth contact point of finger j can also be expressed by utilizing the joint displacement of finger j, we obtain the following relationships:

$$\begin{bmatrix} \Delta\boldsymbol{p}_{Fjk} \\ \Delta\boldsymbol{\phi}_{Fjk} \end{bmatrix} = \boldsymbol{J}_{jk}\Delta\boldsymbol{\theta}_j, \qquad (10)$$

where $\boldsymbol{J}_{jk}$ is the jacobian matrix of the finger link with respect to the joint displacement. Substituting eq.(10) into eq.(5) and aggregating for $k = 1, \cdots c_j$, the following equation is derived:

$$\boldsymbol{D}_{Bj} \begin{bmatrix} \Delta \boldsymbol{p}_{B1} \\ \Delta \boldsymbol{\phi}_{B1} \\ \vdots \\ \Delta \boldsymbol{p}_{Bm} \\ \Delta \boldsymbol{\phi}_{Bm} \end{bmatrix} = \boldsymbol{D}_{Fj} \Delta \boldsymbol{\theta}_j. \tag{11}$$

where $\boldsymbol{D}_{Fj} = \begin{bmatrix} \boldsymbol{D}_{Fj1}\boldsymbol{J}_{j1} \\ \vdots \\ \boldsymbol{D}_{Fjc_j}\boldsymbol{J}_{jc_j} \end{bmatrix}$, and $\boldsymbol{D}_{Bj}$ is the matrix whose components include $\boldsymbol{D}_{Bjk}$ $(k = 1, \cdots, c_j)$. Aggregating eq.(6) for $t = 1, \cdots, r$, the following equation is derived:

$$\boldsymbol{D}_O \begin{bmatrix} \Delta \boldsymbol{p}_{B1} \\ \Delta \boldsymbol{\phi}_{B1} \\ \vdots \\ \Delta \boldsymbol{p}_{Bm} \\ \Delta \boldsymbol{\phi}_{Bm} \end{bmatrix} = \mathbf{o}, \tag{12}$$

where $\boldsymbol{D}_O$ is the matrix whose components include $\boldsymbol{D}_{Oit}$ $(i = 1, \cdots, m,\ t = 1, \cdots, r)$. Using eq.(11) and eq.(12), the following kinematic equation can be obtained:

$$\boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u})\Delta \boldsymbol{x} = \mathbf{o}, \tag{13}$$

where

$$\boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) = \begin{bmatrix} -\boldsymbol{D}_{F1} & \mathbf{o} & \cdots & \cdots & \mathbf{o} & \boldsymbol{D}_{B1} \\ \mathbf{o} & -\boldsymbol{D}_{F2} & \mathbf{o} & \cdots & \mathbf{o} & \boldsymbol{D}_{B2} \\ & & \cdots & \cdots & & \\ \mathbf{o} & \mathbf{o} & \cdots & \cdots & -\boldsymbol{D}_{Fn} & \boldsymbol{D}_{Bn} \\ \mathbf{o} & \cdots & \cdots & \cdots & \mathbf{o} & \boldsymbol{D}_O \end{bmatrix},$$

$$\Delta \boldsymbol{x} = [\Delta \boldsymbol{\theta}_1^T \ \cdots \ \Delta \boldsymbol{\theta}_n^T \ \Delta \boldsymbol{p}_{B1}^T \ \Delta \boldsymbol{\phi}_{B1}^T \ \cdots \ \Delta \boldsymbol{p}_{Bm}^T \ \Delta \boldsymbol{\phi}_{Bm}^T]^T,$$

$$\boldsymbol{u} = [{}^{F11}\boldsymbol{p}_{C11}^T \ {}^{Bi}\boldsymbol{p}_{C11}^T \cdots {}^{Fn}\boldsymbol{p}_{Cnc_n}^T \ {}^{Bi}\boldsymbol{p}_{Cnc_n}^T]^T.$$

We note that eq.(13) shows the constraint condition for the system. In eq.(11), $\dim \boldsymbol{D}_{Bj} = 3c_j \times 6m$ and $\dim \boldsymbol{D}_{Fj} = 3c_j \times s_j$, where s_j denotes the number of joints of finger j. In eq.(12), $\dim \boldsymbol{D}_O = 3r \times 6m$. Therefore, we have $\dim \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) = (3\sum_{j=1}^n c_j + 3r) \times (\sum_{j=1}^n s_j + 6m)$. Additionally, in eq.(13), $\boldsymbol{u}$ is the vector which represents the instantaneous position of contact during rolling motion.

3.2 Motion under Constraint

To evaluate the constraint condition (13), we define the variable α as follows[7]:

$$\alpha = (\sum_{j=1}^n s_j + 6m) - (3\sum_{j=1}^n c_j + 3r). \tag{14}$$

When $\alpha \le 0$ and $\mathrm{rank}\boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) = \sum_{j=1}^n s_j + 6m$, the solution of eq.(13) becomes $\Delta \boldsymbol{x} = \mathbf{o}$, which means that two objects are fully constrained. On the other hand, $\alpha > 0$ implies that there exists some possible degrees of freedom concerning velocity. Hereafter, we assume $\alpha > 0$. We further assume that

$$\mathrm{rank}\boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) = 3\sum_{j=1}^n c_j + 3r. \tag{15}$$

Let us now consider the motion under eqs.(13) and (15). We introduce a new vector $\Delta \boldsymbol{\eta} = \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u})\Delta \boldsymbol{x}$ $(\boldsymbol{\eta} \in R^\alpha)$ such that $\mathrm{rank}\begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u}) \end{bmatrix} = \sum_{j=1}^n s_j + 6m$. By using the newly introduced $\Delta \boldsymbol{\eta}$ and eq.(13), we obtain a set of equation as follows:

$$\begin{bmatrix} \mathbf{o} \\ \Delta \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u}) \end{bmatrix} \Delta \boldsymbol{x}. \tag{16}$$

We can regard that eq.(16) is a coordinate transformation between $\Delta \boldsymbol{x}$ and $\Delta \boldsymbol{\eta}$. We note that the selection of $\Delta \boldsymbol{\eta}$ is not unique. An example of selections of $\Delta \boldsymbol{\eta}$ and $\boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u})$ will be shown in the numerical example. Due to the constrained conditions (13), only the α components of $\Delta \boldsymbol{x}$ can be utilized as independent parameters. Since $\begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u}) \end{bmatrix}$ is nonsingular, we can obtain its inverse. Let $\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{u})$ be the inverse matrix of $\begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{u}) \end{bmatrix}$. As a result, for a given $\Delta \boldsymbol{\eta}$, $\Delta \boldsymbol{x}$ is uniquely determined by

$$\Delta \boldsymbol{x} = \boldsymbol{X}(\boldsymbol{x}, \boldsymbol{u})\Delta \boldsymbol{\eta}. \tag{17}$$

3.3 Displacement of Contact Point

In eq.(13), the vector of contact points is given by $\boldsymbol{u}$. By extending the method proposed by Kerr et al.[1], we derive the velocity of the contact point as follows:

$$\Delta \boldsymbol{u} = \boldsymbol{U}(\boldsymbol{x}, \boldsymbol{u})\Delta \boldsymbol{x}. \tag{18}$$

Substituting eq.(17) into eq.(18), we have the following equation:

$$\Delta \boldsymbol{u} = \boldsymbol{U}(\boldsymbol{x}, \boldsymbol{u})\boldsymbol{X}(\boldsymbol{x}, \boldsymbol{u})\Delta \boldsymbol{\eta}. \tag{19}$$

Thus, for a given $\Delta \boldsymbol{\eta}$, $\Delta \boldsymbol{u}$ is uniquely determined by using eq.(19). Along with eq.(17), all the motions of the system are determined.

Note that, in this section, while we deal with a 3D model, $\dim \boldsymbol{\Omega}(\boldsymbol{x}, \boldsymbol{u}) = (2\sum_{j=1}^n c_j + 2r) \times (\sum_{j=1}^n s_j + 3m)$ for a 2D model, where α is given by

$$\alpha = (\sum_{j=1}^n s_j + 3m) - (2\sum_{j=1}^n c_j + 2r). \tag{20}$$

4 Judgment of Rolling Contacts

Using the results obtained in the previous section, we can derive a condition for judging whether the object can roll over the link surface or not. Using eq.(19), the velocity of the each contact point is rewritten as

$$\begin{bmatrix} {}^{F11}\Delta \boldsymbol{p}_{C11} \\ {}^{Bi}\Delta \boldsymbol{p}_{C11} \end{bmatrix} = \boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u})\Delta\boldsymbol{\eta},$$
$$\vdots$$
$$\begin{bmatrix} {}^{Fjk}\Delta \boldsymbol{p}_{Cjk} \\ {}^{Bi}\Delta \boldsymbol{p}_{Cjk} \end{bmatrix} = \boldsymbol{B}_{jk}(\boldsymbol{x},\boldsymbol{u})\Delta\boldsymbol{\eta},$$
$$\vdots$$
$$\begin{bmatrix} {}^{Bi}\Delta \boldsymbol{p}_{COt} \\ {}^{Bl}\Delta \boldsymbol{p}_{COt} \end{bmatrix} = \boldsymbol{B}_{Ot}(\boldsymbol{x},\boldsymbol{u})\Delta\boldsymbol{\eta},$$
$$\vdots \tag{21}$$

where $\boldsymbol{UX} = [\boldsymbol{B}_{11}^T \cdots \boldsymbol{B}_{nc_n}^T \boldsymbol{B}_{O1}^T \cdots \boldsymbol{B}_{Or}^T]^T$. In eq.(21), if the displacement of a contact point is 0 for any $\Delta\boldsymbol{\eta}$, it is considered that the object cannot roll over the link surface at the point. Now we can find the following theorem:

[Theorem 1]
A necessary and sufficient condition for enabling the objects and contact links to satisfy the rolling condition at each contact point is

$$\boldsymbol{B}_{jk}(\boldsymbol{x},\boldsymbol{u}) \neq \mathbf{o} \quad (j=1,\cdots,s_j \quad k=1,\cdots,c_j), \tag{22}$$
$$\boldsymbol{B}_{Ot}(\boldsymbol{x},\boldsymbol{u}) \neq \mathbf{o} \quad (t=1,\cdots,r). \tag{23}$$

[Proof]
Since sufficiency is obvious, we prove only the necessity. Suppose that the object 1 cannot roll at the 1st contact point of finger 1 even if $\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u}) \neq \mathbf{o}$. Since each velocity component of the left side in the first row of eq.(21) is 0. the first row of eq.(21) can be rewritten as

$$\mathbf{o} = \boldsymbol{b}_{111}\Delta\eta_1 + \cdots + \boldsymbol{b}_{11\alpha}\Delta\eta_\alpha \tag{24}$$

where $\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u}) = [\boldsymbol{b}_{111} \cdots \boldsymbol{b}_{11\alpha}]$. The elements of $\Delta\boldsymbol{\eta}$ can be changed independently. Moreover, since $\Delta\boldsymbol{\eta}$ is not included in the matrix $\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u})$. Thus, to satisfy eq.(24), it is needed that $\boldsymbol{b}_{111} = \cdots = \boldsymbol{b}_{11\alpha} = \mathbf{o}$. The same condition is hold for another contact points. This holds the theorem. □

We note that the vector $\boldsymbol{x}$ is included in the matrices $\boldsymbol{B}_{jk}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{B}_{Ot}(\boldsymbol{x},\boldsymbol{u})$. Moreover, although the matrices $\boldsymbol{B}_{jk}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{B}_{Ot}(\boldsymbol{x},\boldsymbol{u})$ are composed of the matrix $\boldsymbol{X}(\boldsymbol{x},\boldsymbol{u})$, it is generally hard to obtain $\boldsymbol{X}(\boldsymbol{x},\boldsymbol{u})$ symbolically since the dimension of $\boldsymbol{X}(\boldsymbol{x},\boldsymbol{u})$ is large.

5 Numerical Example

To confirm Theorem 1, we show a numerical example. Suppose that two cylindrical objects are enveloped by two planar fingers, as shown in Fig.3, where the finger 1 contacts with object 1 with two different points and finger 2 contacts with object 2 at one contact point. The constraint condition (13) is described as follows:

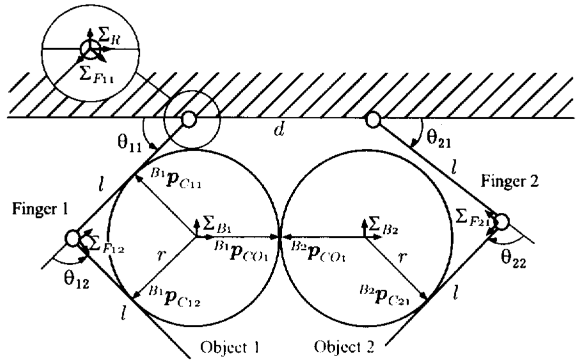


Fig. 3: 2 object system used in numerical example

$$\begin{bmatrix} -\boldsymbol{D}_{F1} & \mathbf{o} & \boldsymbol{D}_{B1} \\ \mathbf{o} & -\boldsymbol{D}_{F2} & \boldsymbol{D}_{B2} \\ \mathbf{o} & \mathbf{o} & \boldsymbol{D}_O \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ -- \\ \Delta\boldsymbol{\theta}_2 \\ -- \\ \Delta\boldsymbol{p}_{B1} \\ \Delta\phi_{B1} \\ \Delta\boldsymbol{p}_{B2} \\ \Delta\phi_{B2} \end{bmatrix} = \mathbf{o}, \tag{25}$$

where $\boldsymbol{\theta}_1 = [\theta_{11} \ \theta_{12}]^T$, $\boldsymbol{\theta}_2 = [\theta_{21} \ \theta_{22}]^T$, $\boldsymbol{p}_{B1} = [x_{B1} \ y_{B2}]^T$ and $\boldsymbol{p}_{B2} = [x_{B2} \ y_{B2}]^T$. The matrix of eq.(25) corresponds to $\boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u})$ in eq.(13). Since $\dim\boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u}) = 8 \times 10$ and $\mathrm{rank}\boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u}) = 8$, $\alpha = 2$ in eq.(20) and the condition of eq.(15) is satisfied. In $\dot{\boldsymbol{x}}$, only the two elements can be independently chosen. In eq.(16), If we set $\boldsymbol{\eta}$ and $\boldsymbol{\Psi}$ as follows:

$$\boldsymbol{\eta} = [\theta_{11} \ \ \theta_{21}]^T$$
$$\boldsymbol{\Psi} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$\begin{bmatrix} \boldsymbol{\Omega}(\boldsymbol{x},\boldsymbol{u}) \\ \boldsymbol{\Psi} \end{bmatrix}$ becomes nonsingular. For numerical computation, we set $r = 1.0$, $l = 2.0$ and $d = 2.0$. The position and orientation of the object and the vector of the contact point are set as $\boldsymbol{p}_{B1} = \begin{bmatrix} 0 \\ -\sqrt{2} \end{bmatrix}$, $\boldsymbol{R}_{B1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{p}_{B2} = \begin{bmatrix} 2 \\ -\sqrt{2} \end{bmatrix}$, $\boldsymbol{R}_{B2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, ${}^{B1}\boldsymbol{p}_{C11} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$, ${}^{B1}\boldsymbol{p}_{C12} = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$ and ${}^{B2}\boldsymbol{p}_{C21} = \begin{bmatrix} \cos(2\pi/9) \\ -\sin(2\pi/9) \end{bmatrix}$. As a result, the joint angles become $\theta_{11} = \pi/4$, $\theta_{12} = \pi/2$, $\theta_{21} = 0.3954$ and $\theta_{22} = 1.8735$. The values of the matrices $\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u})$, $\boldsymbol{B}_{12}(\boldsymbol{x},\boldsymbol{u})$, $\boldsymbol{B}_{21}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{B}_{O1}(\boldsymbol{x},\boldsymbol{u})$ in eq.(21) are computed by using MATLAB as follows:

$$\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u}) = \begin{bmatrix} 0.0 & 0.1220 \times 10^{-15} \\ 0.0 & 0.0 \\ 0.0 & -0.0862 \times 10^{-15} \\ 0.0 & -0.0862 \times 10^{-15} \end{bmatrix},$$

$$\boldsymbol{B}_{12}(\boldsymbol{x},\boldsymbol{u}) = \begin{bmatrix} 0.1148\times10^{-15} & -0.1220\times10^{-15} \\ 0.0 & 0.0 \\ 0.0812\times10^{-15} & -0.0862\times10^{-15} \\ -0.0812\times10^{-15} & 0.0862\times10^{-15} \end{bmatrix},$$

$$\boldsymbol{B}_{21}(\boldsymbol{x},\boldsymbol{u}) = \begin{bmatrix} 0.0 & 0.0 \\ -1.5783 & -1.1396 \\ 1.0116 & 0.7325 \\ 1.2056 & 0.8730 \end{bmatrix},$$

$$\boldsymbol{B}_{O1}(\boldsymbol{x},\boldsymbol{u}) = \begin{bmatrix} 0.0 & 0.0 \\ -0.9742 & -0.5405 \\ 0.0 & 0.0 \\ -0.9742 & -0.5405 \end{bmatrix}.$$

From this results, we can say that the matrices $\boldsymbol{B}_{11}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{B}_{12}(\boldsymbol{x},\boldsymbol{u})$ are almost identical to the zero matrices since all components are either exactly zero or extremely small values. On the other hand, the matrices $\boldsymbol{B}_{21}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{B}_{O1}(\boldsymbol{x},\boldsymbol{u})$ include enough large components. Thus, theorem 1 ensures that object 2 can keep rolling condition at both contact points while object 1 cannot.

6 Condition for Lifting up

In this section, we consider whether two objects can be lifted up by a simple pushing motion(Fig.1). As shown in Fig.4, the common tangential plane of two objects are defined as Π. The plane which is normal to Π and tangent to the gravity vector is defined as Γ. We consider the motion of the objects projected on Γ. The rolling condition is assumed to be satisfied at each

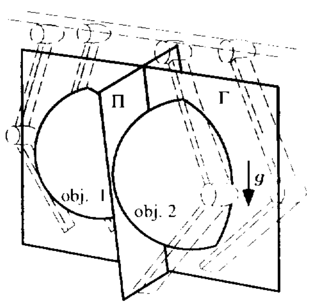


Fig. 4: Definition of the projection plane

contact point. The kinematic relationship between the objects projected on Γ is shown in Fig.5 where the suffix γ denotes a vector on the two dimensional plane Γ. For simplicity, we assume that an object contacts with one finger at one point or that contact points between an object and fingers are overlapped when they are projected on Γ. Now we provide the following definition.

[Definition]
When two objects rotate in opposite direction such that both center of gravity may be close to the palm, we call such a phase palm-reaching phase

For the objects being in palm-reaching phase, object 1 and object 2 have to rotate counter-clockwise and

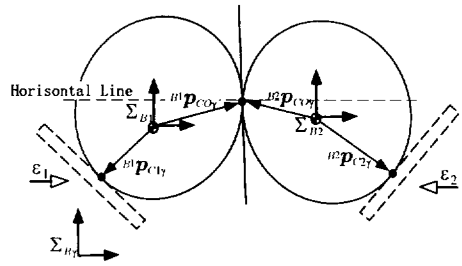


Fig. 5: The projected two object system

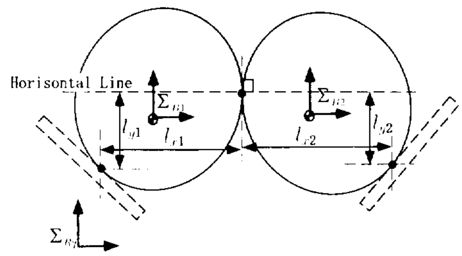


Fig. 6: Variables in eqs.(27),(28),(29) and (30)

clockwise, respectively. Here, we show a sufficient condition for achieving the palm-reaching phase.

[Theorem 2]
Consider that two objects are sandwiched by two finger links. A sufficient condition for achieving the palm-reaching phase by a horizontal pushing motion of finger links is that the contact points between an object and a finger link become lower than the horizontal line including the contact point between two objects.

[Proof]
By using eqs.(11) and (12), the following relation on Γ can be derived as

$$\boldsymbol{D}_{F\gamma}\Delta\boldsymbol{\theta}_{\gamma} = \boldsymbol{D}_{B\gamma}\Delta\boldsymbol{p}_{B\gamma}. \tag{26}$$

$\boldsymbol{D}_{F\gamma}\Delta\boldsymbol{\theta}_{\gamma}$ is defined as

$$\boldsymbol{D}_{F\gamma}\Delta\boldsymbol{\theta}_{\gamma} = [\epsilon_1\ 0\ -\epsilon_2\ 0\ 0\ 0]^T,$$

where $\epsilon_1(>0)$ and $\epsilon_2(>0)$ are the displacements of finger links in the horizontal direction. $\boldsymbol{D}_{B\gamma}$ and $\Delta\boldsymbol{p}_{B\gamma}$ are also defined as

$$\boldsymbol{D}_{B\gamma} = \begin{bmatrix} \boldsymbol{D}_{B11\gamma} & \mathbf{o} \\ \mathbf{o} & \boldsymbol{D}_{B21\gamma} \\ \boldsymbol{D}_{O11\gamma} & -\boldsymbol{D}_{O21\gamma} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & -{}^{B1}y_{C11\gamma} & 0 & 0 & 0 \\ 0 & 1 & {}^{B1}x_{C11\gamma} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -{}^{B2}y_{C21\gamma} \\ 0 & 0 & 0 & 0 & 1 & {}^{B2}x_{C21\gamma} \\ 1 & 0 & -{}^{B1}y_{CO1\gamma} & -1 & 0 & {}^{B2}y_{CO1\gamma} \\ 0 & 1 & {}^{B1}x_{CO1\gamma} & 0 & -1 & -{}^{B2}x_{CO1\gamma} \end{bmatrix},$$

$$\Delta \boldsymbol{p}_{B\gamma} = [\Delta x_{B1\gamma}\ \Delta y_{B1\gamma}\ \Delta \phi_{B1\gamma}\ \Delta x_{B2\gamma}\ \Delta y_{B2\gamma}\ \Delta \phi_{B2\gamma}]^T,$$
$${}^{B1}\boldsymbol{p}_{C11\gamma} = [{}^{B1}x_{C11\gamma}\ {}^{B1}y_{C11\gamma}]^T,$$
$${}^{B2}\boldsymbol{p}_{C21\gamma} = [{}^{B2}x_{C21\gamma}\ {}^{B2}y_{C21\gamma}]^T,$$
$${}^{Bi}\boldsymbol{p}_{CO1\gamma} = [{}^{Bi}x_{CO1\gamma}\ {}^{Bi}y_{CO1\gamma}]^T\ (i = 1, 2).$$

As shown in Fig.6, when the contact points between the object and each finger link are lower than the horizontal line including the contact point between two objects, we can define

$$ {}^{B1}y_{C11\gamma} - {}^{B1}y_{CO1\gamma} \triangleq -l_{y1} < 0, \qquad (27)$$
$$ {}^{B2}y_{C21\gamma} - {}^{B2}y_{CO1\gamma} \triangleq -l_{y2} < 0. \qquad (28)$$

Moreover, from geometrical relationships, the following equations are obtained

$$ {}^{B1}x_{C11\gamma} - {}^{B1}x_{CO1\gamma} \triangleq -l_{x1} < 0, \qquad (29)$$
$$ {}^{B2}x_{C21\gamma} - {}^{B2}x_{CO1\gamma} \triangleq l_{x2} > 0. \qquad (30)$$

Using eqs.(27). (28). (28) and (29). eq.(26) can be solved for $\Delta\phi_{B1\gamma}$ and $\Delta\phi_{B2\gamma}$ as follows:

$$\Delta\phi_{B1\gamma} = (\epsilon_1 + \epsilon_2)l_{x2}/(l_{y1}l_{x2} + l_{x1}l_{y2}) > 0, \quad (31)$$
$$\Delta\phi_{B2\gamma} = -(\epsilon_1 + \epsilon_2)l_{x1}/(l_{y1}l_{x2} + l_{x1}l_{y2}) < 0. \quad (32)$$

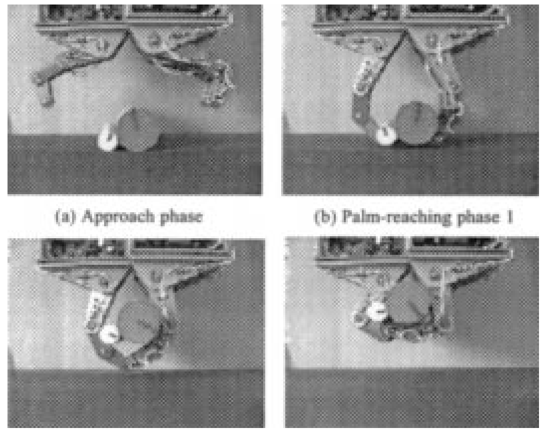
From eqs.(31) and (32), we can show that object 1 rotates counter-clockwise and that object 2 rotates clockwise. This holds the theorem.

□

While the manipulation of multiple objects by rolling contact seems to be very difficult, the above theorem shows that the palm-reaching phase can be achieved by applying a simple pushing motion that can produce both ϵ_1 and ϵ_2.

7 Experiment

To verify how easily the sufficient condition can be satisfied, we execute an experiment by using Hiroshima-Hand. The Hiroshima-Hand is composed of three planar finger units, where each finger has three joints. Two cylindrical objects whose diameters are 33[mm] and 15[mm] are placed on the table. Fig.7 shows continuous photos during the experiment, where the two cylindrical objects on a table are gradually lifted up. After the fingers make contact with the objects, the fingers are driven by constant torque commands. The command torque for the left finger is 0.025[Nm], 0.025[Nm] and 0.0125[Nm] for joint 1, 2 and 3, respectively. Since there are two fingers in the right hand side, the command torque for the right fingers is chosen by the half of those of the left finger. As shown in the figure, although the fingers are driven by using the constant torque commands, the objects are lifted up easily and the enveloping grasp is achieved. It is generally difficult to achieve such a stable manipulation of two cylindrical objects by finger tip contacts.



(c) Palm-reaching phase 2 (d) Grasping phase

Fig. 7: Experimental results

This experimental result shows a potential advantage of enveloping grasp for manipulating multiple objects. Also, the theorem explains an essential principle behind the successful result for enveloping two cylindrical objects.

8 Conclusions

In this paper, we discussed the enveloping grasp for multiple objects. The n finger-m object system with rolling contact is modeled. We showed a necessary and sufficient condition for achieving a rolling motion at each contact point under multiple contacts (Theorem 1). A numerical example was shown to confirm the theorem 1. We also showed a sufficient condition for enabling the two objects to be lifted up(Theorem 2). An experiment was executed to show that an enveloping grasp of cylindrical objects can be achieved easily by utilizing a simple control scheme.

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References

[1] J.Kerr and B.Roth. Analysis of Multifingered Hands. *The Int. J. of Robotics Research, vol.4, no.4, pp.3-17*, 1986.

[2] A.B.A.Cole, J.E.Hauser, and S.S.Sastry. Kinematics and Control of Multifingered Hands with Rolling Contacts. *IEEE Trans. on Automatic Control, vol.34, no.4, pp.398-404*, 1989.

[3] E.Paljug, X.Yun, and V.Kumar. Control of Rolling Contacts in Multi-Arm Manipulation. *IEEE Trans. on Robotics and Automation, vol.10, no.4, pp.441-452*, 1994.

[4] K.Salisbury et al.. Preliminary Design of a Whole-Arm Manipulation Systems(WAMS). *Proc. of 1988*

IEEE Int. Conf. on Robotics and Automation, pp.256-260, 1988.

[5] A.Bicchi. Force Distribution in Multiple Whole-Limb Manipulation. *Proc. of 1993 IEEE Int. Conf. on Robotics and Automation, pp.196-201*, 1993.

[6] X.-Y.Zhang, Y.Nakamura, K.Goda, and K.Yoshimoto. Robustness of Power Grasp. *Proc. of 1994 IEEE Int. Conf. on Robotics and Automation, pp.2828-2835*, 1994.

[7] T.Omata and K.Nagata. Rigid Body Analysis of the Indeterminate Grasp Force in Power Grasps. *Proc. of 1996 IEEE Int. Conf. on Robotics and Automation, pp.1787-1794*, 1996.

[8] M.Kaneko, Y.Hino, and T.Tsuji. On Three Phases for Achieving Enveloping Grasps. *Proc. of 1997 IEEE Int. Conf. on Robotics and Automation*, 1997.

[9] Y.Aiyama, M.Minami, and T.Arai. Operation of Multiple Objects by Dual Manipulator System. *Proc. of the JSME Conf. on Robotics-Mechatronics, pp.457-458*(in Japanese), 1997.

[10] P.Dauchez and X.Delebarre. Force-Controlled Assembly of two Objects with a Two-arm Robot. *Robotica, vol.9, pp.299-306*, 1991.

[11] K.Kosuge, M.Sakai, and K.Kanitani. Decentralized Coordinated Motion Control of Manipulators with Vision and Force Sensors. *Proc. of 1995 IEEE Int. Conf. on Robotics and Automation, pp.2456-2462*, 1995.

[12] R.Mattikalli, D.Baraff, P.Khosla, and B.Repetto. Gravitational Stability of Frictionless Assemblies. *IEEE Trans. on Robotics and Automation, vol.11, no.3, pp.374-388*, 1995.

[13] D.J.Montana. The Kinematics of Contact and Grasp. *The Int. J. of Robotics Research, vol.7, no.3, pp.17-32*, 1988.

[14] Z.Li and J.Canny. Motion of Two Rigid Bodies with Rolling Constraint. *IEEE Trans. on Robotics and Automation, vol.6, no.1, pp.62-72*, 1990.

[15] W.S.Howard and V.Kumar. Modeling and Analysis of Compliance and Stability of Enveloping Grasps. *Proc. of 1995 IEEE Int. Conf. on Robotics and Automation, pp.1367-1372*, 1995.

[16] H.Maekawa, K.Tanie, and K.Komoriya. Kinematics, Statics and Stiffness Effect of 3D Grasp by Multifingered Hand with Rolling Contact at the Fingertip. *Proc. of 1997 IEEE Int. Conf. on Robotics and Automation, pp.78-85*, 1997.

[17] M.Kaneko and T.Tsuji. Realization of Enveloping Grasp. *1997 IEEE Int. Conf. on Robotics and Automation(Video Proceeding)*, 1997.

Appendix

In the appendix, the vector of the point of contact $\boldsymbol{u}$ in eq.(13) is derived. First, the mathematical equations of the surfaces of the objects and the finger links are expressed as follows:

$$S_{Bi}({}^{Bi}\boldsymbol{p}_{Cjk}) = 0, \quad S_{Fjk}({}^{Fjk}\boldsymbol{p}_{Cjk}) = 0, \tag{33}$$

where $S_{Bi}({}^{Bi}\boldsymbol{p}_{Cjk}) < 0$ and $S_{Fjk}({}^{Fjk}\boldsymbol{p}_{Cjk}) < 0$ show the inside of the surfaces. Since the two surfaces touch at the point of contact, they are on opposite sides of a common tangent plane and thus must have equal and opposite outward unit normal vectors,i.e.,

$$\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{e}_{Cjk} = -\boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{e}_{Cjk}, \tag{34}$$

where ${}^{Bi}\boldsymbol{e}_{Cjk} = \frac{\partial S_{Bi}({}^{Bi}\boldsymbol{p}_{Cjk})/\partial^{Bi}\boldsymbol{p}_{Cjk}}{\|\partial S_{Bi}({}^{Bi}\boldsymbol{p}_{Cjk})/\partial^{Bi}\boldsymbol{p}_{Cjk}\|}$ and ${}^{Fjk}\boldsymbol{e}_{Cjk} = \frac{\partial S_{Fjk}({}^{Fjk}\boldsymbol{p}_{Cjk})/\partial^{Fjk}\boldsymbol{p}_{Cjk}}{\|\partial S_{Fjk}({}^{Fjk}\boldsymbol{p}_{Cjk})/\partial^{Fjk}\boldsymbol{p}_{Cjk}\|}$. Differentiation of eq.(34) yields

$$\begin{aligned} &\Delta\boldsymbol{\phi}_{Bi} \times \boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{e}_{Cjk} + \boldsymbol{R}_{Bi}\boldsymbol{A}_{CBjk}\Delta^{Bi}\boldsymbol{p}_{Cjk} \\ &= -\Delta\boldsymbol{\phi}_{Fjk} \times \boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{e}_{Cjk} - \boldsymbol{R}_{Fjk}\boldsymbol{A}_{CFjk}\Delta^{Fjk}\boldsymbol{p}_{Cjk}, \end{aligned} \tag{35}$$

where $\boldsymbol{A}_{CBjk} = \partial^{Bi}\boldsymbol{e}_{Cjk}/\partial^{Bi}\boldsymbol{p}_{Cjk}^T$, $\boldsymbol{A}_{CFjk} = \partial^{Fjk}\boldsymbol{e}_{Cjk}/\partial^{Fjk}\boldsymbol{p}_{Cjk}^T$. Since the contact coordinates ${}^{Bi}\boldsymbol{p}_{Cjk}$ and ${}^{Fjk}\boldsymbol{p}_{Cjk}$ are on the 2 dimensional surfaces, these become functions of 2 dimensional vectors [1] $\boldsymbol{\xi}_{CBjk}$ and $\boldsymbol{\xi}_{CFjk}$ [2] such as

$$\begin{aligned} {}^{Bi}\boldsymbol{p}_{Cjk} &= {}^{Bi}\boldsymbol{p}_{Cjk}(\boldsymbol{\xi}_{CBjk}), && (36) \\ {}^{Fjk}\boldsymbol{p}_{Cjk} &= {}^{Fjk}\boldsymbol{p}_{Cjk}(\boldsymbol{\xi}_{CFjk}), && (37) \\ \Delta^{Bi}\boldsymbol{p}_{Cjk} &= \boldsymbol{L}_{CBjk}\Delta\boldsymbol{\xi}_{CBjk}, && (38) \\ \Delta^{Fjk}\boldsymbol{p}_{Cjk} &= \boldsymbol{L}_{CFjk}\Delta\boldsymbol{\xi}_{CFjk}, && (39) \end{aligned}$$

where $\boldsymbol{L}_{CBjk} = \partial^{Bi}\boldsymbol{p}_{Cjk}/\partial\boldsymbol{\xi}_{CBjk}^T$, $\boldsymbol{L}_{CFjk} = \partial^{Fjk}\boldsymbol{p}_{Cjk}/\partial\boldsymbol{\xi}_{CFjk}^T$. The following equations are derived from eqs.(35), (3), (38) and (39):

$$\boldsymbol{W}_{Cjk}\begin{bmatrix} \Delta\boldsymbol{\xi}_{CBjk} \\ \Delta\boldsymbol{\xi}_{CFjk} \end{bmatrix} = \boldsymbol{Y}_{jk}\begin{bmatrix} \Delta\boldsymbol{\phi}_{Bi} \\ \Delta\boldsymbol{\phi}_{Fjk} \end{bmatrix}, \tag{40}$$

where

$$\boldsymbol{W}_{Cjk} = \begin{bmatrix} \boldsymbol{R}_{Bi}\boldsymbol{L}_{CBjk} & \boldsymbol{R}_{Fjk}\boldsymbol{L}_{CFjk} \\ \boldsymbol{R}_{Bi}\boldsymbol{A}_{CBjk}\boldsymbol{L}_{CBjk} & \boldsymbol{R}_{Fjk}\boldsymbol{A}_{CFjk}\boldsymbol{L}_{CFjk} \end{bmatrix},$$

$$\boldsymbol{Y}_{jk} = \begin{bmatrix} \boldsymbol{o} & \boldsymbol{o} \\ (\boldsymbol{R}_{Bi}{}^{Bi}\boldsymbol{e}_{Cjk}\times) & (\boldsymbol{R}_{Fjk}{}^{Fjk}\boldsymbol{e}_{Cjk}\times) \end{bmatrix}.$$

In eq.(40), the $\dim\boldsymbol{W}_{Cjk} = 6 \times 4$. However, 4 lines of the matrix are independent because normal vectors are of unit length in eq.(34) and because the contact point must lie in the common tangent plane in eq.(3)[2]. Solving eq.(40) with respect to $\Delta\boldsymbol{\xi}_{CBjk}$, $\Delta\boldsymbol{\xi}_{CFjk}$, and substituting into eqs.(38) and (39), we can derive $\Delta^{Bi}\boldsymbol{p}_{Cjk}$ and $\Delta^{Fjk}\boldsymbol{p}_{Cjk}$. $\Delta^{Bi}\boldsymbol{p}_{COt}$ can be derived similarly.

[1] 1 dimensional value for 1 dimensional line